

Grade 12 Physics Kinematics Key

| First Name: | KEY | | | |
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| Last Name: | | | | |

Directions:

- Please answer to 2 decimal points
- \bullet The test is designed to be completed in 75 minutes

For grading use only

| Page: | 2 | 3 | 4 | 5 | Total | | |
|---------|---|---|----|----|-------|--|--|
| Points: | 9 | 9 | 18 | 10 | 46 | | |
| Score: | | | | | | | |

Multiple Choice (10 marks)

1. (1 point) What is the SI unit of momentum? A. Joule (J) B. Newton (N) C. kilogram·meter/second (kg·m/s) D. $kilogram \cdot meter/second^2 (kg \cdot m/s^2)$ 2. (1 point) A 2 kg object moves at 3 m/s. What is its momentum? A. $6 \text{ kg} \cdot \text{m/s}$ B. $3 \text{ kg} \cdot \text{m/s}$ C. $2 \text{ kg} \cdot \text{m/s}$ D. $1.5 \text{ kg} \cdot \text{m/s}$ 3. (1 point) A force of 10 N acts on an object for 5 seconds. What is the impulse imparted? A. 50 N·s B. 2 N·s C. 10 N·s D. $5 \text{ N} \cdot \text{s}$ 4. (1 point) In an inelastic collision, which quantity is conserved? A. Kinetic energy B. Momentum C. Both momentum and kinetic energy D. Neither 5. (1 point) Two ice skaters push off each other. Skater A (60 kg) moves at 2 m/s. What is Skater B's (80 kg) velocity? A. 1.5 m/s B. 2 m/sC. 0.5 m/sD. 1.0 m/s6. (1 point) A ball (0.5 kg) hits a wall at 10 m/s and rebounds at 8 m/s. What is the impulse? A. $1 \text{ kg} \cdot \text{m/s}$ B. $9 \text{ kg} \cdot \text{m/s}$ C. $-9 \text{ kg} \cdot \text{m/s}$ D. $-1 \text{ kg} \cdot \text{m/s}$ 7. (1 point) Which has greater KE if both have same momentum? A. Car B. Truck C. Both same D. Cannot determine 8. (1 point) A 1000 kg car accelerates from rest to 20 m/s in 5 s. Average force? A. 2000 N B. 4000 N C. 1000 N D. 500 N

9. (1 point) Baseball (0.1 kg) thrown at 30 m/s and hit back at 35 m/s. Impulse magnitude?

- A. $0.5 \text{ kg} \cdot \text{m/s}$
- B. $3.0 \text{ kg} \cdot \text{m/s}$
- C. 6.5 kg·m/s
- D. $65 \text{ kg} \cdot \text{m/s}$
- 10. (1 point) Collision where objects stick together is:
 - A. Elastic
 - B. Inelastic
 - C. Perfectly elastic
 - D. Impossible

Long Answer (40 marks)

- 11. A 0.50 kg cart moving at 2.0 m/s collides elastically with a stationary 0.75 kg cart.
 - (a) (4 points) Calculate the total momentum of the system before the collision.
 - (b) (4 points) Determine the velocity of each cart after the collision.

Solution: (a) Total momentum before the collision:

The total momentum of the system before the collision is the sum of the momenta of both carts. The second cart is stationary, so its momentum is zero. The momentum is given by:

$$p = mv$$

Substituting the values:

$$p_{total} = (0.50 \, kg) \cdot (2.0 \, m/s) + (0.75 \, kg) \cdot (0 \, m/s)$$
$$p_{total} = 1.0 \, kg \cdot m/s$$

(b) Velocity of each cart after the collision:

Since the collision is elastic, both momentum and kinetic energy are conserved. The velocities after the collision can be calculated using the following formulas for elastic collisions:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

Substituting the values:

$$v_{1f} = \frac{0.50 - 0.75}{0.50 + 0.75} \cdot 2.0 + \frac{2 \cdot 0.75}{0.50 + 0.75} \cdot 0$$

$$v_{1f} = \frac{-0.25}{1.25} \cdot 2.0 = -0.40 \, m/s$$

$$v_{2f} = \frac{2 \cdot 0.50}{0.50 + 0.75} \cdot 2.0 + \frac{0.75 - 0.50}{0.50 + 0.75} \cdot 0$$

$$v_{2f} = \frac{1.0}{1.25} \cdot 2.0 = 1.6 \, m/s$$

Final velocities:

$$v_{1f} = -0.40\,m/s \quad (The 0.50 kg cart moves in the opposite direction)$$

$$v_{2f} = 1.6\,m/s \quad (The 0.75 kg cart moves forward)$$

12. A rocket with a mass of 5000 kg expels 50 kg of fuel per second at a velocity of 400 m/s relative to the rocket.

- (a) (4 points) Calculate the thrust produced by the rocket.
- (b) (4 points) Determine the rocket's acceleration when its total mass is 4000 kg.

Solution: (a) Thrust produced by the rocket:

The thrust produced by the rocket is the rate at which momentum is expelled. It can be calculated using the following equation:

$$F = \dot{m}v_{ex}$$

where: $\dot{m} = 50 \, kg/s$ (rate of fuel expelled) $v_{ex} = 400 \, m/s$ (velocity of fuel relative to the rocket)

Substituting the values:

$$F = 50 \cdot 400 = 20,000 \, N$$

Thus, the thrust produced by the rocket is 20,000 N.

(b) Rocket's acceleration when its total mass is 4000 kg:

The acceleration of the rocket can be found using Newton's second law:

$$F = ma$$

where: F = 20,000 N (thrust from part (a)) m = 4000 kg (mass of the rocket when fuel is expended)

Solving for a:

$$a = \frac{F}{m} = \frac{20,000}{4000} = 5 \, m/s^2$$

Thus, the rocket's acceleration is $5 m/s^2$.

- 13. Two curling stones collide on ice. Stone A (mass 18 kg) is moving at 4 m/s at 45° north of east, and Stone B (mass 22 kg) is moving at 3 m/s at 60° south of east. After the collision, Stone A moves at 2 m/s at an angle of 30° north of east.
 - (a) (5 points) Determine the velocity (magnitude and direction) of Stone B after the collision.
 - (b) (5 points) Verify if the collision is elastic.

Solution: (13a) Velocity of Stone B after the collision:

To solve this, we use conservation of momentum in both the x- and y-directions. The initial and final velocities of the stones are given. Let's resolve the velocity components for Stone A and Stone B before and after the collision.

Initial velocity of Stone A:

$$v_{A_x} = 4\cos(45^\circ) = 4 \cdot \frac{\sqrt{2}}{2} = 2\sqrt{2} \, m/s, \quad v_{A_y} = 4\sin(45^\circ) = 2\sqrt{2} \, m/s$$

Initial velocity of Stone B:

$$v_{B_x} = 3\cos(60^\circ) = 3 \cdot \frac{1}{2} = 1.5 \, m/s, \quad v_{B_y} = 3\sin(60^\circ) = 3 \cdot \frac{\sqrt{3}}{2} = 2.598 \, m/s$$

Final velocity of Stone A:

$$v_{A_x}' = 2\cos(30^\circ) = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}\,m/s, \quad v_{A_y}' = 2\sin(30^\circ) = 1\,m/s$$

Now, using conservation of momentum in the x- and y-directions:

In the
$$x-direction: m_A v_{A_x} + m_B v_{B_x} = m_A v'_{A_x} + m_B v'_{B_x}$$

$$18 \cdot 2\sqrt{2} + 22 \cdot 1.5 = 18 \cdot \sqrt{3} + 22 \cdot v'_B$$

Solving for v'_{B_x} :

$$v_{B_x}' = \frac{18 \cdot 2\sqrt{2} + 22 \cdot 1.5 - 18 \cdot \sqrt{3}}{22} \approx 2.18 \, m/s$$

$$In the y - direction: \quad m_A v_{A_y} + m_B v_{B_y} = m_A v'_{A_y} + m_B v'_{B_y}$$

$$18 \cdot 2\sqrt{2} + 22 \cdot 2.598 = 18 \cdot 1 + 22 \cdot v_{B_y}'$$

Solving for v'_{B_u} :

$$v_{B_y}' = \frac{18 \cdot 2\sqrt{2} + 22 \cdot 2.598 - 18 \cdot 1}{22} \approx 2.75 \, m/s$$

Thus, the magnitude and direction of Stone B's final velocity are:

$$v_B' = \sqrt{(v_{B_x}')^2 + (v_{B_y}')^2} = \sqrt{(2.18)^2 + (2.75)^2} \approx 3.47 \, m/s$$

The direction is given by:

$$\theta_B' = \tan^{-1}\left(\frac{v_{B_y}'}{v_{B_x}'}\right) \approx \tan^{-1}\left(\frac{2.75}{2.18}\right) \approx 52.5^{\circ} northofeast$$

(13b) Verifying if the collision is elastic:

To check if the collision is elastic, we compare the kinetic energy before and after the collision.

Initial kinetic energy:

$$KE_{initial} = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$KE_{initial} = \frac{1}{2} \cdot 18 \cdot 4^2 + \frac{1}{2} \cdot 22 \cdot 3^2 = 144 + 99 = 243 J$$

Final kinetic energy:

$$KE_{final} = \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_B v_B'^2$$

$$KE_{final} = \frac{1}{2} \cdot 18 \cdot 2^2 + \frac{1}{2} \cdot 22 \cdot 3.47^2 \approx 72 + 134.4 = 206.4 J$$

Since the initial and final kinetic energies are not equal, the collision is inelastic.

- 14. A grenade at rest explodes into three fragments. Fragment 1 (2.5 kg) moves at 6 m/s at 20° north of west, Fragment 2 (3.5 kg) moves at 5 m/s at 40° south of east, and Fragment 3 has a mass of 4 kg.
 - (a) (5 points) Determine the velocity (magnitude and direction) of Fragment 3 after the explosion.
 - (b) (5 points) Calculate the total kinetic energy released in the explosion.

Solution: (a) Velocity of Fragment 3 after the explosion:

Since the grenade was initially at rest, the total momentum before the explosion is zero. After the explosion, the momentum of the system is conserved. We can calculate the velocity of Fragment 3 by applying the conservation of momentum in both the x- and y-directions.

Let the velocity of Fragment 3 be $\mathbf{v}_3 = (v_{3x}, v_{3y})$.

Momentum in the x-direction: The momentum of each fragment in the x-direction is:

$$m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} = 0$$

where: - $m_1 = 2.5 kg$, $v_1 = 6 m/s$ at 20° north of west, so $v_{1x} = -6 \cos(20^{\circ})$ - $m_2 = 3.5 kg$, $v_2 = 5 m/s$ at 40° south of east, so $v_{2x} = 5 \cos(40^{\circ})$

Substitute into the momentum equation:

$$2.5 \cdot (-6\cos(20^\circ)) + 3.5 \cdot (5\cos(40^\circ)) + 4 \cdot v_{3x} = 0$$

Solving for v_{3x} , we get:

$$v_{3x} = \frac{2.5 \cdot 6\cos(20^{\circ}) - 3.5 \cdot 5\cos(40^{\circ})}{4}$$

$$v_{3x} \approx \frac{-2.5 \cdot 5.64 + 3.5 \cdot 3.83}{4} \approx \frac{-14.1 + 13.405}{4} \approx \frac{-0.695}{4} \approx -0.17375 \, m/s$$

Momentum in the y-direction: Similarly, in the y-direction:

$$m_1 v_{1y} + m_2 v_{2y} + m_3 v_{3y} = 0$$

where: $-v_{1y} = 6\sin(20^\circ) - v_{2y} = -5\sin(40^\circ)$

Substitute into the momentum equation:

$$2.5 \cdot (6\sin(20^\circ)) + 3.5 \cdot (-5\sin(40^\circ)) + 4 \cdot v_{3y} = 0$$

Solving for v_{3y} , we get:

$$v_{3y} = \frac{-2.5 \cdot 6\sin(20^\circ) - 3.5 \cdot 5\sin(40^\circ)}{4}$$

$$v_{3y} \approx \frac{-2.5 \cdot 2.06 - 3.5 \cdot 3.21}{4} \approx \frac{-5.15 - 11.23}{4} \approx \frac{-16.38}{4} \approx -4.095 \, m/s$$

Now that we have both components of the velocity of Fragment 3, we can calculate the magnitude and direction of the velocity.

Magnitude of v_3 :

$$|\mathbf{v}_3| = \sqrt{v_{3x}^2 + v_{3y}^2} \approx \sqrt{(-0.17375)^2 + (-4.095)^2} \approx \sqrt{0.0302 + 16.77} \approx \sqrt{16.8002} \approx 4.10 \, m/s$$

Direction of v_3 : The direction is given by:

$$\theta = \tan^{-1}\left(\frac{v_{3y}}{v_{3x}}\right) = \tan^{-1}\left(\frac{-4.095}{-0.17375}\right) \approx \tan^{-1}(23.56) \approx 87.5^{\circ} south of west$$

Thus, the velocity of Fragment 3 is approximately $4.10 \, m/s$ at 87.5° south of west.

(b) Total kinetic energy released in the explosion:

The total kinetic energy is the sum of the kinetic energies of all three fragments. The kinetic energy of each fragment is given by:

$$KE = \frac{1}{2}mv^2$$

For Fragment 1:

$$KE_1 = \frac{1}{2} \cdot 2.5 \cdot 6^2 = 45 J$$

For Fragment 2:

$$KE_2 = \frac{1}{2} \cdot 3.5 \cdot 5^2 = 43.75 \, J$$

For Fragment 3:

$$KE_3 = \frac{1}{2} \cdot 4 \cdot 4.10^2 = 33.56 J$$

Thus, the total kinetic energy released is:

$$KE_{total} = KE_1 + KE_2 + KE_3 = 45 + 43.75 + 33.56 = 122.31 J$$