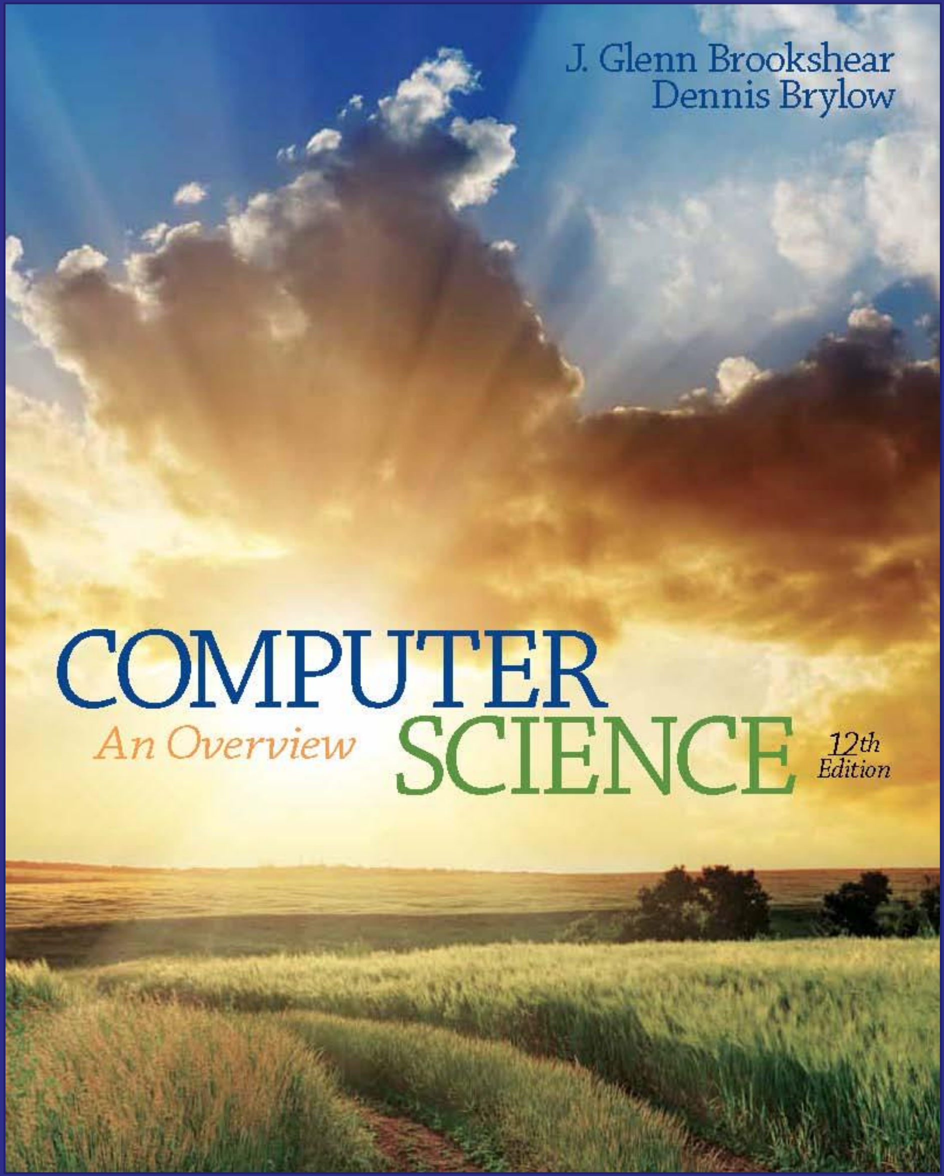
Chapter 5: Algorithms



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Chapter 5: Algorithms

- 5.1 The Concept of an Algorithm
- 5.2 Algorithm Representation
- 5.3 Algorithm Discovery
- 5.4 Iterative Structures
- 5.5 Recursive Structures
- 5.6 Efficiency and Correctness

Definition of Algorithm

An algorithm is an **ordered** set of **unambiguous**, **executable** steps that defines a **terminating** process.

Algorithm Representation

- Requires well-defined primitives
- A collection of primitives constitutes a programming language.

Figure 5.2 **Folding a bird from a square piece of paper**

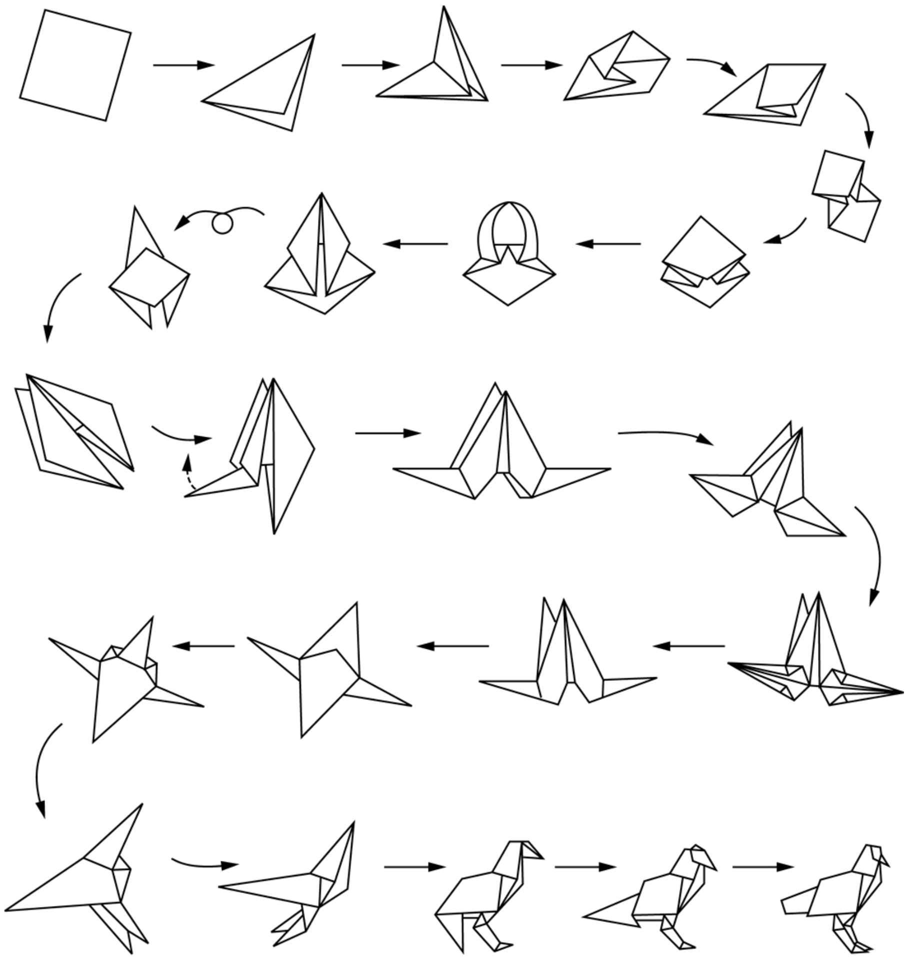
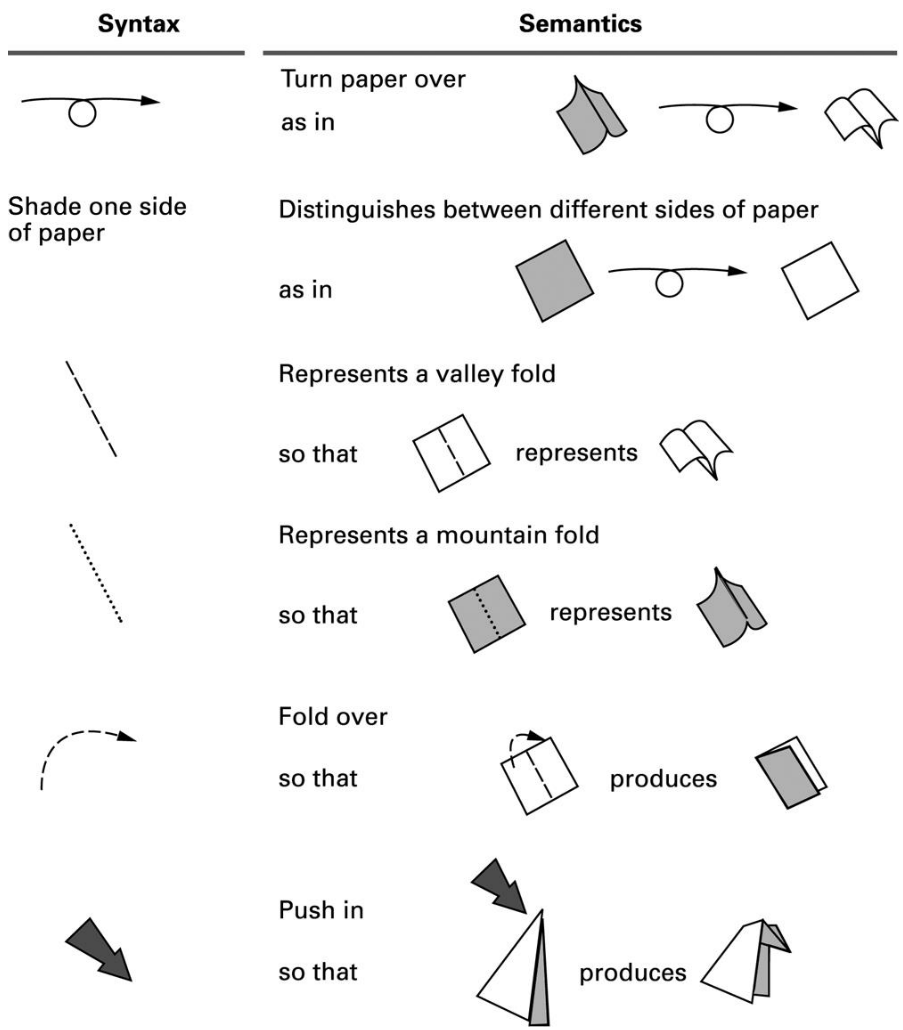


Figure 5.3 **Origami primitives**



Pseudocode Primitives

- Assignment

```
name = expression
```

- Example

```
RemainingFunds = CheckingBalance +
                        SavingsBalance
```

Pseudocode Primitives (continued)

- Conditional selection

```
if (condition):
    activity
```

- Example

```
if (sales have decreased):
    lower the price by 5%
```

Pseudocode Primitives (continued)

- Conditional selection

```
if (condition):
    activity
else:
    activity
```

- Example

```
if (year is leap year):
    daily total = total / 366
else:
    daily total = total / 365
```

Pseudocode Primitives (continued)

- Repeated execution

```
while (condition):
    body
```

- Example

```
while (tickets remain to be sold):
    sell a ticket
```

Pseudocode Primitives (continued)

- Indentation shows **nested** conditions

```
if (not raining):
    if (temperature == hot):
        go swimming
    else:
        play golf
else:
    watch television
```

Pseudocode Primitives (continued)

- Define a function

```
def name():
```

- Example

```
def ProcessLoan():
```

- Executing a function

```
if (. . .):
    ProcessLoan()
else:
    RejectApplication()
```

Figure 5.4 **The procedure Greetings in pseudocode**

```
def Greetings():
    Count = 3
    while (Count > 0):
        print('Hello')
        Count = Count - 1
```

Pseudocode Primitives (continued)

- Using parameters

```
def Sort(List):
      .
      .
```

- Executing Sort on different lists

```
Sort(the membership list)
Sort(the wedding guest list)
```

Polya's Problem Solving Steps

- 1. Understand the problem.
- 2. Devise a plan for solving the problem.
- 3. Carry out the plan.
- 4. Evaluate the solution for accuracy and its potential as a tool for solving other problems.

Polya's Steps in the Context of Program Development

- 1. Understand the problem.
- 2. Get an idea of how an algorithmic function might solve the problem.
- 3. Formulate the algorithm and represent it as a program.
- 4. Evaluate the solution for accuracy and its potential as a tool for solving other problems.

Getting a Foot in the Door

- Try working the problem backwards
- Solve an easier related problem
  - Relax some of the problem constraints
  - Solve pieces of the problem first (bottom up methodology)
- Stepwise refinement: Divide the problem into smaller problems (top-down methodology)

Ages of Children Problem

- Person A is charged with the task of determining the ages of B’s three children.
  - B tells A that the product of the children’s ages is 36.
  - A replies that another clue is required.
  - B tells A the sum of the children’s ages.
  - A replies that another clue is needed.
  - B tells A that the oldest child plays the piano.
  - A tells B the ages of the three children.
- How old are the three children?

Figure 5.5

a. Triples whose product is 36	
(1,1,36)	(1,6,6)
(1,2,18)	(2,2,9)
(1,3,12)	(2,3,6)
(1,4,9)	(3,3,4)

b. Sums of triples from part (a)	
1 + 1 + 36 = 38	1 + 6 + 6 = 13
1 + 2 + 18 = 21	2 + 2 + 9 = 13
1 + 3 + 12 = 16	2 + 3 + 6 = 11
1 + 4 + 9 = 14	3 + 3 + 4 = 10

Figure 5.6 **The sequential search algorithm in pseudocode**

```
def Search (List, TargetValue):
    if (List is empty):
        Declare search a failure
    else:
        Select the first entry in List to be TestEntry
        while (TargetValue > TestEntry and entries remain):
            Select the next entry in List as TestEntry
        if (TargetValue == TestEntry):
            Declare search a success
        else:
            Declare search a failure
```

Figure 5.7 **Components of repetitive control**

Initialize:	Establish an initial state that will be modified toward the termination condition
Test:	Compare the current state to the termination condition and terminate the repetition if equal
Modify:	Change the state in such a way that it moves toward the termination condition

Iterative Structures

- Pretest loop:

```
while (condition):
    body
```

- Posttest loop:

```
repeat:
    body
    until(condition)
```

Figure 5.8 **The while loop structure**

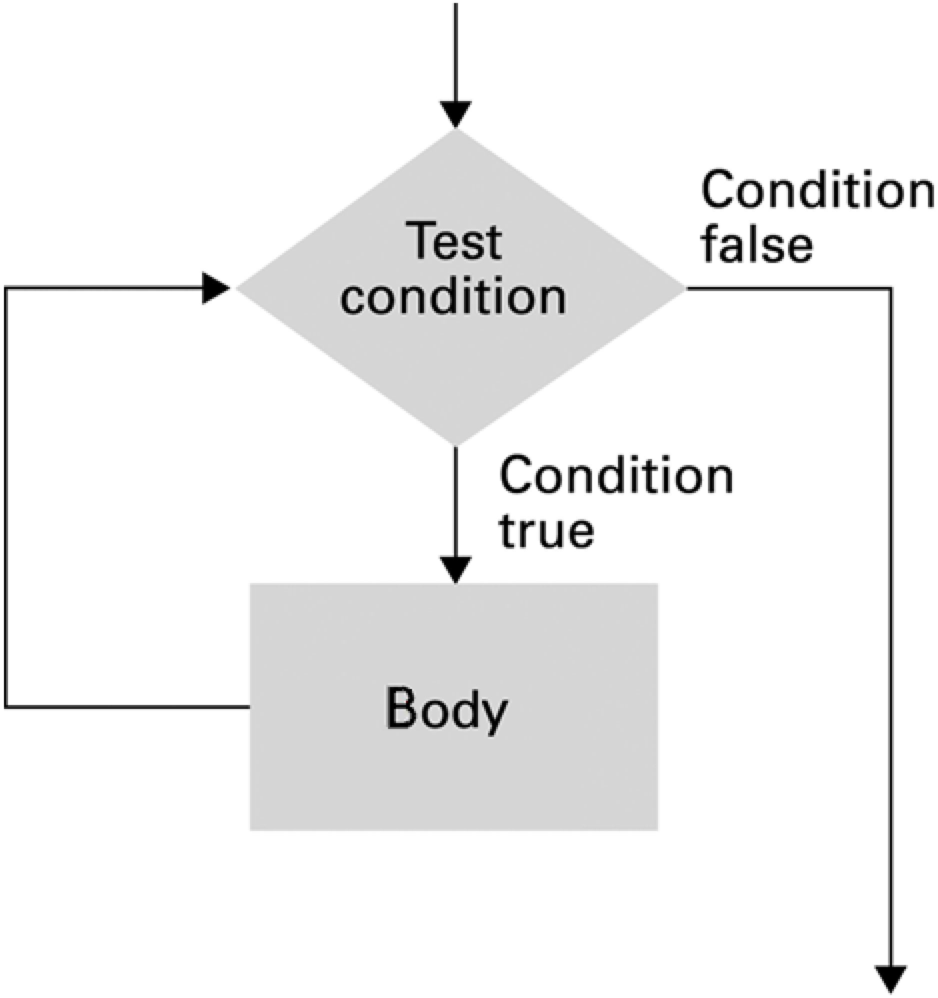


Figure 5.9 **The repeat loop structure**

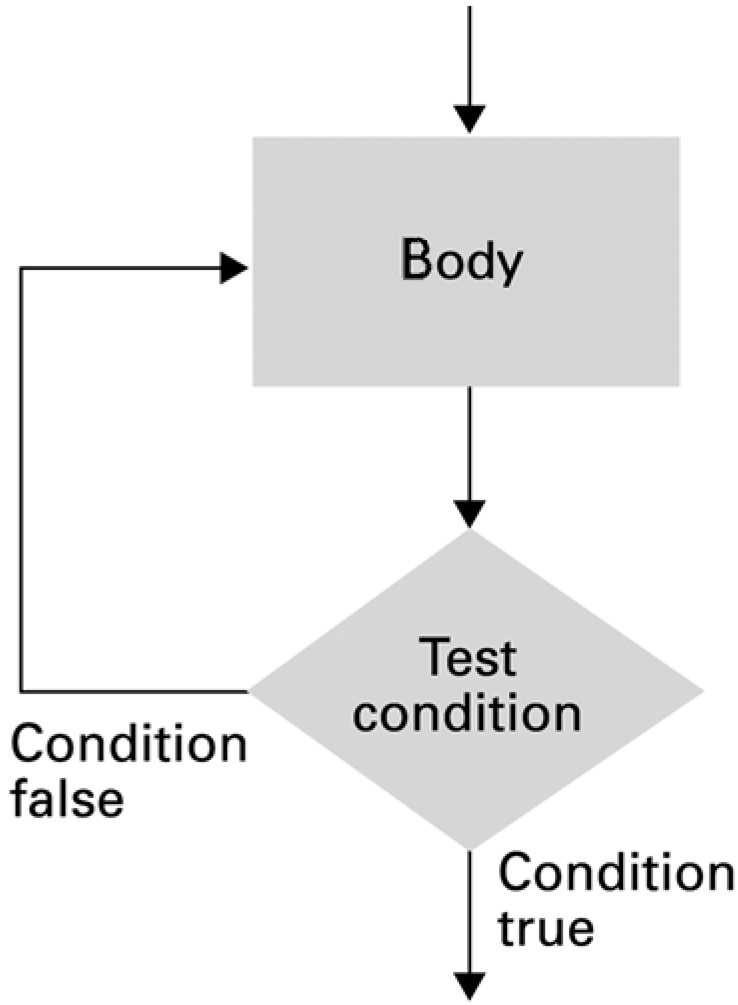


Figure 5.10 **Sorting the list Fred, Alex, Diana, Byron, and Carol alphabetically**

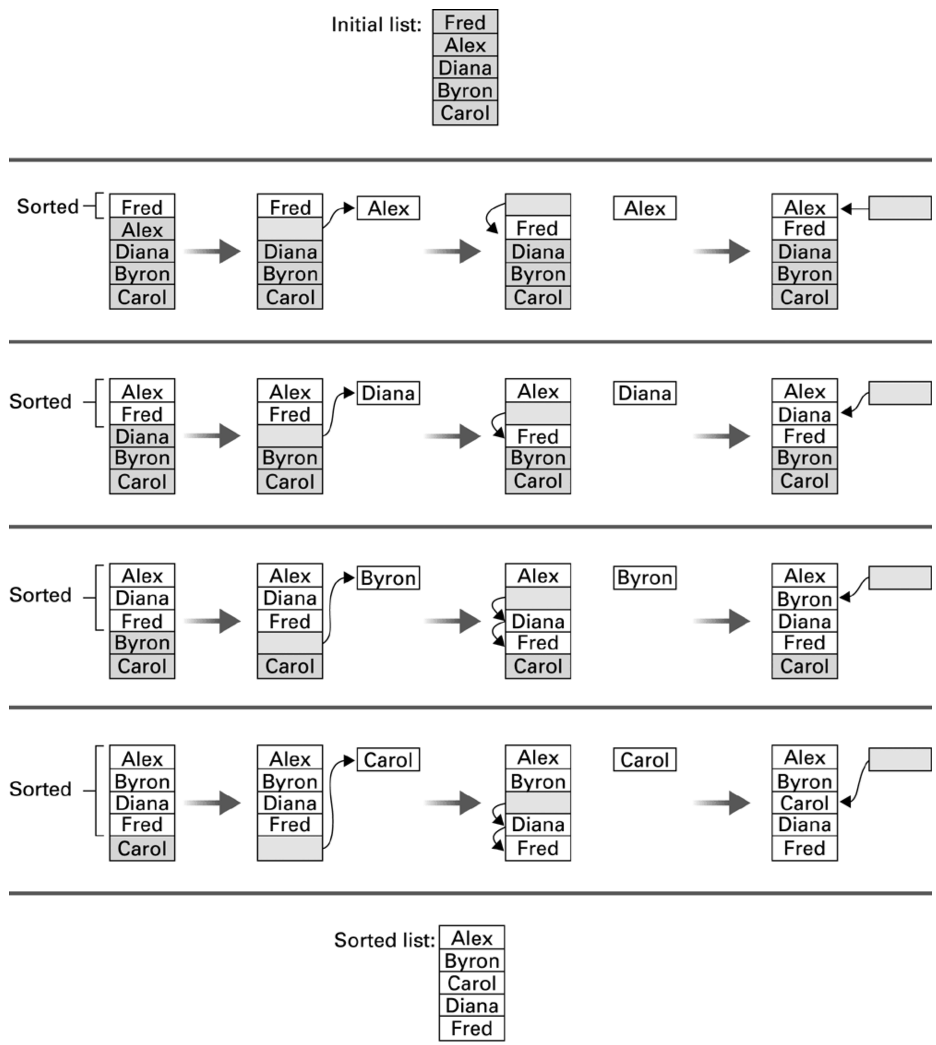


Figure 5.11 **The insertion sort algorithm expressed in pseudocode**

```
def Sort(List):
    N = 2
    while (N <= length of List):
        Pivot = Nth entry in List
        Remove Nth entry leaving a hole in List
        while (there is an Entry above the
                  hole and Entry > Pivot):
            Move Entry down into the hole leaving
            a hole in the list above the Entry
        Move Pivot into the hole
        N = N + 1
```

Recursion

- The execution of a procedure leads to another execution of the procedure.
- Multiple activations of the procedure are formed, all but one of which are waiting for other activations to complete.

Figure 5.12 **Applying our strategy to search a list for the entry John**

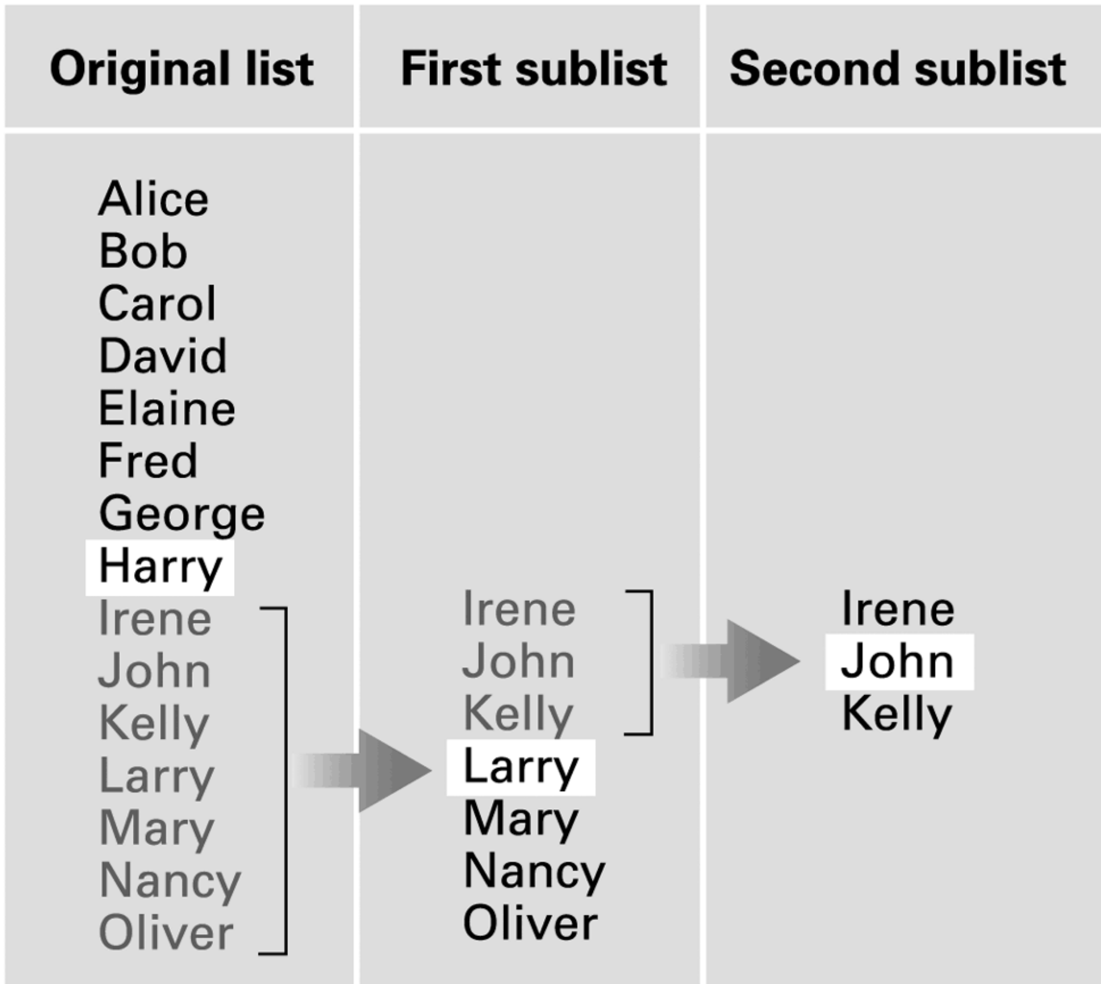


Figure 5.13 **A first draft of the binary search technique**

```
if (List is empty):
    Report that the search failed
else:
    TestEntry = middle entry in the List
    if (TargetValue == TestEntry):
        Report that the search succeeded
    if (TargetValue < TestEntry):
        Search the portion of List preceding TestEntry for
        TargetValue, and report the result of that search
    if (TargetValue > TestEntry):
        Search the portion of List following TestEntry for
        TargetValue, and report the result of that search
```

Figure 5.14 The binary search algorithm in pseudocode

```
def Search(List, TargetValue):
    if (List is empty):
        Report that the search failed
    else:
        TestEntry = middle entry in the List
        if (TargetValue == TestEntry):
            Report that the search succeeded
        if (TargetValue < TestEntry):
            Sublist = portion of List preceding TestEntry
            Search(Sublist, TargetValue)
        if (TargetValue < TestEntry):
            Sublist = portion of List following TestEntry
            Search(Sublist, TargetValue)
```

Figure 5.15

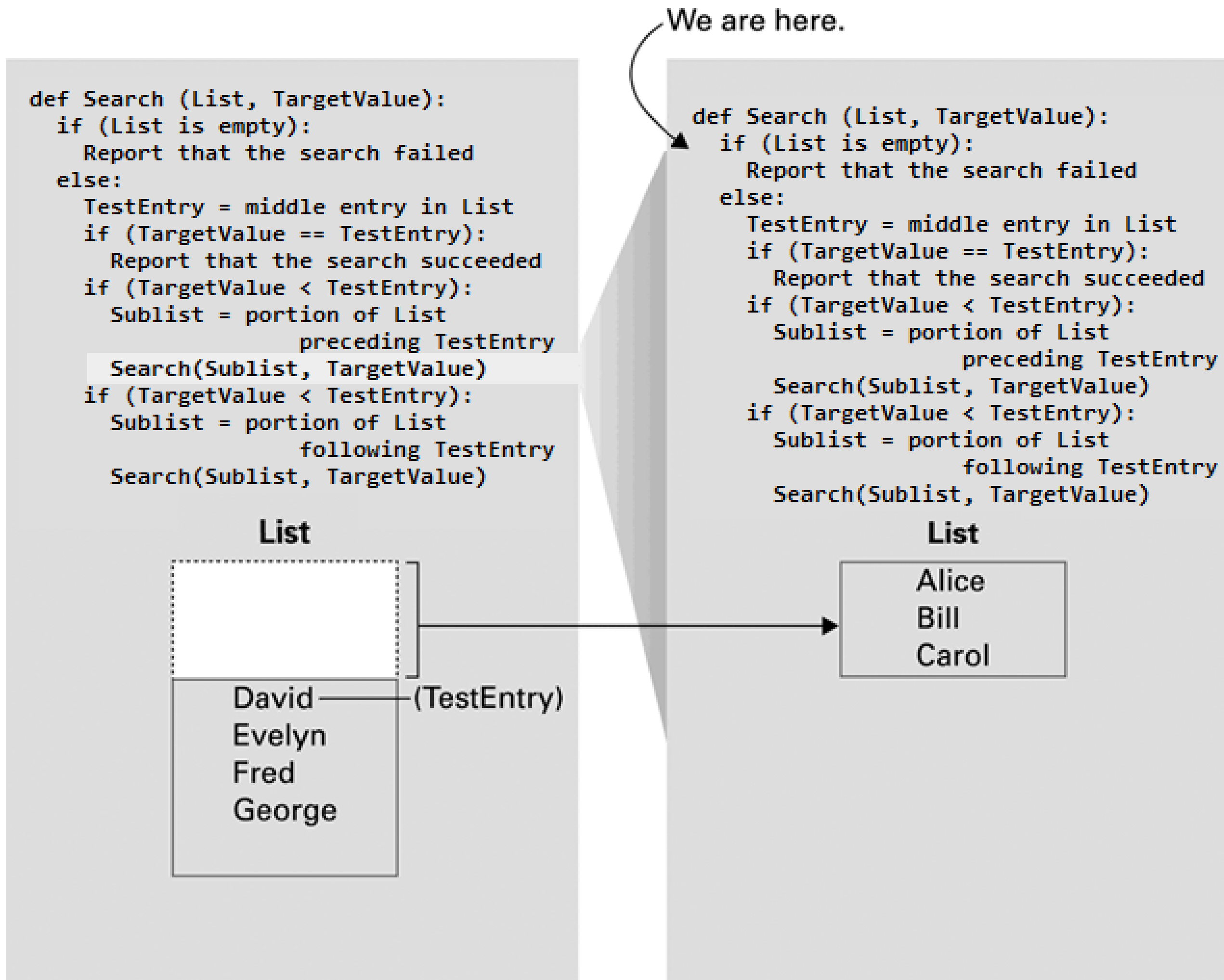
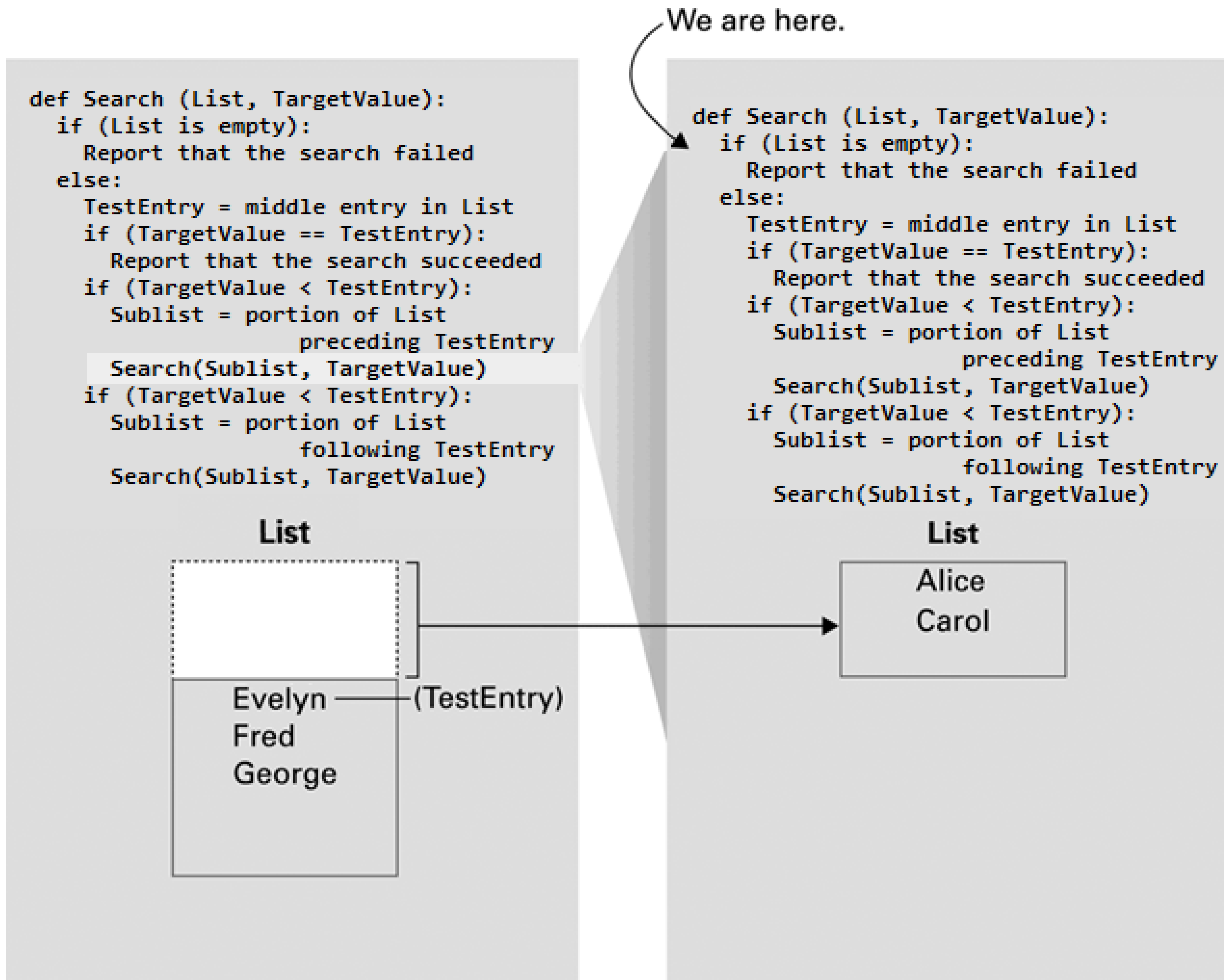
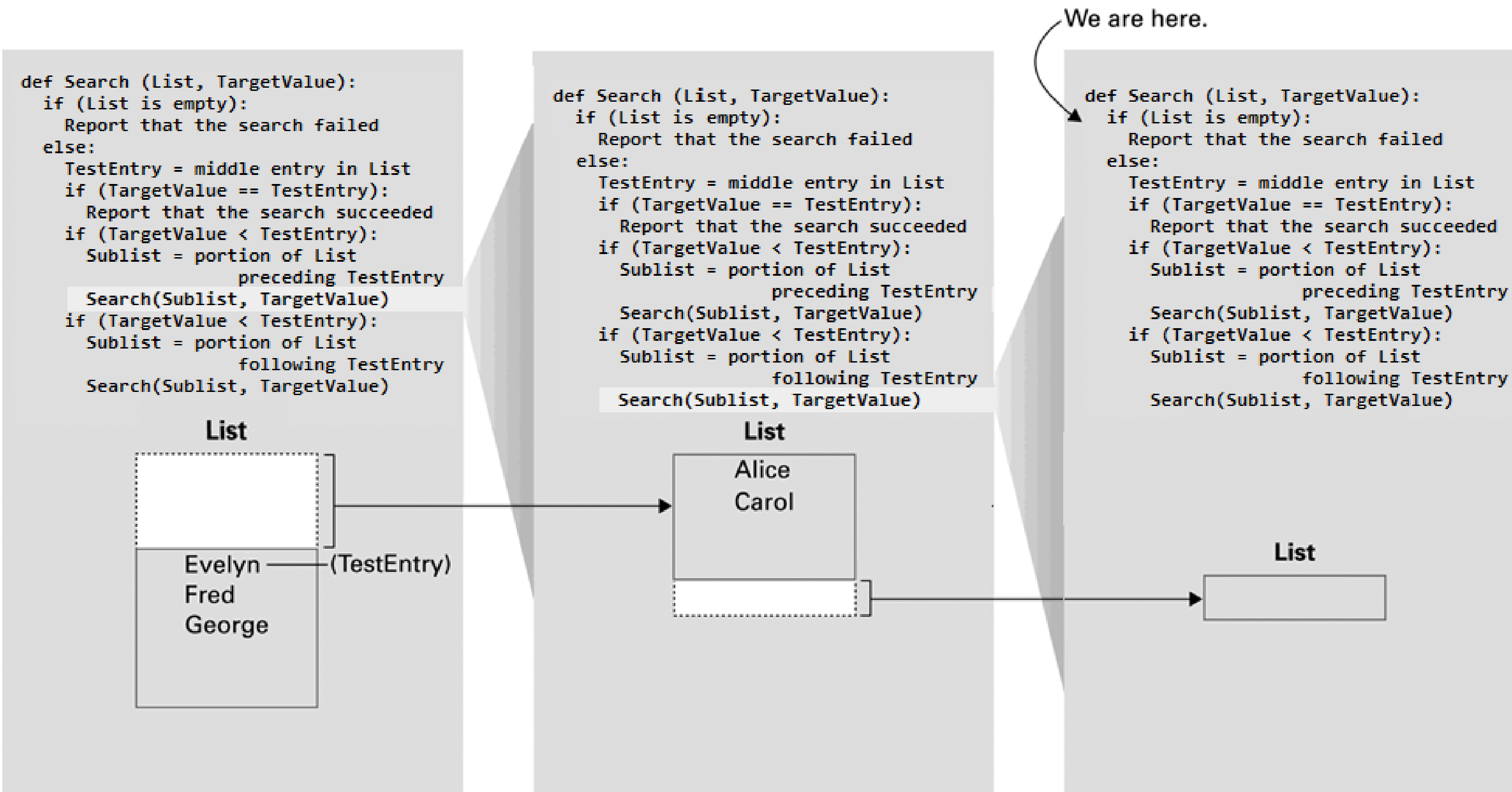

We are here.
def Search (List, TargetValue):
  if (List is empty):
    Report that the search failed
  else:
    TestEntry = middle entry in List
    if (TargetValue == TestEntry):
      Report that the search succeeded
    if (TargetValue < TestEntry):
      Sublist = portion of List
                    preceding TestEntry
      Search(Sublist, TargetValue)
    if (TargetValue < TestEntry):
      Sublist = portion of List
                    following TestEntry
      Search(Sublist, TargetValue)
List
David
Evelyn
Fred
George
(TestEntry)
def Search (List, TargetValue):
  if (List is empty):
    Report that the search failed
  else:
    TestEntry = middle entry in List
    if (TargetValue == TestEntry):
      Report that the search succeeded
    if (TargetValue < TestEntry):
      Sublist = portion of List
                    preceding TestEntry
      Search(Sublist, TargetValue)
    if (TargetValue < TestEntry):
      Sublist = portion of List
                    following TestEntry
      Search(Sublist, TargetValue)
List
Alice
Bill
Carol

Figure 5.16



We are here.
def Search (List, TargetValue):
  if (List is empty):
    Report that the search failed
  else:
    TestEntry = middle entry in List
    if (TargetValue == TestEntry):
      Report that the search succeeded
    if (TargetValue < TestEntry):
      Sublist = portion of List
                preceding TestEntry
      Search(Sublist, TargetValue)
    if (TargetValue < TestEntry):
      Sublist = portion of List
                following TestEntry
      Search(Sublist, TargetValue)
List
Evelyn
Fred
George
(TestEntry)
def Search (List, TargetValue):
  if (List is empty):
    Report that the search failed
  else:
    TestEntry = middle entry in List
    if (TargetValue == TestEntry):
      Report that the search succeeded
    if (TargetValue < TestEntry):
      Sublist = portion of List
                preceding TestEntry
      Search(Sublist, TargetValue)
    if (TargetValue < TestEntry):
      Sublist = portion of List
                following TestEntry
      Search(Sublist, TargetValue)
List
Alice
Carol

Figure 5.17



We are here.
def Search (List, TargetValue):
  if (List is empty):
    Report that the search failed
  else:
    TestEntry = middle entry in List
    if (TargetValue == TestEntry):
      Report that the search succeeded
    if (TargetValue < TestEntry):
      Sublist = portion of List
                    preceding TestEntry
      Search(Sublist, TargetValue)
    if (TargetValue < TestEntry):
      Sublist = portion of List
                    following TestEntry
      Search(Sublist, TargetValue)
List
Evelyn
Fred
George
(TestEntry)
Alice
Carol

Algorithm Efficiency

- Measured as number of instructions executed
- Big theta notation: Used to represent efficiency classes
  - Example: Insertion sort is in $\Theta(n^2)$
- Best, worst, and average case analysis

Figure 5.18 **Applying the insertion sort in a worst-case situation**

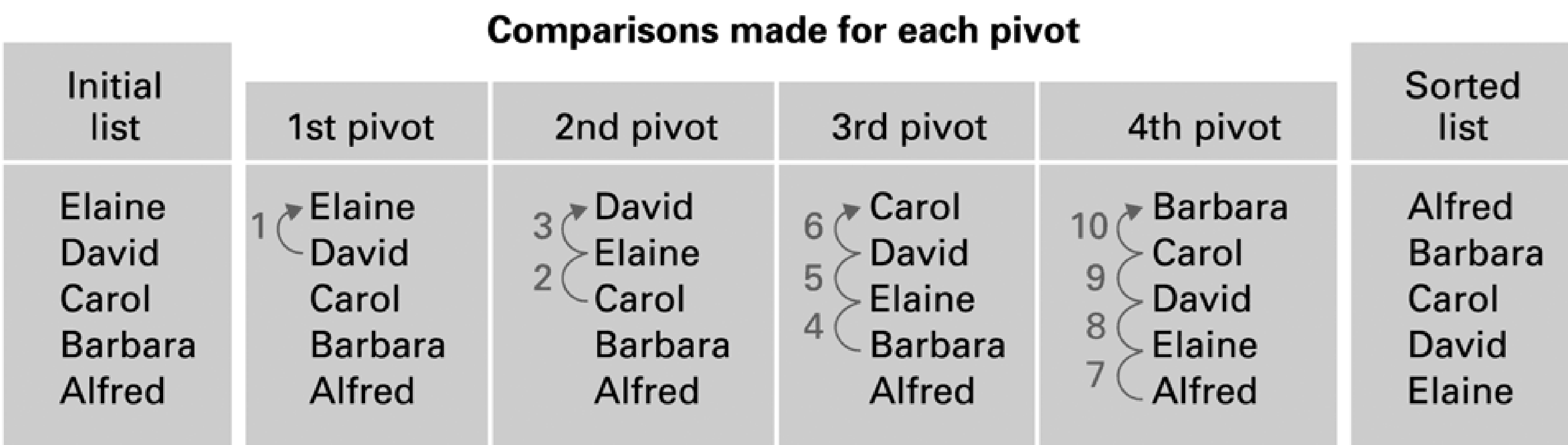


Figure 5.19 **Graph of the worst-case analysis of the insertion sort algorithm**

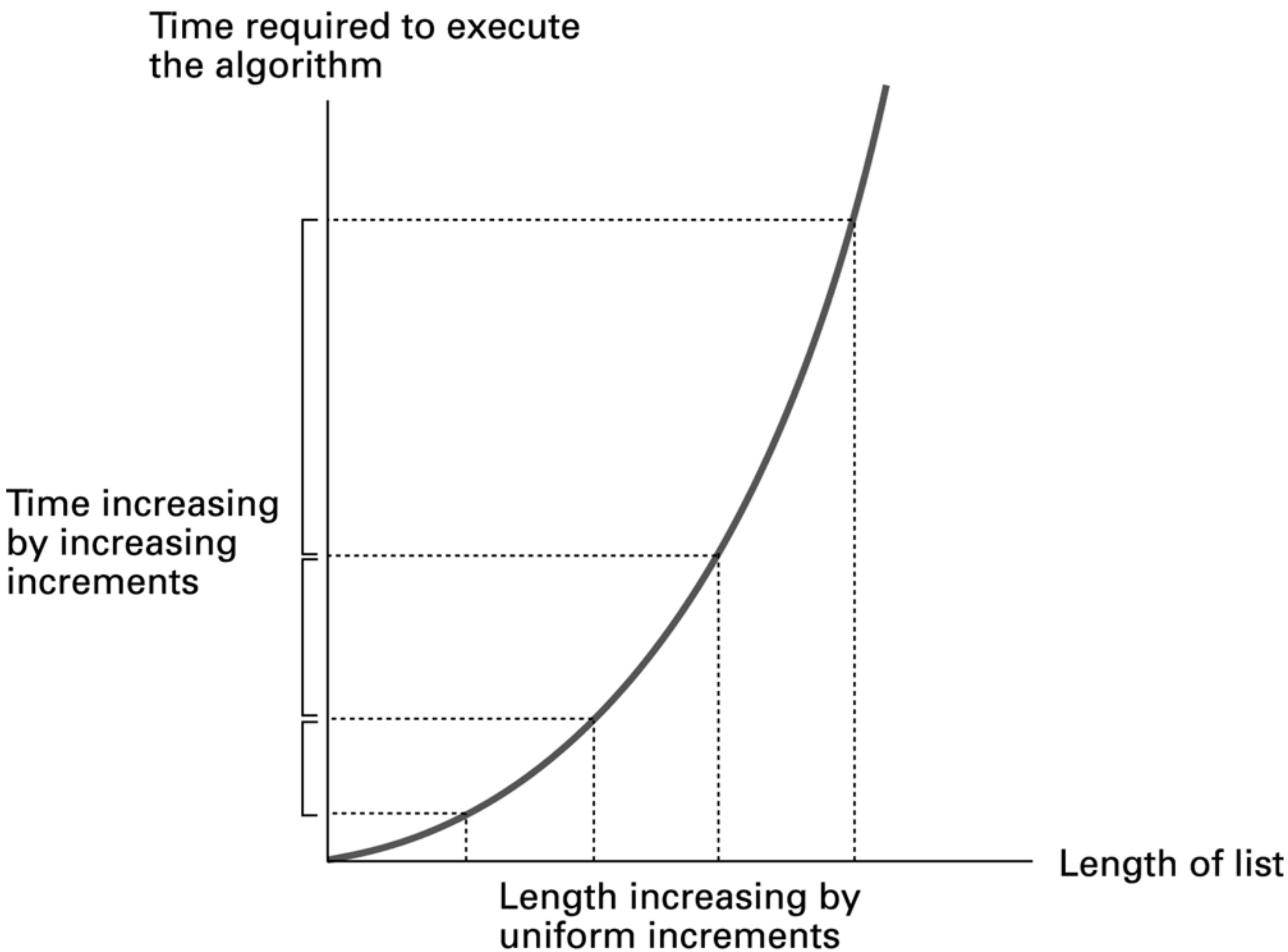
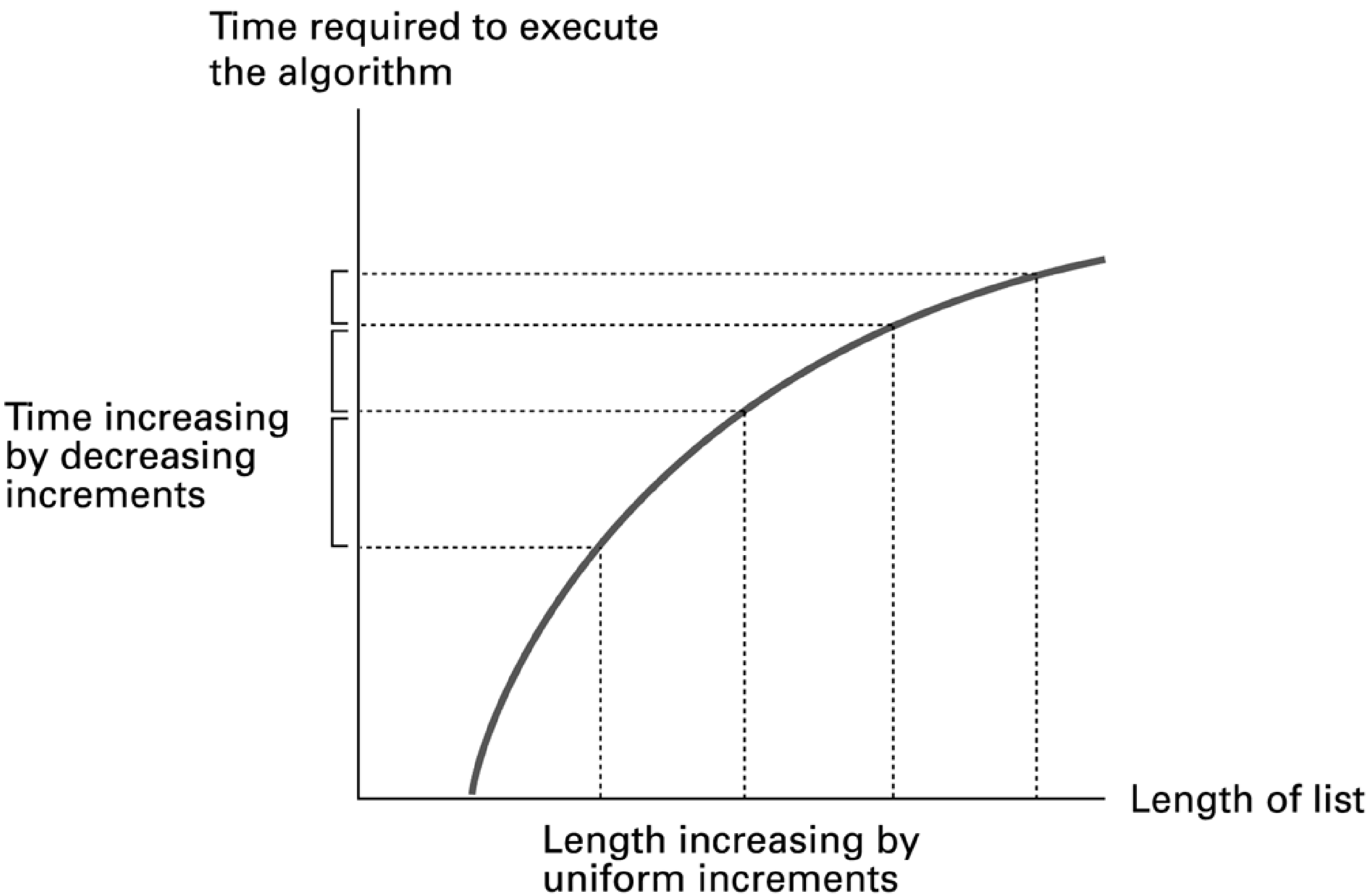


Figure 5.20 **Graph of the worst-case analysis of the binary search algorithm**



Software Verification

- Proof of correctness
  - Assertions
    - Preconditions
    - Loop invariants
- Testing

Chain Separating Problem

- A traveler has a gold chain of seven links.
- He must stay at an isolated hotel for seven nights.
- The rent each night consists of one link from the chain.
- What is the fewest number of links that must be cut so that the traveler can pay the hotel one link of the chain each morning without paying for lodging in advance?

Figure 5.21 **Separating the chain using only three cuts**

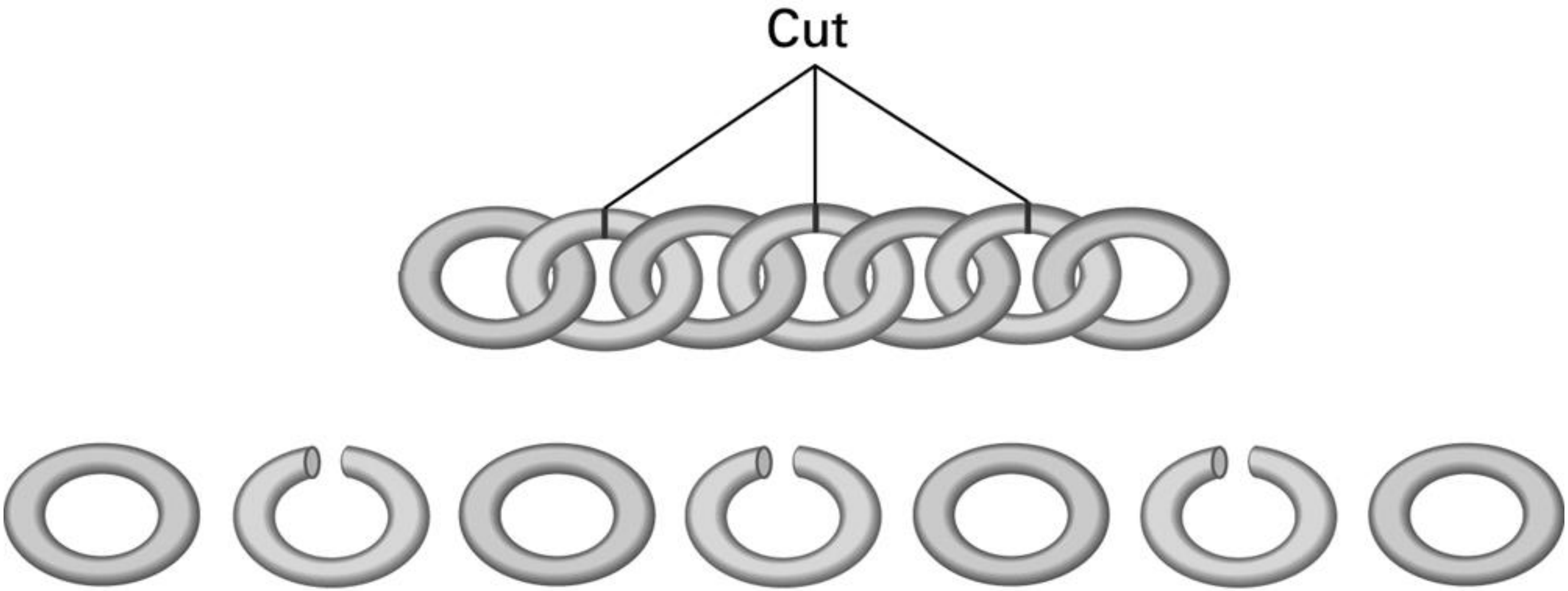


Figure 5.22 **Solving the problem with only one cut**

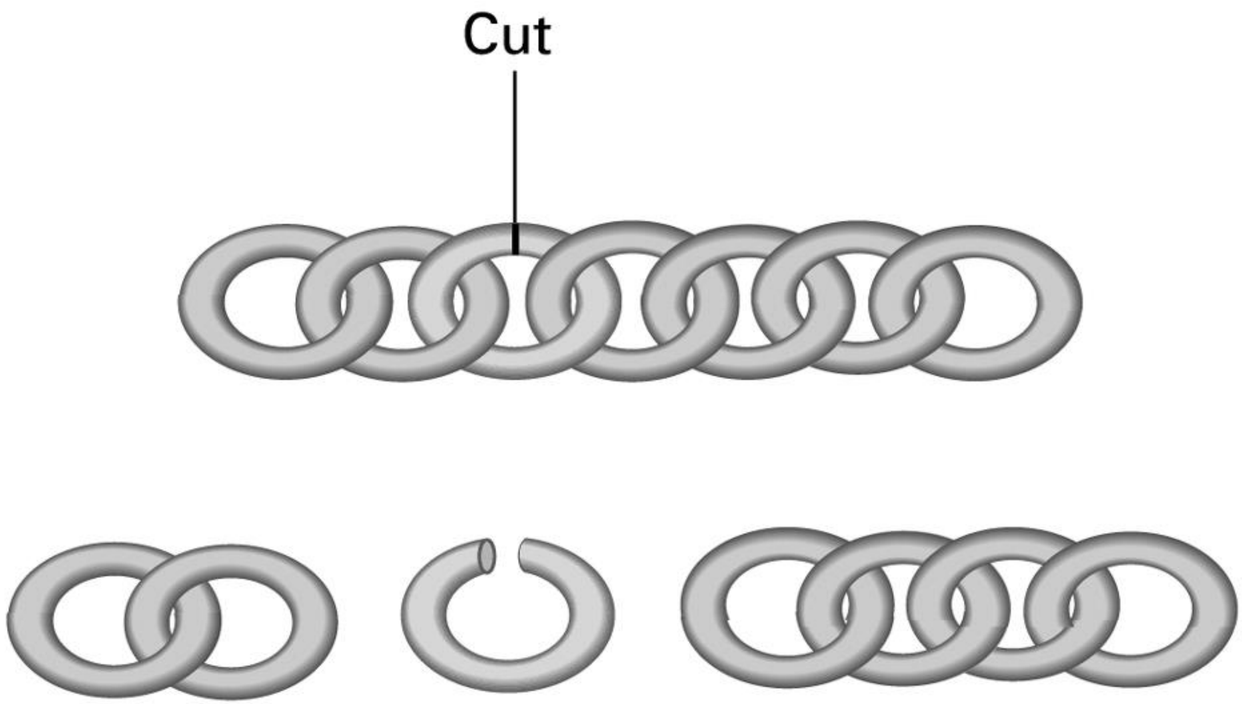
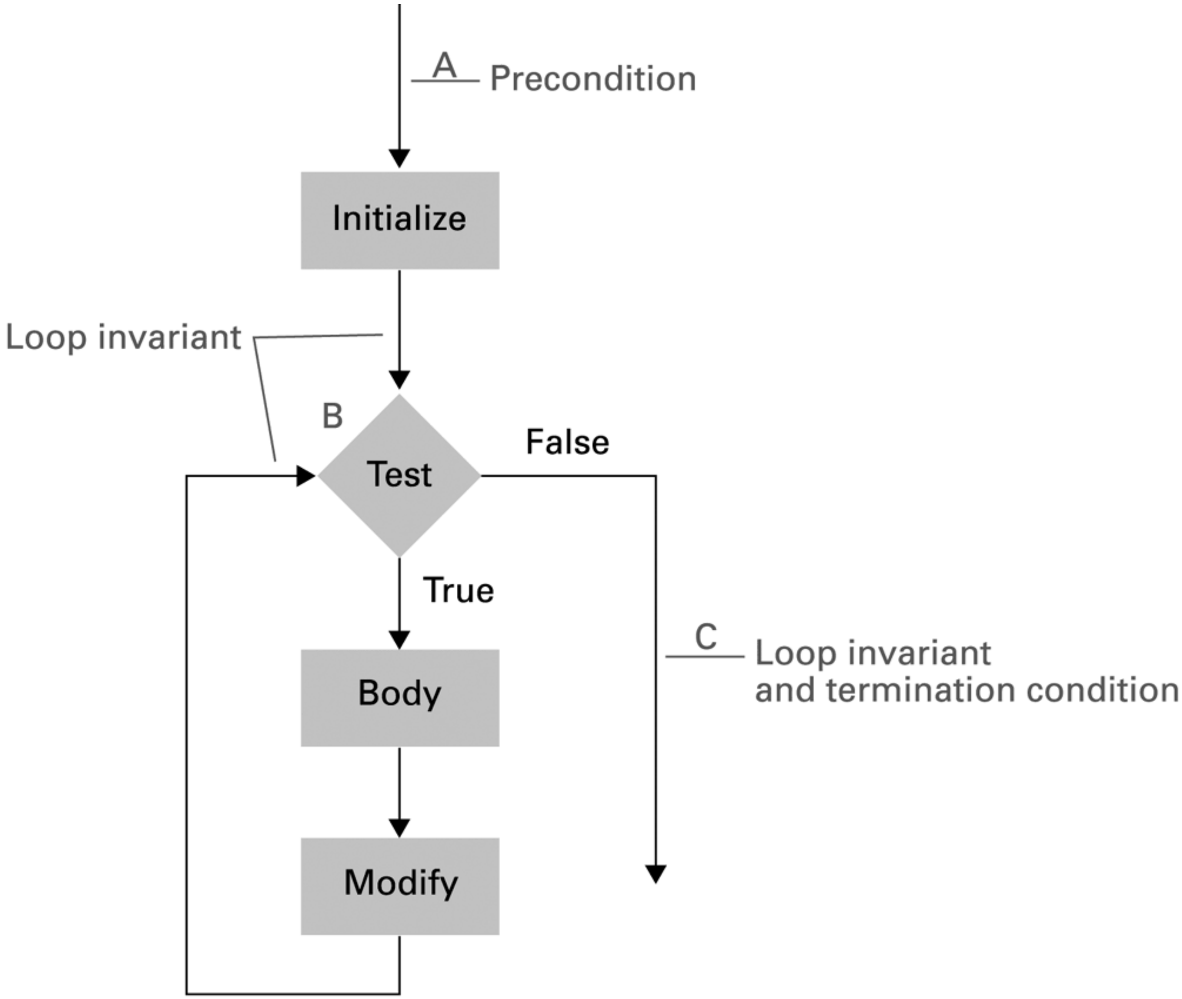
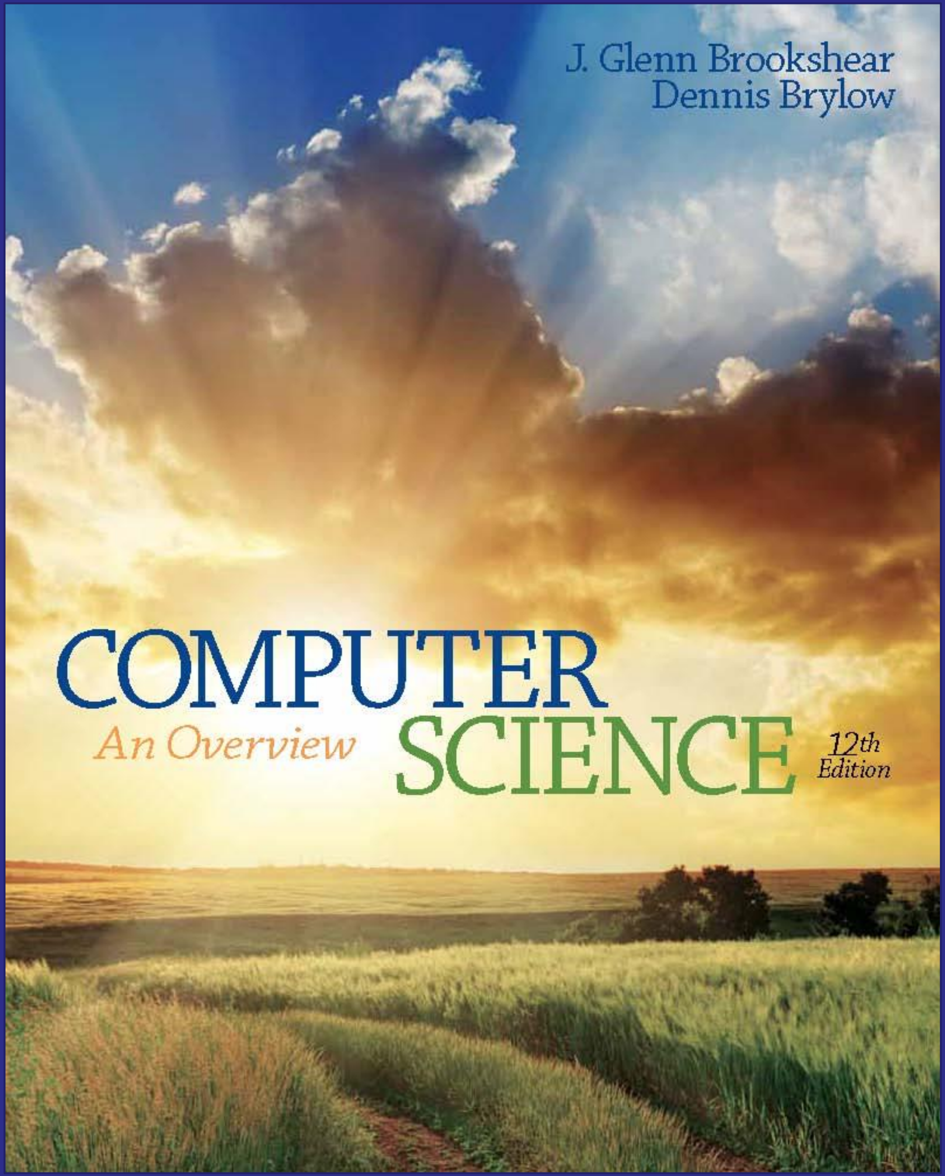


Figure 5.23 **The assertions associated with a typical while structure**



End of Chapter



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