

CSE 426 SYMBOLIC COMPUTATION

Multivariate Polynomial Interpolation

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Overview

- Problem definition
 - What are Sparse polynomials?
 - What are Dense polynomials?
 - What is multivariate polynomial?
 - What are univariate and multivariate polynomial interpolations?
- Example
- Possible solutions for the problem
 - Zippel Algorithm
- Demo
- Challenges
- Resources

What are Sparse polynomials?

- In mathematics, sparse polynomials are polynomials with many zero coefficients and that have far fewer terms than their degree.

Examples:

$$5x^{100} - 16x^8 + 3x$$

$$2x^7 + x^2$$

$$3x^5 + x^3 - 95$$

What are Dense polynomials?

- A multivariate polynomial is dense if "most" of the coefficients of its monomials are non-zero.

Examples:

$$5x^8 - 16x^7 + 3x^6 + x^5 - 12x^4 + 100x^3 + 5x^2 - 16x - 3$$

$$2x^4 + x^3 - 4x^2 - 87x + 24$$

$$x^3 - 4x^2 - 3x + 98$$

What is multivariate polynomial?

- A polynomial in more than one variable.

$$P(x, y) = a_{22} x^2 y^2 + a_{21} x^2 y + a_{12} x y^2 + a_{11} x y + a_{10} x + a_{01} y + a_{00}.$$

Examples:

$$x^4 y + x^2 y^2 z + x y z + 3$$

What are univariate and multivariate polynomial interpolations?

Univariate Polynomial Interpolation :

Let $n+1$ be x_0, x_1, \dots, x_n distinct points on which the values of f , $f(x_0), f(x_1), \dots, f(x_n)$ are known. Find a polynomial $p_n(x)$ of degree n which takes the same values as f at the same points.

Essentially we are looking for a polynomial $p_n(x)$ which satisfies the below **interpolation conditions**

$$p_n(x_i) = f(x_i) \text{ for } i = 0, 1, \dots, n$$

The points x_0, x_1, \dots, x_n are called **interpolation points** and the $p_n(x)$ **interpolation polynomial** of degree n .

Multivariate Polynomial Interpolation :

- When we apply above steps on the polynomials in more than one variable (e.g $x, y, z \dots$), then it's known as Multivariate Polynomial Interpolation.

Example:

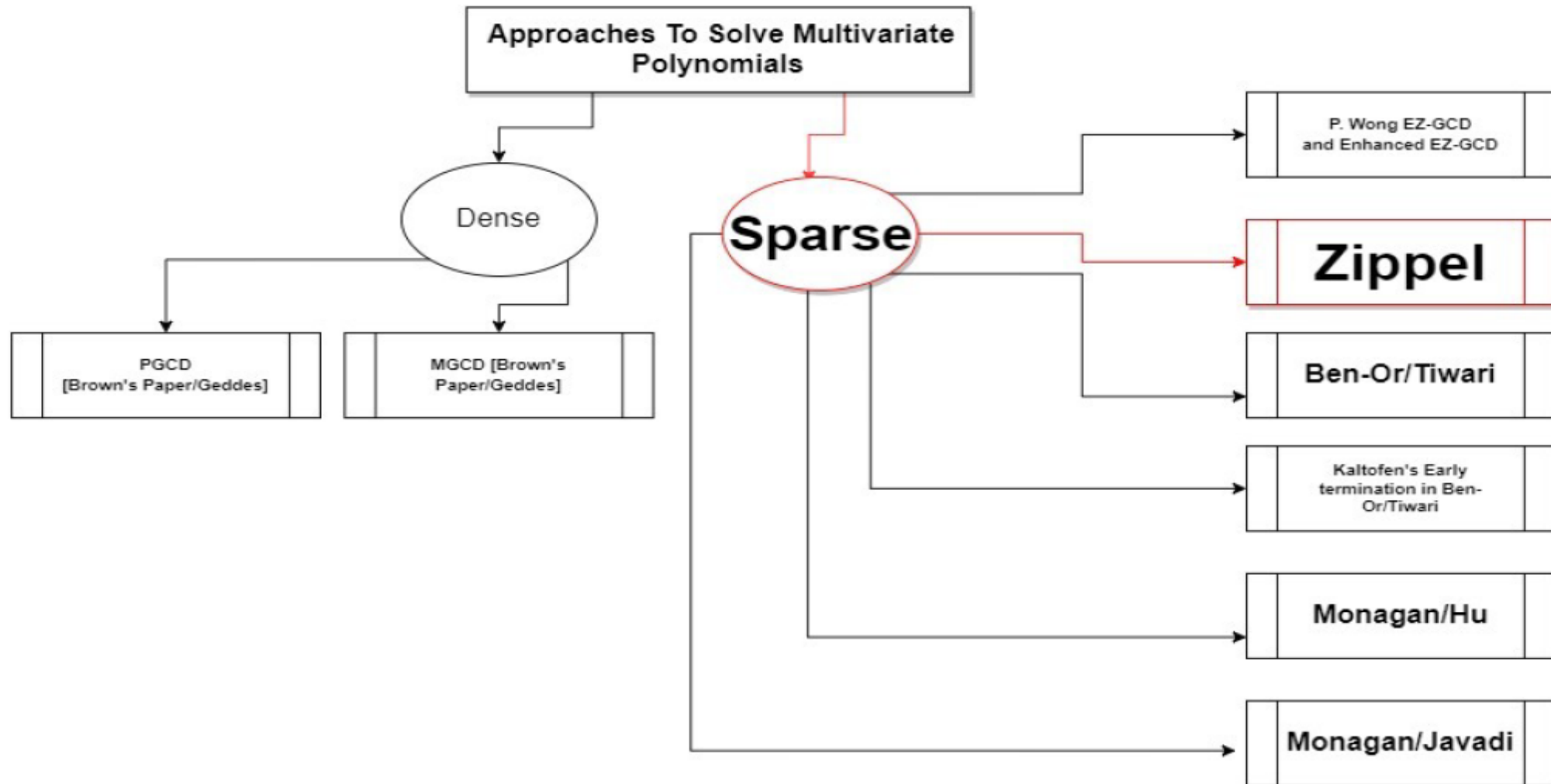
Inputs:

- $f(x,y,z) = x^4 y + x^2 y^2 z + x y z + 3$
- Distinct Points = x_0, x_1, x_2
- Degree Bound = 4 (Used on Zippel's Algorithm)

Output:

$P = x_0^4 x_1 + x_0^2 x_1^2 x_2 + x_0 x_1 x_2 + 3$ (Interpolated Polynomial of $f(x,y,z)$)

Possible solutions for the problem



Zippel's Algorithm 01

- Zippel's algorithm is a probabilistic algorithm which uses a combination of dense and sparse interpolation.
- As an input, Zippel's Algorithm requires a partial degree bound in each variable d (above slide's example).
- The interpolation procedure takes $O(ndt)$ evaluations and takes $O(ndt^3)$
- time, where n is the number of variables, d is the degree in each variable, and t is the number of terms in the final polynomial.

Zippel's Algorithm 02

- Takes a set of variables $\{X_1, \dots, X_v\}$ a degree bound d , a function $F(X_1, \dots, X_v)$.
- A starting point (a_1, \dots, a_v) as arguments. It is assumed that the values F returns are the values of some polynomial of at most v variables and of degree at most d in each variable.
- The algorithm returns a Polynomial $P(X_1, \dots, X_v)$ Where each variable occurs to degree no more than d and $P(b_1, \dots, b_v) = F(b_1, \dots, b_v)$ for all integers b .

Demo 01

- Stored list of functions.
- GUI generates random integer in each click.
- Finds the interpolated polynomial.

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MULTIVARIATE POLYNOMIAL INTERPOLATION

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Input Polynomial: $3x^4y^4z^4 + 5x^4y^2z^3 + x^3yz^2 + 8x^2yz + x$

Interpolated Polynomial: $3x_0^4x_1^4x_2^4 + 5x_0^4x_1^2x_2^3 + x_0^3x_1x_2^2 + 8x_0^2x_1x_2 + x_0$

CALCULATE

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Demo 02

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CSE 426 SYMBOLIC COMPUTATION SEMESTER PROJECT

MULTIVARIATE POLYNOMIAL INTERPOLATION

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Input Polynomial: $x^{**4}*y + x^{**2}*y^{**2}*z + x*y*z + 3$

Interpolated Polynomial: $x0^{**4}*x1 + x0^{**2}*x1^{**2}*x2 + x0*x1*x2 + 3$

CALCULATE

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Demo 03

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CSE 426 SYMBOLIC COMPUTATION SEMESTER PROJECT

MULTIVARIATE POLYNOMIAL INTERPOLATION

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Input Polynomial: $x^{**3}y^{**3}z + x^{**2}y + x*y*z + 5$

Interpolated Polynomial: $x_0^{**3}x_1^{**3}x_2 + x_0^{**2}x_1 + x_0*x_1*x_2 + 5$

CALCULATE

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Challenges

- Implemented in python.
- Faced plenty of Error while Implementing in Julia.
- Asked On Julia-Form, Stack-Over-Flow

KeyError: key coeffs not found

General Usage question math

sherlock_holmes 1 2d Jan 22nd

1/7 Jan 22nd

Hello, I'm getting the error given below continuously. I know that there is a built-in function named "coeffs" in Julia and I want to use it. I'm trying to implement Zippel's Algorithm. I also asked it on stackoverflow but couldn't find a solution. Can you please help me? Thanks in advance.

stackoverflow link: [python - coeffs not found in julia 1.7.1 - Stack Overflow](#)

Here is the error:

```
Warning: `vendor()` is deprecated, use `BLAS.get_config()` and inspect the caller = npyinitialize() at numpy.jl:67
@ PyCall C:\Users\Ashraf\.julia\packages\PyCall\LOfLP\src\numpy.jl:67
ERROR: LoadError: KeyError: key :coeffs not found
```

Stacktrace:

```
[1] __getproperty(o::PyCall.PyObject, s::Symbol)
@ PyCall C:\Users\Ashraf\.julia\packages\PyCall\LOfLP\src\PyCall.jl:307
[2] getproperty
@ C:\Users\Ashraf\.julia\packages\PyCall\LOfLP\src\PyCall.jl:312 [inlined]
[3] getproperty
```

23h ago

"as_coefficients_dict() not found" error in Julia 1.7.1

Asked today Active today Viewed 34 times

I am trying to use `as_coefficients_dict` (a python) function in Julia 1.7.1 but getting the error below:

```
ERROR: LoadError: KeyError: key :as_coefficients_dict not found
Stacktrace:
 [1] __getproperty(o::PyObject, s::Symbol)
@ PyCall ~/.julia/packages/PyCall/LOfLP/src/PyCall.jl:307
 [2] getproperty
@ ~/.julia/packages/PyCall/LOfLP/src/PyCall.jl:312 [inlined]
 [3] getproperty
@ ~/.julia/packages/SymPy/yCtE5/src/matrix.jl:52 [inlined]
 [4] sparse_interpolation(x_var::Tuple{Sym, Sym, Sym}, a::Vector{Int64}, d::Int64)
@ Main ~/Desktop/symbolic_project/zipfel.jl:182
 [5] top-level scope
@ ~/Desktop/symbolic_project/zipfel.jl:219
in expression starting at /home/can/Desktop/symbolic_project/zipfel.jl:219
(base) can@can-ThinkPad-L13:~/Desktop/symbolic_project$
```

The error occurs in the below code in the last line where I have

```
S = [p0].as_coefficients_dict().keys().
```

See the code below:

```
p0 = sympify(dense_interpolation([a[1]] .+ r, [p0] .+ P).subs(Symbol('x'), x_var[1]))
S = [p0].as_coefficients_dict().keys()
```

`as_coefficients_dict()` is a python function. How to represent it in Julia?

Could anyone help me out?

Resources

- https://stackoverflow.com/questions/70808453/i-try-using-sympy-in-julia-1-7-1-and-get-coeffs-not-found-error?noredirect=1#comment125185866_70808453
- https://stackoverflow.com/questions/70818498/as-coefficients-dict-not-found-error-in-julia-1-7-1?noredirect=1#comment125206699_70818498
- <https://discourse.julialang.org/t/keyerror-key-coeffs-not-found/75017>
- <https://www.sciencedirect.com/science/article/pii/S0377042700003538#:~:text=Multivariate%20polynomial%20interpolation%20is%20a,not%20deep%20but%20constant%20attention.>
- https://www.math.univ-toulouse.fr/~calvi/res_fichiers/MPI.pdf
- <https://www.math.auckland.ac.nz/~waldron/Multivariate/multivariate.html>
- https://www-users.cse.umn.edu/~olver/n_mv.pdf
- <https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.723.4396&rep=rep1&type=pdf>
- https://www.youtube.com/watch?v=YwZK6PzVm3Q&ab_channel=Audiopedia
- https://www-users.cse.umn.edu/~olver/n_mv.pdf
- <https://aip.scitation.org/doi/pdf/10.1063/1.168614>
- https://en.wikipedia.org/wiki/Multivariate_interpolation
- <https://link.springer.com/article/10.1007/BF01601941>