CBCS SCHEME

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20MCA14

First Semester MCA Degree Examination, July/August 2021
Mathematical Foundation for Computer Applications

Time: 3 hrs. Max. Marks: 100

Note: *1. Answer any FIVE full questions.*
2. Distribution tables are allowed.

1 a. Define: i) Symmetric difference of two sets ii) Power set, with an illustration for each. **(06 Marks)**

b. State pigeonhole principle. ABC is an equilateral triangle whose sides of length 1m. If we select 10 points inside the triangle, prove that at least two of these points are such that the distance between them is less than 1/3m. **(07 Marks)**

c. Find all the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$$ **(07 Marks)**

2 a. For any two sets A and B, prove that
i) $A - (A \cap B) = A - B$
ii) $A - (A - B) = A \cap B$. **(06 Marks)**

b. Among the integers from 1 to 200, find the number of integers that are
i) Not divisible by 5
ii) Divisible by 2 or 5 or 9
iii) Not divisible by 2 or 5 or 9. **(07 Marks)**

c. State and prove Demorgan laws, distributive laws of set theory. **(07 Marks)**

3 a. State the laws of logic. **(06 Marks)**

b. Prove the following is valid argument:

$$\begin{array}{l} \sim p \leftrightarrow q \\ q \rightarrow r \\ \underline{\sim r} \\ \therefore p \end{array}$$ **(07 Marks)**

c. Negate and simplify each of the followings
i) $\exists x, [p(x) \vee q(x)]$
ii) $\forall x, [p(x) \wedge \sim q(x)]$
iii) $\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$. **(07 Marks)**

4 a. Prove the following logical equivalences without using truth tables:
i) $[p \vee q \vee (\sim p \wedge \sim q \wedge r)] \Leftrightarrow (p \vee q \vee r)$
ii) $[(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$ **(06 Marks)**

b. Define converse, inverse and contra positive of a conditional $p \rightarrow q$. State the converse, inverse and contrapositive of the conditional. "If a quadrilateral is a parallelogram, then its diagonal bisect each other". **(07 Marks)**

c. Define Tautology; contradiction and contingency, prove that, for any propositions p, q, r the compound proposition $\{p \rightarrow (q \rightarrow r)\} \rightarrow \{(p \rightarrow q) \rightarrow (p \rightarrow r)\}$ is a tautology. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

5 a. Define partial order relation R defined on the set A. Let A = {1, 2, 3, 4, 6, 12}, define the relation R by aRb, if and only if a divides b. Prove that R- is a partial order on A, draw Hasse diagram for the relation. **(06 Marks)**

b. Consider A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}. The relation R is defined as $(x, y) \in R$, if and only if $x - y$ is multiple of 5. Verify that R- is an equivalence relation. **(07 Marks)**

c. Let A = {1, 2, 3}, and B = {1, 2, 3, 4}. The relations R and S from A to B are represented by the matrices.

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad M_S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Determine the relations $\overline{R}$, $\overline{S}$, $R \cup S$, $R \cap S$, S^C and their matrix representations. **(07 Marks)**

6 a. Let A = {1, 2, 3, 4, 6} and R be the relation on A defined by aRb if and only if a is multiple of b. Represent the relation R as a matrix and draw its diagraph. **(06 Marks)**

b. Let A = {a, b, c}, and R and S be relations on A whose matrices are given as

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}; \quad M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Find the composite relations ROS, SOR, ROR, SOS and their matrices. **(07 Marks)**

c. Let A = {1, 2, 3, 4, 5}. Define a relation R on A × A by $(x_1, y_1)\, R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$

i) Verify that R- is an equivalence relation on A × A.

ii) Determine the equivalent classes [(1, 3)], [(2, 4)] and [(1, 1)]. **(07 Marks)**

7 a. The probability distribution function P(X) of a variate X is given by the following table.

X:	0	1	2	3	4	5	6
P(X):	K	3K	5K	7K	9K	11K	13K

i) For what value of K, above data represent a valid probability distribution.

ii) Find $P(X < 4)$, $P(X \geq 5)$ and $P(3 < X \leq 6)$. **(06 Marks)**

b. Given 2% of fuses manufactured by a firm are defective. Find the probability that a box containing 200 fuses has

i) At least one ii) 3 or more iii) exactly two, defective fuses. **(07 Marks)**

c. In a test on electric bulbs, it was found that the life of a particular brand was distributed normally with an average life of 2000 hours and standard deviation of 60 hours. If a firm purchases 2500 bulbs find the number of bulbs that are likely to last for

i) More than 2100 hrs

ii) Less than 1950 hrs

iii) Between 1900 to 2100 hrs. **(07 Marks)**

8 a. For the standard normal distribution of a random variable Z, evaluate the followings:

i) $P(0 \leq z \leq 1.45)$ ii) $P(-3.40 \leq z \leq 2.65)$ iii) $P(-2.55 \leq z \leq -0.8)$ iv) $P(z \leq -3.35)$. **(06 Marks)**

b. The length of a telephone conversation has an exponential distribution with a mean of 3-minutes. Find the probability that a call ends.

i) in less than 3-minutes ii) taken between 3 and 5 minutes. **(07 Marks)**

c. A random variable X has the following probability function for various values of x

x:	0	1	2	3	4	5	6	7
p(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

i) Find k ii) evaluate $p(x < 6)$, $p(x \geq 6)$, $p(3 < x \leq 6)$ **(07 Marks)**

9 a. Explain the followings: i) Circuit ii) Euler and Hamiltonian path iii) Konigsberg bridge problem. **(06 Marks)**

b. Prove that whether the two following graphs are isomorphic or not: **(07 Marks)**

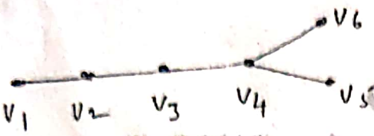

Fig.Q.9(b)(i)

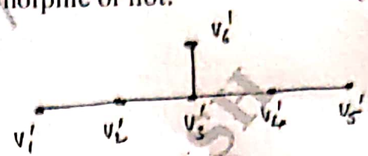

Fig.Q.9(b)(ii)

c. Determine whether the following graphs given are bipartite or not. **(07 Marks)**

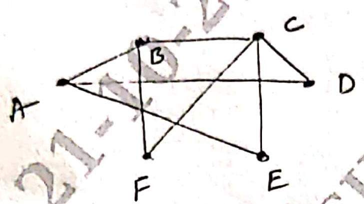

Fig.Q.9(c)(i)

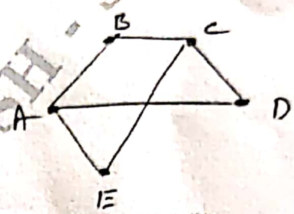

Fig.Q.9(c)(ii)

10 a. Define the terms:
i) Regular graph
ii) K-regular graph
iii) Complete graph **(06 Marks)**

b. Find the in-degree and out-degree of each vertex of each of the following directed graphs. Also verify that the sum of the in-degrees (or the out-degrees) equals the number of edges. **(07 Marks)**

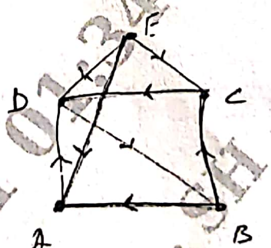

Fig.Q.10(b)

c. Determine whether the graphs shown are isomorphic. **(07 Marks)**

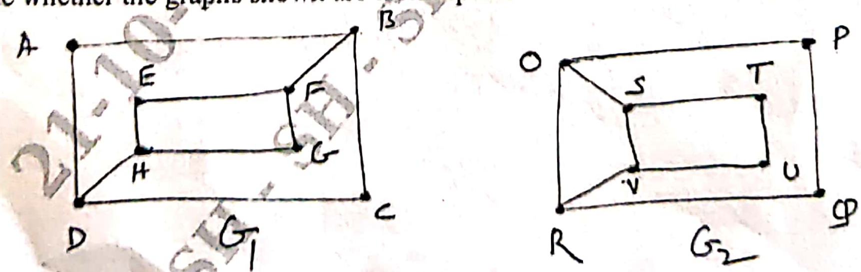

Fig.Q.10(c)

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