MODULE-IV

05/2023

①

Random Variable And Probability Distribution

- Random variable :

In a random experiment, if a real variable is associated with every outcome then it is called a random variable or stochastic variable.

In other words a random variable is a function that assigns a real number to every sample point in the sample space of random experiment.

Random variables usually denoted by X, Y, Z . . .

- Example :

Random experiment : Tosing a coin twice.

Sample space : {HH, HT, TH, TT}

Random variable x : Number of heads in the outcome

: {2, 1, 1, 0}

Range of x : $\{0,1,2\} = \{x_1, x_2, x_3\}$;

where x_1, x_2, x_3 are called values taken by the random variable x.

Imp * • Discrete Probability Distribution:

If for each value x_i of a discrete random variable x, we assign a real numbers $p(x_i)$ such that (i) $p(x_i) \geq 0$ (always positive).

(ii) $\sum_{i=0}^{n} p(x_i) = 1$, then $p(x)$ is said to be probability mass function. The set of values $[x_i, p(x_i)]$ is called a discrete probability distribution.

* Problems :

① A random variable x has the following probability distribution:

x:	0	1	2	3	4	5	6
p(x):	k	3k	5k	7k	9k	11k	13k.

(i) for what value of k this represents a valid probability distribution.

(ii) Find $P(x<4)$, $P(x \geq 3)$, $P(3<x\leq 6)$

(iii) Determine the minimum value of k, so that $P(x\leq 2) \geq 0.3$

pmf - probability mass function.

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

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Soln: (i) $P(x)$ is a valid probability distribution only if $p(x)\geq 0$ and, $\sum p(x)=1$ } (pmf)

we have,

$\sum p(x)=1 \Rightarrow k+3k+5k+7k+9k+11k+13k=1$

$49k=1$; $\boxed{k=1/49}$

(ii)

x	0	1	2	3	4	5	6
$p(x)$	$1/49$	$3/49$	$5/49$	$7/49$	$9/49$	$11/49$	$13/49$

$P(x<4)=P(x=0)+P(x=1)+P(x=2)+P(x=3)=1/49+3/49+5/49+7/49=16/49$

$P(x\geq 5)=P(x=5)+P(x=6)=11/49+13/49=24/49$

$P(3<x\leq 6)=P(x=4)+P(x=5)+P(x=6)=9/49+11/49+13/49=33/49$

(iii). $P(x\leq 2)\geq 0.3$

$k+3k+5k\geq 0.3$

$9k\geq 0.3$

$k\geq \frac{0.3}{9} \Rightarrow k\geq \frac{1}{30}$

(2) A random variable x has the following probability function for various values of x

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

$a=10, b=9, c=-1$

$k=\frac{-9\pm\sqrt{81-4\cdot 10\cdot(-1)}}{2(10)}$

$=\frac{-9\pm\sqrt{121}}{20}$

$k=\frac{-9+11}{20}$

$k=\frac{-9+11}{20}$ or $k=\frac{-9-11}{20}$

$k=1/10$, $k=-1$.

(i) find k.

(ii) Evaluate $P(x<6)$, $P(x\geq 6)$, $P(3<x\leq 6)$

(iii) Find the minimum value of x so that $P(X\leq x)>\frac{1}{2}$.

Soln:

(i) $p(x)\geq 0$ and $\sum p(x)=1$

$0+k+2k+3k+2k+k^2+2k^2+7k^2+k=1$

$10k^2+9k-1=0$

$k=1/10$ & $k=-1$

$k=1/10$

(ii) $[P(x<6)]$

x	0	1	2	3	4	5	6	7
$p(x)$	$0/100$	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$2/100$	$17/100$

- $p(x<6) = \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$

$= \frac{8}{10} + \frac{1}{100} = \frac{80+1}{100} = \frac{81}{100} = 0.81$

- $p(x \geq 6) = P(x=6) + P(x=7) = \frac{2}{100} + \frac{17}{100} = \frac{19}{100} = 0.19$

- $p(3<x\leq 6) = P(x=4) + P(x=5) + P(x=6)$

$= 3/10 + 1/100 + 2/100 = \frac{30+1}{100} = \frac{33}{100} = 0.33$

(iii).

x	0	1	2	3	4	5	6	7
$p(x)$	0	1/10	1/5	1/5	3/10	1/100	1/50	17/100
$p(X\leq x)$	0	1/10	3/10	5/10	8/10	80/100	83/100	1

Note that from the table $x=4$ is the minimum value of x for which,
$\rightarrow P(X\leq x) > 1/2$

(3) The probability distribution of a finite random variable X is given by the following table:

x :	-2	-1	0	1	2	3
$p(x)$:	0.1	k	0.2	2k	0.3	k

Find the value of k, mean, variance and standard deviation (S.D).

Soln: (i) $p(x) \geq 0$ and $\Sigma p(x) = 1$.

$\Rightarrow 0.1 + k + 0.2 + 2k + 0.3 + k = 1$.

$k = 1/10 = 0.1$

x	-2	-1	0	1	2	3
$p(x)$	0.1	0.1	0.2	0.2	0.3	0.1

$x\,p(x)$:	-0.2	-0.1	0	0.2	0.6	0.3	$\Sigma x p(x) = 0.8$
$x^2 p(x)$:	0.4	0.1	0	0.2	1.2	0.9	$\Sigma x^2 p(x) = 2.8$

Mean $(\mu) = EX = \Sigma x p(x)$

$= 0.8$

Variance $V(\sigma^2) = EX^2 - (EX)^2$ X

$= \Sigma x^2 p(x)$

$= 2.8$

$S.D = (\sigma) = \sqrt{v}$.

$= \sqrt{2.86}$

$= 1.46$

Variance, $V(\sigma^2) = EX^2 - (EX)^2$

$= \Sigma x^2 p(x) - (\Sigma x p(x))^2$

$= 2.8 - (0.8)^2$

$= 2.8 - 0.64$

$= 2.16$

HW: (1) Show that the following distribution represents a discrete probability distribution. find the mean and variance

x :	10	20	30	40
$p(x)$:	$1/8$	$3/8$	$3/8$	$1/8$

(3) Find the value of k such that the following distribution represents a finite probability distribution. Hence find its mean & standard deviation also find $P(x \leq 1), P(x > 1)$ & $P(-1 < x \leq 2)$

x :	-3	-2	-1	0	1	2	3
$p(x)$:	k	2k	3k	4k	3k	2k	k

it should be . poison $n > 30$

① v.imp ★★★ • **Binomial Distribution : (for $n \leq 30$)** (It is used for small samples)

Defination: A discrete random variable x is said to follow the binomial distribution if its probability mass function is of the form $p(x) = {}^nC_x p^x q^{n-x}$; where

$x = 0, 1, 2, \ldots n$

n = sample size or size of the sample.

p = probability of sucess.

q = probability of failure.

$p + q = 1$

(1 denotes the total probability)

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

$$^nP_r = \frac{n!}{(n-r)!}$$

atleast - min
atmost - max.

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Mean, variance and standard deviation of Binomial distribution:

Mean, $\mu = EX = np$

Variance, $V(x) = npq$.

Standard deviation, S.D, $\sigma = \sqrt{V(x)}$

$= \sqrt{npq}$

Problems:

1) The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured find the probability that

(i) exactly 2 will be defective.

(ii) at least 2 will be defective

(iii) none will be defective.

Soln: Let x denotes the defective pens

Probability of sucess = probability defective pens = $1/10 = 0.1 = p$.

Probability of failure = probability of non-defective pens = $q = 1 - p$.

$= 1 - 0.1 = 0.9$.

$n = 12$, $p = 0.1$, $q = 0.9$

$P(x) = {}^{12}C_x (0.1)^x (0.9)^{12-x}$

(i) $P(x=2)$

$= {}^{12}C_2 (0.1)^2 (0.9)^{12-2}$

$= 66 \times 0.01 \times 0.34$

$= 0.224$

(2) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - [{}^{12}C_0 (0.1)^0 (0.9)^{12-0} + {}^{12}C_1 (0.1)(0.9)^{12-1}]$

$= 1 - [1 \times 1 \times \quad + 12 \times 0.1 \times \quad]$

(3) $P(X=0) = {}^{12}C_0 (0.1)^0 . (0.9)^{12-0}$

$= 1 \times 1 \times 0.28 \qquad = 0.28$

(2) When a coin is tossed 4 times find using binomial distribution the probability of getting

(i) exactly one head.

(ii) atmost 3 heads

(iii) atleast 3 heads

Soln:

X : No of getting heads in the toss.

$n = 4$

$a^m . a^n = a^{m+n}$

p - probability of getting head $= \frac{1}{2} = 0.5$

$\frac{a^m}{a^n} = a^{m-n}$

q - probability of not getting head $= 1 - p = 0.5$

$p(x) = {}^nC_x p^x q^{n-x}$

$p(x) = {}^4C_x (0.5)^x (0.5)^{4-x} = {}^4C_x (0.5)^4 = 0.0625 \, {}^4C_x$.

(i) P (exacty one head) $= P(X=1) = {}^4C_1 (0.5)^4 = 0.0625 \, {}^4C_1 = 0.25$

(ii) P (atmost 3 heads) $= P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 1 - P(X > 3)$

$= 1 - P(X=4)$

$= 1 - 0.0625 \, {}^4C_4$ $\quad ({}^nC_n = 1)$

$= 0.9375$

$P(X \geq 3) =$

(iii) P (atleast 3 heads) $= P(X=3) + P(X=4)$

$= 0.0625 \, {}^4C_3 + 0.0625 \, {}^4C_4$.

$= 0.3125$

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(3) In 800 families with five children each how many families would be expected to have
(i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys.
(iv) atmost 2 girls by assuming probability for boys and girls to be equal.

Soln: X - Number of boys in the family.

p = probability of expecting boy in the family. = 0.5

n = 5

N = 800

q = probability of expecting girl in the family. = 0.5

$P(x) = {}^5C_x (0.5)^x \cdot (0.5)^{5-x}$

$a^m \cdot a^n = a^{m+n}$

$= {}^5C_x (0.5)^5$

$= 0.03125 \; {}^5C_x$

(i)
$P(x=3) = 0.03125 \; {}^5C_3$

$= 0.03125 \times \frac{5 \times 4 \times 3!}{(5-3)!\,3!} = 0.03125 \times \frac{5 \times 4 \times 3!}{2! \times 3!}$

$= 0.03125 \times \frac{5 \times \cancel{4}^{2}}{2 \times 1} = 0.03125 \times 10$

$= 0.3125$

∴ probability of expecting 3 boys in 800 families = $800 \times 0.3125 = 250$

(ii) $P(x=0) = 0.03125 \; \underbrace{{}^5C_0}_{=1} = 0.03125$

∴ probability of 5 girls in 800 families = $800 \times 0.03125 = 25$

(iii) $P(x=2) + P(x=3) = 0.03125 \; {}^5C_2 + 0.03125 \; {}^5C_3$

$= 0.625$

∴ probability of expecting either 2 or 3 boys = $800 \times 0.625 = 500$

(iv) $P(x=3) + P(x=4) + P(x=5) =$

$= 0.03125 \; {}^5C_3 + 0.03125 \; {}^5C_4 + 0.03125 \; {}^5C_5$

$= 0.03125 \times 10 + 0.03125 + 0.03125$

$= 0.5$

B	G
3	2
4	1
5	0

∴ probability of getting atmost 2 girls = $0.5 \times 800 = 400$

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06/05/2022. Discrete probability distribution:

(4) A random variable x takes the values $-3, -2, -1, 0, 1, 2, 3$ such that

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(i) $P(x=0) = P(x<0)$

(ii) $P(x=-3) = P(x=-2) = P(x=-1) = P(x=1) = P(x=2) = P(x=3)$.

find its probability distribution.

Soln: Let

x:	-3	-2	-1	0	1	2	3
$P(x)$	p_1	p_2	p_3	p_4	p_5	p_6	p_7.

(i) $P(x=0) = P(x<0)$

$P(x=0) = P(x=-3) + P(x=-2) + P(x=-1)$

$p_4 = p_1 + p_2 + p_3 \longrightarrow (1)$

(2) $P(x=-3) = P(x=-2) = P(x=-1) = P(x=1) = P(x=2) = P(x=3)$.

$p_1 = p_2 = p_3 = p_5 = p_6 = p_7 \longrightarrow (2)$

Now (1)

$p_4 = p_1 + p_1 + p_1$

$\underline{p_4 = 3p_1}$

w.k.T $\Sigma p(x) = 1 \Rightarrow p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 = 1$

$\Rightarrow p_1 + p_1 + p_1 + 3p_1 + p_1 + p_1 + p_1 = 1$

$9p_1 = 1$

$\underline{p_1 = 1/9}$

$p_4 = 3\left(\frac{1}{9}\right) = \frac{1}{3}$

x	-3	-2	-1	0	1	2	3
$P(x)$	$1/9$	$1/9$	$1/9$	$1/3$	$1/9$	$1/9$	$1/9$.

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(4) In a quix contest of answering "YES" or "No" what is the probability of guessing atleast 6 answers correctly out of 10qns asked? Also find the probability of the same if there are 4 options for a correct answer.

Soln: x = is guessing correct answer.

p - is probability of saying YES $= \frac{1}{2} = 0.5$

q - is probability of saying No $= \frac{1}{2} = 0.5$

$n = 10$

$P(x) = {}^{10}C_x (0.5)^x . (0.5)^{10-x}$

$= {}^{10}C_x (0.5)^{10}$

$= 9.765625 \times 10^{-4} \; {}^{10}C_x$

If there are 4 options for a correct answer then $p = 1/4$ $q = 1-p$.

i.e. $1 - 1/4 = 3/4$

$P(x) = {}^{10}C_x (0.25)^x (0.75)^{10-x}$.

=

(5) In a sampling of a large number of parts manufactured by a company, the mean number of defectives in sample of 20 is 2. out of 1000 such samples how many would be expected to contain atleast 3 defective parts:

(4)

Soln: $n = 20$ $\mu = 2$ X = no of defective parts.

mean $= np = 2$

$20p = 2$

$p = \frac{2}{20} = 0.1$ $q = 1 - p$

$= 1 - 0.1$

$= 0.9$

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$p(x) = {}^{20}C_x \; 0.1^x . 0.9^{20-x}$

$P(X \geq 3) = 1 - P(X<3)$

$= 1 - [P(X=2)+P(X=1)+P(X=0)]$

$= 1 - [{}^{20}C_2 (0.1)^2 (0.9)^{20-2} + {}^{20}C_1 (0.1)^1 (0.9)^{20-1} + {}^{20}C_0 (0.1)^0 (0.9)^{20-0})$

~~(∴ The no of samp)~~

$= 1 - [190 (0.01)(0.150) + 20 (0.1)(0.135) + (1)(1)(0.121)]$

$= 1 - 0.676$

$= 0.324$

∴ The no of samples having atleast 3 defective parts out of 1000 samples is

$= 1000 \times 0.324$

$= 324$

(6) The mean and variance of a binomial distribution are 4 & 4/3 respectively. Find $P(x \geq 1)$

Soln:

$M=4 \quad V = 4/3 \quad P(X \geq 1)$

$np = 4 \quad npq = 4/3$

$4q = 4/3$

$q = 1/3 = 0.3333$

$p = 1-q = 1 - 0.3333 = 0.6666$

$np = 4 \;, \; n(0.6666) = 4$

$n = \frac{4}{0.6666} \quad 6.0066 \approx 6.$

$p(x) = {}^nC_x \, p^x q^{n-x}$

$= {}^6C_x (0.6666)^x (0.3333)^{6-x}$

$P(X \geq 1) = 1 - P(X<1)$

$= 1 - P(X=0)$

$= 1 - {}^6C_0 (0.6666)^0 (0.3333)^{6-0}$

$= 1 - 1(1)(1.3709 \times 10^{-3})$

$= 0.9986$

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(7) If the mean and standard deviation of the number of correctly answered questions in a test given to 4096 students are 2.5 and $\sqrt{1.875}$. Find an estimate number of candidates answer correctly
(i) 8 or more qns
(ii) 2 or less qns
(iii) 5 qns.

Soln: Let x = no of correctly answered questions.

$\mu = 2.5$

$S.D = \sqrt{1.875}$

$np = 2.5$

$S.D = \sqrt{npq}$

$\sqrt{npq} = \sqrt{1.875}$

$npq = 1.875$

$2.5q = 1.875$

$q = \frac{1.875}{2.5} = 0.75$

$p = 1 - q = 1 - 0.75 = 0.25$

$np = 2.5$

$n(0.25) = 2.5$

$n = \frac{2.5}{0.25}$

$= 10$

$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{10}C_{x} (0.25)^{x} (0.75)^{10-x}$.

(i) $P(X \geq 8) = P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_{8}(0.25)^{8}(0.75)^{2} + {}^{10}C_{9}(0.25)^{9}(0.75)^{1} + {}^{10}C_{10}(0.25)^{10}(0.75)^{0}$

$= 45\,(1.5258 \times 10^{-5})\, 0.5625 + 10\,(3.8146 \times 10^{-6})(0.75) + 1\,(9.536 \times 10^{-7})\, 1.$

$= 3.86218 \times 10^{-4} + 2.860 \times 10^{-5} + 9.536 \times 10^{-7}$

$= 4.1577 \times 10^{-4}$

$\therefore$ The no of candidates answering 8 or more question correctly out of 4096 students $= 4.1577 \times 10^{-4} \times 4096$.

$= 1.70300 \approx 2$

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(8) It is conjectured that an impurity exists in 30 % of all drinking wells in a certain rural community. In order to gain some insight into the true extent of the problem, it is determined that some testing is necessary. It is too expensive to test all of the wells in the area, so 10 are randomly selected for testing. Find using B.D

(1) probability that exactly 3 wells have the impurity.

(2) probability that more than 3 wells are impure.

Soln: X = no of impure wells.

$p = 30\% = \frac{30}{100} = 0.3$

$q = 1 - 0.3 = 0.7$

$n = 10$

$p(x) = {}^{n}C_x \, p^x q^{n-x}$

$= {}^{10}C_x (0.3)^x (0.7)^{10-x}$

(i) $p(x=3) = {}^{10}C_3 \, (0.3)^3 (0.7)^{10-3}$

$= 120(0.027)(0.082)$

$= \underline{0.2668}$

0.3504 (ii) $p(x > 3)$

H.W (9) A box contains 100 transistors, 20 of which are defective & 10 are selected at random. Find the probability that

(1) all are defective

(2) atleast one is defective

(3) all are good

(4) atmost 3 are defective.

6) The probability that a patient recovers from a rare blood disease is 0.6. If the 15 people are known to have contracted this disease, what is the probability that

a) all least 10 survive.

b) from 3 to 8 survive

c) exactly 5 survive

(Hint X: number of people surviving $p = 0.4$, $n = 15$)

(11) The number of telephones lines busy at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are chosen at random what is the probability that (i) no line is busy

(ii) all lines are busy

(iii) atleast one line is busy.

(iv) almost 2 lines are busy

(Hint: $p = 0.1$, $n = 10$).

② Poisson Distribution: (for $n > 30$)

Poisson distribution can be derived as limiting case of the binomial distribution by making $\boxed{n \text{ very large}}$ $(n \to \infty)$ and p $\boxed{\text{very small } (p \to 0)}$ keeping

$\boxed{np = m}$ a finite constant.

The poisson probability function or the poisson distribution of the random variable is given by

$$\boxed{p(x) = \frac{m^x e^{-m}}{x!}}$$

where x is called poisson variate.

Poisson Distribution: (for $n > 30$)

- Mean, Variance and standard deviation of the poisson distribution.

Mean, $\mu = EX = m$.

Variance, $V(x) = m$.

Standard Deviation, S.D, $\sigma = \sqrt{V(x)}$

$= \sqrt{m}$.

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Problems :

(1) If the probability of a bad reaction from a certain injection is 0.001. Determine the chance that out of 2000 individuals more than two will get a bad reaction.

Soln: → X - Number of individuals having bad reaction.

$p = 0.001$

$n = 2000$

$m = np = 2000 \times 0.001 = 2$.

$$p(x) = \frac{m^x e^{-m}}{x!}$$

$$= \frac{2^x e^{-2}}{x!}$$

$$P(x>2) = 1 - P(x \leq 2) = 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$P(x>2) = 1 - \left[\frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!}\right]$$

$$= 1 - [0.1353 + 0.2706 + 0.2706]$$

$$= 1 - [0.6765]$$

$$= 0.3233$$

(2) In a certain factory turning out razor blade there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use poisson distribution to calculate the approximate number of packets containing no defective, one defective and 2 defective blades respectively in a consignment of 10,0000 packets.

Soln:

X = Number of defective blades

$p = 0.02$

$n = 10$

$m = np = 0.002 \times 10 = 0.02$

$$p(x) = \frac{m^x e^{-m}}{x!} = \frac{(0.02)^x e^{-0.02}}{x!}$$

$$= 0.9801\left(\frac{0.02^x}{x!}\right)$$

(i) $P(x=0) = 0.9801\left(\frac{0.02^0}{0!}\right)$

$= 0.9801$

$\therefore$ No of packets containing no defective blades out of 10,000 packets =

$= 10000 \times 0.9801$

$= 9801$

(ii) $P(x=1) = 0.9801\left(\frac{0.02^1}{1!}\right)$

$= 0.01960$

$\therefore$ No of packets containing 1 defective blades out of 10,000 packets $= 10000 \times 0.01960$

$= 196.02$

(iii) $P(x=2) = 0.9801\left(\frac{0.02^2}{2!}\right)$

$= 1.9602 \times 10^{-4}$.

$\therefore$ No of packets containg 2 defective blades out of 10.000 packets $= 10000 \times 1.9602 \times 10^{-4}$

$= 1.9602$

(3) Alpha particles are emitted by a radio active source at an average of 5 emissions is 20 min. what is the probability that there will be

(i) exactly 2 emissions.

(2) atleast 2 emissions in 20 minutes.

Soln: X: Number of emission $\mu = np = m$.

average $= m = 5$

$$P(x) = \frac{m^x e^{-m}}{x!} = \frac{5^x e^{-5}}{x!} = \frac{6.7379 \times 10^{-3}\ 5^x}{x!}$$

(i.) $P(x=2)$

$$P(x) = \frac{6.7379 \times 10^{-3}\ 5^x}{x!} \left[\frac{5^2}{2!}\right]$$

$$= 0.08422$$

(ii.) $P(X \geq 2) = 1 - [P(X<2)]$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - \left[\frac{6.7379 \times 10^{-3} \times 5^0}{0!} + \frac{6.7379 \times 10^{-3} \times 5^1}{1!}\right]$$

$$= 1 - 6.7379 \times 10^{-3} + 0.03368$$

$$= 0.95958$$

(4) A car hire from has 2 cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson distribution with mean 1.5. Calculate the proportion of days.

(i) on which there is no demand.

(ii.) on which demand is refused.

Soln: X = number of demands for a car.

Mean $= m = 1.5$

$$P(x) = \frac{m^x e^{-m}}{x!} = \frac{1.5^x e^{-1.5}}{x!} = \frac{0.2231\ 1.5^x}{x!} = \frac{0.2231\ (1.5^x)}{x!}$$

(i) $P(X=0)$

$$P(x) = \frac{0.2231 \; 1.5^{0}}{0!}$$

$$= 0.2231$$

(ii) $P(X>2)$

$$\cancel{P(x) = \frac{0.2231 \; 1.5^{2}}{2!}}$$

$$P(x) = 1 - P(X \leq 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[0.2331 \left[\frac{1.5^{0}}{0!}\right] + 0.2331 \left[\frac{1.5^{1}}{1!}\right] + 0.2331 \left(\frac{1.5^{2}}{2!}\right)\right]$$

$$= 1 - [0.2331 + 0.3346 + 0.25098]$$

5) During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is probability that 6 particles enter the counter in a given millisecond?

Soln: X: Number of radioactive particles passing through the counter.

Mean $= m = 4$

$$p(x) = \frac{m^{x} e^{-m}}{x!}$$

$$= \frac{4^{x} e^{-4}}{x!} = \frac{0.01831 \; 4^{x}}{x!}$$

$P(X=6)$

$$= \frac{0.01831 \; 4^{6}}{6!} = 0.1041$$

(6) Ten is the average number of oil tankers arriving eachday at a certain post. The facilities at a port can handle atmost 15 tankers per day. What is the probability that on a given day tankers have to be turned away?

Soln: X = number of tanker

$m = 10$.

$$p(x) = \frac{m^x e^{-m}}{x!}$$

$$= \frac{10^x e^{-10}}{x!}$$

$p(x > 15)$

Answer : 0.0487

(7) In a certain industrial facility, accidents occur infrequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other.

(a) What is the probability that in any given perold of 400days there will be an accident on one day.

(b) What is the probability that there are atmost 3 days with an accident?

Soln: X = Number of ~~(accidents per)~~ days with accident.

$p = 0.005$

$n = 400$

$m = np$

$= 400 \times 0.005$

$= 2$

$$p(x) = \frac{m^x e^{-m}}{x!} = \frac{2^x e^{-2}}{x!} = 0.1353 \frac{2^x}{x!}$$

(a) $p(x=1)$

$$= \frac{2^1 \, 0.1353}{1!} = 0.2706$$

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(b) $P(x \leq 3)$.

$= P(x=0) + P(x=1) + P(x=2) + P(x=3)$.

$= \frac{0.1353(2^0)}{0!} + \frac{0.1353(2^1)}{1!} + \frac{0.1353(2^2)}{2!} + \frac{0.1353(2^3)}{3!}$

$= 0.1353 + 0.2706 + 0.2706 + 0.1804$.

$= 0.857$

(8) In a manufacturing process where glass products are made, defects or bubbles occur occasionally rendering the piece undesirable for marketing. It is known that, on average 1, in every 1000 of these items produced has one or more bubbles. What is the probability that a random sample of 8000 will yield fewer than 7 items possessing window?

Solution.

x: Number of last product having bubbles.

$n = 8000$

$p = \frac{1}{1000} = 0.001$

$m = np = 8000 \times 0.001 = 8$

$p(x) = \frac{m^x e^{-m}}{x!}$

$p(x) = \frac{8^x e^{-8}}{x!} = \frac{8^x (3.3546 \times 10^{-4})}{x!}$

$P(x<7)$.

$= P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6) + P(x=7)$.

$= \frac{8^0 e^{-8}}{0!} + 8^1 e^-$

$= \frac{8^0(3.3546 \times 10^{-4})}{0!} + \frac{8^1 (3.3546 \times 10^{-4})}{1!} + \frac{8^2 (3.3546 \times 10^{-4})}{2!} + \frac{8^3 (3.354 \times 10^{-4})}{3!}$

$+ \frac{8^4 (3.3546 \times 10^{-4})}{4!} + \frac{8^5 (3.3546 \times 10^{-4})}{5!} + \frac{8^6 (3.3546 \times 10^{-4})}{6!} + \frac{8^7 (3.3546 \times 10^{-4})}{7!}$

S.D $= \sqrt{v}$.

$v = EX^2 - (EX)^2 = \int x^2 f(x)dx - \left[\int x f(x) dx\right]^2$.

$f(x) \geq 0$

$\int_{-\infty}^{\infty} f(x) = 1$

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Problems:

$e^{-\infty} = 0$.

$e^0 = 1$

(1) Is the following function a density function.

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

If so, determine the probability that a variable having this density will fall in the interval $(1,2)$

Soln:

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0 & x < 0. \end{cases}$$

clearly $f(x) \geq 0$ (e^x is a +ve function)

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} e^{-x} dx$$

$$= 0 + \left[\frac{e^{-x}}{-1}\right]_0^{\infty}$$

$$= e^{-x}\Big[_0^{\infty} = -\left[e^{-\infty} - e^0\right]$$

$$= -[0-1] = 1$$

$\therefore f(x)$ is a density function. (pdf)

The probability that the variate having this density following in a interval $(1,2)$ is to compute $P(1<x<2)$

i.e $P(1<x<2) = \int_1^2 f(x)dx$

$$= \int_1^2 e^{-x} dx$$

$$= \left[\frac{e^{-x}}{-1}\right]_1^2$$

$$= -\left[e^{-2} - e^{-1}\right]$$

$$= [0.1353 - 0.3678]$$

$$= 0.2325$$

$f(x) \geq 0$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$e^{\infty} = 1.$

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(2) If x is a continous random variable with probability function given by

$$f(x) = \begin{cases} kx & , 0 < x \leq 2 \\ 2k & , 2 \leq x \leq 4 \\ -kx + 6k & , 4 \leq x \leq 6 \end{cases}$$

find k & mean value of x

$\int e^{ax}\,dx = \frac{e^{ax}}{a}$

$\int e^{-ax}\,dx = \frac{e^{-ax}}{-a}$

$\int x^n\,dx = \frac{x^{n+1}}{n+1}$

$\int x\,dx = \frac{x^2}{2}$

$\int x^0\,dx = \frac{x^1}{1} = x$

Soln: Since f(x) is pdf

$f(x) \geq 0$ & $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx + 6k)\,dx = 1$$

$$= k\frac{x^2}{2}\Big|_0^2 + 2kx\Big|_2^4 - k\frac{x^2}{2}\Big|_4^6 + 6kx\Big|_4^6 = 1$$

$$= k\left[\frac{4}{2} - 0\right] + 2k\left[4-2\right] - k\left[\frac{36}{2} - \frac{16}{2}\right] + 6k\left[6-4\right] = 1$$

$$= 2k + 4k - 10k + 12k = 1$$

$$8k = 1$$

$$k = 1/8$$

$$\mu = EX = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^2 x \cdot kx\,dx + \int_2^4 x \cdot 2k\,dx + \int_4^6 x(-kx + 6k)\,dx$$

$$= \int_0^2 kx^2\,dx + \int_2^4 2kx\,dx + \int_4^6 (-kx^2 + 6kx)\,dx$$

$$= k\frac{x^3}{3}\Big|_0^2 + 2k\frac{x^2}{2}\Big|_2^4 - k\frac{x^3}{3}\Big|_4^6 + 6k\frac{x^2}{2}\Big|_4^6$$

$$= \frac{1}{8}\left[\frac{8}{3} - 0\right] + \frac{2}{8}\left[\frac{16}{2} - \frac{4}{2}\right] - \frac{1}{8}\left[\frac{216}{3} - \frac{64}{3}\right] + \frac{6}{8}\left[\frac{36}{2} - \frac{16}{2}\right]$$

$$= \frac{1}{3} + \frac{1}{4}\left[6\right] - \frac{1}{8}\left[\frac{152}{3}\right] + \frac{3}{4}\left[10\right]$$

$$= \frac{1}{3} + \frac{6}{4} - \frac{153}{24} + \frac{30}{4} = 3$$

(3) Random variable x has density function.

$$p(x) = \begin{cases} kx^2 & , -3 \le x \le 3 \\ 0 & , elsewhere \end{cases}$$

euvaluate k and find

(i) $P(x \le 1)$ (ii) $P(1 \le x \le 2)$ (iii) $P(x \le 2)$ (iv) $P(x > 1)$ (v) $P(x > 2)$

(*) $p(x) \ge 0$ & $\int_{-\infty}^{\infty} p(x)dx = 1$

$= \int_{-\infty}^{\infty} p(x)dx = 1$

$= \int_{-3}^{3} kx^2 dx = 1$

$= k \frac{x^3}{3}\Big|_{-3}^{3} = 1$

$= k\left[\frac{27}{3} - \left(\frac{-27}{3}\right)\right] = 1$

$= k[18] = 1$

$= k = 1/18$

(1) $p(x \le 1) = \int_{-3}^{1} p(x)dx$

$= \int_{-3}^{1} kx^2 dx$

$= \int_{-3}^{1} \frac{1}{18} x^2 dx.$

$= \frac{1}{18}\left[\frac{x^3}{3}\right]_{3}^{1}$

$= \frac{1}{18}\left[\frac{1}{3} - \left(\frac{-27}{3}\right)\right]$

$= \frac{1}{18}\left[\frac{28}{3}\right]$

$= 0.5185$

(2) $p(1 \le x \le 2) = \int_{1}^{2} p(x)dx$

$= \int_{1}^{2} \frac{x^2}{18} dx = \frac{1}{18}\left[\frac{x^3}{3}\right]^2$

$= \frac{1}{18}\left[\frac{8}{3} - \frac{1}{3}\right]$

$= \frac{1}{18}\left[\frac{7}{3}\right]$

$= 0.1296$

(3) $p(x \le 2) = \int_{-3}^{2} p(x)dx$

$= \int_{-3}^{2} \frac{x^2}{18} dx$

$= \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^{2}$

$= \frac{1}{18}\left[\frac{8}{3} - -\frac{27}{3}\right]$

$= \frac{1}{18}\left[\frac{35}{9}\right]$

$= 0.6481$

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(4). $p(x>1) = \int_2^3 p(x)dx$

$= \int_2^3 \frac{x^2}{18} dx$

$= \frac{1}{18}\left[\frac{x^3}{3}\right]_2^3$

$= \frac{1}{18}\left[\frac{27}{3} - \frac{8}{3}\right]$

$= \underline{\underline{0.3518}}$

(5) $p(x>2) = 1 - P(x \le 2)$

$= 1 - \int_{-3}^{2} \frac{x^2}{18} dx$

$= 1 - \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^{2}$

$= 1 - \frac{1}{18}\left[\frac{8}{3} - \frac{27}{3}\right]$

$= \underline{\underline{0.3518}}$

4) The diameter of an electrical cable is assumed to be a continous variate with probability function $f(x) = 6x(1-x) = 6x - 6x^2$, $0 \le x \le 1$.
Verify that $f(x)$ is probability density function & also find mean, variance

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(9) Exponential Distribution:

The continous probability distribution having probability density function given by $f(x) = \begin{cases} \alpha e^{-\alpha x}, & 0 < x < \infty \\ 0 & , \text{elsewhere} \end{cases}$

this is known as exponential distribution.

Mean, variance and S.D for Exponential distribution.

Mean, $\mu = \frac{1}{\alpha}$.

Variance, $V(x) = \frac{1}{\alpha^2}$

S.D, $\sigma =$

$$\sqrt{V(x)} = \sqrt{\frac{1}{\alpha^2}} = \frac{1}{\alpha}.$$

Problems:

(1) If x is an exponential variate with mean 5, euvaluate

(1) $P(0 < x < 1)$

(2) $P(-\infty < x < 10)$

(3) $P(x \leq 0 \text{ or } x \geq 1)$

(4.) $P(x > 1)$.

Soln: Mean = 5

$$\frac{1}{\alpha} = 5 \qquad f(x) = 0.2e^{-0.2x}, \ 0 < x < \infty$$

$$\alpha = \frac{1}{5} = 0.2$$

(i) $P(0 < x < 1) = \int_0^1 f(x)\,dx$

$$= \int_0^1 0.2e^{-0.2x}\,dx.$$

$$= 0.2 \left[\frac{e^{-0.2x}}{-0.2} \right]_0^1$$

$= -\left[e^{-0.2} - e^{0}\right]$ $\qquad$ $\int \text{constant} \cdot 1\, dx = c \cdot x.$

$= -\left[0.81 - 1\right]$ $\qquad$ $\int 0\, dx = 0 \cdot x = 0$

$= 0.1812$

(2) $P(-\infty < x < 10) = \int_{-\infty}^{10} f(x)\,dx.$

$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{10} f(x)\,dx.$

$= \int_{-\infty}^{0} 0 \cdot dx + \int_{0}^{10} 0.2\, e^{-0.2x}\,dx.$

$= 0 + \cancel{0.2}\left[\frac{e^{-0.2x}}{-\cancel{0.2}}\right]_{0}^{10}$

$= -\left[e^{-2} - e^{0}\right]$

$= \cancel{(0.1353)}\ 0.8646$

(3) $P(x \leq 0$ or $x \geq 1) = \int_{-\infty}^{0} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx.$

$= 0 + \int_{1}^{\infty} e^{-0.2x}\,dx$

$= \cancel{0.2}\left[\frac{e^{-0.2x}}{-\cancel{0.2}}\right]_{1}^{\infty}$

$= -\left[e^{-\infty} - e^{-0.2}\right]$

$= 0.8187$

(4) $P(x > 1) = \int_{1}^{\infty} f(x)\,dx$

$= \int_{2}^{\infty} 0.2\, e^{-0.2x}\,dx$

$$= 0.2 \left[\frac{e^{-0.2x}}{-0.2} \right]_{2}^{\infty}$$

$$= - \left[e^{-\infty} - e^{-0.4} \right]$$

$$= \cancel{(-1)} \; 0.6703 \quad = 0.8187$$

(2) The length of telephone conversation in a both has been exponentially distributed and found an average to be 5 min. Find the probability that a random call (i) ends in less than 5 mints. and.

(ii) takes between 5 and 10 min.

Soln:

x : length of telephone conversation.

mean = 5

$$\frac{1}{\alpha} = 5 \Rightarrow \alpha = \frac{1}{5} = 0.2$$

$$f(x) = \alpha e^{-\alpha x}, \; 0 < x < \infty$$

$$f(x) = 0.2 e^{-0.2x}, \; 0 < x < \infty$$

$$(i) \; P(x<5) = \int_{-\infty}^{5} f(x)\,dx$$

$$= \underbrace{\int_{-\infty}^{0} f(x)\,dx}_{=0} + \int_{0}^{5} f(x)\,dx$$

$$= \int_{0}^{5} 0.2\, e^{-0.2x}\,dx$$

$$= \cancel{0.2} \left[\frac{e^{-0.2x}}{-\cancel{0.2}} \right]_{0}^{5} = - \left[e^{-1} - e^{0} \right]$$

$$= \cancel{0.55067} \; 0.6321$$

$$(2) \; P(5<x<10) = \int_{5}^{10} f(x)\,dx.$$

$$= \int_{5}^{10} 0.2 \; e^{-0.2x} \; dx$$

$$= 0.2 \left[\frac{e^{-0.2x}}{-0.2} \right]_5^{10}$$

$$= -\left[e^{-2} - e^{-1} \right]$$

$$= \underline{\underline{0.2325}}$$

(3) In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. what is the probability that shower will end for.

(i) 10 min or more

(ii) less than 10 min

(iii) btween 10 and 12 min

Soln:

$\mu = 5$

$\frac{1}{\alpha} = 5$, $\alpha = 0.2$ $\qquad f(x) =$

x : duration of shower.

(i) $P(X \geq 10) = \int_{10}^{\infty} f(x)\,dx$.

(ii) $P(x < 10) = \int_{-\infty}^{10} f(x)\,dx$

$$= \int_{-\infty}^{10} f(x)\,dx + \int_{0}^{10} f(x)\,dx.$$

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(iii) $P(10 < X < 12) = \int_{10}^{12} f(x)\,dx.$

(4) The average daily turnout in a medical store is Rs 10,000/- & the net profit is 8%. If the turnout has an exponential distribution, find the probability that the net profit will exceed Rs 3000 each on 2 consecutive days.

Soln:

$\mu = \frac{1}{\alpha} = 10{,}000$

$\alpha = \frac{1}{10000} = 0.0001$

net profit = 8 %.

$f(x) = \alpha e^{-\alpha x}, \quad 0 < x < \infty$

$f(x) = 0.0001\, e^{-0.0001x}, \quad 0 < x < \infty$

Let A be the amount for which net profit is 8 %.

$8\% \cdot A = 3000$

$\frac{8}{100} A = 3000$

$\Rightarrow 3000 \times 100 / 8 = 37{,}500 = A$

$p(\text{sale amount} > 37{,}500) = p(x > 37500)$

$$= \int_{37500}^{\infty} f(x)\,dx = \int_{37500}^{\infty} 0.0001\ e^{-0.0001x}\,dx$$

$$= 0.0001 \left[\frac{e^{-0.0001x}}{-0.0001} \right]_{37500}^{\infty}$$

$$= -\left[e^{-\infty} - e^{3.75} \right]$$

$$= \underline{\underline{0.02351}}$$

(4) Normal distribution :

A continous random variable having probability density function $f(x)$ given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \longrightarrow (1)$$

where $-\infty < x < \infty$, $-\infty < \mu < \infty$ and $\sigma > 0$ is known as normal distribution

Take $\mu = 0, \sigma = 1$ in (1)

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

which is nothing but standard normal distribution for continous random variable x

Thus we can say that the normal pdf with,

$\mu = 0, \sigma = 1$ is std normal pdf.

The continous random variable x is said to have standard normal distribution if the probability density function is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} , \; -\infty < z < \infty , \; -\infty < \mu < \infty$$

and

$$z = \frac{x-\mu}{\sigma}$$

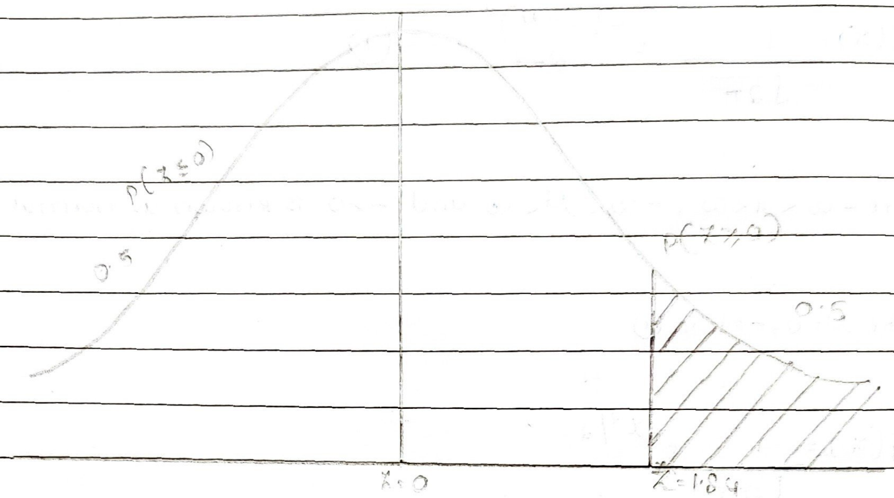
is called standard normal variate

① Give a standard normal distribution to find the area under the curve that lies

(1) to the right of $z = 1.84$

(2) between $z = -1.97$ & $z = 0.86$

Soln:

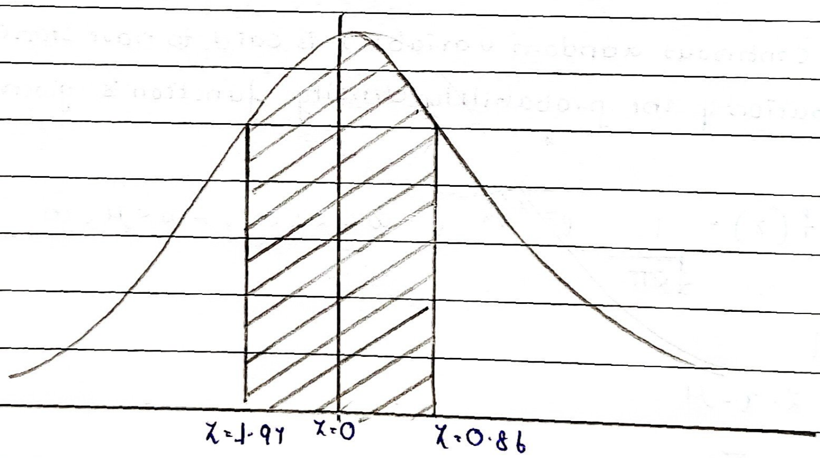


(a) $P(Z \geq 1.84) = 0.5 - P(0 \leq Z \leq 1.84)$

$= 0.5 - 0.4671$

$= 0.0329$

~~(b) p(x~~



(b) $P(-1.97 < Z < 0.86) = P(-1.97 \leq Z \leq 0) + P(0 \leq Z \leq 0.86)$.

$= P(1.97) + P(0.86)$

$= 0.4756 + 0.3051$

$= 0.7807$

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(2) If x is a normal variate with mean 30, standard deviation 5 find the probability that

(1) $26 \le x \le 40$

(2) $x \ge 45$

(3) $|x-30| > 5$.

Soln: $\mu = 30$ $\sigma = 5$

$z = \frac{x-\mu}{\sigma}$ standard normal variate.

(i) $P(26 \le x \le 40)$

$= P\left(\frac{26-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{40-\mu}{\sigma}\right)$

$= P\left(\frac{26-30}{5} \le z \le \frac{40-30}{5}\right)$

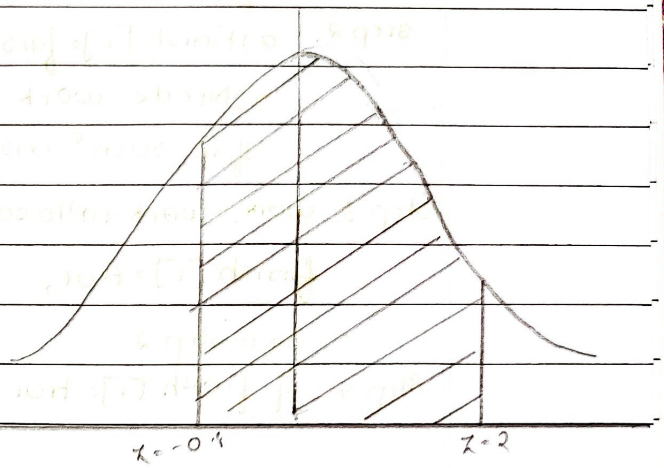
$= P(-0.8 \le z \le 2)$

$= P(-0.8 \le z \le 0) + P(0 \le z \le 2)$

$= P(-0.8) + P(2)$

$= 0.2881 + 0.4772.$

$= 0.7653$



(ii) $P(x \ge 45)$

$= P\left(\frac{x-\mu}{\sigma} \ge \frac{45-\mu}{\sigma}\right)$

$= P\left(z \ge \frac{45-30}{5}\right)$

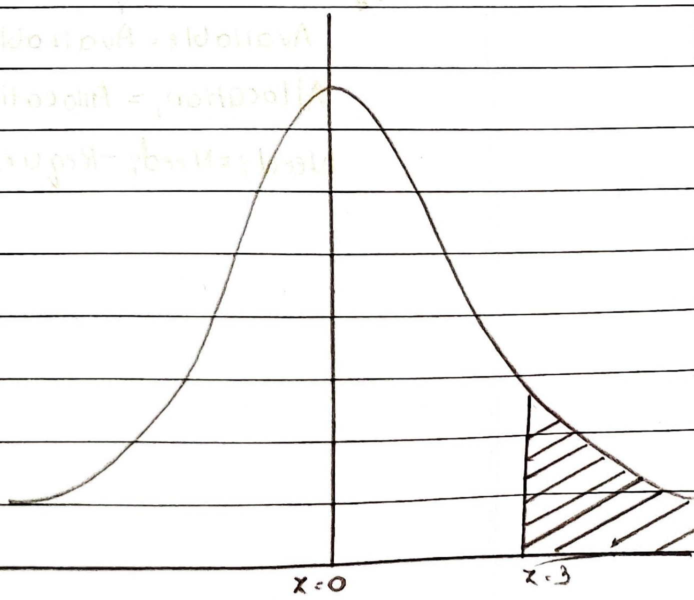
$= P\left(z \ge \frac{15}{5}\right) = P(z \ge 3)$

$= 0.5 - P(0 \le z \le 3)$

$= 0.5 - P(3)$

$= 0.5 - 0.4987$

$= 1.3 \times 10^{-03}$



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② (iii) $|X-30|>5$ $\qquad \mu = 30,\ \sigma = 5.$

$P(|X-30|>5)$

$= 1 - P(|X-30| \le 5) \qquad (\because |x| \le a = -a \le x \le a).$

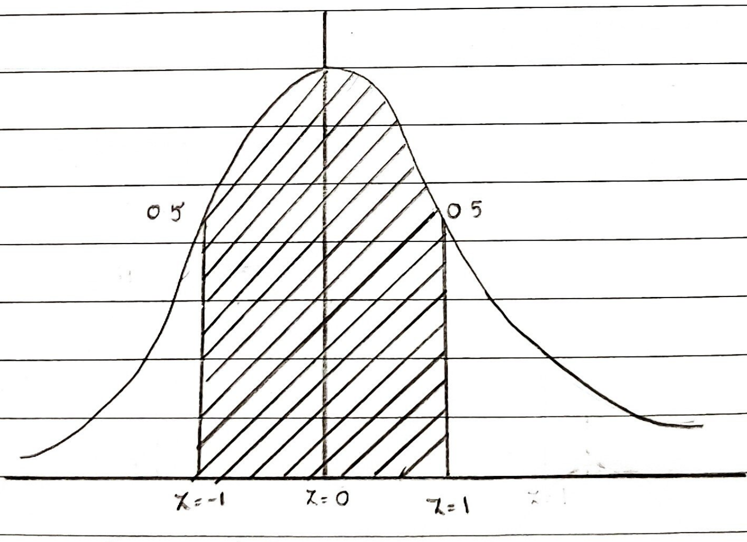
$= 1 - P(-5 \le X - 30 \le 5)$

$= 1 - P(-5+30 \le X \le 5+30)$

$= 1 - P(25 \le X \le 35)$

$= 1 - P\left(\frac{25-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{35-\mu}{\sigma}\right)$

$= 1 - P(-1 \le Z \le 1).$



$= 1 - [P(-1 \le Z \le 0) + P(0 \le Z \le 1)]$

$= 1 - 2\,P(0 \le Z \le 1)$

$= 1 - 2(0.3413)$

$= \underline{0.3174}$

(3) The marks of 1000 students in an examination follows a normal distribution with mean 70 and S.D 5. find the no of students whose marks will be.

(i) less than 65.

(ii) more than 75.

(iii) between 65 and 75

Soln: $\mu = 70$, $\sigma = 5$

Standard normal variate = $z = \dfrac{x-\mu}{\sigma}$

X: marks of students.

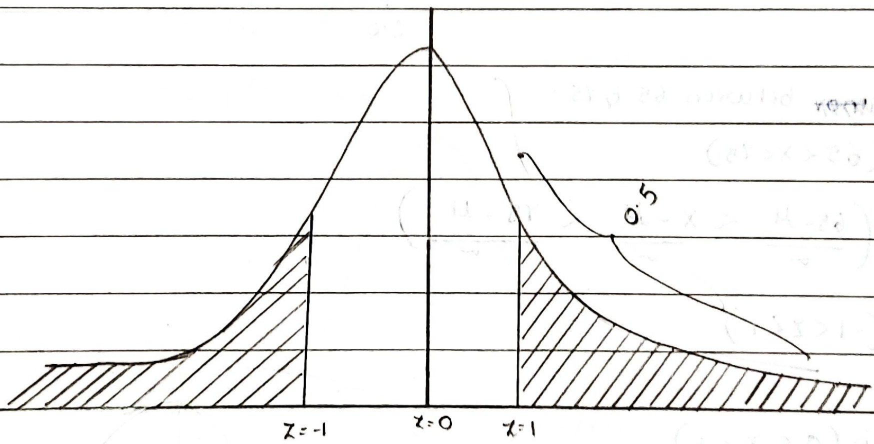
(i) less than 65

$P(X < 65) = P\left(\dfrac{x-\mu}{\sigma}\right)$

$= P\left(\dfrac{x-\mu}{\sigma} < \dfrac{65-\mu}{\sigma}\right)$

$= P\left(z < \dfrac{65-70}{5}\right)$

$= P(z < -1)$.



$= P(z > 1)$

$= P(z > 1)$

$= 0.5 - P(0 \le z \le 1)$

$= 0.5 - (0.3413)$

$= 0.1587$

$\therefore$ The no of students having mark less than 65 = 1000×0.1587

$= 158.7 \simeq 159$

(ii) more than 75.

$P(x>75) = P\left(\frac{x-\mu}{\sigma}\right)$

$= P\left(\frac{x-\mu}{\sigma} > \frac{75-\mu}{\sigma}\right)$

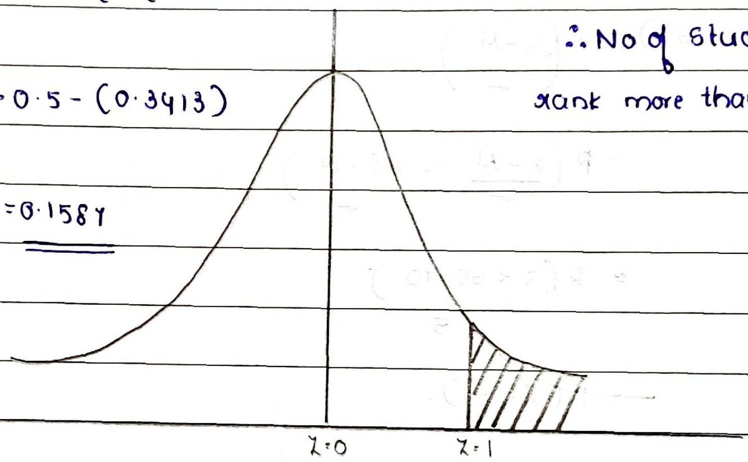
$= P\left(z > \frac{75-70}{5}\right)$

$= P(z>1)$.

$= 0.5 - (P(0 \le z \le 1)$

$= 0.5 - (0.3413)$

$= 0.1587$



∴ No of students having rank more than 75 = 1000×0.1587

$= 158.72,159$

(iii) between 65 & 75.

$P(65<x<75)$

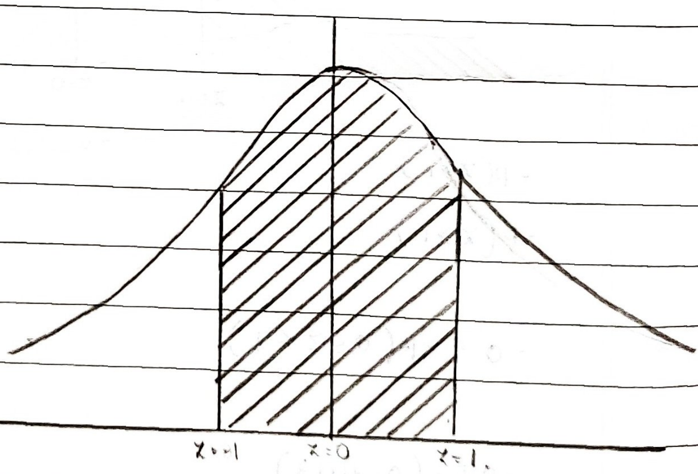
$= P\left(\frac{65-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{75-\mu}{\sigma}\right)$

$= P(-1<z<1)$

$= 2P(0 \le z \le 1)$

$= 2(0.3413)$

$= 0.6826$



∴ No of students having rank between 65 and 75 = 1000×0.6826

$= 682.6 \approx 683$

(4) In a test on 2000 electric bulbs it was found that the life of a particular make was normally distributed with an average life of 2040 hours & S.D of 60 hrs. Estimate the no of bulbs likely to burn for.

(i) more than 2150 hrs

(ii) less than 1950 hrs.

(iii) more than 1920 hrs & less than 2160 hrs.

Soln:

X: life of bulbs.

$\mu = 2040$, $\sigma = 60$

$Z = \frac{x-\mu}{\sigma}$, standard normal variate.

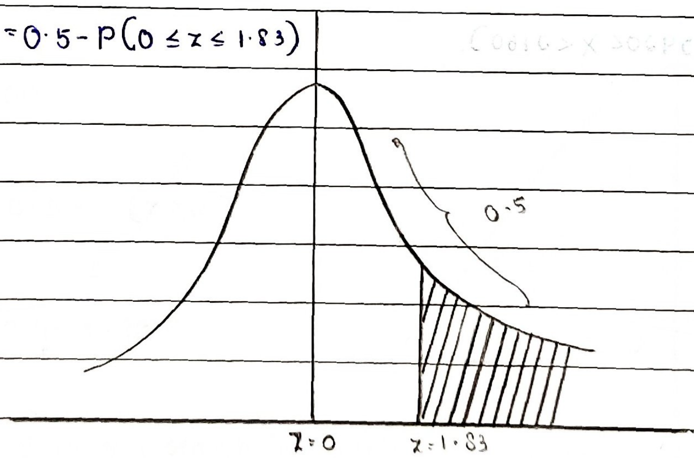
(i) more than 2150 hrs.

$P(X > 2150) = P\left(\frac{X-\mu}{\sigma} > \frac{2150-\mu}{\sigma}\right)$

$= P\left(Z > \frac{2150-2040}{60}\right)$

$= P(Z > 1.83)$

$= 0.5 - P(0 \le Z \le 1.83)$



$= 0.5 - (0.4664)$

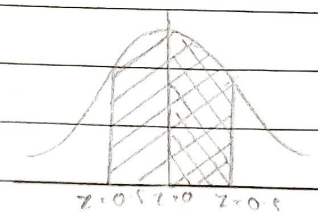
$= 0.0336$

∴ The required no of bulbs likely to burn for more than 2150 hrs = 2000 x 0.0336.

= 67.2 ≈ 67

(5) Given Random variable x having a normal distribution with $\mu = 50$, & $\sigma = 10$. find the probability that x assumes a value between 45 and 62.

Soln: $\mu = 50$, $\sigma = 10$ $\quad Z = \frac{X-\mu}{\sigma}$



$P(45 < X < 62) = P\left(\frac{45-50}{10} < \frac{X-\mu}{\sigma} < \frac{62-50}{10}\right)$.

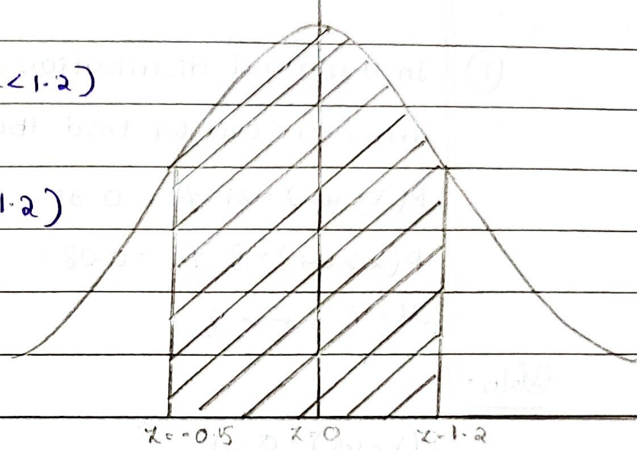
$= P(-0.5 < Z < 1.2)$

$= P(-0.5 < Z < 0) + P(0 < Z < 1.2)$

$= P(0 < Z < 0.5) + P(0 < Z < 1.2)$

$= 0.1915 + 0.3849$

$= \underline{\underline{0.5764}}$



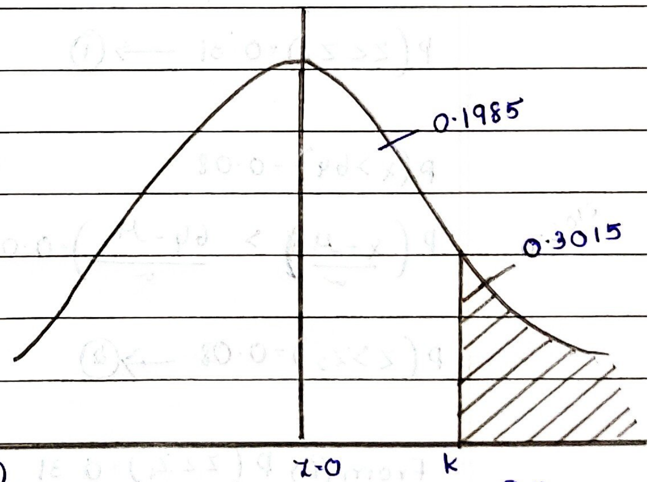
(6) Given standard normal distribution find the value of k such that

(a) $P(Z > k) = 0.3015$

(b) $P(k < Z < -0.18) = 0.4197$

Soln:

(a) $P(Z > K) = 0.3015$



$P(0 < Z < K) = 0.5 - P(Z > K)$

$= 0.5 - 0.3015$

$P(0 < Z < k) = 0.1985$ (search in table).

$\underline{Z = 0.52}$

0.5
0.02
$\underline{0.52}$

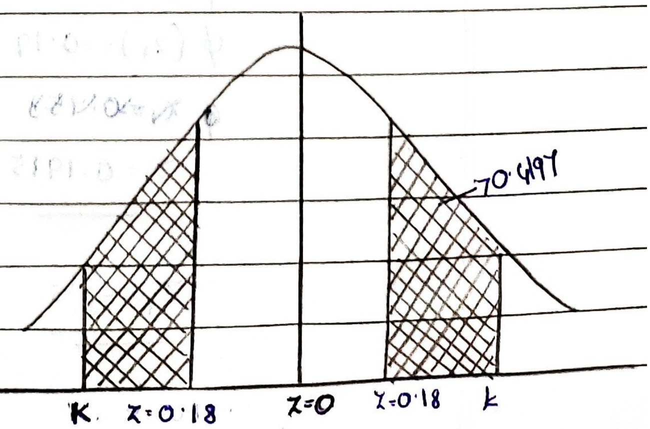
(b) $P(k < Z < -0.18) = 0.4197$.

$P(0 < Z < k) = P(k < Z < -0.18) + P(0.18 < Z < k)$

$= 0.0714 + 0.4197$

$= 0.4911$

$P(0 < Z < K) = \underline{\underline{0.4911}}$



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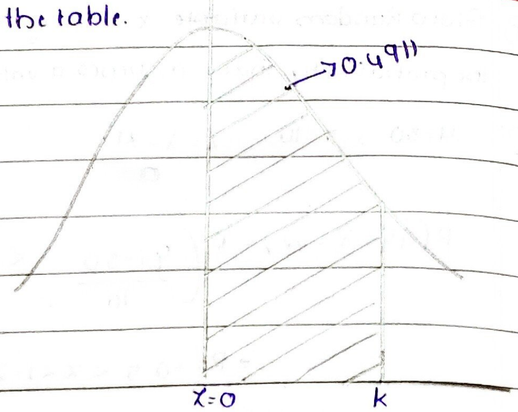
$P(0<z<k) = 0.4911$ (Search in the table.

$k = 2.37.$



⑦ In a normal distribution 31 % of the items are under 45 and 8% of the items are over 64. find the mean and S.D of the distribution.

$P(x<45) = 31\% = 0.31$

$P(x>64) = 8\% = 0.08.$

$\mu = ?,\ \sigma = ?$

Soln:

$P(x<45) = 0.31$

$P\left(\frac{x-\mu}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.31$

$P(z<z_1) = 0.31 \longrightarrow ①$

$P(x>64) = 0.08$

$P\left(\frac{x-\mu}{\sigma} > \frac{64-\mu}{\sigma}\right) = 0.08$

$P(z>z_2) = 0.08 \longrightarrow ②$

From ① $P(z<z_1) = 0.31$

$0.5 + \phi(z_1) = 0.31$

$\phi(z_1) = 0.31 - 0.5$

$\phi(z_1) = -0.19$ (Search in table.) → nearest

$z_1 = 0.1915$

$z_1 = -0.5$

Imp formula

$P(z>$

from ②

$z_1 = -0.5$

$\frac{45-\mu}{\sigma} =$

$45-\mu =$

$-0.5\sigma +$

$z_2 = 1.4$

$\frac{64-\mu}{\sigma}$

$64-\mu =$

$1.41\sigma +$

④ →

$(1.41$

Substitu

$-0.$

$-0.$

Imp formula

Note : $P(z<z_1) = 0.5 + \phi(z_1)$ $\phi(z_1) \rightarrow$ area under the curve from $0 \rightarrow z$.

$P(z>z_1) = 0.5 - \phi(z_2)$

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from (2) $P(z>z_2) = 0.08$

$0.5 - \phi(z_2) = 0.08$

$\phi(z_2) = 0.5 - 0.08$

$\phi(z_2) = 0.42$ (search in table) $\rightarrow$ approx.

$z_2 = 1.41$

$z_1 = -0.5$

$\frac{45 - \mu}{\sigma} = -0.5$

$45 - \mu = -0.5\sigma$

$-0.5\sigma + \mu = 45 \rightarrow$ (3)

$z_2 = 1.41$

$\frac{64 - \mu}{\sigma} = 1.41$

$64 - \mu = 1.41\sigma$

$1.41\sigma + \mu = 64 \rightarrow$ (4)

(4) $\rightarrow$ (3)

$(1.41\sigma + \cancel{\mu}) - (-0.5\sigma + \cancel{\mu}) = 64 - 45$

$1.91\sigma = 19$

$\sigma = \frac{19}{1.91} = 9.94$

$\boxed{\sigma = 10}$

Substituting in (3)

$-0.5\sigma + \mu = 45$

$-0.5(10) + \mu = 45$

$\mu = 45 + 5$

$\mu = 50$

(8) Given a normal distribution where $\mu = 40$, $\sigma = 6$. find the value of x that has

(a) 45% of the area to the left.

(b) 14% of the area to the

Soln: $\mu = 40, \sigma = 6$

(a) we need to find x such that x has 0.45 area to the left

$P(z<0) - P(z_1$