LC Electromagnetism I Lecture Notes

Year 1 Semester 2

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Table of Lectures

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Lecture 1 - EM1 Intro and Electric Fields

1   Course Intro

Course Materials:

- Background material and derivations etc on PowerPoint.

- Worked examples etc are handwritten on visualiser, these are the bits we really need to know.

Why is EM important?

- Foundations of modern technology and the modern world.

- What gives elements their properties.

- Responsible for life itself.

- Everyday materials are held together by EM forces.

- Optics can only be understood through EM theory.

The course aim is to lay down the foundations, eventually leading us to Maxell's Laws.

1.1   Maxwell's Laws

Maxwell's four equations are:

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \wedge \boldsymbol{B} = \mu_0 \left(\boldsymbol{J} + \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \right)$$

Where:

$$\nabla = \boldsymbol{i} \frac{\partial}{\partial x} + \boldsymbol{j} \frac{\partial}{\partial y} + \boldsymbol{k} \frac{\partial}{\partial z}$$

Together, these show that the electric and magnetic fields are related and are two aspects of a single force, the electromagnetic force. We don't have to properly understand them yet, but cannot learn them in EMII unless we fundamentally understand E and B fields from this module.

1.2   Course Structure

Part I: Electric Fields

- Charge and Coulomb's Law.

- The electric field.

- Gauss' Law.

- Capacitors.

Part II: Magnetic Fields

- Magnetic Fields

- Charged Particles in B-Fields

- Electromagnetic Induction.

- Magnetic Dipoles

In this lecture:

- Introduction to EM.

- Electric charge.

- Force between charges.

- The concept of the Electric Field (E-Field).

2   Electric Charge

First attributed to Thales circa. 624 - 546 BC. Experiments by Franklin and Coulomb expanded and showed that there was two types of charge, which they called positive and negative.

The "positive electricity" came from rubbing a glass rod with silk, and the negative from rubbing an ebonite (early plastic) rod with fur. They found that like charges repel and opposite charges attract.

We know that the elementary unit charge is the magnitude of charge of an electron/proton and everything else is a multiple of this [1]:

$$e = 1.6 \times 10^{-19} C$$

This has units of the Coulomb.

2.1   Charge Conservation

Electrons and protons are both stable (protons decay with a life greater than 10^{31} years). This means that the total charge of an isolated system is constant and can be conserved.

They have the same magnitude of charge, exactly:

$$|q_p| = |q_e| = e$$

2.2   Electrostatic Force

Like charges repeal and opposite charges attract, along the line of action given by a line drawn between the two charges. The force is proportional to the product of charges so:

$$F \propto q_1 q_2$$

Here, a negative force means attraction and a positive force means repulsion. Newton called this "force at a distance". Like gravity, two charges will exert a force on each other at a distance without any contact.

There must, therefore, be something between them that mediates this force. Later physics gives this as "virtual particles" which isn't a Y1 topic, so classically we say that this medium is the Electric Field.

2.3   Electric Field

A charge produces a field around it. Another charge also interacts with this field, and this interaction is what causes a force:

$$\underline{F} = \underline{E}q$$

Where F is the force exerted on a test charge of charge q by a charge Q producing a field E. The magnitude of the electric field has units NC^{-1} (force per units charge).

$$|\underline{E}| \propto Q \qquad |\underline{F}| \propto Qq$$

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[1]While quarks have fractional charge, we don't get free quarks

Consider a point charge with a spherical electric field spreading out around it. As the distance from the charge increases, the surface area increases as $4\pi r^2$. Therefore the magnitude of the electric field must decrease with $4\pi r^2$

Therefore:

$$|\underline{E}| \propto \frac{Q}{4\pi r^2}$$

We need a (inverse) constant of proportionality. This depends on the medium, but for a vacuum we call it the permittivity of free space ϵ_0:

$$\epsilon_0 = 8.854 \times 10^{-12} C^2 m^{-2} N^{-1}$$

Hence:

$$\boxed{E = |\underline{E}| = \frac{Q}{4\pi\epsilon_0 r^2}}$$

2.4   Direction of the E-Field

Force is a vector, so the E-field must be too. Consider an E-field from a charge Q at distance r. We give E components E_r, E_θ, E_ϕ, where, since it's a sphere ϕ and θ represent the unit vectors in the two possible tangential directions.

If there was a component in E_θ this would be clockwise from one perspective, but anticlockwise from another (walking behind it). This is not possible, as the field must behave in the same manner from all viewpoints. Therefore:

$$E_\theta = E_\phi = 0$$

$$\underline{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{t}$$

2.5   Force between two charges

Consider two charges q_1 and q_2. The force on q_2 due to the e-field from q_1 is:

$$\underline{F_1} = \underline{E_1}q_2$$

This is equal to the force on q_1 due to the e-field from q_2, given by:

$$\underline{F_2} = \underline{E_2}q_1$$

So the force between two charges is:

$$\underline{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}_{12}$$

2.6   Force between many charges

If we have more than two positive charges, we use the "principle of superposition". Effectively, you consider each pair of charges at the same time and vector sum of the forces together. I.e. if we have three points and we care about the net force on one, we take the vector sum of the two vectors from that point to the others.

In general:

$$\underline{F} = \underline{F_1} + \underline{F_2} + \underline{F_3} + \cdots$$

$$= q\sum_i \frac{q_i}{4\pi\epsilon_0 r^2}\hat{r}_j$$

Where r_j is the distance between q_i and q, with unit vector $\hat{r}_j$ between them.

Since $\underline{F} = q\underline{E}$, the electric field at a test charge q must be:

$$\underline{E} = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i^2}\hat{r}_j$$

2.7 Example

Say we have a square of side length a. Clockwise, these corners have charge $Q, q, -2Q, 3Q$.

What is the net force exerted on the q charge?

Thu 26 Feb 2026 11:00

Lecture 12 - Currents and Magnetic Force

Today we start Part II, Magnetism:

- Definition of current and current density.
- Magnetic force on a moving charge.
- The Lorentz Force.
- Field lines for a magnetic field.

1 Current

1.1 Definition of Current

Suppose a current carries a current I, this is defined in terms of the rate of flow of charge past a given cross section:

$$I = \frac{dQ}{dt}$$

The SI unit of current is the *ampere*, defined as the flow of one coulomb per second.

1.2 Nature of Current

There's two experimentally observed effects of a current flow:

- Heating
- Creation of magnetic fields.

Consider a current through a conductor. This is caused by electrons moving (with average drift velocity v) as a result of the induced E-field.

In some time, they travel $v\Delta t$ through the cross section, and if this cross section has area A they sweep out an area of $Av\Delta t$.

If a unit volume has n conducting free electrons, the number of charges is given by:

$$N = n(Av\Delta t)$$

And hence the total charge is:

$$\Delta Q = nAv\Delta T(-e)$$

Hence:

$$\hat{\imath} = \frac{\Delta Q}{\Delta t} = -nAe\underline{v}$$

1.3 Current Density

More usefully, we talk about the current per unit cross-section, which is the current density J:

$$\hat{\jmath} = \frac{\hat{\imath}}{\underline{A}} = -ne\underline{v}$$

The negative sign arises as we define current as flowing from positive to negative. Electrons, being negatively charged, actually physically flow from negative to positive. Current flow and actual electron flow are therefore in the opposite direction. This is because current was initially understood and defined before the electron was defined.

Typically, $v < 1\text{mms}^{-1}$ when a current is flowing.

2 Magnetism

2.1 Magnetic Force on a Charge

A magnetic field exerts a force on a moving charge that is present in the magnetic field.

Suppose a particle of charge $+q$ moving with some velocity $\underline{v}$ in a magnetic field $\underline{B}$ experiences some force $\underline{F}_m$.

We cannot derive from first principles, but experimentally have observed:

- $\underline{F}_m \perp \underline{v}, \underline{B}$.
- $\underline{F}_m \propto \underline{v}$.
- $\underline{F}_m \propto \underline{B}$.
- $\underline{F}_m \propto q$

If θ is the angle between the velocity and the B-field direction, we hae:

$$\boxed{\underline{F}_m = Bqv\sin\theta}$$

In vector form:

$$\boxed{\underline{F}_m = q\underline{v} \wedge \underline{B}}$$

Here, $\underline{B}$ has the unit Tesla, T. In base units this is $\text{NC}^{-1}\text{m}^{-1}\text{s}$. A one Tesla magnetic field is really quite powerful, so we also define the Gauss, $1\text{G} = 10^{-4}\text{T}$ to move to a nicer scale.

By definition, if a 1C charge moving at 1m/s perpendicular to a magnetic field experiences a force 1N, the field is 1T.

- Earths magnetic field is $\sim 0.5\text{G}$
- Poles of a large electromagnetic: $\sim 2\text{T}$.
- Surface of a neutron star: $\sim 10^8\text{T}$.
- The maximum pulsed magnetic field that can create in a lab: $\sim 50\text{T}$.

The Earth's field is believed to be generated by electron currents in the iron alloys in its core. It's not completely understood yet, but convection currents causing these conductive alloys to flow and move is the working theory. This movement causes the North and South poles to swap places on average every $300,000$ years.

2.2 The Lorentz Force

In a region where we have both an E-field and a B-field, we have the total force as the vector sum of both:

$$\underline{F} = q\left(\underline{E} + \underline{v} \wedge B\right)$$

This is called the Lorentz force.

Suppose Earth's magnetic field is given by:

$$\underline{B} = B\cos 70°\,\hat{\jmath} - B\sin 70°\,\hat{k} \qquad \text{where: } B = 5 \times 10^{-5}\text{T}$$

A proton moves in this field with:

$$\underline{v} = 10^7\,\hat{\jmath}\,\text{ms}^{-1}$$

$$\underline{F}_m = +e\underline{v} \wedge \underline{B} = 1.6 \times 10^{19} \times 10^7 \times \det \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 0 \\ 0 & B_y & B_z \end{bmatrix}$$

$$= 1.6 \times 10^{-12} \left[\hat{\imath}(B_z - 0) + \hat{\jmath}(0 - 0) + \hat{k}(0 - 0) \right]$$

$$= 1.6 \times 10^{-12} B_Z \hat{\imath}$$

$$= -1.6 \times 10^{-12} \times 5 \times 10^{-5} \sin 70° \hat{\imath}$$

$$-7.5 \times 10^{-17} \hat{\imath} \text{N}$$
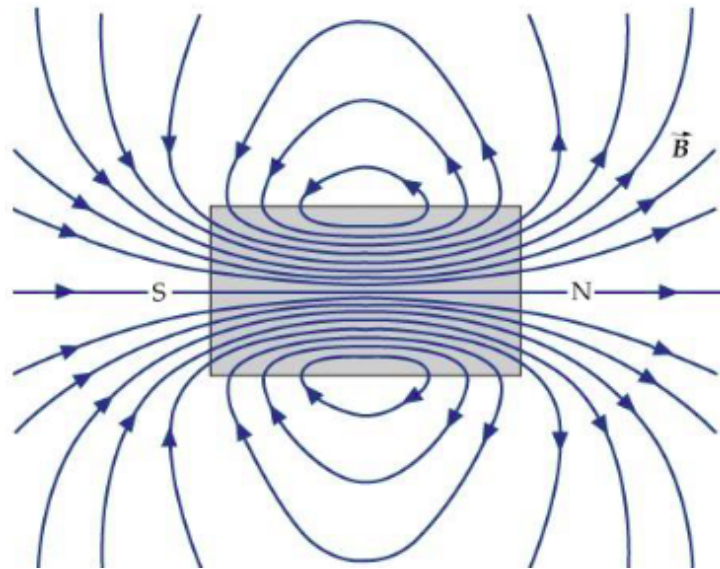
2.3   Magnetic Field Lines

Figure 2.1: Magnetic Field Lines Around a Bar Magnetic

Magnetic field lines are different to electrical field lines. They do not point in the direction of force and travel out of North poles and into South poles. Magnetic field lines are always continuous so there is no magnetic monopoles.

Physicists have searched for a potential magnetic monopoles, and the current upper limit for the number of magnetic monopoles per body is $< 10^{-29}$.

The tangent to a field line at some point gives the direction of B at that point. The number of field lines drawn per unit cross section $\propto$ B.

2.4   Magnetic Flux

The magnetic flux $\Delta\phi_B$ has the same definition as electric flux in an E-field. For a small area ΔA:

$$\Delta\phi_B = B\cos\theta\Delta A$$

Where θ is the angle between the B-field and the surface normal vector.

The total magnetic flux is therefore the integral of this:

$$\phi_B = \int_S \underline{B} \cdot d\underline{S}$$

As there is no monopoles, and exiting flux must also return. Therefore, the net flux over any enclosed surface is zero:

$$\int_S \underline{B} \cdot d\underline{S} = 0$$

While this does form one of Maxwell's equations and is very useful for EMII next year, it's not useful in EMI...