LC Electromagnetism I Lecture Notes

Year 1 Semester 2
MSci Physics with Particle Physics and Cosmology

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Lecture Index

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Lecture 1 - EM1 Intro and Electric Fields

1 Course Intro

Course Materials:

- Background material and derivations etc on PowerPoint.
- Worked examples etc are handwritten on visualiser, these are the bits we really need to know.

Why is EM important?

- Foundations of modern technology and the modern world.
- What gives elements their properties.
- Responsible for life itself.
- Everyday materials are held together by EM forces.
- Optics can only be understood through EM theory.

The course aim is to lay down the foundations, eventually leading us to Maxell's Laws.

1.1 Maxwell's Laws

Maxwell's four equations are:

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$
$$\nabla \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \wedge \boldsymbol{B} = \mu_0 \left(\boldsymbol{J} + \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \right)$$

Where:

$$\nabla = \boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z}$$

Together, these show that the electric and magnetic fields are related and are two aspects of a single force, the electromagnetic force. We don't have to properly understand them yet, but cannot learn them in EMII unless we fundamentally understand E and B fields from this module.

1.2 Course Structure

Part I: Electric Fields

- Charge and Coulomb's Law.
- The electric field.
- Gauss' Law.
- Capacitors.

Part II: Magnetic Fields

- Magnetic Fields
- Charged Particles in B-Fields
- Electromagnetic Induction.
- Magnetic Dipoles

In this lecture:

- Introduction to EM.
- Electric charge.
- Force between charges.
- The concept of the Electric Field (E-Field).

2 Electric Charge

First attributed to Thales circa. 624 - 546 BC. Experiments by Franklin and Coulomb expanded and showed that there was two types of charge, which they called positive and negative.

The "positive electricity" came from rubbing a glass rod with silk, and the negative from rubbing an ebonite (early plastic) rod with fur. They found that like charges repel and opposite charges attract.

We know that the elementary unit charge is the magnitude of charge of an electron/proton and everything else is a multiple of this [1]:

$$e = 1.6 \times 10^{-19} C$$

This has units of the Coulomb.

2.1 Charge Conservation

Electrons and protons are both stable (protons decay with a life greater than 10^{31} years). This means that the total charge of an isolated system is constant and can be conserved.

They have the same magnitude of charge, exactly:

$$|q_p| = |q_e| = e$$

2.2 Electrostatic Force

Like charges repeal and opposite charges attract, along the line of action given by a line drawn between the two charges. The force is proportional to the product of charges so:

$$F \propto q_1 q_2$$

Here, a negative force means attraction and a positive force means repulsion. Newton called this "force at a distance". Like gravity, two charges will exert a force on each other at a distance without any contact.

There must, therefore, be something between them that mediates this force. Later physics gives this as "virtual particles" which isn't a Y1 topic, so classically we say that this medium is the Electric Field.

2.3 Electric Field

A charge produces a field around it. Another charge also interacts with this field, and this interaction is what causes a force:

$$\underline{F} = \underline{E} q$$

Where F is the force exerted on a test charge of charge q by a charge Q producing a field E. The magnitude of the electric field has units NC^{-1} (force per units charge).

$$|\underline{E}| \propto Q \qquad |\underline{F}| \propto Qq$$

[1] While quarks have fractional charge, we don't get free quarks

Consider a point charge with a spherical electric field spreading out around it. As the distance from the charge increases, the surface area increases as $4\pi r^2$. Therefore the magnitude of the electric field must decrease with $4\pi r^2$

Therefore:

$$|\underline{E}| \propto \frac{Q}{4\pi r^2}$$

We need a (inverse) constant of proportionality. This depends on the medium, but for a vacuum we call it the permittivity of free space ϵ_0:

$$\epsilon_0 = 8.854 \times 10^{-12} C^2 m^{-2} N^{-1}$$

Hence:

$$\boxed{E = |\underline{E}| = \frac{Q}{4\pi\epsilon_0 r^2}}$$

2.4 Direction of the E-Field

Force is a vector, so the E-field must be too. Consider an E-field from a charge Q at distance r. We give E components E_r, E_θ, E_ϕ, where, since it's a sphere ϕ and θ represent the unit vectors in the two possible tangential directions.

If there was a component in E_θ this would be clockwise from one perspective, but anticlockwise from another (walking behind it). This is not possible, as the field must behave in the same manner from all viewpoints. Therefore:

$$E_\theta = E_\phi = 0$$

$$\underline{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{t}$$

2.5 Force between two charges

Consider two charges q_1 and q_2. The force on q_2 due to the e-field from q_1 is:

$$\underline{F_1} = \underline{E_1} q_2$$

This is equal to the force on q_1 due to the e-field from q_2, given by:

$$\underline{F_2} = \underline{E_2} q_1$$

So the force between two charges is:

$$\underline{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}_{12}$$

2.6 Force between many charges

If we have more than two positive charges, we use the "principle of superposition". Effectively, you consider each pair of charges at the same time and vector sum of the forces together. I.e. if we have three points and we care about the net force on one, we take the vector sum of the two vectors from that point to the others.

In general:

$$\begin{aligned}\underline{F} &= \underline{F_1} + \underline{F_2} + \underline{F_3} + \cdots \\ &= q\sum_i \frac{q_i}{4\pi\epsilon_0 r^2}\hat{r}_j\end{aligned}$$

Where r_j is the distance between q_i and q, with unit vector $\hat{r}_j$ between them.

Since $\underline{F} = q\underline{E}$, the electric field at a test charge q must be:

$$\underline{E} = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i^2}\hat{r}_j$$

2.7 Example

Say we have a square of side length a. Clockwise, these corners have charge Q, q, $-2Q$, $3Q$.

What is the net force exerted on the q charge?