

# Lecture Notes

MSci Physics w/ Particle Physics and Cosmology  
University of Birmingham

Year 1, Semester 1  
Ash Stewart

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# LC Classical Mechanics and Relativity 1

Thu 02 Oct 2025 14:15

# Lecture 1 - Orders of Magnitude and Dimensional Analysis

TODO

Thu 02 Oct 2025 15:00

## Lecture 2 - Dimensional Analysis (contd.) and Vectors

### Continuation of Dimensional Analysis

What if, in theory, we could build a system of units entirely from  $c$ , the speed of light,  $G$ , Newton's constant and  $h$ , the Plank Constant?

Cont. from Lec01, we can try to use this to work out the earliest possible cosmic time.

$$\begin{aligned}h &= 6.6 \times 10^{-34} Js \\ G &= 6.67 \times 10^{-11} Nm^2/kg^2 \\ c &= 3 \times 10^8 m/s\end{aligned}$$

Dimensionally:

$$\begin{aligned}[h] &= \frac{ML^2}{T} \\ [G] &= \frac{L^3}{T^2 M} \\ [c] &= \frac{L}{T}\end{aligned}$$

We want to use these to build out a time unit, so:

$$\begin{aligned}[h^u G^v c^z] &= T \\ \left(\frac{ML^2}{T}\right)^u \left(\frac{L^3}{T^2 M}\right)^v \left(\frac{L}{T}\right)^z &= T \\ M^{u-v} L^{2u+3v+z} T^{-u-2v-z} &= T\end{aligned}$$

Solving for:

$$u - v = 0$$

$$2u + 3v + z = 0$$

$$-u - 2v - z = 1$$

Gives us:

$$u = \frac{1}{2} \tag{1}$$

$$v = \frac{1}{2} \tag{2}$$

$$z = \frac{-5}{2} \tag{3}$$

$t_p = \sqrt{\frac{Gh}{c^5}}$  and plugging in the values for  $G$ ,  $h$ ,  $c$  gives us a value of time, which the earliest possible cosmic time equal to about  $10^{-43}s$

## Plank Energy

Doing the same process for energy gives us (this time, the plank energy is the energy at which traditional theories of physics break down):

$$E_p = \frac{hc^5}{G} \approx 10^9 J$$

On the other hand, the LHC manages about 10TeV, which is orders of magnitude smaller than this, so the LHC cannot accurately simulate energies of this magnitude.

## More Vectors

Again, vector notation will be  $\vec{a}$ . We define the x, y, z unit vectors as  $\hat{e}_x, \hat{e}_y, \hat{e}_z$ .

We can therefore define any vector as:

$$\vec{a} = a_x \hat{e}_x + a_y \hat{e}_y + a_z \hat{e}_z.$$

The length of a vector is again  $|\vec{a}|$ .

## Vector Multiplication

Given  $\vec{a}$  and  $\vec{b}$  we can define the dot (scalar) product and the cross (vector) product

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

Say we want to know the component of a vector along an axis, we can do the following (eg for x):

$$\vec{a} \cdot \hat{e}_x = a_x$$

For the vector product, we can define:

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin(\theta) \hat{j}$$

As the vector perpendicular to the plane containing a and b. It is in the direction such that<sup>1</sup>. Theta is the angle between a and b, while j is the unit vector in the direction the new vector will point.

## Solar Energy Example

The world yearly energy usage is about 180,000TWh, which is about  $5 \times 10^{20} J$  total. Is it (theoretically) possible to get this all from solar energy? We can check using an approximate order of magnitude calculation.

The Sun's total luminosity is  $L_\odot = 3.8 \times 10^{26}$ . This energy is radiated in a spherically symmetric way (we assume). Therefore the energy per time, per unit surface is (using 1AU for distance):

$$\frac{L_\odot}{4\pi \times (1.5 \times 10^6)^2}$$

Which is approximately (using order of magnitude):

$$\frac{3.8 \times 10^{26} W}{10 \times 10^{22} m^2} \approx \frac{1 kW}{m^2}$$

This is true in ideal conditions, and real energy supply is lower (due to clouds, atmosphere etc).

If we totally covered the earth's surface area ( $A_{\text{surface}} \approx \pi R_\oplus^2$ ) which is approximately:

$$A_{\text{surface}} \approx \pi \times (6 \times 10^3 \times 10^3 m)^2 \approx 10^{14} m^2$$

Therefore total energy received is approximately:

$$P = \frac{1 kW}{m^2} \times 10^{14} m^2 \approx 10^{17} W$$

And to power the world:

$$E = \frac{5 \times 10^{20} J}{3 \times 10^7 s} \approx 10^{13} W$$

So, it's theoretically possible, if we could cover enough of the world in solar panels and if we could perfectly capture the sun's energy without losing some to sources such as clouds, atmosphere, areas of the ocean we cannot cover in solar panels etc.

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<sup>1</sup>TODO, fix

Thu 04 Sep 2025 12:00

## Lecture 3 - Kinematics Introduction

For kinematics, we'll treat all objects as points and disregard aspects like rotation/the physical size of the body etc.

Given some point, we can define its position as a function of time  $\vec{r}(t)$ , and velocity as the derivative wrt time of this:

$$\vec{v}(t) = \frac{d\vec{r}}{dt}$$

And acceleration:

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

### Position from Unit Vectors

We can define:

$$\vec{r}(t) = r_x(t)\hat{e}_x + r_y(t)\hat{e}_y + r_z(t)\hat{e}_z = \sum_{j=1}^3 r_j(t)\hat{e}_j$$

So:

$$\begin{aligned} \frac{d\vec{r}}{dt} &= \frac{d}{dt} \left( \sum_{j=1}^3 r_j \hat{e}_j \right) \\ &= \sum_j \frac{d}{dt} (r_j \hat{e}_j) \end{aligned}$$

Note: Taking the derivative of a vector wrt time is looking at how the variable changes in some infinitesimal time. This can be a change in direction, and/or a change in magnitude. To differentiate a vector we can differentiate it component-wise.

### Cartesian and Polar

Instead of representing a point as x and y components (in 2D), we can instead define it as a distance from the origin  $r$  and the angle this distance line forms with the positive x-axis  $\theta$ .

Therefore (by basic right angle trig)  $x = r\cos\theta$ ,  $y = r\sin\theta$ , and hence:

$$\vec{r} = r \cos \theta \hat{e}_x + r \sin \theta \hat{e}_y$$

So:

$$\begin{aligned} \vec{u}(t) &= \frac{d\vec{r}}{dt} = \frac{d}{dt} (r \cos \theta) \hat{e}_x + \frac{d}{dt} (r \sin \theta) \hat{e}_y \\ &= \left( \dot{r} \cos \theta + r(-\sin \theta) \dot{\theta} \right) \hat{e}_x + \left( \dot{r} \sin \theta + r(\cos \theta) \dot{\theta} \right) \hat{e}_y \\ &= r(\cos \theta \hat{e}_x + \sin \theta \hat{e}_y) + r \dot{\theta} (-\sin \theta \hat{e}_x + \cos \theta \hat{e}_y) \end{aligned}$$

### Example

Lets model a particle, in a single dimension moving with constant acceleration ( $a_0$ ) along a line. What is  $x(t)$ ?

# LC Introduction to Probability and Statistics



Wed 01 Oct 2025 12:00

# Lecture 1 - Introduction and Descriptive Statistics

## Course Welcome

- First half of the semester: Statistics
- Second half of the semester: Probability
- All slides and notes on Canvas.

**Why Descriptive Statistics?** If we want to share an interesting bit of data, sharing the whole data is going to be confusing. Instead, we can share a small number of stats which describe and summarise the data.

## Sample Statistics

One of the most simple is the number of samples ( $N$ ), and the sample mean:

$$\text{Sample Mean: } \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

We can also calculate the sample standard deviation as the average of mean squared error across the points in the sample:

$$\text{Sample STDev: } s_n^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2$$

We can also use median or mode as measures of central tendency. The mode is a poor estimator however (as it massively depends on how binning is done, for a continuous measurement), while the median is more resistant to outliers.

Thu 02 Oct 2025 09:00

## Lecture 2 - Population Statistics

### Accuracy and Precision

We usually take measurements to determine some kind of true value. Usually, we can't actually know what this true value is, but if we could there are two bits of terminology that is particularly important:

**Accuracy:** Accuracy is the 'closeness' between our value and the 'true' value.

**Precision:** Precision is the 'closeness' between our measurements, i.e. how spread out are our various measurements.

### Error

**Random Error:** is uncertainty related to the fact that our measurements are only a finite sample, so is not going to be immediately representative of the true value. The smaller this error, the more precise the measurement is.

**Systematic Error:** is related to some kind of issue with the measurement or the equipment. This shifts all values, and negatively affects accuracy (but leaves precision unchanged)

Taking many repeat measurements decreases the effects of random error, but the effects of systematic error are much harder to combat.

Ideally, we want to be both precise and accurate, however accuracy is arguably more important. This is because a value which is precise, but not accurate may lead to false conclusions around the inaccurate value.

# LC Mathematics for Physicists 1A

Thu 02 Oct 2025 15:53

# Lecture 1

Thu 02 Oct 2025 15:53

## Lecture 2

Thu 02 Oct 2025 16:00

## Lecture 3

# LC Optics and Waves

Wed 01 Oct 2025 11:00

# Lecture 1 - Intro to Waves and SHM Recap

## Course Objectives

- Have a sound understanding of basic wave properties
- Have a basic understanding of interference effects, inc diffraction
- Be able to use simple geometric optics and understand the fundamentals of optical instruments.

## Recommended Textbooks

1. University Physics, Young and Freedman (Ch 15, 16 for Waves, Ch 33-36 for Optics)
2. Physics for Scientists and Engineers (Ch 20, 21 for Waves, Ch 22-24 for Optics)
3. 5e, Tipler and Mosca, (Ch 15, 16 for Waves, 31-33 for Optics)
4. Fundamentals of Optics, Jenkins and White
5. Optics, Hecht and Zajac

**What is a wave?** Waves occur when a system is disturbed from equilibrium and the disturbance can travel from one region to another region. Waves carry energy, but do not move mass. The course aim is to derive basic equations for describing waves, and learn their physical properties.

## Periodic Motion

Waves are very linked to periodic motion. Therefore we recap periodic motion first.

It has these characteristics:

- A period,  $T$  (the time for one cycle)
- A frequency,  $f$ , the number of cycles per unit time ( $f = \frac{1}{T}$ )
- An amplitude,  $A$ , the maximum displacement from equilibrium.

Periodic motion continues due to the restoring force. When an object is displaced from equilibrium, the restoring force acts back towards the equi point. The object reaches equi with a non-zero speed, so the motion continues past the equi point and continues forever.

## Energy

Periodic motion is an exchange between potential and kinetic energy, with no energy loss. Energy is conserved.

## Simple Harmonic Motion

If the restoring force is directly proportional to the displacement  $F = -kx$ , then the periodic motion becomes Simple Harmonic Motion and the object is called a harmonic oscillator.

In a single dimension, displacement is given by:

$$x = A \cos(\omega t + \phi)$$

Where  $\omega = 2\pi f$  is the angular velocity, and  $\phi$  is the phase angle. In cases like this, where the phase angle is 90 deg we can simplify to  $x = -A \sin(\omega t)$



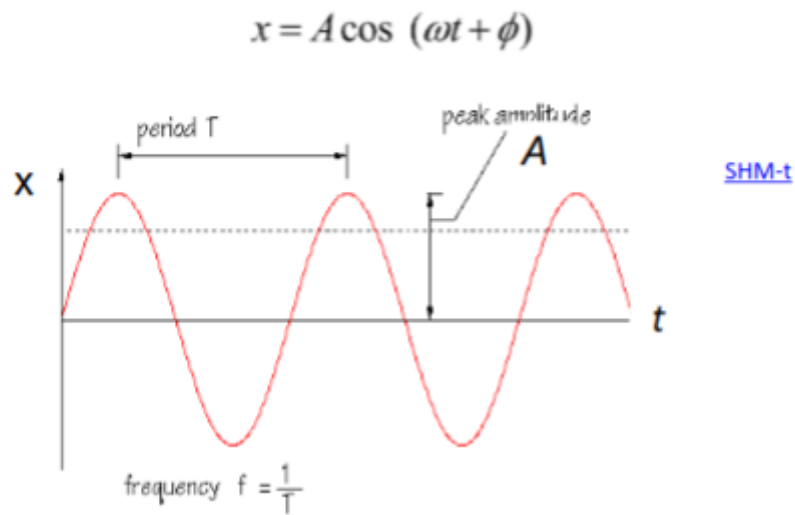


Figure 1: A Phase Angle of 90

## More SHM Equations

### Velocity

$$v_x = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

### Acceleration

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

Both properties are signed to indicate direction, as they are both vectors.

Thu 02 Oct 2025 13:00

## Lecture 2 - Wave Functions

### Sine Waves

**Mechanical Waves** A mechanical wave is a disturbance through a medium. It's formed of a single wave pulse or a periodic wave.

Mechanical Waves have the following properties:

1. Transverse: Where displacement of the medium is perpendicular to the direction of propagation.
2. Longitudinal, displacement of the medium is in the same direction as propagation.
3. Propagation depends on the medium the wave moves through (i.e. density, rigidity)
4. The medium does not travel with the wave.
5. Waves have a magnitude and a direction.
6. The disturbance travels with a known exact speed.
7. Waves transport energy but not matter throughout the medium.

### Wave Functions

We want to define a wave function in terms of two variables,  $x$  and  $t$ . In any given moment, if we consider a single point on the wave (i.e.  $t = 0$ ), and wait a short while, the wave will have travelled to some  $t = t_1 > 0$ .

In order to quantify displacement, we therefore want to specify both the time, and the displacement. This will let us find the wave speed, acceleration and the (new) wave number.

We are also able to talk about the velocity and acceleration of individual particles on the wave.

### Wave Function for a Sine Wave

Consider a sine wave. We want to find a wave function in the form  $y(x, t)$ . Consider the particle at  $x = 0$ .

We can express the wave function at this point as  $y(x = 0, t) = A \cos \omega t$ . However we want to expand this to any general point. Now consider a point (2) which is one wavelength away. We know the behaviour of particle 1 is mirrored by particle 2 (with a time lag).

Since the string is initially at rest, it takes on period ( $T$ ) for the propagation of the wave to reach point 2, therefore point 2 is lagging behind the motion of point 1. The wave equation is therefore (if particle two has  $x = \lambda$ )  $y(x = \lambda, t) = A \cos(\omega t - 2\pi)$ .

For arbitrary  $x$ ,  $y(x, t) = A \cos(\omega t - \frac{x}{\lambda} \cdot 2\pi)$  to account for this delay. This quantity is called the wave number:

$$\text{Wave Number: } k = \frac{2\pi}{\lambda}$$

So:

$$\begin{aligned} y(x, t) &= A \cos(\omega t - kx) \\ &= A \cos(kx - \omega t) \end{aligned}$$

Note the second step is possible as  $\cos$  is an even function.  $k$  can also be signed to indicate direction: if  $k > 0$ , the wave travels in the positive  $x$ . If  $k < 0$ , the wave travels in the negative  $x$  direction. Again,  $\omega = 2\pi f$

## Displacement Stuff

Considering a point (starting at equi), the time taken for the particle on the sin wave to reach maximum displacement, minimum displacement and back takes the time period  $T$ . The speed of the wave is distance travelled over the time taken. We take the distance to be the wavelength  $\lambda$ , as we know the time by definition this takes is one time period  $T$ . Therefore wave speed  $v$  is:

$$v = \frac{\lambda}{T} = \lambda f$$

Since  $\lambda = \frac{2\pi}{k}$  and  $f = \frac{\omega}{2\pi}$  (as  $\omega$  is defined as  $\frac{2\pi}{T}$ ), we can also write:

$$v = \frac{2\pi}{k} \cdot \frac{\omega}{2\pi} = \frac{\omega}{k}$$

## Particle Velocity

We can also determine the velocity of individual particles in the medium. We can use this to determine the acceleration.

We know that

$$y(x, t) = A \cos(kx - \omega t)$$

The vertical velocity  $v_y$  is therefore given by:

$$v_y = \frac{dy(x, t)}{dt}$$

Which is unhelpful (as we can't differentiate two variables at once), we can slightly cheat this by looking at purely a certain value of  $x$ , and therefore treating  $x$  as constant (to get a single variable derivative).

$$v_y = \left. \frac{dy(x, t)}{dt} \right|_{x=\text{const.}}$$

However this is notationally yucky, so we therefore use the notation:

$$\frac{\partial y(x, t)}{\partial t}$$

To represent the same thing. Finally (carrying out the partial derivative):

$$v_y(x, t) = \frac{\partial y(x, t)}{\partial t} = \omega A \sin(kx - \omega t)$$

## Particle Acceleration

We can work out particle acceleration (transverse acceleration) by differentiating in the same manner again:

$$a_y(x, t) = \frac{\partial^2 y(x, t)}{\partial t^2} = \frac{\partial}{\partial t} \left( \frac{\partial y(x, t)}{\partial t} \right) = -\omega^2 A \cos(kx - \omega t) = -\omega^2 y(x, t).$$

# LC Quantum Mechanics

Fri 03 Oct 2025 12:00

## Lecture 1 - Atomic Structure

### What is the course?

- Quantum mech is weird and unintuitive, we will build up a case in the course for why this weird theory was necessary and why we're confident it works.
- Each week will be a self-contained concept and/or historical experiment, working up to the Shroedinger Equation and wave-particle duality.
- Names and dates do not need to be memorised.
- Recommended text: University Physics (Young and Freedman).
- Office hours: 13:00 – 13:50 Fridays (immediately post-lecture), Physics East Rm 207.

### Atomic Structure

What actually is an atom? What does it actually look like inside?

#### Early Clues

- Periodic Table (Mendeleev, 1869), periodic patterns in elements properties.
- Radioactivity (Becquerel, 1896, Curie 1898)
- Atoms emit and absorb specific discrete wavelengths, (Balmer, 1884)
- Discovery of the Electron (Thompson 1897). Cathode rays - heating metal in a vacuum with an electric field above it, to strip away electrons from the metal.
  - This showed electrons were negatively charged and extremely light (1/2000th of the atomic mass).

## Atoms emit/absorb light at discrete wavelengths

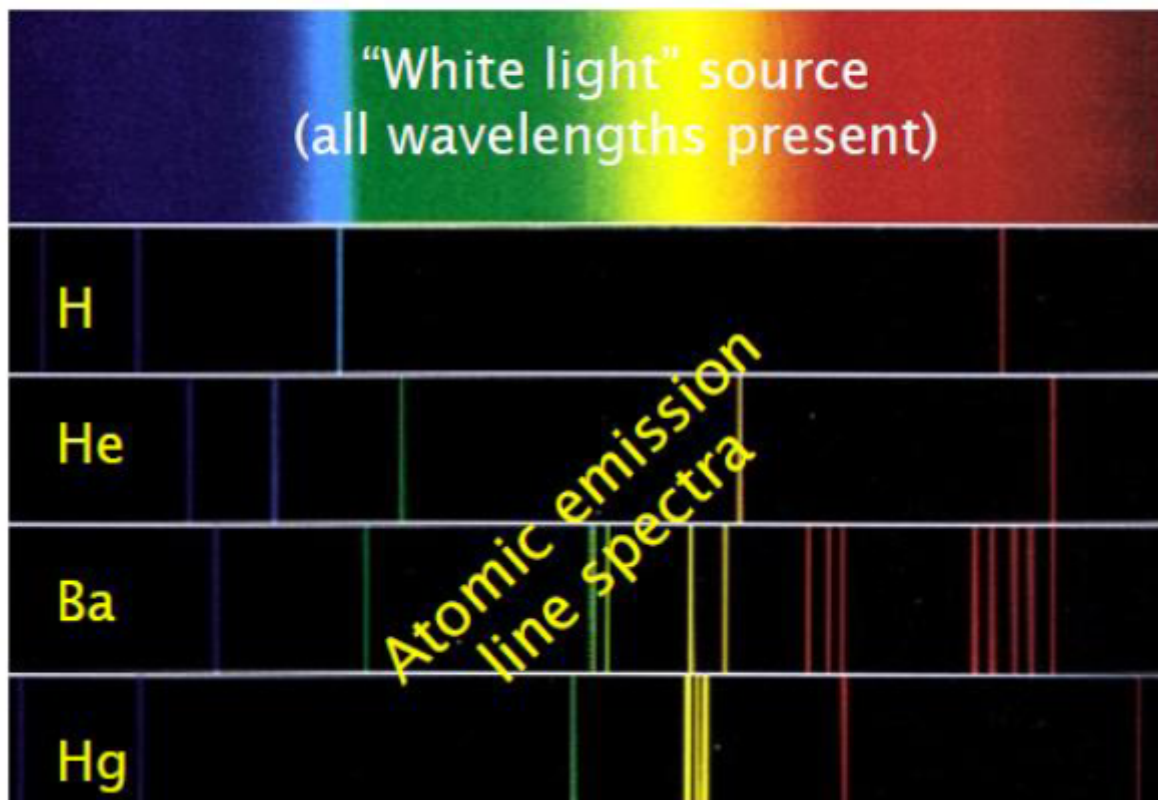


Figure 2: Absorption Spectra

### Plum Pudding Model

A solid, uniform lump of positively charged matter, approximately  $10^{-10}\text{m}$  across. This had evenly distributed negative charges (electrons) scattered throughout.

### Discovery of the Nucleus

Geiger and Marsden (1908-1913), fired alpha particles (He Nucleus) at thin gold foil and measured the deflection/scattering.

The alpha particles had a mass of  $4u$ , a charge of  $+2e$  and an energy of approximately  $5\text{MeV}$ .

They found that most  $\alpha$  were scattered only by small angles, but (surprisingly) a small number were scattered right back towards to emitter (through  $\theta > 90^\circ$ ). The distribution of the angles is approximately Normally distributed, with a mean of 0. Only approximately 1 in 8,000 fired  $\alpha$ s were scattered by  $\theta > 90^\circ$  ("back-scattering").

Can this be explained with the Plum Pudding Model? No, it cannot. This was used to demonstrate that atoms cannot be evenly distributed.

**Demonstrating by Calculation** Lets work out the work done to take an  $\alpha$  from infnity to the pudding centre. If the electrostatic repulsion is not enough to overcome this, we cannot stop the  $\alpha$  and cannot back scatter.

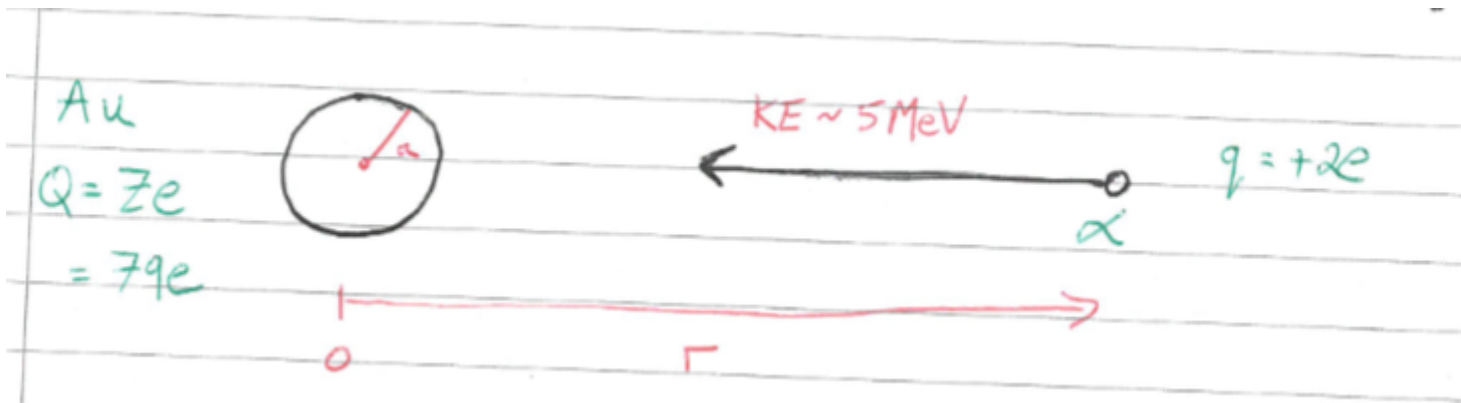


Figure 3: The experiment

### Assumptions

- The atom stays still.
- Ignore the gold electrons (this is fine, as they would cancel some positive charge and make repulsion weaker, which would be even worse. If we can't do it without them, it would be equally impossible to do it with.)

### Recap and Eqns

Coulomb Potential Energy is:

$$u(r) = \frac{qQ}{4\pi\epsilon_0 r}$$

Force is:

$$F(r) = -\frac{du}{dr} = \frac{qQ}{4\pi\epsilon_0 r^2}$$

Change in potential energy ( $u_2 - u_1$ ) is work done:

$$\int_{u_1}^{u_2} du = - \int_{r_1}^{r_2} F(r) dr$$

From outside the atomic radius, we treat the atomic pudding as a point charge of charge  $Q$ . From inside the atomic radius, we treat it as a smaller point charge  $Q'(r)$ , where we only consider the charge inside the portion of the pudding where  $r < a$ .

If charge is spread uniformly, the total charge is proportional to the volume of the sphere. So:

$$\frac{Q'}{Q} = \frac{4\pi r^3}{4\pi a^3}$$

$$Q' = Q \frac{r^3}{a^3}$$

### Inside the Pudding

$$F = \frac{qQ}{4\pi\epsilon_0 r^2}$$

### Next Idea: The Solar System Model