LC Optics and Waves

MSci Physics w/ Particle Physics and Cosmology
University of Birmingham

Year 1, Semester 1
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Lectures Index

Wed 01 Oct 2025 11:00

Lecture 1 - Intro to Waves and SHM Recap

Course Objectives

- Have a sound understanding of basic wave properties
- Have a basic understanding of interference effects, inc diffraction
- Be able to use simple geometric optics and understand the fundamentals of optical instruments.

Recommended Textbooks

1. University Physics, Young and Freedman (Ch 15, 16 for Waves, Ch 33-36 for Optics)
2. Physics for Scientists and Engineers (Ch 20, 21 for Waves, Ch 22-24 for Optics)
3. 5e, Tipler and Mosca, (Ch 15, 16 for Waves, 31-33 for Optics)
4. Fundamentals of Optics, Jenkins and White
5. Optics, Hecht and Zajac

What is a wave? Waves occur when a system is disturbed from equilibrium and the disturbance can travel from one region to another region. Waves carry energy, but do not move mass. The course aim is to derive basic equations for describing waves, and learn their physical properties.

Periodic Motion

Waves are very linked to periodic motion. Therefore we recap periodic motion first.

It has these characterstics:

- A period, T (the time for one cycle)
- A frequency, f, the number of cycles per unit time ($f = \frac{1}{T}$)
- An amplitude, A, the maximum displacement from equilibrium.

Periodic motion continues due to the restoring force. When an object is displaced from equilibrium, the restoring force acts back towards the equi point. The object reaches equi with a non-zero speed, so the motion continues past the equi point and continues forever.

Energy

Periodic motion is an exchange between potential and kinetic energy, with no energy loss. Energy is conserved.

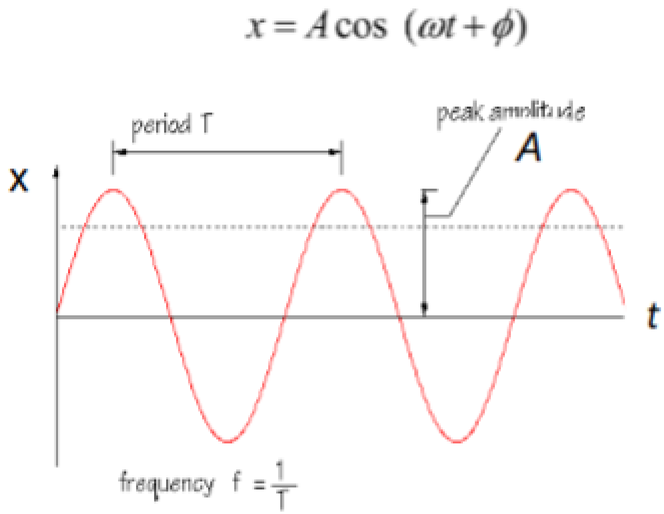
Simple Harmonic Motion

If the restoring force is directly proportional to the displacement $F = -kx$, then the periodic motion becomes Simple Harmonic Motion and the object is called a harmonic oscillator.

In a single dimension, displacement is given by:

$$x = A\cos(\omega t + \phi)$$

Where $\omega = 2\pi f$ is the angular velocity, and ϕ is the phase angle. In cases like this, where the phase angle is $90\,\text{deg}$ we can simplify to $x = -A\sin(\omega t)$



SHM-t

Figure 1: A Phase Angle of 90

More SHM Equations

Velocity

$$v_x = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

Acceleration

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

Both properties are signed to indicate direction, as they are both vectors.

Thu 02 Oct 2025 13:00

Lecture 2 - Wave Functions

Sine Waves

Mechanical Waves A mechanical wave is a disturbance through a medium. It's formed of a single wave pulse or a periodic wave.

Mechanical Waves have the following properties:

1. Transverse: Where displacement of the medium is perpendicular to the direction of propogation.
2. Longitudinal, displaccement of hte medium is in the same direction as propogation.
3. Propogation depends on the medium the wave moves through (i.e. density, rigidity)
4. The medium does not travel with the wave.
5. Waves have a magnitude and a direction.
6. The disturbance travels with a known exact speed.
7. Waves transport energy but not matter throughout the medium.

Wave Functions

We want to define a wave function in terms of two variables, x and t. In any given moment, if we consider a single point on the wave (i.e. $t = 0$), and wait a short while, the wave will have travelled to some $t = t_1 > 0$.

In order to quantify displacement, we therefore want to specify both the time, and the displacement. This will let us find the wave speed, acceleration and the (new) wave number.

We are also able to talk about the velocity and acceleration of individual particles on the wave.

Wave Function for a Sine Wave

Consider a sine wave. We want to find a wave funtion in the form $y(x, t)$. Consider the particle at $x = 0$.

We can express the wave function at this point as $y(x = 0, t) = A \cos \omega t$. However we want to expand this to any general point. Now consider a point (2) which is one wavelength away. We know the behaviour of particle 1 is mirrored by particle 2 (with a time lag).

Since the string is initially at rest, it takes on period (T) for the propogation of the wave to reach point 2, therefore point 2 is lagging behind the motion of point 1. The wave equation is therefore (if particle two has $x = \lambda$) $y(x = \lambda, t) = A \cos(\omega t - 2\pi)$.

For arbitrary x, $y(x, t) = A \cos(\omega t - \frac{x}{\lambda} \cdot 2\pi)$ to account for this delay. This quantity is called the wave number:

$$\text{Wave Number: } k = \frac{2\pi}{\lambda}$$

So:

$$\begin{aligned} y(x, t) &= A \cos(\omega t - kx) \\ &= A \cos(kx - \omega t) \end{aligned}$$

Note the second step is possible as cos is an even function. k can also be signed to indicate direction: if $k > 0$, the wave travels in the positive x. If $k < 0$, the wave travels in the negative x direction. Again, $\omega = 2\pi f$

Displacement Stuff

Considering a point (starting at equi), the time taken for the particle on the sin wave to reach maximum displacement, minimum displacement and back takes the time period T. The speed of the wave is distance travelled over the time taken. We take the distance to be the wavelength λ, as we know the time by definition this takes is one time period T. Therefore wave speed v is:

$$v = \frac{\lambda}{T} = \lambda f$$

Since $\lambda = \frac{2\pi}{k}$ and $f = \frac{\omega}{2\pi}$ (as ω is defined as $\frac{2\pi}{T}$), we can also write:

$$v = \frac{2\pi}{k} \cdot \frac{\omega}{2\pi} = \frac{\omega}{k}$$

Particle Velocity

We can also determine the velocity of individual particles in the medium. We can use this to determine the acceleration.

We know that

$$y(x,t) = A\cos(kx - \omega t)$$

The vertical velocity v_y is therefore given by:

$$v_y = \frac{dy(x,t)}{dt}$$

Which is unhelpful (as we can't differentiate two variables at once), we can slightly cheat this by looking at purely a certain value of x, and therefore treating x as constant (to get a single variable derivative).

$$v_y = \frac{dy(x,t)}{dt}\bigg|_{x=\text{const.}}$$

However this is notationally yucky, so we therefore use the notation:

$$\frac{\partial y(x,t)}{\partial t}$$

To represent the same thing. Finally (carrying out the partial derivative):

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$

Particle Acceleration

We can work out particle acceleration (transverse acceleration) by differentiating in the same manner again:

$$a_y(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial y(x,t)}{\partial t}\right) = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y(x,t).$$

Wed 08 Oct 2025 11:00

Lecture 3 - Generalised Wavefunctions

Recap: For a sine wave, the wavefunction is:

$$y(x,t) = A\cos(kx - \omega t)$$

Where k is the wave number, $k = \frac{2\pi}{\lambda}$ and omega is $\frac{2\pi}{T} = 2\pi f$

More Wavefunctions

What is the general form of a triangular shaped wave? What about a square stepped wave? TODO: Diagram

We know that each particle (i.e. along a string) copies the motion of its immediate left-hand neighbour (for a particle moving in the positive x), with a time delay proportional to their distance. Every wave is describable $y(x,t)$ wave function, and every mechanical wave relies on a medium (i.e. a piece of string or water, concrete etc) to travel.

Ideally, we want to be able to find a general form of a wave function.

In a single fixed instant of time, the moving wave pulse is stationary, so purely a function of $y = f(x)$. We want a wave function where we can input any value of t, so we need a moving frame of reference. We define this frame of reference as O' (for the origin) and x', y' axes. This frame of reference moves with the wave pulse and at the same speed, therefore y' is a function of x' only, independant of speed.

New System

x is the distance from the origin O to the relevant point, while x' is the distance from O'.

$$x = x' + vt$$

$$x' = x - vt$$

$$y' = f(x') = f(x - vt)$$

However, as the wave is moving purely in one direction (along x), $y = y'$, so:

$$y = f(x - vt)$$

Back to Basics

Going back to:

$$\begin{aligned} y(x,t) &= A\cos(kx - \omega t) \\ &= A\cos\left[k\left(x - \frac{\omega}{k}t\right)\right] \\ &= A\cos\left[k\left(x - vt\right)\right] \end{aligned}$$

(Note, this is true for a wave in the positive x, for a wave moving in the negative x this would be x + vt)

Equivalent Rrepresentations

There are some equivalent representations for a sine wave:

$$y(x,t) = A\cos\left(2\pi f\frac{x}{v} - \omega t\right) = A\cos\left(2\pi\left[\frac{x}{\lambda} - \frac{t}{T}\right]\right) = TODO$$

More Differentiation

We've already looked at differentiating with respect to t, but what about x? This would give us the slope of the string at that point:

$$\frac{\partial y(x,t)}{\partial x}$$

And the curvature of the string:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 A\cos(kx - \omega t) = -k^2 y(x,t)$$

Note the similarities here with the equation for transverse acceleration:

$$(1)\frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 y(x,t)$$

$$(2)\frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 y(x,t)$$

Dividing (1) by (2):

$$\frac{\partial^2 y(x,t)/\partial t^2}{\partial^2 y(x,t)/\partial x^2} = \frac{\omega^2}{k^2} = v^2$$

Therefore:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

We call this the 'wave equation' as every wave function y(x, t) must satisfy it, regardless of whether or not it is periodic or its direction of travel. If y(x, t) does not satisfy this, it is not a wave function.

An Example

$$y(x,t) = \frac{x^3 - vt^2}{e^t}$$

TODO, the example in our own time

Wave Equation for a String

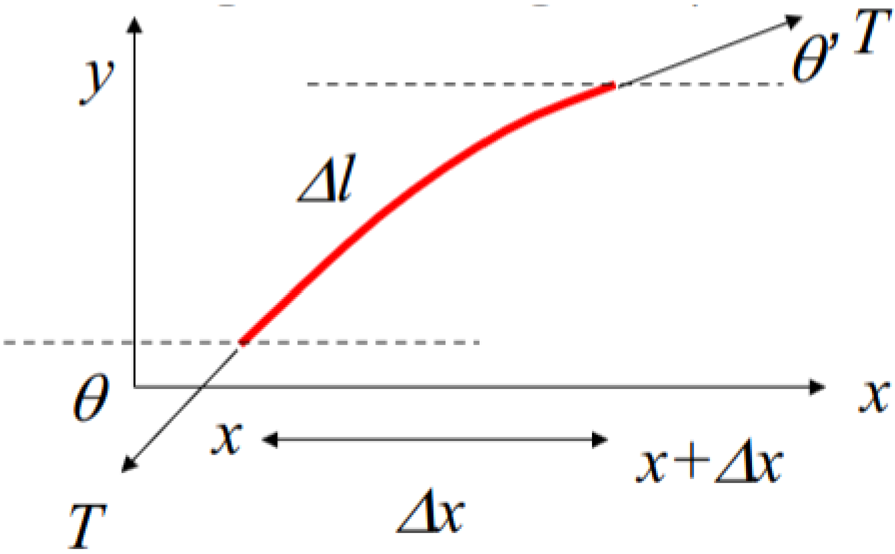


Figure 2: A snapshot of the wave.

Lets say we have some string, suspended horizontally under tension. We generate a single wave pulse and allow it to propogate down the string.

We assume the string is 1D, under tension T (constant throughout) and has mass per unit length μ.

Consider a small segment of string of length ΔL from x to $x + \Delta x$. This string makes angle θ with the horizontal at the bottom of the string, and angle θ' with the horizontal at the top of the string.

Net force (transverse in y) is:

$$F_y = T\sin\theta' - T\sin\theta$$

And using the small angle approx $\sin\theta \approx \tan\theta = \frac{dy}{dx}$:

$$F_y = T\left(\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}\right)$$

Using differentiation by first principles:

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

We can say:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \lim_{\Delta x\to 0}\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x}$$

Therefore:

$$F_y = T\frac{d^2y}{dx}\Delta x$$

The mass of this section of string is $\mu\Delta x$, and considering the acceleration in the y direction we can plug into $F = ma$ to get:

$$F = ma$$

$$T\frac{d^2y}{dx^2}\Delta x = \mu\Delta x\frac{d^2y}{dt^2}$$

$$\frac{d^2y}{dx^2} = \frac{\mu}{T}\frac{d^2y}{dt^2}$$

The LHS is the rate of change of the string's gradient, which as mentioned is the curvature of the string. The RHS includes the transverse acceleration of the string, therefore acceleration is proportional to curvature. To evaluate this at a fixed time/position we should write:

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}$$

And comparing to the wave equation we get:

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}$$

$$\frac{1}{v^2} = \frac{\mu}{T}$$

$$v = \sqrt{\frac{T}{\mu}}$$

So a wave travels faster under a higher tension with a lower mass per unit length.

Thu 09 Oct 2025 13:00

Lecture 4 - Waves at Boundaries

Recap

We previously derived:

$$v = \sqrt{\frac{T}{\mu}}$$

And we also have:

$$v = \frac{\omega}{k}$$

The former is useful explictly for a wave travelling over a string, while the latter is applicable to the movement of any wave. On a string, a higher tension yields a higher restoring force and therefore a higher speed, while a higher mass per unit length gives a higher mass for some arbitrary length of string, therefore a lower acceleration and lower speed.

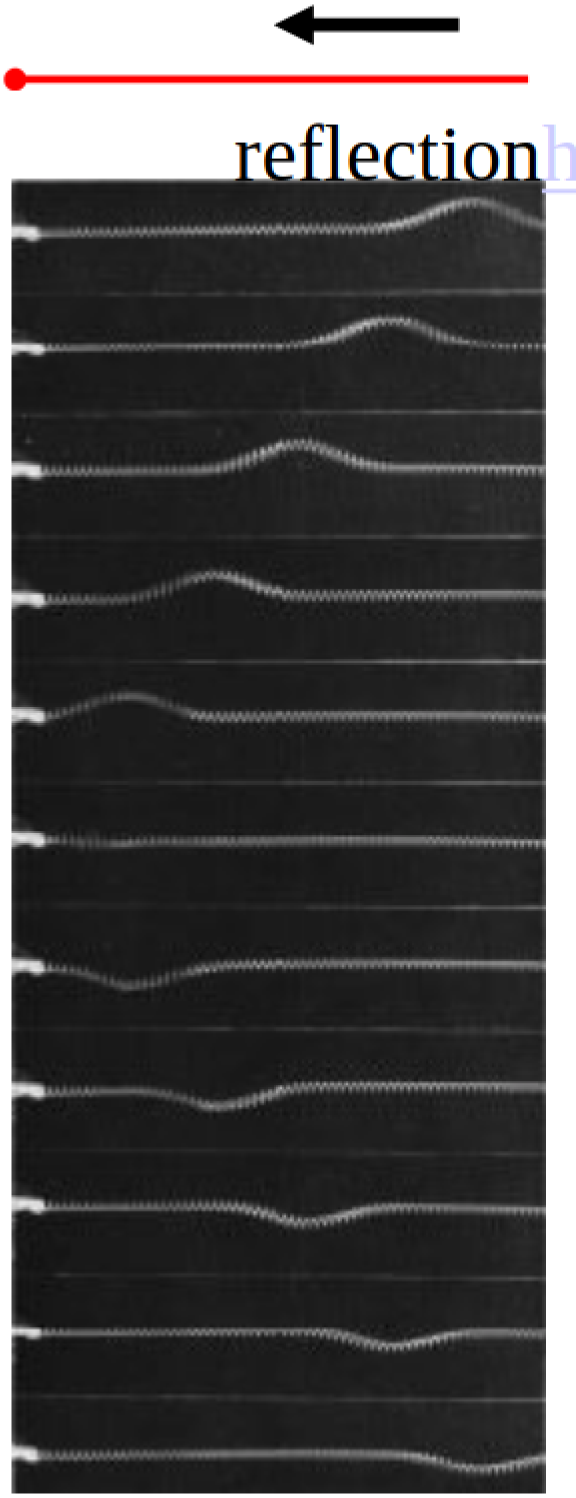
A Quick Interlude

The speed of a mechanical wave has the general form:

$$v = \sqrt{\frac{\text{Restoring force returning to equilibrium}}{\text{Inertia resisting return to equilibrium}}}$$

Reflection

When a wave hits a fixed boundary, it is reflected and inverted. Lets consider a case where a string is fixed on the LHS and is reflected back:


reflection

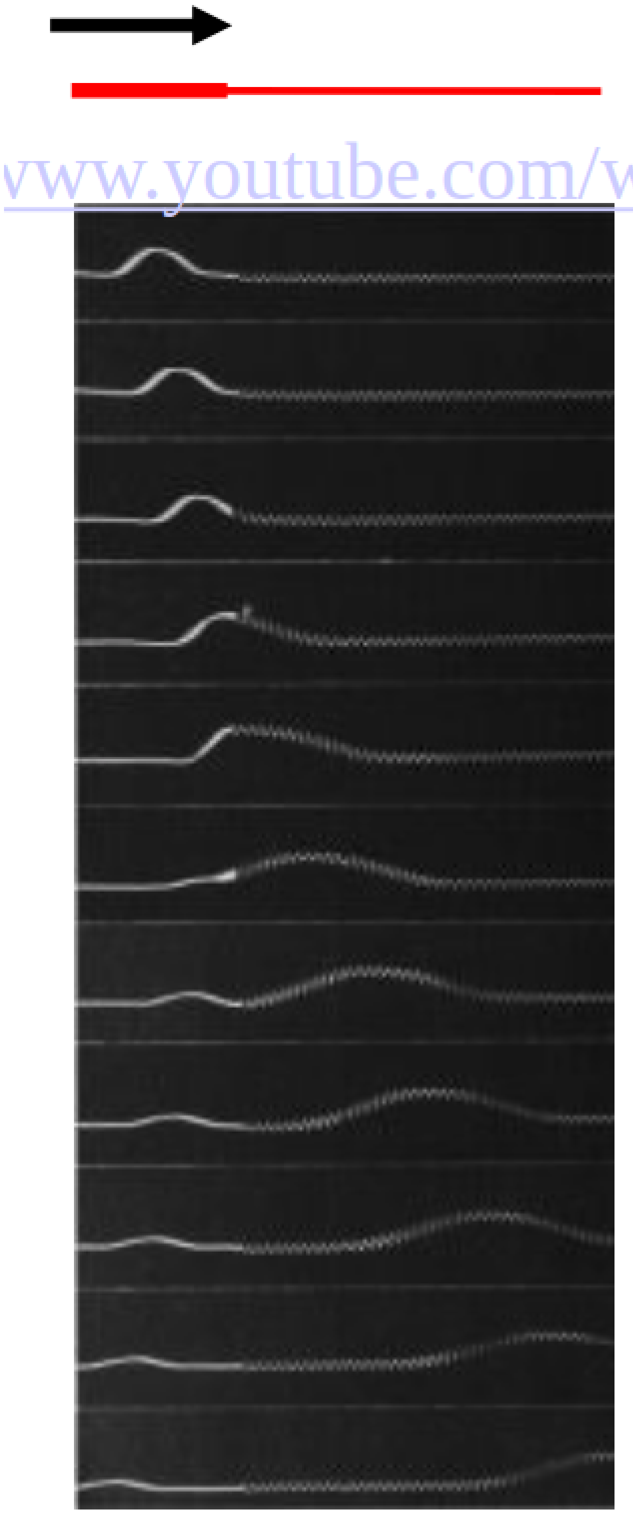


Figure 3: Or with two strings, going from thick to thin:

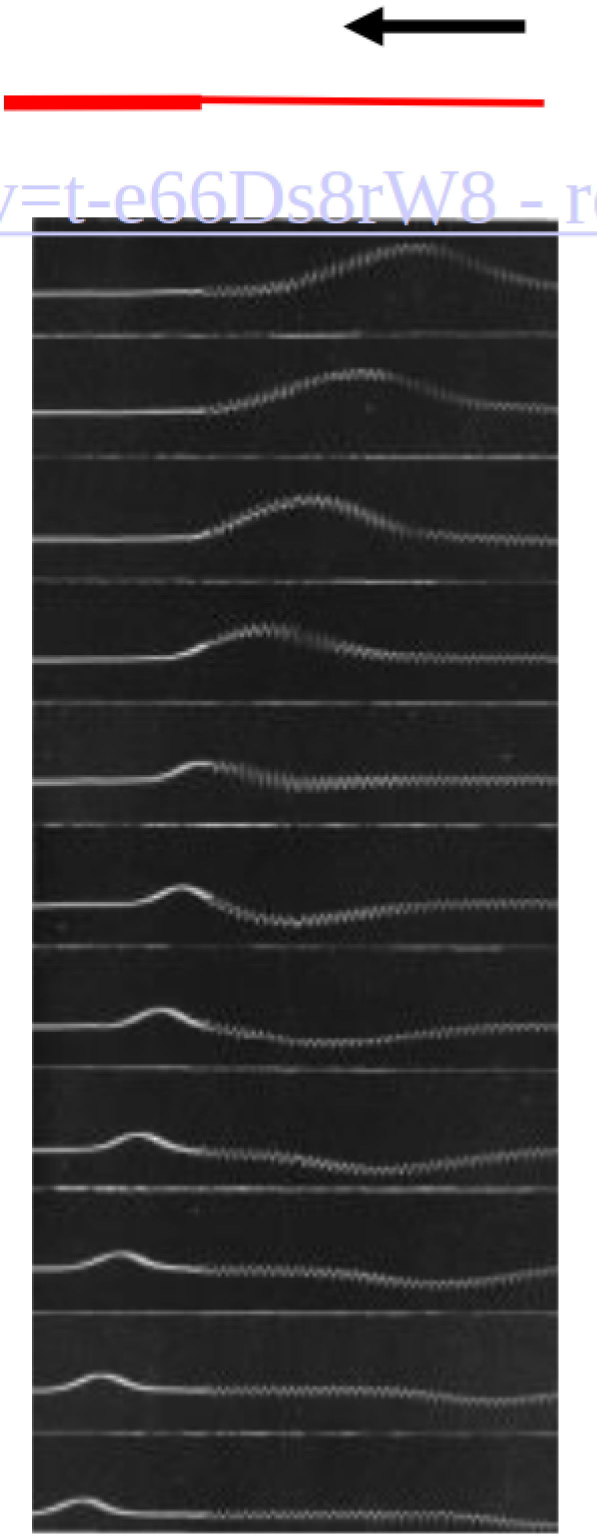


Figure 4: Or from thin to thick.

Waves Interacting at Boundaries

Say we have two pieces of string connected to each other, one thin string with mass per unit length μ_1 and a thicker string with μ_2 (both under the same tension, T). If a wave pulse is passed along from the thin string to the thicker string, whath happens at the point of connection, P?

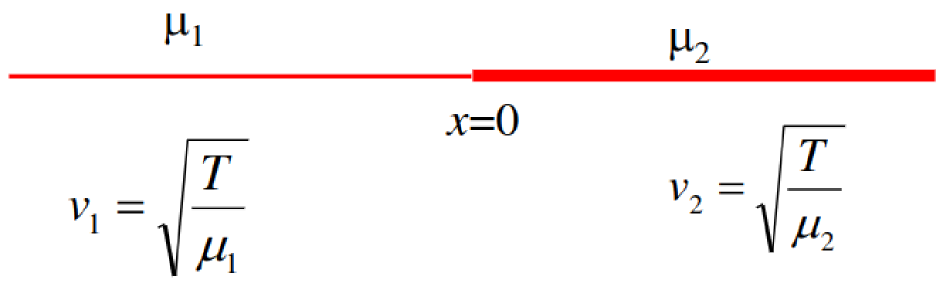


Figure 5: The two connected strings.

Consider a travelling wave coming from the left:

$$y_1 = A\cos(k_1 x - \omega_1 t)$$

$$y_2 = B\cos(k_2 x - \omega_2 t)$$

At the point of connection, we'll define this as $x = 0$. Here (as the string is not broken, so it must be connected):

$$y_1 = y_2 \tag{1}$$

Also, force is finite, therefore curvature must be finite (as force is proportional to curvature). Therefore we cannot have any discontinuities in curvature.

$$\frac{\partial y_1}{\partial x_1} = \frac{\partial y_2}{\partial x_2} \tag{2}$$

Disregarding non-linear effects, so assuming that the frequency with which the waves travel down the string is the same for both parts of the string, $\omega_1 = \omega_2 = \omega$.

From (1) and noting $x = 0$:

$$A\cos(-\omega t) = B\cos(\omega t) \tag{3}$$

And from (2) with the same note:

$$-k_1 A\sin -\omega t = -k_2 B\sin -\omega t \tag{4}$$

However this suggests that $A = B = 0$. This is technically a solution, but not really - it doesn't represent an actual wave (just two flat lines with no amplitude ever)

Including Reflection

The previous case did not work as we disregarded reflection at the wave boundary, P. Lets add an extra wave (the C term) to represent the reflection back into the light string:

$$y_1 = A\cos(k_1 x - \omega t) + C\cos(k_1 x + \omega t) = B\cos(k_2 x - \omega t)$$

From y1 = y2 at x = 0 we get:

$$y_1 = A\cos(-\omega t) + C\cos(-\omega t) = B\cos(-\omega t)$$

So: $A + C = B$

From (2) we get:

$$-k_1 A\sin(-\omega t) - k_1 C\sin(\omega t) = -k_2 B\sin(-\omega t)$$

Which has solutions:

$$B = \frac{2k_1}{k_1 + k_2} A$$

$$C = \frac{k_1 - k_2}{k_1 + k_2} A$$

We have the incident wave:

$$y_1 = Acos(k_1 x - \omega t)$$

The transmitted wave:

$$y_2 = B\cos(k_2 x - \omega t) = \frac{2k_1}{k_1 + k_2} A\cos(k_2 x - \omega t)$$

And lastly the reflected wave:

$$y_3 = C\cos(k_1 x + \omega t) = \frac{k_1 - k_2}{k_1 + k_2} A\cos(k_1 x - \omega t)$$

We note:

k_1 is the wave number in the medium where the incident wave comes from.

k_2 is the wave number in the medium where the transmitted wave goes into.

Example

If the wave comes in from the left, then $\mu_2 > \mu_1$ per example diagram, then $k_2 > k_1$, and C is negative.

Note that if the single pulse as a postive y amplitude, then the transmitted pulse in the heavier string will also have a positive y amplitude, but will have a smaller magnitude. The reflected wave will have a negative y-amplitude as the reflection inverts it.

If the wave comes from the right (from thick to thin), then $\mu_1 > \mu_2$, $k_1 > k_2$ and C is positive.

Fri 17 Oct 2025 12:56

Lecture 5

Fri 17 Oct 2025 13:00

Lecture 6 - Standing Waves 2 Electric Boogaloo

Recap

$$\frac{\partial y}{\partial x} = 2Ak\cos(kx)\sin(\omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = 2A(-k^2)\sin(kx)\sin(\omega t)$$

$$\frac{\partial y}{\partial t} = 2A\omega\sin(kx)\cos(\omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = 2A(-\omega^2)\sin(kx)\sin(\omega t)$$

Therefore:

$$\frac{\frac{\partial^2 y}{\partial t^2}}{\frac{\partial^2 y}{\partial x^2}} = \frac{2A(-\omega^2)\sin(kx)\sin(\omega t)}{2A(-k^2)\sin(kx)\sin(\omega t)} = \frac{\omega^2}{k^2} = v^2$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Therefore a standing wave still obeys the wave equation, as it must.

Standing Wave Properties

Wavelength

Consider a horizontal string from $x = 0$ to $x = L$, with both ends fixed. We generate a sinusoidal wave pulse, which must satisfy:

$$y(x,t) = 2A\sin(kx)\sin(\omega t)$$

We know at $x = L$ and $x = 0$, $y = 0$ at all times as fixed at this point. Therefore:

$$kL = n\pi, (n \in \mathbb{N})$$

For $n = 1$, we have half a wavelength on the string:

$$\lambda = \frac{2L}{1} = 2L$$

And this has a general form: $\lambda = \frac{2L}{n}$.

Frequency

$$f_n = \frac{v}{\lambda_n} = \frac{v}{\left(\frac{2L}{n}\right)} = \frac{nv}{2L}$$

Crucially:

$$f_1 = \frac{v}{2L}$$

Is the first harmonic (or fundamental). $f_2 = 2f_1$ is the second harmonic, or first overtone, etc. All of these, f_n where $n \in \mathbb{N}$ are called "normal modes". For each normal mode, the corresponding frequency is called the resonant frequency (natural frequency of the system).

What happens if we try to create a standing wave where $\lambda \neq \frac{2L}{n}$? In short, we cannot. The system will reject any attempts to do so.

Energy

Energy is proportional to ω^2. Energy can only take certain discrete values (corresponding to $f_1, f_2, \ldots, f_n$), we find that the system has quantised possible values for energy.

To generate a wave with a higher frequency we either have to use a shorter L, or a higher v. A higher v is achieved by using a lighter string or placing the system under higher tension.

Sound Waves

Notation

Displacement of a sound wave is denoted:

$$s(x,t) = S_m \cos(kx - \omega t)$$

And pressure is given by:

$$\Delta P(x,t) = \Delta P_m \sin(kx - \omega t)$$

Different Boundary Conditions

The equations for standing waves given is only true for the boundary conditions of both ends fixed. If we vary these (for example left end fixed, right end not, wave initially travelling left) we get a different solution. For example, the first harmonic:

$$L = \frac{1}{4\lambda_1}$$

$$f_1 = \frac{v}{4L}$$

Where the left end forms a node (as required by boundary conditions) and the right end forms an antinode, as it is free to move. For the third harmonic:

$$L = \frac{3}{4\lambda_3}$$

$$f_3 = \frac{3v}{4L} = 3f_1$$

And fifth:

$$L = \frac{5}{4\lambda_5}$$

$$f_5 = \frac{5v}{4L} = 5f_1$$

Notably, this system cannot support even harmonics.