LC Quantum Mechanics

MSci Physics w/ Particle Physics and Cosmology
University of Birmingham

Year 1, Semester 1

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Lectures Index

Fri 03 Oct 2025 12:00

Lecture 1 - Atomic Structure

What is the course?

- Quantum mech is weird and unintuative, we will build up a case in the course for why this weird theory was necessary and why we're confident it works.
- Each week will be a self-contained concept and/or historial experiment, working up to the Shroedinger Equation and wave-particle duality.
- Names and dates do not need to be memorised.
- Recommended text: University Physics (Young and Freedman).
- Office hours: 13:00 – 13:50 Fridays (immediately post-lecture), Physics East Rm 207.

Atomic Structure

What actually is an atom? What does it actually look like inside?

Early Clues

- Periodic Table (Mendelev, 1989), periodic patterns in elements properties.
- Radioactivity (Becquerel, 1896, Curie 1898)
- Atoms emit and absorb specific discrete wavelengths, (Balmer, 1884)
- Discovery of the Electron (Thompson 1897). Cathode rays - heating metal in a vacuum with an electric field above it, to strip away electrons from the metal.
  - This showed electrons were negatively charged and extremely light (1/2000th of the atomic mass).

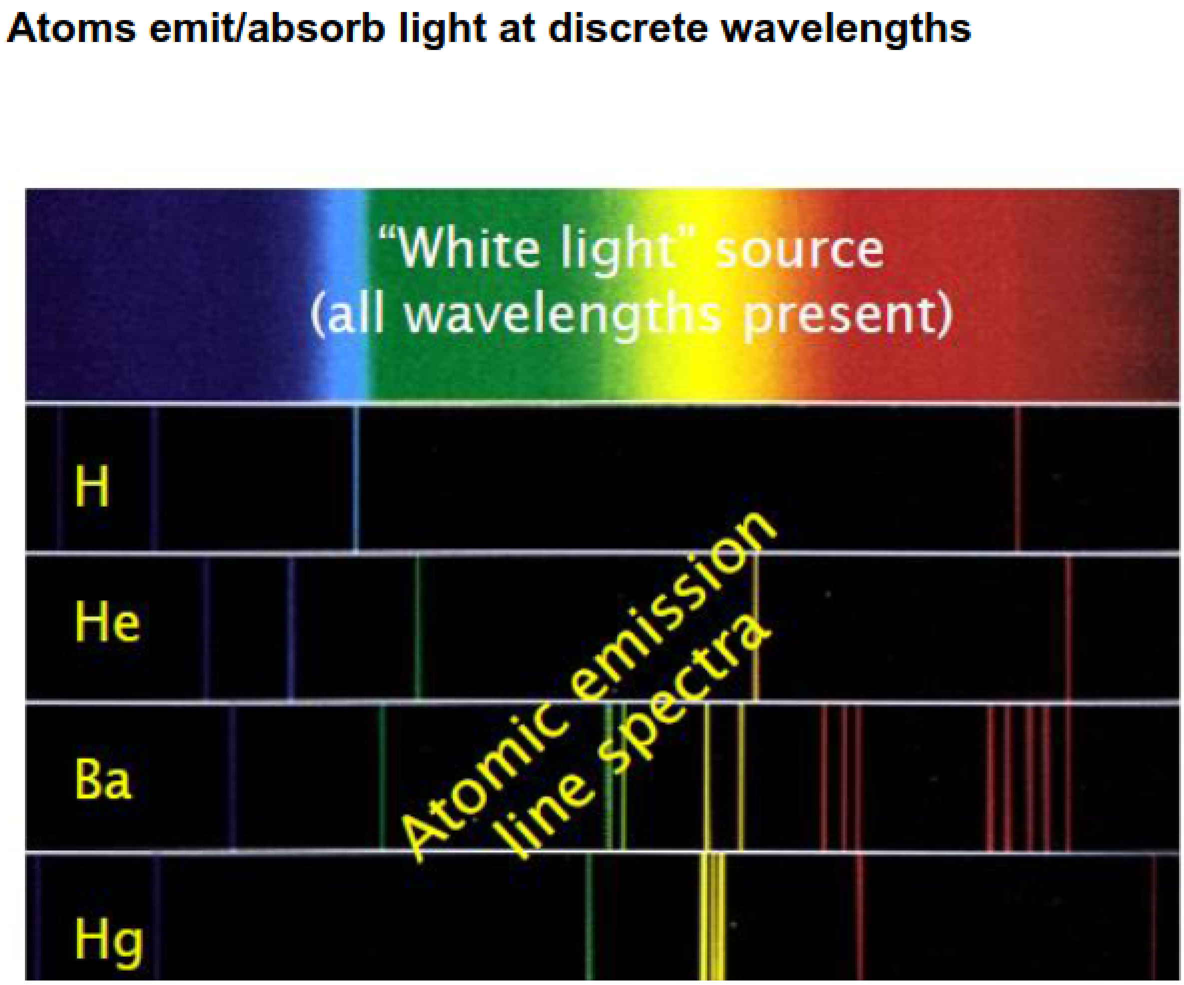

Figure 1: Absorption Spectra

Plum Pudding Model

A solid, uniform lump of positively charged matter, approximately 10^{-10}m across. This had evenly distributed negative charges (electrons) scattered throughout.

Discovery of the Nucleus

Geiger and Marsden (1908-1913), fired alpha particles (He nuclei) at thin gold foil and measured the deflection / scattering.

The alpha particles had a mass of 4u, a charge of +2e and an energy of approximately 5MeV.

They found that most α were scattered only by small angles, but (surprisingly) a small number were scattered right back towards to emitter (through $\theta > 90\,\text{deg}$). The distribution of the angles is approximately Normally distributed, with a mean of 0. Only approximately 1 in 8,000 fired αs were scattered by $\theta > 90$ ("back-scattering").

Can this be explained with the Plum Pudding Model? No, it cannot. This was used to demonstrate that atoms cannot be evenly distributed.

Demonstrating by Calculation

Lets work out the work done to take an α from infty to the pudding centre. If the electrostatic repulsion is not enough to overcome this, we cannot stop the α and cannot back scatter.

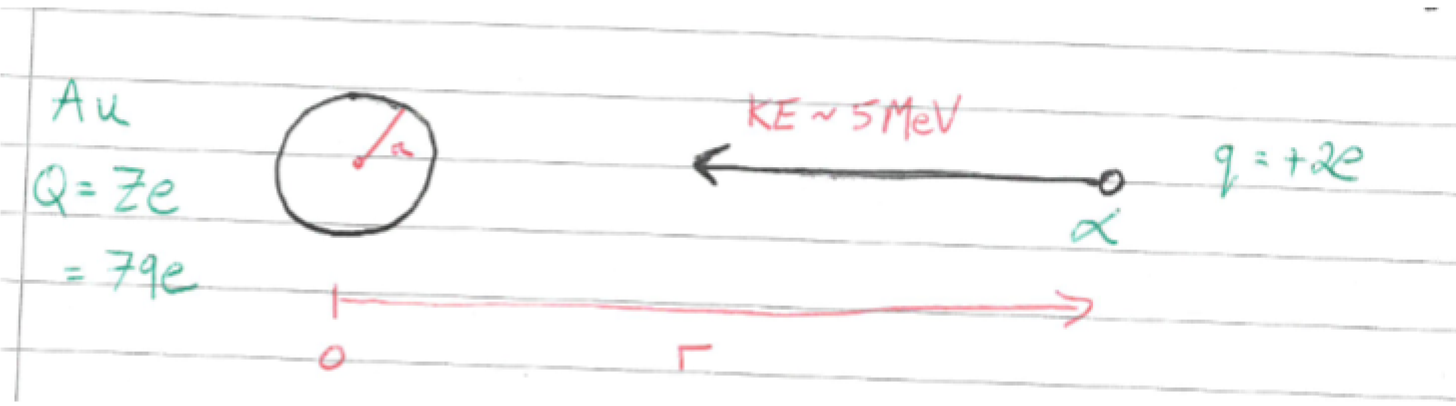

Figure 2: The experiment

Assumptions

- The atom stays still.
- Ignore the gold electrons (this is fine, as they would cancel some positive charge and make repulsion weaker, which would be even worse. If we can't do it without them, it would be equally impossible to do it with.)

Recap and Eqns

Coulomb Potential Energy is:

$$u(r) = \frac{qQ}{4\pi\epsilon_0 r}$$

Force is:

$$F(r) = -\frac{du}{dr} = \frac{qQ}{4\pi\epsilon_0 r^2}$$

Change in potential energy $(u_2 - u_1)$ is work done:

$$\int_{u_1}^{u_2} du = -\int_{r_1}^{r_2} F(r)dr$$

From outside the atomic radius, we treat the atomic pudding as a point charge of charge Q. From inside the atomic radius, we treat it as a smaller point charge $Q'(r)$, where we only consider the charge inside the portion of the pudding where $r < a$, where a is the current position inside the sphere.

If charge is spread uniformly, the total charge is proportional to the volume of the sphere. So:

$$\frac{Q'}{Q} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3}$$

$$Q' = Q\frac{r^3}{a^3}$$

Inside the Pudding

$$F = \frac{qQ'}{4\pi\epsilon_0 r^2}$$

$$F = \frac{qQr^3}{4\pi\epsilon_0 r^2 a^3}$$

$$F = \frac{qQr}{4\pi\epsilon_0 a^3}$$

$$F = \frac{qQ}{4\pi\epsilon_0 a^3} \times r$$

Hence inside, $F \propto r$

Outside the Pudding

$$F = \frac{Qq}{4\pi\epsilon_0 r^2}$$

$$F = \frac{Qq}{4\pi\epsilon_0} \times \frac{1}{r^2}$$

Hence outside, $F \propto \frac{1}{r^2}$

To integrate, we are therefore integrating the area under this (almost) triangle:

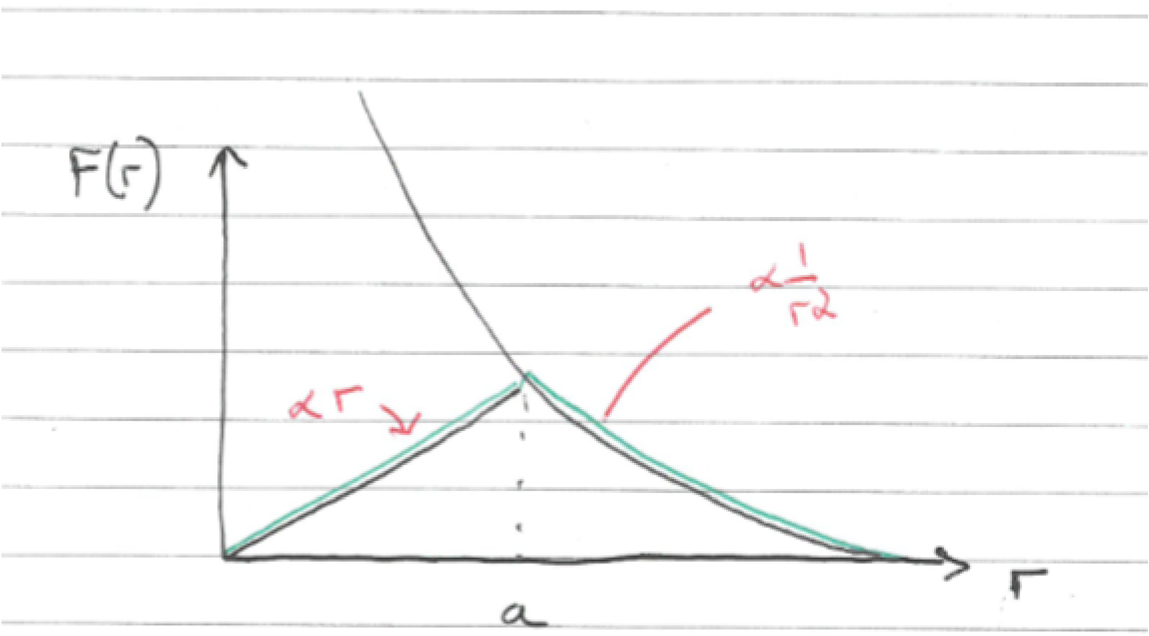

Figure 3: Radius vs electrostatic repulsion force

$$\Delta u = -\int_{r_1}^{r_2} F(r)\, dx$$

Splitting into two sections (prior to, and after the point where r = atomic radius, $r = a$), and integrating across all r (as we are attempting to work out the work done to bring an α from infinity to the charge, at which poind distance is 0):

$$= -\int_{\infty}^{a} \frac{qQ}{4\pi\epsilon_0 r^2}\, dx - \int_{a}^{0} \frac{qQr}{4\pi\epsilon_0 a^3}\, dx$$

$$= -\frac{qQ}{4\pi\epsilon_0}\int_{\infty}^{a} \frac{1}{r^2}\, dx - \frac{qQ}{4\pi\epsilon_0 a^3}\int_{a}^{0} r\, dx$$

$$= -\frac{qQ}{4\pi\epsilon_0}\lim_{x\to\infty}\int_{x}^{a} \frac{1}{r^2}\, dx - \frac{qQ}{4\pi\epsilon_0 a^3}\int_{a}^{0} r\, dx$$

$$= -\frac{qQ}{4\pi\epsilon_0}\lim_{x\to\infty}\left[-\frac{1}{r}\right]_{x}^{a} - \frac{qQ}{4\pi\epsilon_0 a^3}\left[\frac{1}{2}r^2\right]_{a}^{0}$$

$$= -\frac{qQ}{4\pi\epsilon_0}\left(-\frac{1}{a} - 0\right) - \frac{qQ}{4\pi\epsilon_0 a^3}\left(\frac{1}{2}0^2 - \frac{1}{2}a^2\right)$$

$$= -\frac{qQ}{4\pi\epsilon_0}\left(-\frac{1}{a}\right) - \frac{qQ}{4\pi\epsilon_0 a^3}\left(-\frac{1}{2}a^2\right)$$

$$= \frac{qQ}{4\pi\epsilon_0 a} + \frac{qQa^2}{8\pi\epsilon_0 a^3}$$

$$= \frac{qQ}{4\pi\epsilon_0 a} + \frac{qQ}{8\pi\epsilon_0 a}$$

$$= \frac{qQ}{4\pi\epsilon_0 a} + \frac{1}{2}\frac{qQ}{4\pi\epsilon_0 a}$$

$$= \frac{3}{2}\frac{qQ}{4\pi\epsilon_0}$$

As required! Plugging in values gives us:

$$\Delta u = \frac{3}{2}\frac{(2e)(79e)}{4\pi(8.854 \times 10^{-12}) \times 10^{-10}}$$

$$= 5.45 \times 10^{-16} J = 3.41 KeV$$

This is much less than the kinetic energy of the 5MeV alpha particle, therefore (as this value is maximum work done against the repulsive force) a plum pudding could not backscatter a 5MeV alpha particle. However, since $\Delta u \propto 1/a$, a smaller volume of charge could. How small, however?

$$\Delta u = 5\text{MeV} = -\int_{\infty}^{r_{max}} \frac{qQ}{4\pi\epsilon_0 r^2}\, dr$$

$$-5\text{MeV} = \frac{qQ}{4\pi\epsilon_0}\int_{\infty}^{r_{max}} \frac{1}{r^2}\, dr$$

$$-5\text{MeV} = \frac{qQ}{4\pi\epsilon_0}\lim_{x\to\infty}\left[-\frac{1}{r}\right]_x^{r_{max}}$$

$$-5\text{MeV} = \frac{qQ}{4\pi\epsilon_0}\left[-\frac{1}{r_{max}} - \lim_{x\to\infty}\frac{1}{x}\right]$$

$$-5\text{MeV} = \frac{qQ}{4\pi\epsilon_0}\left[-\frac{1}{r_{max}}\right]$$

$$5\text{MeV} = \frac{qQ}{4\pi\epsilon_0 r_{\text{max}}}$$

Subsitution and rearrangement gives $r_{\text{max}} = 4.5 \times 10^{-14} m = 45 fm$. We accept the 10^{-10}m $= 100,000$fm figure as the total width of the atom, but this demonstrates that there must exist a nucleus of no larger than 45fm.

Next Idea: The Solar System Model

Therefore, the next idea was an orbiting solar system model, where electrons orbit in fixed paths around a central nucleus. However, accelerating charges (i.e. a charge in circular motion) radiates energy, so this orbiting electron would be on a decaying path to crash into the nucleus. We can observe this does not happen, so need another idea...

Bohr made two postulates:

- The electron in hydrogen moves in a set non-radiating circular orbit.
- Radiation is only emitted or abosrbed when an electron moves from one orbit to another.

This works (at least for hydrogen) and explains the absoption spectra, but for now lacks a physical grounding.

Fri 10 Oct 2025 12:00

Lecture 2 - The Ultraviolet Catastrophe

In this lecture:

- How classical theories fail to explain black body radiation ("The Ultraviolet Catastrophe").
- How quantising light into photons gives predictions that fit this observation.

Black Body Radiation

A 'black body' is an idealised perfect object, that does not reflect, and absorbs internally all light (regardless of wavelength) incident upon it. No light is transmitted, so nothing shines out the other side. The object is perfectly black.

All bodies emit electromagnetic energy, usually outside the visible portion of the spectrum. For example, Paul Hollywood (and other humans) emit at about 300 Kelvin, which is infrared (at the temperature which night vision goggles are tuned to).

For the black body, emission spectrum is **only** from this thermal emission (no reflection, no flourescence, etc). Hotter objects are brighter and bluer (hotter means higher energy, and therefore a shorter wavelength)

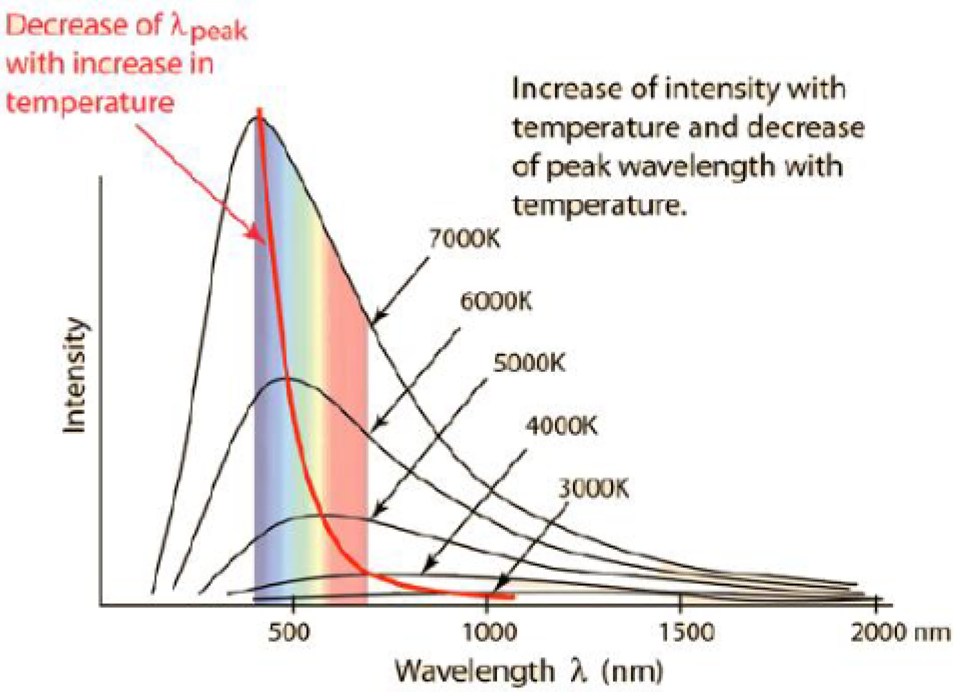

Figure 4: Observed Emission Spectra

However, we run into a problem. If we plot the spectra predicted by classical thermodynamics, vs the observed spectra for a given temperature object, the classical prediction gets it totally wrong, especially at shorter wavelengths.

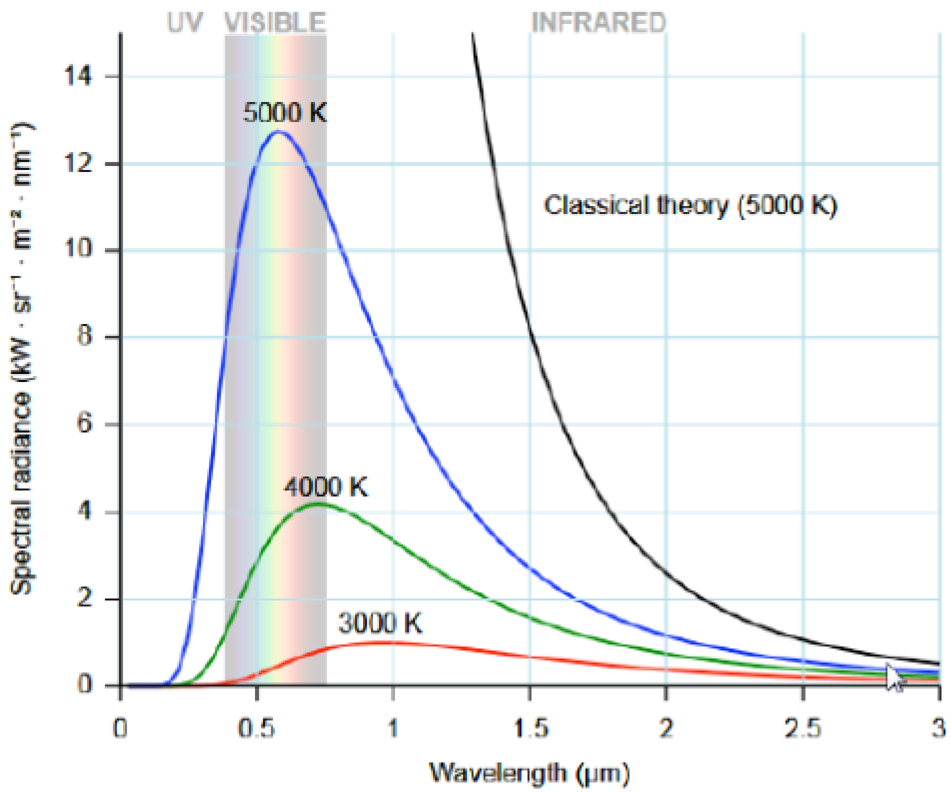

Figure 5: Predicted and Observed Spectra for 5000K (and Observed for 4k K and 3k K)

Notation

$I(\lambda)$ is the intensity per wavelength for an emitted wavelength λ. I is the total intensity across all wavelengths per unit time (in W/m^2, power per unit area).

$$I = \int_0^\infty I(\lambda)\, d\lambda$$

I is the total area under the $I(\lambda)$ curve, i.e. the sum of intensity per wavelength, across every wavelength.

The Ultraviolet Catastrophe

Empirical Results

The Stefan-Boltzmann Law gives $I = \sigma T^4$, where σ is the Stefan-Boltzmann constant, $\sigma = 6.57 \times 10^{-8} W m^{-2} K^{-4}$

Wien's Displacement Law gives $\lambda_{\text{peak}} = \frac{b}{T}$, where $b = 2.898 \times 10^{-3} Km$.

Why does classical mechanics break?

Lets model the $I(\lambda)$ spectrum by slotting standing waves into a cavity. Inside the blackbody, EM waves form standing waves (in a limited number of possible configurations).

We can simplify by considering a 1D cavity of length L. We can consider 'cavity modes' as the possible standing waves that can exist in this cavity. As we know the wave is bound at each end, the displacement at each end of the cavity must be 0. Therefore, the only possible waves must obey this, and these are cavity modes.

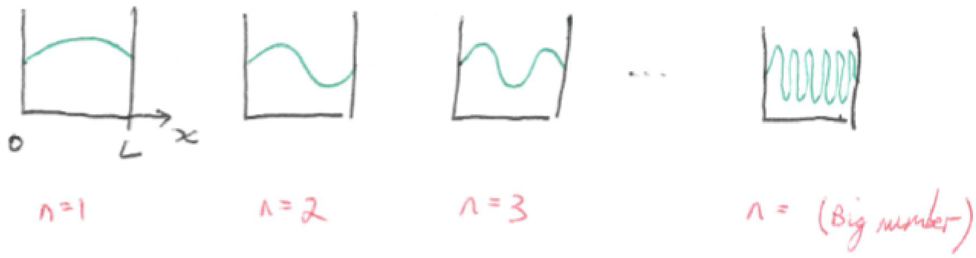

Figure 6: Possible cavity modes

The amplitude $a(x)$ can be given by this:

$$a(x) = \sin\left(\frac{n\pi x}{L}\right), \qquad n = 1, 2, 3, \dots n \quad \text{(mode number)}$$

And by inspection from the figures:

$$\lambda = \frac{2L}{n}$$

And therefore the number of nodes per wavelength is:

$$n(\lambda) = \frac{2L}{\lambda}$$

So, classically:

$$I(\lambda) \propto \frac{n(\lambda)}{\lambda} \times k_B T \propto \frac{1}{\lambda^2}$$

Where the first term is the density of nodes at lamda, and the second is the average energy of nodes. As we head to UV and $\lambda \to 0$, $I(\lambda) \to \infty$... which is not accurate. This is the UV Catastrophe!

Where did it go wrong?

The issue was assuming that all cavity modes have average energy $k_B T$ - the "Equipartition Theorem" (which we'll meet in later courses).

In brief: the probability distribution of energies is a "Boltzmann Distribution":

$$p(E) = \frac{\exp\left(-\frac{E}{k_B T}\right)}{k_B T}$$

Average energy:

$$\bar{E} = \int_0^\infty E p(E)\, dE = k_B T$$

Which the UV Catastrophe says is incorrect... We therefore need another model to replace the idea of the Boltzmann Distribution.

Plank's Hypothesis

A rather desperate Plank hypothesised that energy was quantised, i.e. it comes in discrete packets, called quanta. The energy of these quanta is proportional to frequency. This was radical at the time, even though we accept it now.

$$\Delta E = hf = \frac{hc}{\lambda}$$

Where $h = 6.626 \times 10^{-34} Js$ is Plank's constant.

Sticking this into the Partition Function from statistical mechanics (which we will properly encounter later on, for now don't worry!), we get an average energy:

$$\bar{E}(\lambda) = \frac{hc/\lambda}{\exp(hc/\lambda k_B T) - 1}$$

Looking at limits:

$$\text{For } \bar{E}(\lambda \to \infty): \quad \frac{hc}{\lambda k_B T} << 1$$

And the Taylor Series of e^x:

$$\exp\left(\frac{hc}{\lambda k_B T}\right) \approx 1 + \frac{hc}{\lambda k_B T} + \dots$$

Yields:

$$\bar{E}(\lambda) \approx \frac{hc/\lambda}{1 + (hc/\lambda k_B T) - 1} = k_B T$$

This is a good sign, because it means that Plank's Hypothesis holds the correct classically predicted and empirically observed behaviour for higher wavelengths.

Now what about lower wavelengths, where the classical behaviour broke?

$$\text{For } \bar{E}(\lambda \to 0): \quad \exp\left(\frac{hc}{\lambda k_B T}\right) \to \infty \quad \text{(very quickly)}$$

$$\text{For } \bar{E}(\lambda \to 0): \quad \frac{hc}{\lambda} \to \infty \quad \text{(slower)}$$

Therefore, in the expression:

$$\bar{E}(\lambda) = \frac{hc/\lambda}{\exp(hc/\lambda k_B T) - 1}$$

The numerator and denominator both tend to infinity, but the denominator does so much faster. Therefore (and this can be done in a less handwavy manner via L'Hopital):

$$\bar{E}(\lambda \to 0) \to \frac{1}{\infty} \to 0$$

Which recovers the behaviour at UV wavelengths, so no Catastrophe!

Conclusion

- This strange quantisation hypothesis actually fits the data.
- Quantising energy means that the average energy of each cavity mode is wavelength dependant, and not fixed $k_B T$ as seen at larger wavelengths.
- This solves the UV Catastrophe!

Thu 17 Oct 2025 12:00

Lecture 3 - Particle Nature of Light

In this lecture:

- The photoelectric effect.
- Compton scattering.

Which are two examples where classical theory (light as a wave) break down.

The Photoelectric Effect

When shining ultraviolet light on a metal surface, electrons are emitted. This is the photoelectric effect.

Why are we not bombarded by electrons in daily life? For the electron to fly off, we must be in a vacuum. Otherwise, it'll immediately strike an air molecule and be absorbed.

Photoelectric Effect Background

- Discovered by Hertz, 1887
- Thomson (1889) went further, so did Lenard (1902) and others.
- Einstein won his Nobel Prize for explaining this, not from relativity.

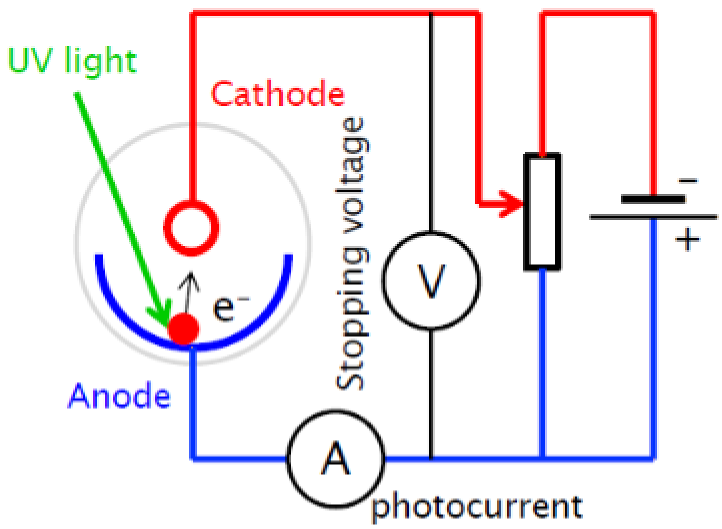

Figure 7: A circuit diagram for measuring the photoelectric effect.

The above setup would be encased in a glass ball (containing a vacuum), with a setup like this:

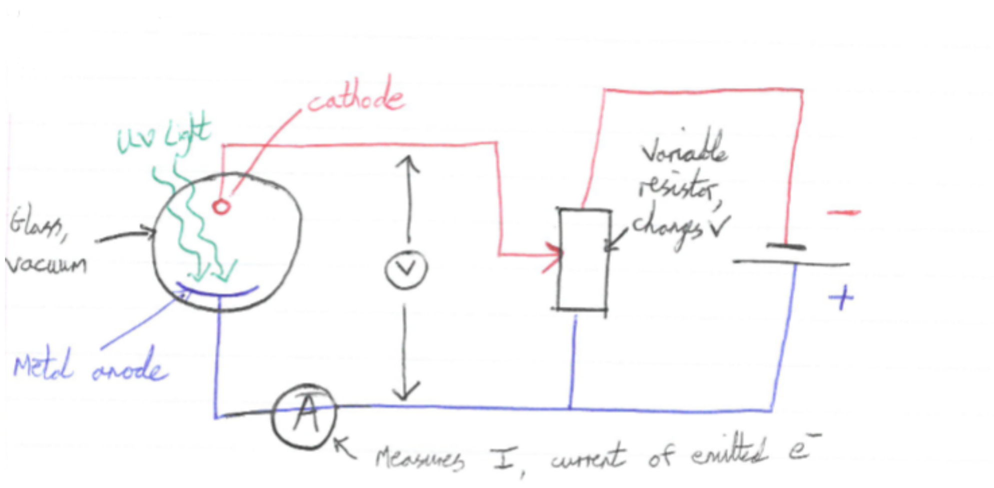

Figure 8: Experimental Setup

Results

Result One - Changing Intensity

For fixed UV wavelength, increasing the intensity of light increases the measured photocurrent:

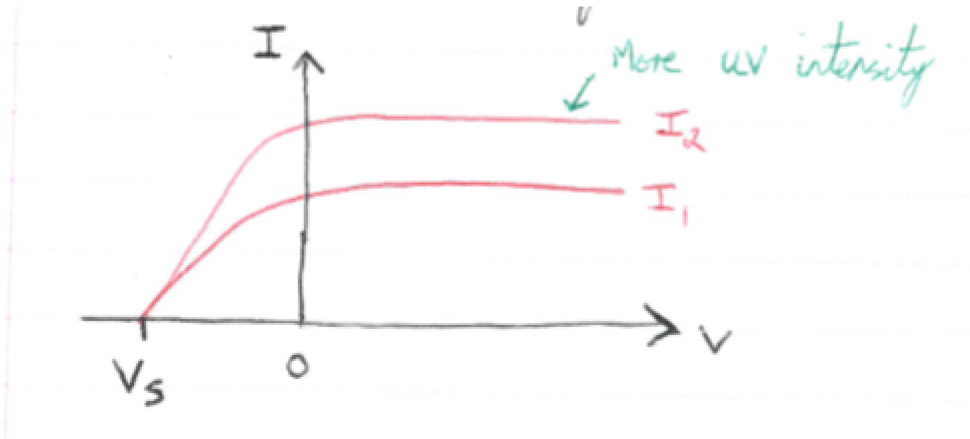

Figure 9

- Increasingly negative potential the cathode decreases photocurrent. At some potential v_s (the "stopping potential") this current drops to zero.
- Potential does not affect electron emission, however adding potential causes an electric field which effectively blows electrons back towards the anode. The stopping potential is when this electric field is perfectly strong to prevent electrons from reaching the cathode and causing a current.
- The fact this can happen consistently (i.e. no current means no electrons made it through) implies that there must be some maximum kinetic energy these electrons can have ($KE_{max} = eV_S$).
- The stopping potential is independant of UV intensity. More UV makes current increase, but does not change stopping potential (i.e. it does not give more energy to each electron, they each have the same energy). This does not make sense classically. Classically we would expect adding more energy to cause emitted electrons to have more energy, therefore changing the stopping potential.

Result Two - Changing Wavelength

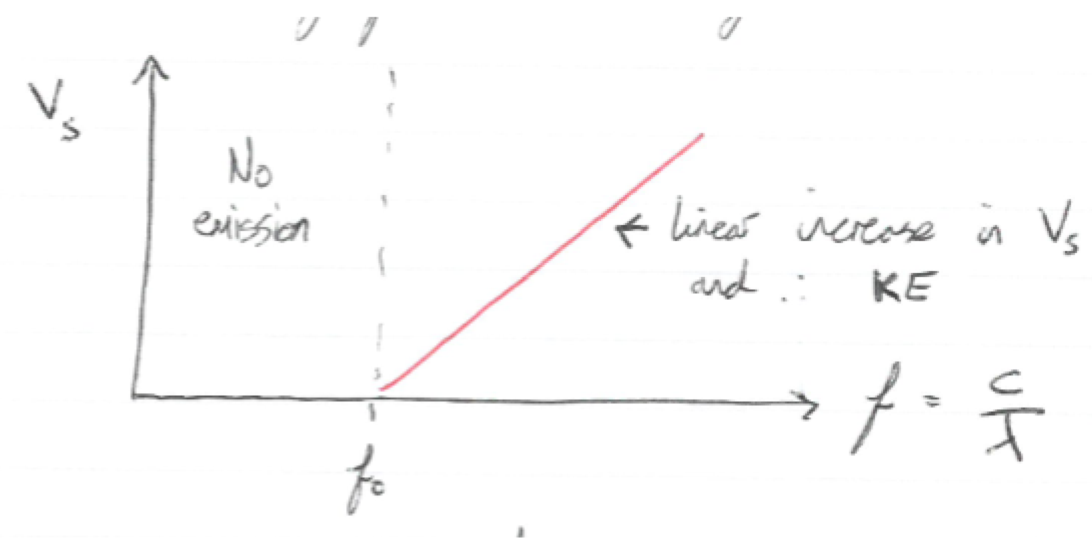

Figure 10

We have to reach some baseline threshold frequency f_0 before we see any photocurrent. After this, increasing wavelength increases photocurrent (and hence KE of emitted electrons) linearly.

- For a given metal, we find the threshold frequency f_0, below which there is no emission of electrons (no current). If below the frequency f_0, intensity is irrelevant. This contradicts classical mechanics which would suggest that turning up the light intensity would supply more (and potentially sufficient) energy.
- Above the threshold, the energy of individual emitted photons depends on UV frequency and not intensity (by result one).

Conclusions

Classically

Classically, we expect energy to be proportional to intensity, $\therefore v_s$ should increase with greater intensity. We also expect there to be no link between between frequency and energy, hence no threshold frequency. We'd expect no threshold frequency, instead being a time delay as electrons "soak up" energy to reach the required threshold.

In theory, great, in practice *this is not observed.*

Einstein's Proposal

Energy in light comes from phots with energy $E = hf$. There is a minimum energy required for an electron to be able to escape from the metal. This minimum energy is called the work function ϕ.

$$KE_{\text{max}} = hf - \phi = eV_s$$

Now:

- Higher intensity means more of the same particles (more photons), but the energy of each is unchanged.
- $E = hf$ so frequency changes energy (as observed).
- The Bohr model says that an electron can only have certain electron energy transitions when the correct energy is supplied (an electron cannot gradually soak up energy). This explains why there is a cutoff below the work function, and no observed time delay (as the "soaking up" that casues the delay does not happen). Either an incoming photon has sufficient energy, or it does not. Having more photons does not help.
- The first incoming photons immediately releases an electron (assuming the incoming light has sufficient energy), therefore there's no time delay.

In Practice

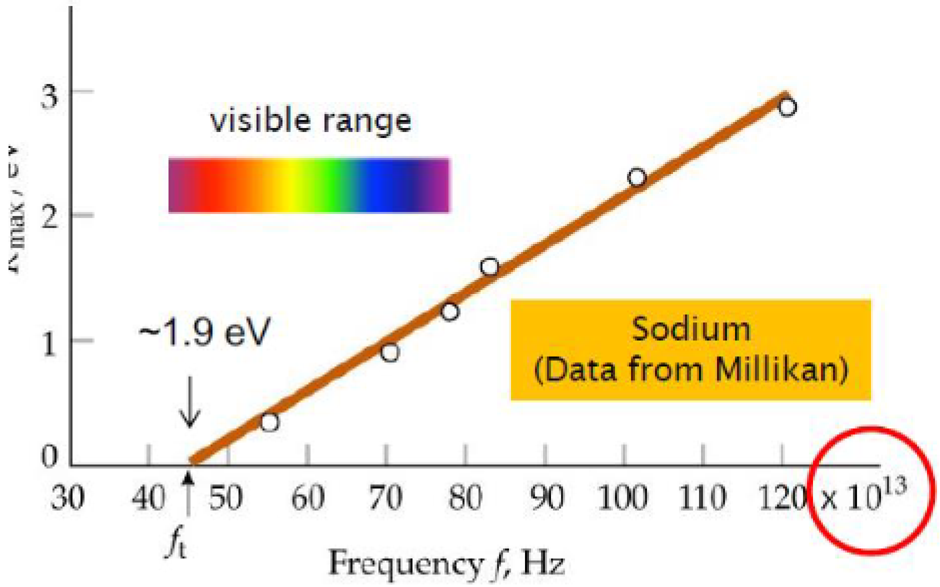

Figure 11: Sodium photocurrent measurements by Robert Millikan

Compton Scattering

Compton Scattering is the scattering of x-rays off carbon atoms.

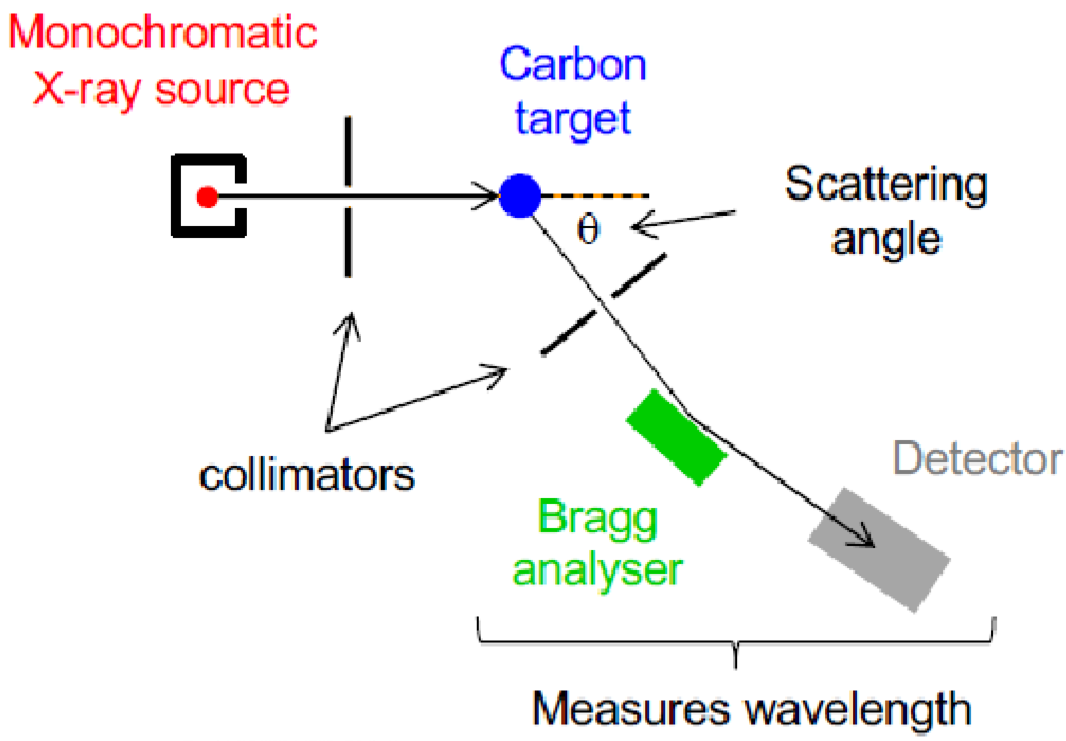

Figure 12: A Compton Scattering experimental setup.

The surprising result is that two wavelengths were observed (not just the original) - λ_1, λ_2, where λ_1 is the original and λ_2 is different. Classically this is hard to exlain and λ should not change.

The difference between these two wavelengths increases with scattering angle θ. This can be explained if the x-ray beam is a stream of photons, but not classically.

Possible options when a photon collides:

- Elastic: No loss of energy, hence no change in wavelength.
- Inelastic: Change of energy, so the wavelength also changes. An electron is fired out of the target, carrying energy and momentum, so the photon loses energy.

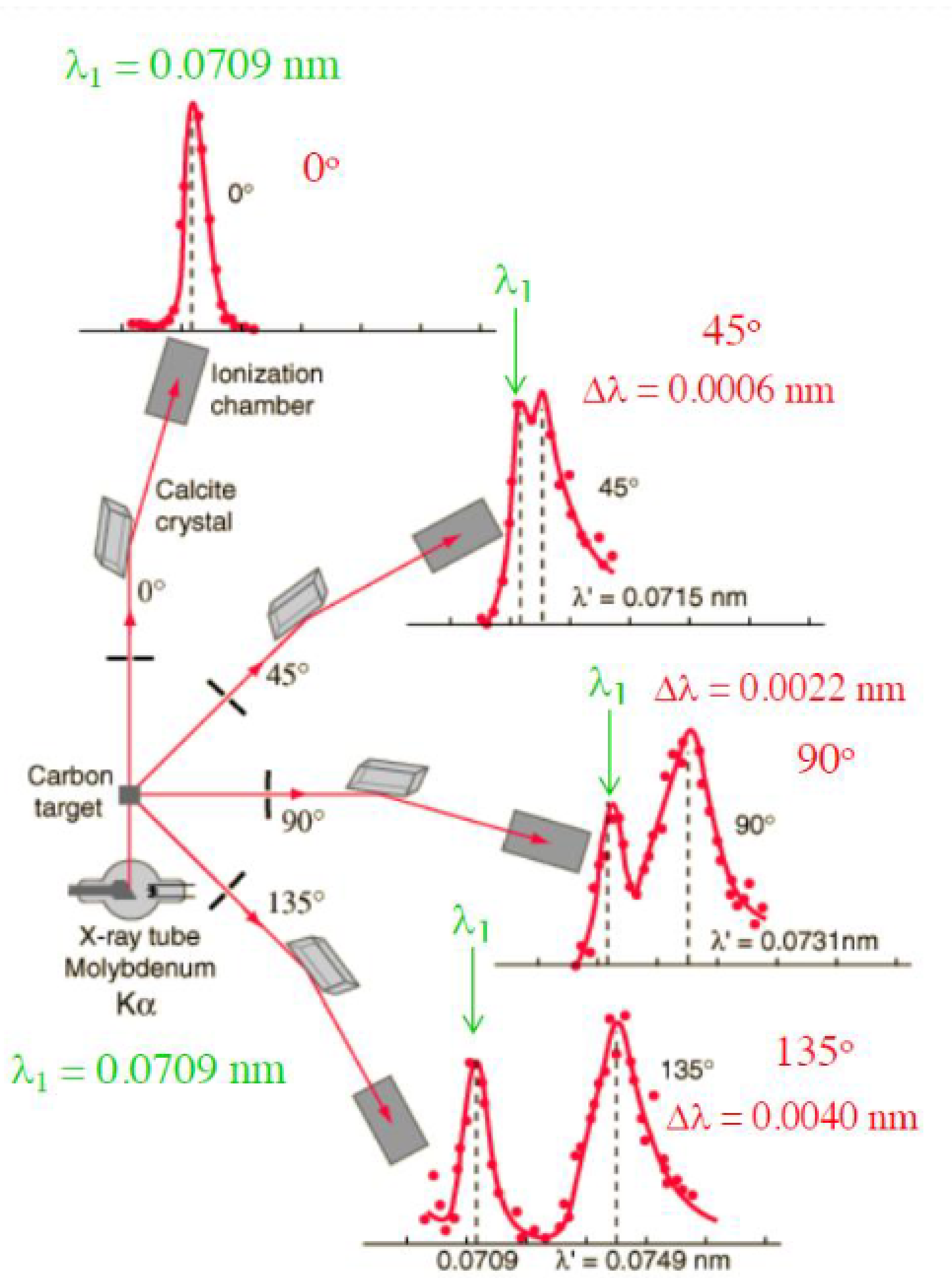

Figure 13: The observed results

Deriving Compton's Equation

Given an incoming photon with with energy E_1, wavelength λ_1 and momentum $\underline{p}_1$. This strikes a carbon atom and is deflected by angle θ. There is also an emitted electron at angle ϕ which must be in the oppsite direction (angled up vs down) to conserve momentum. The new deflected photon has E_2, λ_2, $\underline{p}_2$.

We must consider relativistic effects here given the high speed ($E^2 = p^2c^2 + m^2c^4$)

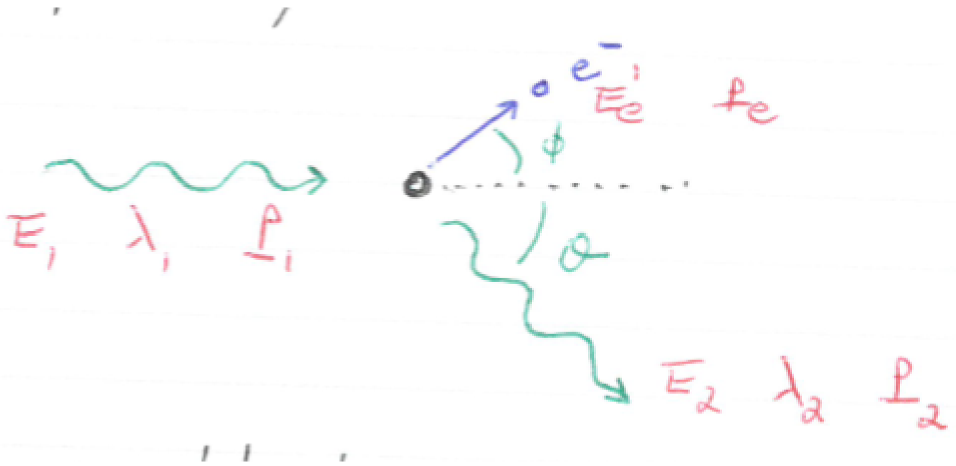

Figure 14

Setup

For a massless photon ($m = 0$):

$$E = pc$$

And:

$$E = \frac{hc}{\lambda}$$

So:

$$p = \frac{h}{\lambda} \tag{1}$$

Conservation and Relativity

Conserving momentum (underlines ommitted for speed):

$$p_1 = p_e + p_2$$

$$p_e = p_1 - p_2$$

Squaring both sides:

$$p_e^2 = p_1^2 + p_2^2 - 2p_1 \cdot p_2$$
$$p_e^2 = p_1^2 + p_2^2 - 2p_1 p_2 \cos\theta \tag{2}$$

And then by conservation of energy:

$$E_1 + E_e = E_2 + E_e'$$

Where E_1 is the incoming photon energy, E_e is the energy of the electron at rest in atom before collision, E_2 is the deflected photon energy and finally E_e' is the deflected electron's energy.

Using this:

$$p_1 c + m_e c^2 = p_2 c + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$
$$\implies p_1 - p_2 + m_e c = \sqrt{p_e^2 + m_e^2 c^2}$$

And squaring both sides:

$$(p_1 - p_2)^2 + \cancel{m_e^2 c^2} + 2m_e c(p_1 - p_2) = p_e^2 + \cancel{m_e^2 c^2}$$

Subsituting in Eqn 2 for p_e^2

$$(p_1 - p_2)^2 + 2m_e c(p_1 - p_2) = p_e^2$$
$$(p_1 - p_2)^2 + 2m_e c(p_1 - p_2) = p_1^2 + p_2^2 - 2p_1 p_2 \cos\theta$$

And rearranging:

$$\begin{aligned}
(p_1 - p_2)^2 + 2m_e c(p_1 - p_2) &= p_1^2 + p_2^2 - 2p_1p_2\cos\theta \\
p_1^2 + p_2^2 - 2p_1p_2 + 2m_e c(p_1 - p_2) &= p_1^2 + p_2^2 - 2p_1p_2\cos\theta \\
-2p_1p_2 + 2m_e c(p_1 - p_2) &= -2p_1p_2\cos\theta \\
-p_1p_2 + m_e c(p_1 - p_2) &= -p_1p_2\cos\theta \\
m_e c(p_1 - p_2) &= -p_1p_2\cos\theta + p_1p_2 \\
m_e c(p_1 - p_2) &= p_1p_2(1 - \cos\theta)
\end{aligned}$$

Subsituting Eqn 1:

$$\begin{aligned}
m_e c(p_1 - p_2) &= p_1p_2(1 - \cos\theta) \\
m_e c\left(\frac{h}{\lambda_1} - \frac{h}{\lambda_2}\right) &= \frac{h}{\lambda_1}\frac{h}{\lambda_2}(1 - \cos\theta) \\
m_e c\left(\frac{h}{\lambda_1} - \frac{h}{\lambda_2}\right) &= \frac{h^2}{\lambda_1\lambda_2}(1 - \cos\theta) \\
m_e c\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) &= \frac{h}{\lambda_1\lambda_2}(1 - \cos\theta) \\
m_e c\left(\frac{\lambda_1\lambda_2}{\lambda_1} - \frac{\lambda_1\lambda_2}{\lambda_2}\right) &= h(1 - \cos\theta) \\
m_e c\,(\lambda_2 - \lambda_1) &= h(1 - \cos\theta) \\
(\lambda_2 - \lambda_1) &= \frac{h}{m_e c}(1 - \cos\theta)
\end{aligned}$$

Which is the Compton Equation. This shows that the change in wavelength is proportional to $1 - \cos\theta$.

Conclusions

The photoelectric effect and Compton scattering are two more physical phenomena that cannot be explained using traditional classical mechancics with EM waves alone. They both require assuming photons of energy $E = hf$ to be adequately explained.