

LC Introduction to Probability and Statistics

MSci Physics w/ Particle Physics and Cosmology
University of Birmingham

Year 1, Semester 1
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Thu 09 Oct 2025 09:00

Lecture 4 - Covariance and Correlation

Office Hours: Thursday 11am - 1pm, Physics West Rm 222 (b.becs@bham.ac.uk)

Previous, when looking at two or more variables for error propagation/combinations etc, we assumed that they were independent of one another. Today we look at how to handle multiple variables which may be correlated.

Covariance

Covariance is a measure that indicates how much two variables fluctuate together:

$$\text{Cov}(x, y) = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})$$

Covariance matrices represent all combinations of covariance (noting $\text{Cov}(x, y) = \text{Cov}(y, x)$ and $\text{Cov}(x, y) = \text{Var}(x)$)

$$\Sigma = \begin{pmatrix} \text{Cov}(x, x) & \text{Cov}(x, y) \\ \text{Cov}(y, x) & \text{Cov}(y, y) \end{pmatrix}$$

We can then define correlation:

$$\text{Corr}(x, y) = \frac{\text{Cov}(x, y)}{\sqrt{\text{Var}(x)\text{Var}(y)}} = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^N (x_i - \bar{x})^2 \sum_{i=1}^N (y_i - \bar{y})^2}}$$

This is bounded between -1 ($x = -y$), 1 ($x = y$) and zero for no correlation. We can again put this in a matrix, noting it is symmetrical:

$$\begin{pmatrix} 1 & \text{Corr}(x, y) \\ \text{Corr}(y, x) & 1 \end{pmatrix}$$

Variable Combinations

Now, with correlated variables, we can say:

$$\langle x + y \rangle = \langle x \rangle + \langle y \rangle$$

$$\text{Var}(x, y) = \text{Var}(x) + \text{Var}(y) + 2\text{Cov}(x, y)$$

And (noting the mean slightly increases with correlated variables):

$$\langle xy \rangle = \langle x \rangle \langle y \rangle + \text{Cov}(x, y)$$

And the one formula to rule them all

$$\text{Var}(f) \approx \frac{\partial f}{\partial A}^2 \text{Var}(A) + \frac{\partial f}{\partial B}^2 \text{Var}(B) + 2 \frac{\partial f}{\partial A} \frac{\partial f}{\partial B} \text{Cov}(A, B)$$

Fri 07 Nov 2025 11:00

Lecture 10 - Introduction to Probability

What is probability? Probability is the pure mathematical description of randomness.

Set Theory

Say we want to group trees into four sets:

- Tall, or not.
- Variegated (has a lighter coloured leaf border) or not.

In a park of 142 trees, we observe