LC Classical Mechanics and Relativity 1

MSci Physics w/ Particle Physics and Cosmology
University of Birmingham

Year 1, Semester 1

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Lectures Index

Thu 02 Oct 2025 14:15

Lecture 1 - Orders of Magnitude and Dimensional Analysis

TODO

Thu 02 Oct 2025 15:00

Lecture 2 - Dimensional Analysis (contd.) and Vectors

Continuation of Dimensional Analysis

What if, in theory, we could build a system of units entirely from c, the speed of light, G, Newton's constant and h, the Plank Constant?

Cont. from Lec01, we can try to use this to work out the earliest possible cosmic time.

$$h = 6.6 \times 10^{-34} Js$$
$$G = 6.67 \times 10^{-11} Nm^2/kg^2$$
$$c = 3 \times 10^8 m/s$$

Dimensionally:

$$[h] = \frac{ML^2}{T}$$
$$[G] = \frac{L^3}{T^2 M}$$
$$[c] = \frac{L}{T}$$

We want to use these to build out a time unit, so:

$$[h^u G^v c^z] = T$$
$$\left(\frac{ML^2}{T}\right)^u \left(\frac{L^3}{T^2 M}\right)^v \left(\frac{L}{T}\right)^z = T$$

$$M^{u-v} L^{2u+3v+z} T^{-u-2v-z} = T$$

Solving for:

$$u - v = 0$$

$$2u + 3v + z = 0$$

$$-u - 2v - z = 1$$

Gives us:

$$u = \frac{1}{2} \tag{1}$$
$$v = \frac{1}{2} \tag{2}$$
$$z = \frac{-5}{2} \tag{3}$$

$t_p = \sqrt{\frac{Gh}{c^5}}$ and plugging in the values for G, h, c gives us a value of time, which the earliest possible cosmic time equal to about $10^{-43}s$

Plank Energy

Doing the same process for energy gives us (this time, the plank energy is the energy at which traditional theories of physics break down):

$$E_p = \frac{hc^5}{G}^{0.5} \approx 10^9 J$$

On the other hand, the LHC manages about 10TeV, which is orders of magnitude smaller than this, so the LHC cannot accurate simulate energies of this magnitude.

More Vectors

Again, vector notation will be $\vec{a}$. We define the x, y, z unit vectors as $\underline{\hat{e_x}}, \underline{\hat{e_y}}, \underline{\hat{e_z}}$.

We can therefore define any vector as:

$$\vec{a} = a_x \underline{\hat{e_x}} + a_y \underline{\hat{e_y}} + a_z \underline{\hat{e_z}}.$$

The length of a vector is again $|\vec{a}|$.

Vector Multiplication

Given $\vec{a}$ and $\vec{b}$ we can define the dot (scalar) product and the cross (vector) product

$$\vec{a} \cdot \vec{b} = |a||b| \cos\theta$$

Say we want to know the component of a vector along an axis, we can do the following (eg for x):

$$\vec{a} \cdot \underline{\hat{e_x}} = a_x$$

For the vector product, we can define:

$$\vec{a} \times \vec{b} = |a||b| \sin(\theta) \hat{\jmath}$$

As the vector perpendicular to the plane containing a and b. It is in the direction such that[1]. Theta is the angle between a and b, while j is the unit vector in the direction the new vector will point.

Solar Energy Example

The world yearly energy usage is about 180,000TWh, which is about $5 \times 10^{20} J$ total. Is it (theoretically) possible to get this all from solar energy? We can check using an approximate order of magnitude calculation.

The Sun's total luminosity is $L_\odot = 3.8 \times 10^{26}$. This energy is radiated in a spherically symmetric way (we assume). Therefore the energy per time, per unit surface is (using 1AU for distance):

$$\frac{L_\odot}{4\pi \times (1.5 \times 10^6)^2}$$

Which is approximately (using order of magnitude):

$$\frac{3.8 \times 10^{26} W}{10 \times 10^{22} m^2} \approx \frac{1kW}{m^2}$$

This is true in ideal conditions, and real energy supply is lower (due to clouds, atmosphere etc).

If we totally covered the earth's surface area ($A_{\text{surface}} \approx \pi R_\odot^2$) which is approximately:

$$A_{\text{surface}} \approx \pi \times (6 \times 10^3 \times 10^3 m)^2 \approx 10^{14} m^2$$

Therefore total energy recieved is approximately:

$$P = \frac{1kW}{m^2} \times 10^{14} m^2 \approx 10^{17} W$$

And to power the world:

$$E = \frac{5 \times 10^{20} J}{3 \times 10^7 s} \approx 10^{13} W$$

So, it's theoretically possible, if we could cover enough of the world in solar panels and if we could perfectly capture the sun's energy without losing some to sources such as clouds, atmosphere, areas of the ocean we cannot cover in solar panels etc.

[1] TODO, fix

Thu 04 Sep 2025 12:00

Lecture 3 - Kinematics Introduction

For kinematics, we'll treat all objects as points and disregard aspects like rotation/the physical sie of the body etc.

Given some point, we can define its position as a fucntion of time $\vec{r}(t)$, and velocity as the derivative wrt time of this:

$$\vec{v}(t) = \frac{d\vec{r}}{dt}$$

And acceleration:

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

Position from Unit Vectors

We can define:

$$\vec{r}(t) = r_x(t)\hat{e_x} + r_y(t)\hat{e_y} + r_z(t)\hat{e_z} = \sum_{j=1}^{3} r_j(t)\hat{e}_j$$

So:

$$\frac{d\vec{r}}{dt} = \frac{d}{dt}\left(\sum_{j=1}^{3} r_j\hat{e}_j\right)$$

$$= \sum_j \frac{d}{dt}(r_j\hat{e}_J)$$

Note: Taking the derivative of a vector wrt time is looking at how the variable changes in some infinitestimal time. This can be a change in direction, and/or a change in magnitude. To differentiate a vector we can differentiate it component-wise.

Cartestian and Polar

Instead of representing a point as x and y components (in 2D), we can instead define it as a distance from the origin r and the angle this distance line forms with the positive x-axis θ.

Therefore (by basic right angle trig) $x = rcos\theta$, $y = rsin\theta$, and hence:

$$\vec{r} = r\cos\theta\hat{e}_x + r\sin\theta\hat{e}_y$$

So:

$$\vec{u(t)} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\cos\theta)\hat{e}_x + \frac{d}{dt}(r\sin\theta)\hat{e}_y$$

$$= \left(\dot{r}\cos\theta + r(-\sin\theta)\dot{\theta}\right)\hat{e}_x + \left(\dot{r}\sin\theta + r(\cos\theta)\dot{\theta}\right)\hat{e}_y$$

$$= r\left(\cos\theta\hat{e}_x + \sin\theta\hat{e}_y\right) + r\dot{\theta}\left(-\sin\theta\hat{e}_x + \cos\theta\hat{e}_y\right)$$

Example

Lets model a particle, in a single dimension moving with constant acceleration (a_0) along a line. What is x(t)?

Going Backwards

Lets say we have some body with $a(t) = kt^3$. What is the position function $x(t)$?

$$a = \frac{dv}{dt} = kt^3$$

$$v = \int_a^b kt^3 \, dx$$

Thu 09 Oct 2025 15:00

Lecture 4 - Projectile Motion

Projectile Motion: The motion of a particle subject to gravitational acceleration, $g \approx 9.81\text{m/s}$

Projectile Motion

For this to hold, the height of the particle above the ground must be $m << R_e \approxeq 6 \times 10^3\text{km}$.

$$x(t) = x_0 + v_0(t - t_0) + \frac{1}{2}a_0(t - t_0)^2$$

Lets begin soley by considering motion in the vertical axis (called z here, for some strange reason). This particle is falling from height hm, to the ground at $h = 0$, with constant acceleration $g\text{m/s}^2$. It has been dropped at time $t = t_0$

At time t_0, $v = 0, z = h$

$$z(t) = h + 0 - \frac{1}{2}gt^2$$
$$\frac{1}{2}gt^2 = h$$
$$\implies t = \sqrt{\frac{2h}{g}}$$

What about 2D?

Now we can expand our example to (rather than drop the particle from rest) give the particle some initial velocity v_0m/s parallel to the ground. We now want two position functions, $x(t)$ and $z(t)$. As previously calculated:

$$z(t) = h + 0 + \frac{1}{2}(-g)t^2$$

And horizontally:

$$x(t) = 0 + v_0(t) + 0$$

So:

$$\begin{cases} z = h - \frac{1}{2}gt^2 \\ x = v_0 t \end{cases}$$

Rearranging:

$$t = \frac{x}{v_0}$$
$$z = h - \frac{1}{2}g\left(\frac{x}{v_0}\right)^2$$

Since h, g, v_0 are all constants, this is an x^2 parabola.

Interplanetary Example

Lets consider some planet, with $g_{\text{planet}} = 5m/s^2$. You (denoted Y) fall into the atmosphere at some distance h from the ground, and some horizontal distance d from $O(x = 0)$. There is an alien who wants to kill you, by shooting you down. This "gun" can throw pebbles at some constant speed v_0. The only degree of freedom the alien has to target you is change the shooting angle wrt the horizontal, θ. From the alien's persepctive, what is the required θ to hit the incoming spacecraft?

To hit you, there is some time t, when the position of the bullet B, with initial velocity v where B is in the same position as Y

Consider B

$$x_B(t) = v_0 \cos(\theta)t$$

$$z_B(t) = v_0 \sin(\theta)t - \frac{1}{2}g_p t^2$$

Consider Y

$$x_Y(t) = d$$

$$z_Y(t) = h - \frac{1}{2}g_p t^2$$

We want to find a θ where $x_B = x_Y$ and $z_B = z_Y$ at the same t:

$$v_0 \cos(\theta)t = d \tag{4}$$

$$v_0 \sin(\theta)t - \frac{1}{2}g_p t^2 = h - \frac{1}{2}g_p t^2 \tag{5}$$

From 2:

$$v_0 \sin(\theta)t = h$$

$$\implies t = \frac{h}{v_0 \sin\theta}$$

And subsituting:

$$v_0 \cos(\theta)\left(\frac{h}{v_0 \sin\theta}\right) = d$$

$$\frac{\cos(\theta)h}{\sin(\theta)} = d$$

$$\frac{\cos\theta}{\sin\theta} = \frac{d}{h}$$

$$\tan\theta = \frac{h}{d}$$

Since we have the value of θ in terms of two constants, yes, the alien can always hit the spaceship provided it correctly selects the angle corresponding to the value of these two constants. This means that the required angle does not depend on velocity, in this example.

Frames of Reference

"Observer" represents a frame of reference.