LC Mathematics for Physicists 1A Lecture Notes

Year 1 Semester 1
MSci Physics with Particle Physics and Cosmology

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Lecture Index

Mon 29 Sep 2025 12:00

Lecture 1 - Course Welcome and Introduction to Vectors

Recommended Course Books

- *Mathematical Techniques...* by DW Jordan and Smith, 3rd Edition
  - Closest to the course.
- *Engineering Mathematics* by Stroud.
  - Lots of examples and extremely clear.
- *Mathematical Methods for the Physical Sciences* by Mary Boas (Wiley).
  - Succinct and faster.
- *Elementary Vector Analysis* by Weatherburn.
  - Vector specific book.

1 Intro To Vectors

Some physical quantities can be represented by numbers, i.e. charge, speed, time, etc. Some other quantities have an associated direction as well as a magnitude, for example velocity, acceleration, position, electric fields, etc.

We call the single-number quantities scalars, and the directional quantities vectors. Some quantities (such as moment of inertia, covered in CMR2) depend on more than one direction, we call these *tensors*.

We will initially deal with vectors geometrically, in terms of points and the vectors which arise from them. Consider two points (in 2D for now). We label one as the origin O and one as point A, we define the vector $\overrightarrow{OA}$ as being the distance and direction *from O to A*.

The vector has a "sense" from O to A, we could also have the vector $\overrightarrow{AO}$ which would point in the opposite direction:

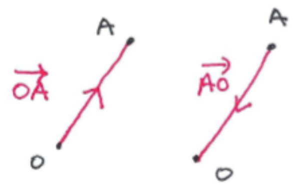

Figure 1.1

We denote the magnitude (length) of a vector as $|\overrightarrow{OA}|$ which is the distance from O to A.

Once we define a vector, we can move it anywhere we want, and it is not constrained as having to actually start at O and end at A. We can 'liberate' the vector from its initial points and translate it anywhere we want in the plane, and it is still the same vector provided the magnitude and direction do not change. Once we've used O and A to define $\overrightarrow{OA}$, we can copy the vector anywhere.

2 Vector Operations

We now want to define standard mathematical operations for vectors, starting with addition.

2.1 Vector Addition

Consider three points, O, A, B. We can naturally define $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{AB}$

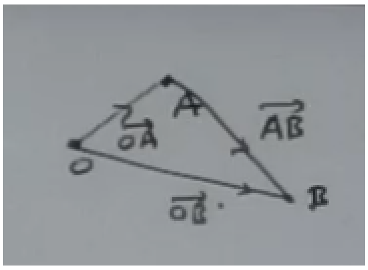

Figure 1.2

We define the following:

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

2.2 Vector Subtraction

We note that $\overrightarrow{OA}$ and $\overrightarrow{AO}$ are the same vector with opposite directions, therefore:

$$\overrightarrow{OA} + \overrightarrow{AO} = \overrightarrow{O}$$

$$\overrightarrow{OA} = -\overrightarrow{AO}$$

Hence $\overrightarrow{OA}$ and $\overrightarrow{AO}$ are inverses of each other, just like 2 and -2 are for numbers. We consider our equation from addition and add $\overrightarrow{BO}$ to both sides:

$$\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BO} = \overrightarrow{OB} + \overrightarrow{BO}$$

$$\overrightarrow{OA} + \overrightarrow{AB} - \overrightarrow{OB} = 0$$

Which gives us an example of subtraction.

3 Better Notation

We use the following improved and more compact notation, which is especially useful if we have some number of several points relative to an origin.

Given the points A, B, C we have so far written $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$. Instead, we now write:

$$\overrightarrow{OA} = \boldsymbol{a}$$

$$\overrightarrow{OB} = \boldsymbol{b}$$

$$\overrightarrow{OC} = \boldsymbol{c}$$

Note that vectors are handwritten with an underline, so $\underline{a}$, but may instead when typed be in bold font, so $\boldsymbol{a}$.

Say O is the centre of mass of n particles, we can also use the notation $r_1, r_2, r_2, \ldots, r_n$ to denote the position vectors of these particles wrt the origin O.

3.1 Physical Example

Recall the definition of equilibrium:

- $\boldsymbol{F} = m\boldsymbol{a}$, for equilibrium $\boldsymbol{F} = 0$.
- Zero net moment.

Say we have a particle of mass m hanging by a single spring. The spring exerts some tension force $\boldsymbol{T}$, while the mass has a weight force $\boldsymbol{F}_g = m\boldsymbol{g}$.

For this particle to be in equilibrium, we must have:

$$\boldsymbol{T} + m\boldsymbol{g} = 0$$

$$\boldsymbol{T} = -m\boldsymbol{g}$$

Simple enough! Consider an example where two springs connect to the particle and connect to some ceiling. We now have:

$$\boldsymbol{T}_1 + \boldsymbol{T}_2 + m\boldsymbol{g} = 0$$

Again, simple.

4 Midpoints

Consider a triangle with points (clockwise), O, A, and B. Say we want the position vector of the midpoint of AB, denoted M.

Wed 01 Oct 2025 09:00

Lecture 2

Thu 02 Oct 2025 16:00

Lecture 3

Mon 06 Oct 2025 12:00

Lecture 4

Wed 08 Oct 2025 09:00

Lecture 5

Thu 09 Oct 2025 16:00

Lecture 6

Mon 13 Oct 2025 12:00

Lecture 7

Wed 15 Oct 2025 09:00

Lecture 8

Thu 16 Oct 2025 16:00

Lecture 9

Mon 20 Oct 2025 12:00

Lecture 10

Wed 22 Oct 2025 09:00

Lecture 11

Thu 23 Oct 2025 16:00

Lecture 12

Mon 27 Oct 2025 12:00

Lecture 13

Wed 29 Oct 2025 09:00

Lecture 14

Thu 30 Oct 2025 16:00

Lecture 15

Mon 03 Nov 2025 12:00

Lecture 16

Wed 05 Nov 2025 09:00

Lecture 17

Thu 06 Nov 2025 16:00

Lecture 18

Mon 10 Nov 2025 12:00

Lecture 19

Wed 12 Nov 2025 09:00

Lecture 20

Thu 13 Nov 2025 16:00

Lecture 21

Mon 17 Nov 2025 12:00

Lecture 22

Wed 19 Nov 2025 09:00

Lecture 23

Thu 20 Nov 2025 16:00

Lecture 24

Mon 24 Nov 2025 12:00

Lecture 25

Wed 26 Nov 2025 09:00

Lecture 26

Thu 27 Nov 2025 16:00

Lecture 27

Mon 01 Dec 2025 12:00

Lecture 28

Wed 03 Dec 2025 09:00

Lecture 29

Thu 04 Dec 2025 16:00

Lecture 30

Mon 08 Dec 2025 12:00

Lecture 31

Wed 10 Dec 2025 09:00

Lecture 32

Thu 11 Dec 2025 16:00

Lecture 33