

LC Optics and Waves

MSci Physics w/ Particle Physics and Cosmology
University of Birmingham

Year 1, Semester 1
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Wed 01 Oct 2025 11:00

Lecture 1 - Intro to Waves and SHM Recap

Course Objectives

- Have a sound understanding of basic wave properties
- Have a basic understanding of interference effects, inc diffraction
- Be able to use simple geometric optics and understand the fundamentals of optical instruments.

Recommended Textbooks

1. University Physics, Young and Freedman (Ch 15, 16 for Waves, Ch 33-36 for Optics)
2. Physics for Scientists and Engineers (Ch 20, 21 for Waves, Ch 22-24 for Optics)
3. 5e, Tipler and Mosca, (Ch 15, 16 for Waves, 31-33 for Optics)
4. Fundamentals of Optics, Jenkins and White
5. Optics, Hecht and Zajac

What is a wave? Waves occur when a system is disturbed from equilibrium and the disturbance can travel from one region to another region. Waves carry energy, but do not move mass. The course aim is to derive basic equations for describing waves, and learn their physical properties.

Periodic Motion

Waves are very linked to periodic motion. Therefore we recap periodic motion first.

It has these characteristics:

- A period, T (the time for one cycle)
- A frequency, f , the number of cycles per unit time ($f = \frac{1}{T}$)
- An amplitude, A , the maximum displacement from equilibrium.

Periodic motion continues due to the restoring force. When an object is displaced from equilibrium, the restoring force acts back towards the equi point. The object reaches equi with a non-zero speed, so the motion continues past the equi point and continues forever.

Energy

Periodic motion is an exchange between potential and kinetic energy, with no energy loss. Energy is conserved.

Simple Harmonic Motion

If the restoring force is directly proportional to the displacement $F = -kx$, then the periodic motion becomes Simple Harmonic Motion and the object is called a harmonic oscillator.

In a single dimension, displacement is given by:

$$x = A \cos(\omega t + \phi)$$

Where $\omega = 2\pi f$ is the angular velocity, and ϕ is the phase angle. In cases like this, where the phase angle is 90 deg we can simplify to $x = -A \sin(\omega t)$

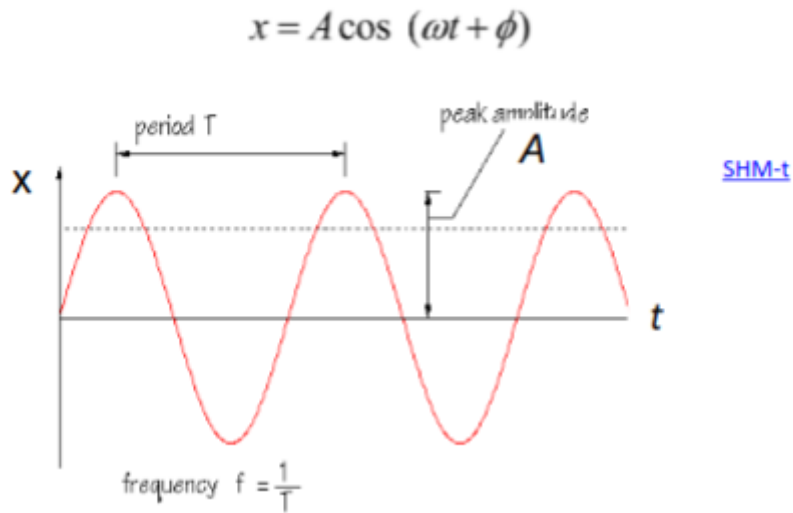


Figure 1: A Phase Angle of 90

More SHM Equations

Velocity

$$v_x = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

Acceleration

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

Both properties are signed to indicate direction, as they are both vectors.

Thu 02 Oct 2025 13:00

Lecture 2 - Wave Functions

Sine Waves

Mechanical Waves A mechanical wave is a disturbance through a medium. It's formed of a single wave pulse or a periodic wave.

Mechanical Waves have the following properties:

1. Transverse: Where displacement of the medium is perpendicular to the direction of propagation.
2. Longitudinal, displacement of the medium is in the same direction as propagation.
3. Propagation depends on the medium the wave moves through (i.e. density, rigidity)
4. The medium does not travel with the wave.
5. Waves have a magnitude and a direction.
6. The disturbance travels with a known exact speed.
7. Waves transport energy but not matter throughout the medium.

Wave Functions

We want to define a wave function in terms of two variables, x and t . In any given moment, if we consider a single point on the wave (i.e. $t = 0$), and wait a short while, the wave will have travelled to some $t = t_1 > 0$.

In order to quantify displacement, we therefore want to specify both the time, and the displacement. This will let us find the wave speed, acceleration and the (new) wave number.

We are also able to talk about the velocity and acceleration of individual particles on the wave.

Wave Function for a Sine Wave

Consider a sine wave. We want to find a wave function in the form $y(x, t)$. Consider the particle at $x = 0$.

We can express the wave function at this point as $y(x = 0, t) = A \cos \omega t$. However we want to expand this to any general point. Now consider a point (2) which is one wavelength away. We know the behaviour of particle 1 is mirrored by particle 2 (with a time lag).

Since the string is initially at rest, it takes on period (T) for the propagation of the wave to reach point 2, therefore point 2 is lagging behind the motion of point 1. The wave equation is therefore (if particle two has $x = \lambda$) $y(x = \lambda, t) = A \cos(\omega t - 2\pi)$.

For arbitrary x , $y(x, t) = A \cos(\omega t - \frac{x}{\lambda} \cdot 2\pi)$ to account for this delay. This quantity is called the wave number:

$$\text{Wave Number: } k = \frac{2\pi}{\lambda}$$

So:

$$\begin{aligned} y(x, t) &= A \cos(\omega t - kx) \\ &= A \cos(kx - \omega t) \end{aligned}$$

Note the second step is possible as \cos is an even function. k can also be signed to indicate direction: if $k > 0$, the wave travels in the positive x . If $k < 0$, the wave travels in the negative x direction. Again, $\omega = 2\pi f$

Displacement Stuff

Considering a point (starting at equi), the time taken for the particle on the sin wave to reach maximum displacement, minimum displacement and back takes the time period T . The speed of the wave is distance travelled over the time taken. We take the distance to be the wavelength λ , as we know the time by definition this takes is one time period T . Therefore wave speed v is:

$$v = \frac{\lambda}{T} = \lambda f$$

Since $\lambda = \frac{2\pi}{k}$ and $f = \frac{\omega}{2\pi}$ (as ω is defined as $\frac{2\pi}{T}$), we can also write:

$$v = \frac{2\pi}{k} \cdot \frac{\omega}{2\pi} = \frac{\omega}{k}$$

Particle Velocity

We can also determine the velocity of individual particles in the medium. We can use this to determine the acceleration.

We know that

$$y(x, t) = A \cos(kx - \omega t)$$

The vertical velocity v_y is therefore given by:

$$v_y = \frac{dy(x, t)}{dt}$$

Which is unhelpful (as we can't differentiate two variables at once), we can slightly cheat this by looking at purely a certain value of x , and therefore treating x as constant (to get a single variable derivative).

$$v_y = \left. \frac{dy(x, t)}{dt} \right|_{x=\text{const.}}$$

However this is notationally yucky, so we therefore use the notation:

$$\frac{\partial y(x, t)}{\partial t}$$

To represent the same thing. Finally (carrying out the partial derivative):

$$v_y(x, t) = \frac{\partial y(x, t)}{\partial t} = \omega A \sin(kx - \omega t)$$

Particle Acceleration

We can work out particle acceleration (transverse acceleration) by differentiating in the same manner again:

$$a_y(x, t) = \frac{\partial^2 y(x, t)}{\partial t^2} = \frac{\partial}{\partial t} \left(\frac{\partial y(x, t)}{\partial t} \right) = -\omega^2 A \cos(kx - \omega t) = -\omega^2 y(x, t).$$