Lecture Notes

Year 1 Semester 2
MSci Physics with Particle Physics and Cosmology

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Lecture Index

LC Classical Mechanics and Relativity 2

LC Electric Circuits

Fri 13 Feb 2026 11:00

Lecture 3

LC Electromagnetism I

Mon 19 Jan 2026 11:00

Lecture 1 - EM1 Intro and Electric Fields

1 Course Intro

Course Materials:

- Background material and derivations etc on PowerPoint.
- Worked examples etc are handwritten on visualiser, these are the bits we really need to know.

Why is EM important?

- Foundations of modern technology and the modern world.
- What gives elements their properties.
- Responsible for life itself.
- Everyday materials are held together by EM forces.
- Optics can only be understood through EM theory.

The course aim is to lay down the foundations, eventually leading us to Maxell's Laws.

1.1 Maxwell's Laws

Maxwell's four equations are:

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$
$$\nabla \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \wedge \boldsymbol{B} = \mu_0 \left(\boldsymbol{J} + \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \right)$$

Where:

$$\nabla = \boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z}$$

Together, these show that the electric and magnetic fields are related and are two aspects of a single force, the electromagnetic force. We don't have to properly understand them yet, but cannot learn them in EMII unless we fundamentally understand E and B fields from this module.

1.2 Course Structure

Part I: Electric Fields

- Charge and Coulomb's Law.
- The electric field.
- Gauss' Law.
- Capacitors.

Part II: Magnetic Fields

- Magnetic Fields
- Charged Particles in B-Fields
- Electromagnetic Induction.
- Magnetic Dipoles

In this lecture:

- Introduction to EM.
- Electric charge.
- Force between charges.
- The concept of the Electric Field (E-Field).

2 Electric Charge

First attributed to Thales circa. 624 - 546 BC. Experiments by Franklin and Coulomb expanded and showed that there was two types of charge, which they called positive and negative.

The "positive electricity" came from rubbing a glass rod with silk, and the negative from rubbing an ebonite (early plastic) rod with fur. They found that like charges repel and opposite charges attract.

We know that the elementary unit charge is the magnitude of charge of an electron/proton and everything else is a multiple of this [1]:

$$e = 1.6 \times 10^{-19} C$$

This has units of the Coulomb.

2.1 Charge Conservation

Electrons and protons are both stable (protons decay with a life greater than 10^{31} years). This means that the total charge of an isolated system is constant and can be conserved.

They have the same magnitude of charge, exactly:

$$|q_p| = |q_e| = e$$

2.2 Electrostatic Force

Like charges repeal and opposite charges attract, along the line of action given by a line drawn between the two charges. The force is proportional to the product of charges so:

$$F \propto q_1 q_2$$

Here, a negative force means attraction and a positive force means repulsion. Newton called this "force at a distance". Like gravity, two charges will exert a force on each other at a distance without any contact.

There must, therefore, be something between them that mediates this force. Later physics gives this as "virtual particles" which isn't a Y1 topic, so classically we say that this medium is the Electric Field.

2.3 Electric Field

A charge produces a field around it. Another charge also interacts with this field, and this interaction is what causes a force:

$$\underline{F} = \underline{E} q$$

Where F is the force exerted on a test charge of charge q by a charge Q producing a field E. The magnitude of the electric field has units NC^{-1} (force per units charge).

$$|\underline{E}| \propto Q \qquad |\underline{F}| \propto Qq$$

[1] While quarks have fractional charge, we don't get free quarks

Consider a point charge with a spherical electric field spreading out around it. As the distance from the charge increases, the surface area increases as $4\pi r^2$. Therefore the magnitude of the electric field must decrease with $4\pi r^2$

Therefore:

$$|\underline{E}| \propto \frac{Q}{4\pi r^2}$$

We need a (inverse) constant of proportionality. This depends on the medium, but for a vacuum we call it the permittivity of free space ϵ_0:

$$\epsilon_0 = 8.854 \times 10^{-12} C^2 m^{-2} N^{-1}$$

Hence:

$$\boxed{E = |\underline{E}| = \frac{Q}{4\pi\epsilon_0 r^2}}$$

2.4 Direction of the E-Field

Force is a vector, so the E-field must be too. Consider an E-field from a charge Q at distance r. We give E components E_r, E_θ, E_ϕ, where, since it's a sphere ϕ and θ represent the unit vectors in the two possible tangential directions.

If there was a component in E_θ this would be clockwise from one perspective, but anticlockwise from another (walking behind it). This is not possible, as the field must behave in the same manner from all viewpoints. Therefore:

$$E_\theta = E_\phi = 0$$

$$\underline{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{t}$$

2.5 Force between two charges

Consider two charges q_1 and q_2. The force on q_2 due to the e-field from q_1 is:

$$\underline{F_1} = \underline{E_1} q_2$$

This is equal to the force on q_1 due to the e-field from q_2, given by:

$$\underline{F_2} = \underline{E_2} q_1$$

So the force between two charges is:

$$\underline{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}_{12}$$

2.6 Force between many charges

If we have more than two positive charges, we use the "principle of superposition". Effectively, you consider each pair of charges at the same time and vector sum of the forces together. I.e. if we have three points and we care about the net force on one, we take the vector sum of the two vectors from that point to the others.

In general:

$$\begin{aligned}\underline{F} &= \underline{F_1} + \underline{F_2} + \underline{F_3} + \cdots \\ &= q\sum_i \frac{q_i}{4\pi\epsilon_0 r^2}\hat{r}_j\end{aligned}$$

Where r_j is the distance between q_i and q, with unit vector $\hat{r}_j$ between them.

Since $\underline{F} = q\underline{E}$, the electric field at a test charge q must be:

$$\underline{E} = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i^2}\hat{r}_j$$

2.7 Example

Say we have a square of side length a. Clockwise, these corners have charge Q, q, $-2Q$, $3Q$.

What is the net force exerted on the q charge?

LC Introduction to Particle Physics and Cosmology

LC Mathematics for Physicists 1B

Fri 13 Feb 2026 12:00

Lecture 12 - End of Partial Differentiation & Start of ODEs

Recap of lecture 11:

- The tangent plane to a surface $f(x,y,z)=0$ at (x_0,y_0,z_0) is given by:

$$\left(\frac{\partial f}{\partial x}\right)_0(x-x_0)+\left(\frac{\partial f}{\partial y}\right)_0(y-y_0)+\left(\frac{\partial f}{\partial z}\right)_0(z-z_0)=0$$

  Such that $\underline{\nabla} f(x_0,y_0,z_0)$ is the normal vector to the plane
- The parametric representation of a curve $\underline{r}(t)$ has:
  - Unit tangent: $\underline{\hat{T}}=\frac{dr}{dt}/\left|\frac{dr}{dt}\right|$
  - Arc length $s(t)$: $\frac{ds}{dt}=\left|\frac{dr}{dt}\right|\rightarrow\underline{\hat{T}}=\frac{dr}{ds}$.
  - Unit normal and curvature:

1 Orthonormal Triads

We can create an *orthonormal triad* by introducing a new normal vector called the unit binormal, $\underline{\hat{B}}=\underline{\hat{T}}\times\underline{\hat{N}}$. Since $\underline{\hat{N}}\times\underline{\hat{N}}$, differentiating wrt s gives:

$$\underline{\hat{N}}\cdot\frac{d\underline{\hat{N}}}{ds}=0$$

TODO

We have:

$$\frac{d\underline{\hat{T}}}{ds}=\kappa\underline{\hat{N}}$$

$$\frac{d\underline{\hat{N}}}{ds}=-\kappa\underline{\hat{T}}+\tau\underline{\hat{B}}$$

Hence:

$$\begin{aligned}\frac{d\underline{\hat{B}}}{ds}&=\frac{d}{ds}\left(\underline{T}\hat{\times}\underline{\hat{N}}\right)\\&=\frac{d\underline{\hat{T}}}{ds}\times\underline{\hat{N}}+\underline{\hat{T}}\times\frac{d\underline{\hat{N}}}{ds}\\&=\kappa\underline{\hat{N}}\times\underline{\hat{N}}+\underline{\hat{T}}\times\left(-\kappa\underline{\hat{T}}+\tau\underline{\hat{B}}\right)\\&=\tau\underline{\hat{T}}\times\underline{\hat{B}}=\tau\underline{\hat{T}}\times\left(\underline{\hat{T}}\times\underline{\hat{N}}\right)\\&=\tau\left[\left(\underline{\hat{T}}\cdot\underline{\hat{N}}\right)\underline{\hat{T}}-\left(\underline{\hat{T}}\cdot\underline{\hat{T}}\right)\underline{\hat{N}}\right]\end{aligned}$$

$$=\tau\underline{\hat{N}}$$

This gives the Frenet-Serret Formulae:

This concludes partial differentiation! :D

2 Ordinary Differential Equations

A differential equation is any equation that involves derivatives. We care, because most laws of physics manifest themselves in the form of differential equations. For example:

$$\text{Newton's Second Law:} \qquad \underline{F} = m\frac{d^2\underline{r}}{dt^2}$$

$$\text{3D Time-Independent Schrödinger Eqn:} \qquad -\frac{\hbar}{2m}\left(\frac{\partial^2\psi}{dx^2}+\frac{\partial^2\psi}{dy^2}+\frac{\partial^2\psi}{dz^2}\right)+V(x,y,z)\psi = E\psi$$

$$\text{3D Wave Eqn:} \qquad \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$
$$= \nabla^2\phi = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

$$\text{Gauss' Law:} \qquad \frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z} = \frac{\rho}{\epsilon_0}$$

$$\text{Navier-Stokes Eqn:} \qquad \rho\left(\frac{\partial \underline{v}}{\partial t}+\left(\underline{v}\cdot\underline{\nabla}\right)\underline{v}\right) = -\underline{\nabla}p+\rho\underline{g}+\mu\nabla^2\underline{v}$$

In this course, we will only solve DEs of a single variable, i.e. Ordinary Differential Equations (ODEs). We don't look at Partial DEs of multiple variables yet.

In order to think about solving these, we need to classify them. Most DEs aren't soluble in closed form with elementary functions and need to be solved numerically. Here, we only consider nice soluble functions, but this is a vast minority in reality. We want to identify classes of DEs we can reasonable solve with a method for each.

We can generally solve linear equations by breaking them into small chunks and solving them individually, for example.

3 Types of DEs

3.1 Partial vs. Ordinary

In the examples above, only the first was an ODE, and the rest PDEs. Ordinary Differential Equations (ODEs) involve only a single variable.

Consider a vector $\underline{r}(t) = (x(t), y(t), z(t))$. t is called the independent variable, with x, y, z being dependant variables. While we have 3 dependant variables, we only have one independent variable (so only one thing to differentiate wrt), so this would end up being ordinary.

PDEs involve equations of two or more variables and hence involve partial derivatives.

3.2 Order

The order of a DE is given by the order of the highest derivative involved, so Newton's 2nd Law is a second order DE, as the highest order derivative is a second derivative.

3.3 Degree

The degree of a DE is a less important measure than the others. It is given by the highest power of the highest order derivative. For example, Newton's 2nd is a first order, while an equation containing a^3 would be third degree (and second order, as a is a second derivative).

Ideally, we want this to be 1 for ease of solving, and higher degrees are rare but they do exist. For example, from Lagrangian Mechanics we have:

$$\frac{1}{2m}\left[\left(\frac{\partial s}{\partial x}\right)^2+\left(\frac{\partial s}{\partial y}\right)^2+\left(\frac{\partial s}{\partial z}\right)^2\right]+V(x,y,z)=\frac{ds}{dt}$$

3.4 Homogenous and Inhomogeneous

A homogenous DE is a DE that does not have any terms of only the independent variable(s), while an inhomogeneous DE does.

For example, Newton's 2nd is homogenous as there is no term that involves t alone. This would be inhomogeneous:

$$\frac{\partial^2 x}{\partial t^2}=t+x$$

While this would be homogenous:

$$\frac{\partial^2 x}{\partial t^2}=tx$$

As t is a coefficient and not a pure term in its own right.

3.5 Linear and Non-Linear

A DE is linear if the dependant variable(s) and all of its/their derivatives occur purely as linear functions. For example:

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$

This is linear, as the dependant variable y never has a coefficient greater than 1.

$$\frac{dy}{dx}+xy=0$$

Is also linear, while this is not:

$$\frac{dy}{dx}+xy^2=0$$

This is also non-linear (as shown by the Taylor Expansion of sine):

$$\frac{d^2\theta}{dt^2}=-\frac{g}{l}\sin\theta$$

3.6 Examples

$$(1)\quad \left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=u^2$$

Homogenous first-order second-degree non-linear PDE.

$$(2)\quad \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=x^2+y^2+z^2$$

Inhomogeneous second-order first-degree linear PDE.

$$(3) \quad \frac{\partial y}{\partial x} + y^2 = x$$

Inhomogeneous first-order second-degree non-linear ODE.

LC Temperature and Matter

Tue 20 Jan 2026 12:00

Lecture 1