LC Introduction to Particle Physics and Cosmology Lecture Notes

Year 1 Semester 2

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Table of Lectures

Thu 22 Jan 2026 16:00

Lecture 1 - Start of Particle Physics: The Standard Model of Particle Physics

1 Course Introduction

Course Structure

- Particle Physics: 6 lectures in weeks 1 to 6.
    1. Introduction and the standard model.
    2. Experimental measurements.
    3. Interactions with matter
    4. Tracking detectors I
    5. Tracking detectors II
    6. Calorimeters and Particle Identification.
- Cosmology: 4 lectures in weeks 7 to 11.

Course Aims

- Overview of current methods in Particle Physics experiments.
- An emphasis placed on the questions and challenges.

For example, the LHC has already been programmed with experiments all the way up to 2041. Therefore, any detectors we design today only become relevant in over a decade, which makes good detector design decisions incredibly important. This course will equip us to understand what drives those design choices.

The course is assessed by a single one hour long exam, weighted half particle physics and half cosmology.

Recommended Texts

- Detectors for particle radiation (2nd edition), K. Kleinknecht (1998)
- Particle Physics, Martin and Shaw.
- High Energy Physics, D. H. Perkins (2nd through 4th editions)
- Feynman Lectures.
- Modern Particle Physics, M. Thomson.

2 Matter Particles

Fermions all have quantum spin 1/2. Spin is a purely inherent quantum property (like mass or charge) and has no classical representation, but is analogous to angular momentum. They are subject to Fermi-Dirac statistics, which means that no identical fermion in a system of fermions can have the same quantum number as any other. Fermions are divided into two types, quarks and leptons.

2.1 Quarks

There are three generations of quarks:

First Generation

- Up Quark (u), mass of $\approx$ 0.001GeV
- Down Quark (d), mass of $\approx$ 0.001GeV

Second Generation

- Charm Quark (c), mass of $\approx$ 1.3GeV
- Strange Quark (s) mass of $\approx$ 4.3GeV

Third Generation

- Top Quark (t), mass of $\approx$ 175GeV
- Bottom Quark (b), mass of $\approx$ 4.3GeV

"up-type quarks", u, c, t have electromagnetic charge $+2/3$ and "down-type quarks", d, s, b, have electromagnetic charge $-1/3$ (charges in units of e). Quarks do not ever exist alone in isolation.

2.2 Leptons

There are three generations of leptons, given by the electron e^-, muon, μ^- and tau τ^-. These all have charge of -1 (in units of e). These have masses (in MeV) of approximately $0.5, 105, 1800$.

These also have associated neutrinos, the electron neutrino, the mu neutrino and the tau neutrino, ν_e, ν_μ, ν_τ. These are not massless, but have a tiny mass many orders of magnitude smaller than their corresponding non-neutrino counterparts. These are neutrally charged.

Leptons are not subject to the strong interaction.

2.3 Hadrons

Quarks do not exist in isolation, but form bound states subject to the strong force called hadrons. There are two types of hadrons - baryons and mesons.

Baryons are formed from three quarks, which may be the same or different $q_1 q_2 q_2$.

Mesons are formed from a quark-antiquark pair, $q_1 \bar{q}_2$. Examples of baryons include the proton and the neutrino, given by uud and udd.

3 Forces

As far as we know, there are four fundamental forces:

- Gravity
- Electromagnetism
- Strong
- Weak

For considering particle interactions, we disregard gravity as it becomes incredibly weak for small masses. Creating a complete theory that incorporates all four is an open question in physics. It's okay to neglect it, but it is unsatisfying.

We consider these forces as arising by the exchange (between two particles subject to a force between them) of particles called bosons. These have spin-1, so are called "gauge bosons". These are subject to Bose-Einstein statistics, which does not impose the same restriction as Fermi-Dirac for quantum numbers in a system.

For EM: The exchange particle is a photon, γ. This is represented on a Feynman diagram as a wiggly line. They are massless and couple to electric charge.

For Weak: The exchange particle is a $W^\pm$ or Z^0 boson. This is represented by a wiggly line or a dotted straight line. They are not massless, and have masses of approx. 80 and 90GeV respectively.

For Strong: The exchange particle is a gluon g. This is represented by a series of curls on a Feynman diagram. They are massless. They couple to "colour charge" which is just another quantum number analogous to electric charge. Just like electric charge has values ±, colour charge has values we denote r, g, b

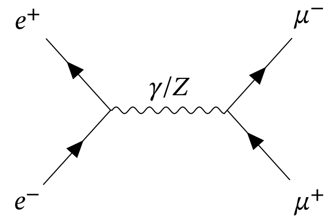
Quarks are subject to the strong, electromagnetic and weak interactions

Leptons are not subject to the strong interaction, but the e, μ, τ are subject to EM (ν is not as it is neutrally charged), and all are subject to the weak interaction. This makes neutrinos very difficult to detect as they are only affected by the weak interaction.

4 Feynman Diagrams

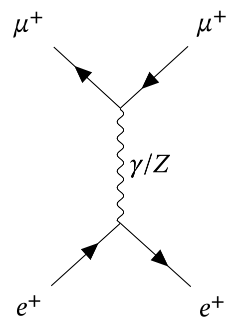
Feynman diagrams are space (y-axis), time (x-axis) diagrams to show allowed interactions between particles.

Consider a simple example of electron-positron annihilation. They travel towards each other, meeting and annihilating into either a photon or a Z boson. This is called a 'time-like exchange'. The boson then decays and we see pair production of two muons (one μ^- muon and one μ^+ antimuon).



Arrows in Feynman diagrams convey "fermion flow". This means that for a matter particle, the arrows aligns to the time axis. For antimatter particles, they antialign. Some conservation laws (i.e. charge) apply at the vertex level, while others only apply across whole processes.

We now consider a space-like exchange where the exchange is aligned with the vertical (space) axis. An antimuon scatters off a position like such:



5 Luminosity

We can determine the rate of interactions with the following:

$$W = \mathcal{L}\sigma$$

Where $W\,(\mathrm{s}^{-1})$ is interaction rate, $\mathcal{L}\,(\mathrm{cm}^{-2}\mathrm{s}^{-1})$ represents the luminosity (an attribute of the accelerator being used), and $\sigma\,(\mathrm{cm}^{-2})$ is the cross section, representing the underlying physics of the interaction.

These are investigated in greater detail in Lecture 03.

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Lecture 2 - Luminosity and Particle Signatures

1 Cross Sections

Consider a proton-proton interaction, producing some unknown particle X:

$$pp \to ppX$$

We have said that the rate of interaction is given by:

$$W = \mathcal{L}\sigma$$

Where $\mathcal{L}$ in $\mathrm{cm}^{-2}\mathrm{s}^{-1}$ is (coarsely) a parameter of the accelerator, describing its ability to produce collisions, and σ in cm^{-2} is a measure of interaction probability. Even though the particles are point-like, we treat them as having an effective area, and the magnitude of that area dictates how likely an interaction is to take place.

In this interaction, we have two protons (modelled as solid balls) passing immediately next to each other (one travelling clockwise and one counter-clockwise) around the accelerator. Assuming they pass immediately next to each other, and we model them as having radius 10^{-15}m, we have a a separation between the centres of each proton as 2×10^{-15}m, therefore a cross section of:

$$\pi \left(2 \times 10^{-28}\mathrm{m}^2\right) \sim 0.12 \times 10^{-28}\mathrm{m}^2$$

To move this to a less annoying length scale, we define a new unit, the barn:

$$1\mathrm{barn} \equiv 10^{-28}\mathrm{m}^2 = 10^{-24}cm^2$$

In reality, this model may approximate a cross section, but it's not accurate. In reality, there's a much wider variation:

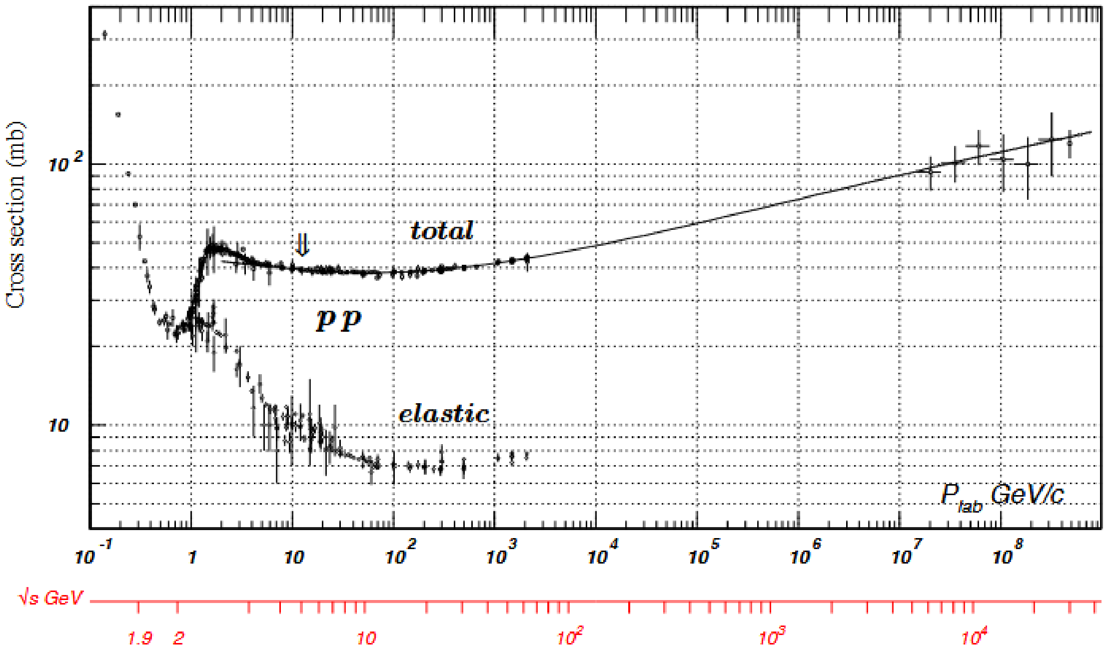

Figure 2.1

Here we have two x-axes running in parallel - the black axis is the momentum in a lab frame (as it hits some fixed target) while the red axis is the corresponding "centre of mass energy". How do we relate these two?

We want to know what the maximum mass of the particle we can generate is. In the lab frame, this requires us to take the momentum of the incoming and generated particle into account. We then have to take the final momentum of the system into account to conserve momentum, as the whole system must continue moving in the direction of motion for conservation.

Translating into the frame of reference given by the system centre of mass gives us a system where the two masses can be thought of as approaching each other with equal and opposite momenta. Since the total momentum is zero, the objects (incoming particle and the target) can theoretically hit each other and come to a complete stop. Because the system does not have to keep moving after the collision, all of the energy in this frame is free to be converted into the mass of a new particle.

While this may not be accurate in practice, it gives us a hard upper maximum for the possible energy available for production.

We can show relativistically that the energy in the centre of mass frame (labelled $\sqrt{s}$, the energy available for particle production) is:

$$E_{\text{com}} = \sqrt{s} \propto \sqrt{p_{\text{lab}}}$$

This can be seen in the final values of the x-axis, which (both starting approx. 1) are 10^8 and 10^4. This tells us that we reach diminishing returns with a fixed target collider - increasing energies by 8 orders of magnitude only increases the energy available for particle production by 4 orders. This is an inherent inefficiency of a fixed target collider.

This is much cheaper as its easier to align, we just fire a beam at a block of (for example) lead. It also makes it quite easy to change the target material. Changing materials in a colliding-beam collider (where two beams are fired in opposite directions, one clockwise and one anticlockwise, and collide with each other), i.e. to fire lead nuclei requires an extensive recalibration process.

In a dual-beam collider, only a very small proportion of the accelerated material from each beam actually interact with each other. In a fixed target collider, the target is much more dense, so we see a higher rate of interactions.

In summary, the advantages of a fixed target collider are:

1. Easier to collide.
2. Easier to change the target.

3. Very high density

2 Luminosity

2.1 Fixed Target Case

Consider a fixed target collider. We want to build an expression for the luminosity of this setup.

We have some incoming flux of particles (per second per unit area), J, incident on the block of material with density ρ, thickness t and mass of one nucleus m_A. The beam, modelled by a cylinder, illuminates some circular portion of the block, with area A.

Consider our interaction rate W. This is given by (where V is the volume of a cylinder from the illuminated beam, of thickness equal to the target):

$$W = \underbrace{JA}_{\substack{\text{incident particles}\\ \text{per unit time}}} \times \underbrace{\frac{\rho}{m_A}V}_{\substack{\text{total number of}\\ \text{target particles}}} \times \underbrace{\frac{\sigma}{A}}_{\substack{\text{probablity of}\\ \text{interaction}}}$$

$$W = JA \times \frac{\rho}{m_A}At \times \frac{\sigma}{A}$$

$$W = J \times \frac{\rho}{m_A}At \times \sigma$$

$$W = \frac{J\rho At}{m_A}\sigma$$

Comparing to $W = \mathcal{L}\sigma$ gives the luminosity as:

$$\mathcal{L} = \frac{J\rho At}{m_A}$$

2.2 Colliding Beam Case

In a colliding beam case, the derivation is more (and too) complex. It is equal to:

$$\mathcal{L} = \frac{f_{\text{rep}} n_b N_1 N_2}{4\pi\sigma_x\sigma_y}$$

Where:

- f_{rep} is the repetition frequency, i.e the rate of the beam passing the collision point.
- n_b is the number of bunches in the beam.
  - A beam can be thought of as as string of pearls, rather than a single discrete constant beam - i.e. clusters of particles “bunches”, followed by empty space between them.
- N_1, N_2 are the number of particles per bunch for each beam
- σ_x, σ_y are the dimensions of the beam in the x- and y-direction, not a cross section as previously.

3 Examples of Detectors

We have some interaction point producing a spray of particles, and surround this with a series of different detector layers. Each produced particle will trigger a different subset of these layers, defining a unique signature we can use to identify produced particles.

Broadly, in some generic detector, we have:

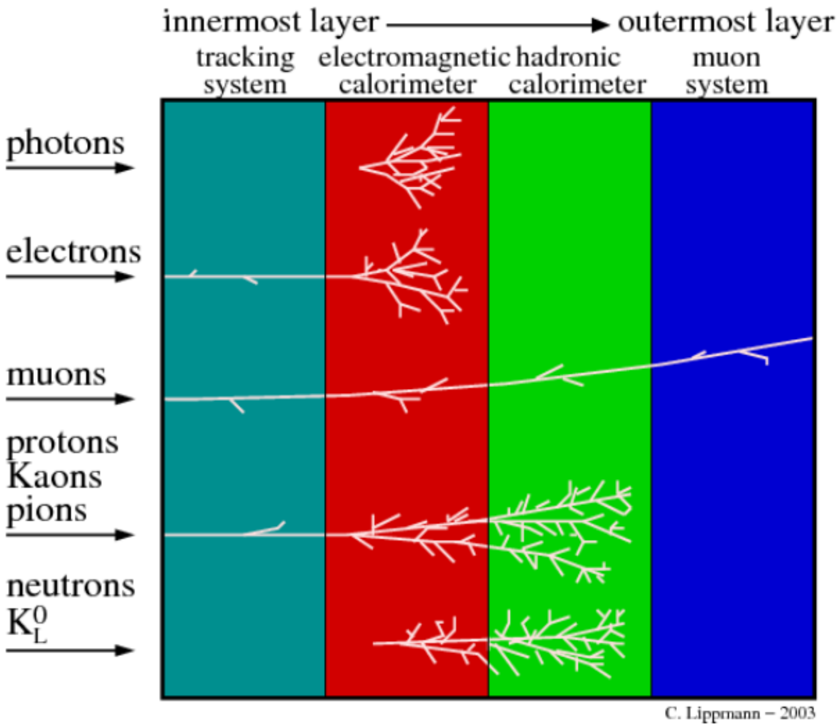

Figure 2.2

The tracking system is non-destructive. Formed of layers of silicon, a charged particle with ionise small portions of each layer. This can be turned into a signal. Neutral hadrons and photons will pass straight through, but charged particles will leave a deposit of charge and pass through unaffected.

We then move to destructive layers. Particles leave tree-like structures as they pass through these layers and create a shower of particles.

3.1 Decays

Most decays take place over a very low time scale, $< 10^{-8}$s and produce a final state made up of some subset of the following: $\gamma, \pi^+, \pi^-, \kappa^+, \kappa^-, p, n, \pi^0, e^+, e^-, \mu^+, \mu^-$.

So, in order to detect some exotic particle, we assume that it will either persist long enough to be detected itself, or decay into some subset of these known particles which will reach out detector.

Consider a parent particle A, for example B^0 ($\bar{b}d$) decaying into two child particles B, C, given by a "J Psi" ($c\bar{c}$), J/ψ and a "K Short", K_s^0 ($d\bar{s}$):

$$\begin{aligned} B &\to J/\psi \quad K_s^0 \\ \bar{b}d &\to c\bar{c} \quad d\bar{s} \end{aligned}$$

This decay has a lifetime of 10^{-12}s, via the weak interaction due to the change in quark flavour. The J Psi decays into e^+e^- or $\mu^+\mu^-$ via the EM interaction very rapidly in 10^{-21}s (its lifetime is governed by the strong interaction, which it may also use to decay via, even though we detect it via the EM decay path). The K Short decays into $\pi^+\pi^-$ or $\pi^0\pi^0$ again via the weak interaction with lifetime in 10^{-10}s.

The range of a particle is given by:

$$\text{Range} = \beta\gamma c\tau$$

Where τ is a time scale (lifetime), c is the speed of light, γ is the Lorentz Factor and β scales the range based on the speed actually being travelled. We also have:

$$E = \gamma m$$

$$p = \beta\gamma m$$

And familiarly:

$$E^2 - p^2 = m^2$$

If the B^0 has energy 20GeV and mass 5GeV, we have $\gamma = 4$ and this gives a range of $\approx$ 1mm. This is so small we will never observe it directly. The K_s^0 however has range $\approx$ 30cm, so is detectable.

Crucially:

- SI lifetimes: $\sim 10^{-21} - 10^{-24}$s
- EM lifetimes: $\sim 10^{-16} - 10^{-20}$s
- WI lifetimes: $\sim 10^{-12}$s

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Lecture 3 - Particle/Matter Interactions

In order to look at specific detectors and how they work, we need to consider how particles interact with matter. We'll consider categories of particles and their standard interactions, and use this to build a model for how we can build a detector.

We've looked at this previously:

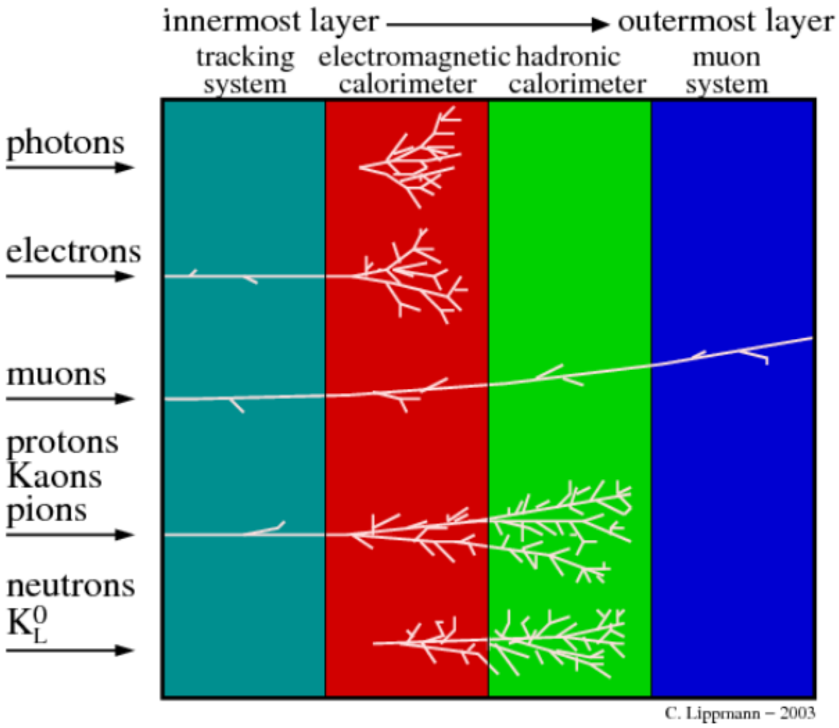


Figure 3.1

For example:

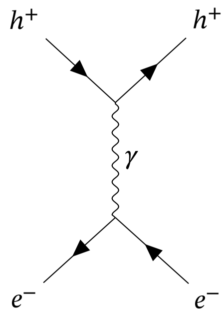
- Charged particles produce ionisation in the non-destructive tracking system, leaving deposits we can detect as signal.
- Electrons and photons leave distinctive signatures (shape of the shower of particles) in the electromagnetic calorimeters.
- Hadrons leave deposits in the electromagnetic calorimeter, but they dump all their energy (and are mostly identifiable by) the hadronic calorimeter.
- Muons make it through all of the previous layers, and are picked up at the very end by the outermost muon system.

1 Charged Particles

1.1 Ionisation

This is the process that powers the tracking system. A charged particle, h comes into the system (it could be a hadron or a lepton). It exchanges a photon with an electron in the system and excites the electron, ejecting it from the atom.

Strictly:



We can to try to characterise the rate of energy loss of the charged particle. The average rate of energy loss per unit distance is:

$$-\left\langle \frac{dE}{dl} \right\rangle \propto \ln E$$

Where E is energy, and the sign is negative as energy is being lost here.

For example, a real detector here might be two charged plates with a high voltage sandwiching a gas mixture. As the gas mixture is ionised, the ionised portion drifts towards one of the electrodes where it induces a detectable current.

1.2 Bremsstrahlung

Translates to “braking radiation”. Consider a free electron radiating a photon. This is impossible for a free particle (as if we consider the electron’s rest frame, emission would violate conservation of energy). We therefore need a source of external interference, in this case matter.

An electron is accelerated by nuclear change as it passes through material and is scattered. This scattering causes bremsstrahlung photon emission. The average rate of energy loss is given by:

$$-\left\langle \frac{dE}{dx} \right\rangle \propto \frac{E}{m^2} \propto \frac{E}{X_0}$$

An electron will then generate more bremsstrahlung than a muon, due to its much smaller mass. X_0 is called radiation length, and is covered in future lectures.

1.3 Cherenkov Radiation

Consider a charged particle moving through a (non-vacuum) material. It emits photons at some angle θ_c if the particle is moving faster than the speed of light in the medium (note this does not violate relativity, as the speed of light in a medium is less than the speed of light in a vacuum).

These emitted photons cause a coherent wavefront to form around the particle’s trajectory, forming a cone around the direction of travel. The angle θ_c is given by:

Geometrically, after time t, the emitted photon has travelled ct/n and the particle vt, hence:

$$\cos\theta_c = \frac{ct/n}{vt} = \frac{c}{nv} = \frac{1}{n\beta}$$

Where n is the refractive index of the material. This is analogous to shock waves forming when an object goes faster than the speed of sound.

A planar detector will take a single cross-section through this cone, detecting rings around the point the charged particle passed through the material. By measuring this ring, we can work out the speed of the particle, and use this along with a measured momentum (in a tracking detector) to work out the mass of the particle.

Again:

$$-\left\langle \frac{dE}{dx} \right\rangle \propto z^2 \sin^2 \theta_c$$

Where z is the particle charge in units of $|e|$. It is important to note that this is a very small energy loss for the particle. It may emit $10^3\gamma$/cm, and only lose a few keV/cm

2 Photons

2.1 Photoelectric Effect

We have a photon strike a atom, transferring energy and forcing an electron to be ejected. The max kinetic energy of the electron is given by the photon energy minus some amount of work to eject it:

$$E_{kmax} = hf - \phi$$

Where ϕ, the work function, is the energy required to liberate one electron from the atom's surface.

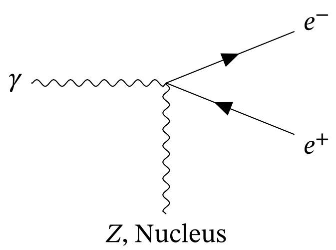
2.2 Compton Scattering

See QM1. A photon scatters off a quasi-free electron in an atom. The photon and electron are scattered with a change in energy:

$$\gamma + e^- \rightarrow \gamma' + e^{-\prime}$$

Thomson scattering is a low-energy form of scattering where the energies do not change, and Rayleigh scattering is a very low energy limit, where the interaction takes place between a photon and multiple atomic electrons.

2.3 Pair Production



A photon interacting with a nucleus can (leaving the nucleus unscathed) produce a particle and the corresponding antiparticle, typically a positron and an electron. This has a minimum photon energy of $E_\gamma > 2m_ec^2$[1] to ensure conservation of energy isn't violated and must occur in the presence of matter (for the same reason as Bremsstrahlung). The photon is not present in the final state.

However, say a photon has precisely the energy required to create a pair. This would (as it stands) create a pair of electrons with zero momentum (or very small momentum). However, a photon always has a momentum given by its energy, so momentum is not conserved. If the pair travel to attempt to resolve this, they now have some kinetic energy too, which means the photon must have a higher energy, and hence a higher momentum. This creates mismatch, we cannot conserve both energy and momentum in this situation.

By this interaction taking place in the presence of a nucleus, the nucleus can absorb some recoil (via exchange of a virtual photon) to ensure conservation of energy and momentum are satisfied.

This must take place in a Coulomb field to contribute this photon. The present nucleus has charge $z|e|$.

Lets consider the cross section of this interaction:

$$\sigma_{\text{pprod}} \propto \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0}$$

[1] $m_ec^2 = 0.511$MeV

X_0 is the radiation length. It is a complex property of the material but is proportional to $1/z^2$.

3 Neutrinos

Neutrinos are really difficult to detect, as they are only effected by the weak interaction and have no ionisation/pair production etc - they are effectively non-interacting. We detect them by detecting the products of a weak force interaction.

For example:

- A muon neutrino can exchange a W boson with a proton, which becomes a neutron. The muon neutrino becomes a muon and the up quark in the proton becomes a down quark in the neutron.
  - We can detect the muon that's produced as the neutrino passes through the detector and interacts.
  - We build a massive multi-tonne detector that puts a large amount of mass in the way of the neutrino, often water.
  - We do this in the hope that it will interact with some of the matter and produce the more-detectable muon.
- The neutrino can scatter off a nucleus.
  - A mall amount of energy is exchanged with the nucleus, depositing a small amount of heat energy in the detector's matter.
  - Nothing changes/decays/is produced etc other than a small amount of heat.
  - We build a very sensitive detector capable of detecting this very small amount of heat.

4 Hadrons

Hadrons are really complex in their interactions. They can have inelastic nuclear interactions, with large energy deposits at a small number of sites (compared to an ECal shower where we'd expect to see many smaller deposits). This arises as a result of the nucleus becoming excited or breaking apart entirely and producing a chain of secondary hadrons/particles and a change in the nucleus.

These secondary particles can go on to interact again. This are complex and messy objects which can fragment off to cause many secondary impacts.

5 Conclusion

In summary:

- The key interactions we care about are ionisation, Bremsstrahlung and Cherenkov radiation for charged particles.
- For photons, we care about pair production.
- For hadrons, hadron showers are messy and complex. We add inelastic nuclear interactions on top of an electromagnetic component from ionisation and everything becomes rather tricky.

Mon 09 Feb 2026 12:00

Lecture 4 - Vertexing and Tracking Systems

In our familiar layered model:

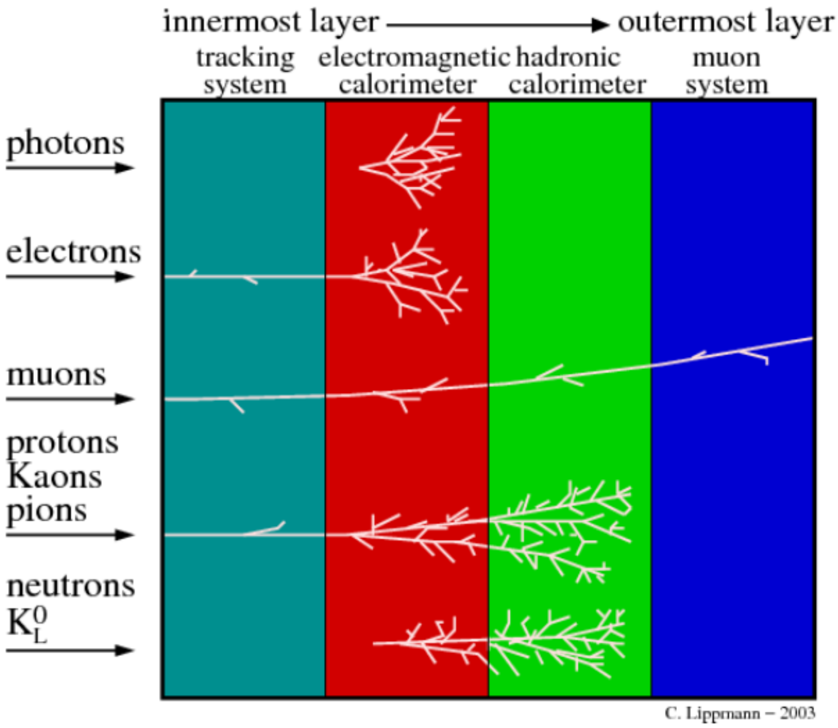


Figure 4.1

The first subdetector type is the initial tracking system which provides a non-destructive estimate of a particle's trajectory.

1 Tracking Systems

Purpose: To determine the trajectory of charged particles (usually in the presence of a magnetic field), in order to infer the momentum of the particle. This happens by ionisation, whereby the particle leaves small deposits of charge in either a gas or layers of a semiconductor sensor.

Position: As the following layers are destructive (i.e. calorimeters will absorb the particle entirely in order to make an energy measurement, while a muon system will block any other particle), we need to place the tracking system first for it to be effective.

We also want it placed near the primary interaction point. Since we want to use position measurements in the tracking system to extrapolate backwards and determine the origin point (where the collision took place), keeping them as close as possible to this origin point reduces the overall uncertainty of the track.

For a long path, a small uncertainty in position points propagates to a much larger uncertainty in track the further away we are from the initial collision point.

1.1 Material Budget

- In an ideal world, the tracking detector would have a perfectly massless and lightweight material, in order to reduce the risk of scattering. However, we need some mass in order to measure ionisation, so it's a constant trade off between the two.
- We want as small a number of radiation lengths X_0s within / upstream of the tracking system as possible.
- Some material is unavoidable, for example in the LHC there needs to be a conductive shield around the beam to prevent the large magnetic fields from inducing currents that would interfere with the sensitive measurement equipment. It also separates the highly pure vacuum of the beam pipe from the slightly less pure vacuum of the outer portion containing the LHCs electronics.

Radiation Length This is an inherent property of each material and is a measure of energy loss. A particle of energy E_0 passes through a distance of one radiation length and loses a factor of energy $1/e$.

$$E(x) = E_o \exp\left(\frac{-x}{X_0}\right)$$

For example:

- For Cu: $X_0 = 15\text{mm}$.
- For Be: $X_0 = 35.2\text{cm}$.

The processes by which energy is lost depends on the particle species in question. For example, an electron loses energy by Bremsstrahlung much more rapidly compared to a muon, due to the differences in mass (average energy loss $\propto 1/m^2$). To define radiation length, we use an electron as a scale.

This lets us talk about "material budget", as adding more material causes a higher uncertainty. For each particle path we care about, we want the fewest radiation lengths possible.

2 Measuring Trajectories

For a particle passing through a tracking system, we reconstruct its trajectory by measuring individual energy deposits (called "hits") caused by ionisation in a large volume (typically $1\text{m} \times 1\text{m} \times 1\text{m}$)

This volume is made up often of many layers, so we can gain an idea of the particles position as it passes through each layer and use this to extrapolate. A tracking system has a resolution of a few $100\mu\text{m}$ and may leave $10 - 100$ hits.

In a "vertex detector", we use a smaller system to reconstruct tracks at/around the interaction point with a higher precision - typically aiming for $\sim 10\mu\text{m}$. It is typically a silicon detector (layers of silicon detector sheets) and generally records fewer hits (< 10).

In a "tracking detector" we use a much larger system commonly further away from the primary interaction point. In a vertex system, there's no magnetic field so while we can use it to extrapolate back to find the PIP, we cannot use it to estimate momentum.

The LHC-b experiment for example has both a vertexer very close to the PIP (to determine the PIP location), and a much more substantial tracking system behind a magnet further away (to determine the deflected track in a magnetic field and hence the momentum.)

3 Measuring Momentum

The motion of a charged particle in a magnetic arises due to the Lorentz force and is proportional to charge:

$$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$$

Where $\underline{F}$ is the force, $\underline{E}$ is the electric field, $\underline{v}$ is the particle velocity and $\underline{B}$ is the magnetic field. Assuming we are only dealing with a magnetic field, and taking magnitudes, we have:

$$\boxed{\underbrace{p}_{\text{GeV}} = 0.3 \times \underbrace{B}_{\text{T}} \times \underbrace{q}_{|e|} \times \underbrace{r}_{\text{m}}}$$

Where the magnetic field produces curvature of radius r in the plane perpendicular to the B-field. Motion parallel to $\underline{B}$ is unchanged.

In 3D, and assuming we are contained within the magnetic field, the particle will follow a helix of constant radius of curvature. This assumes that there are no non-conservative forces (i.e. scattering). In the ideal case, there is no work done, meaning in our force:

$$\underline{F} = q\underline{v} \times \underline{B}$$

The force and the velocity are perpendicular, so $\underline{F} \cdot \underline{v} = 0$.

3.1 Quantitatively

Consider a particle path with a constant radius of curvature R. Consider three hits, with vertical separation between the top and the bottom being $L/2$. The distance between the top/bottom hit and the origin is ℓ, and the final distance between this straight line and the deflected path is the "sagitta" s.

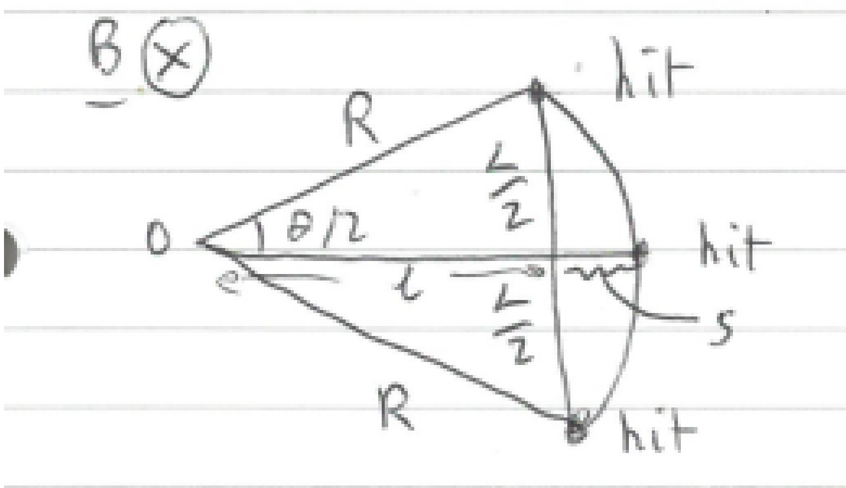


Figure 4.2

We know $L/2$ (the vertical separation between hits) as we've build the detector so know the resolution, and we want to measure the sagitta, the deviation from a straight line.

We know that:

$$p = 0.3BqR$$

And from the diagram:

$$R = \ell + s$$

$$l = R\cos\left(\frac{\theta}{2}\right)$$

Putting these together:

$$s = R\left(1 - \cos\frac{\theta}{2}\right)$$

Assuming the bending is gentle in a detector, so θ and s are small. Hence we apply a small angle approximation:

$$\cos\frac{\theta}{2} \approxeq 1 - \left(\frac{\theta}{2}\right)^2 \frac{1}{2!} + \left(\frac{\theta}{2}\right)^4 \frac{1}{4!} + \ldots$$

Taking the first order:

$$s \approxeq \left(\left(\frac{\theta}{2}\right)^2 \frac{1}{2!}\right) = \frac{R\theta^2}{8}$$

We can also use the small angle approximation for sine:

$$\sin\frac{\theta}{2} = \frac{L}{2R}$$

For small theta:

$$\sin\frac{\theta}{2} \approxeq \frac{\theta}{2}$$

$$\theta \approxeq \frac{L}{R}$$

So:

$$s = \frac{L^2}{8R}$$

Hence, finally, we have:

$$p = 0.3Bq\frac{L^2}{8s}$$

3.2 Uncertainties

We want to find the uncertainty on momentum, σ_p/p:

$$\frac{\sigma_p}{p} = \frac{\sigma_s}{s} = \frac{8p}{0.3BqL^2}\sigma_s$$

However we want to use our uncertainties on individual hits, x_1, x_2, x_3 etc. We have (and will not derive):

$$\frac{\sigma_p}{p} = \frac{\sigma_{xy}P}{0.3BL^2}\sqrt{\frac{720}{N+4}}$$

Mon 09 Feb 2026 12:00

Lecture 5 - Calorimetry

Now that we've finished talking about determining the principal interaction point and the momentum of produced particles in a tracking detector, we move onto calorimeters:

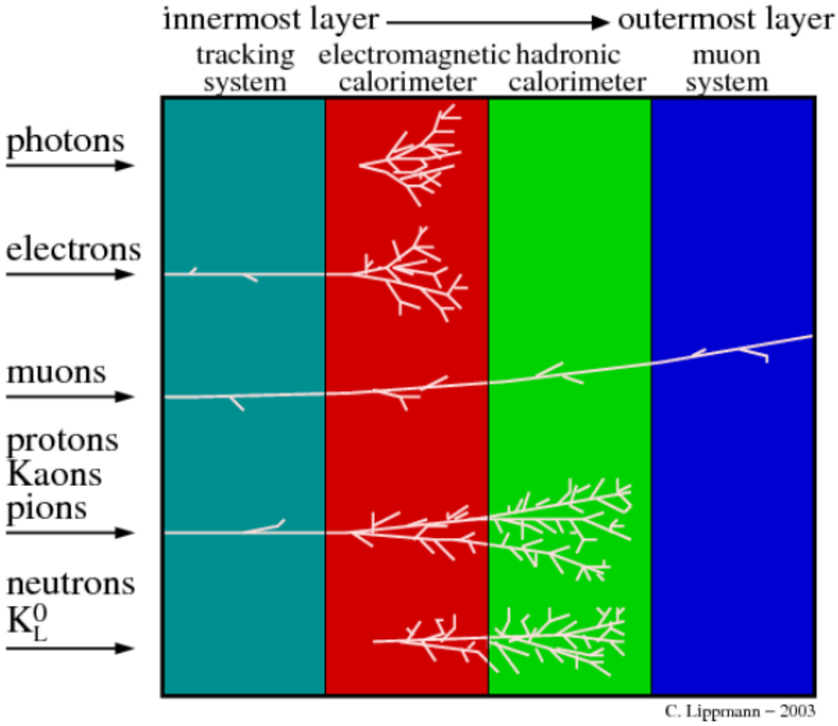


Figure 5.1

1 Materials and Energy Loss

Energy loss in a material is described by the Bethe formula, which gives mean energy loss by ionisation for unit path length:

$$\left\langle -\frac{dE}{dx}\right\rangle = Kz^2\frac{1}{\beta^2}\left[\frac{1}{2}\ln\frac{2m_ec^2\beta^2\gamma^2W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

This is:

- Valid for $\beta\gamma < 1000$ within a few percent precision.
- Includes dependence on the medium, I, Z, A etc.

The good news is that we do not need to know this formula. We do however need to know some key features:

- $\frac{dE}{dx} \propto \frac{1}{\beta^2}$ below some minimal value of $\frac{dE}{dx}$
- $\frac{dE}{dx} \propto \ln(\beta^2\gamma^2)$ above this minimal value of $\frac{dE}{dx}$. This is called "relativistic rise".
- This minima happens at approximately $\beta\gamma \sim 3 - 4$.

- At large $\beta\gamma$, polarisation of the medium causes saturation (effectively a plateau).

Broadly, we expect to see a fall up until some minimal value, then a relativistic rise, and then a plateau:

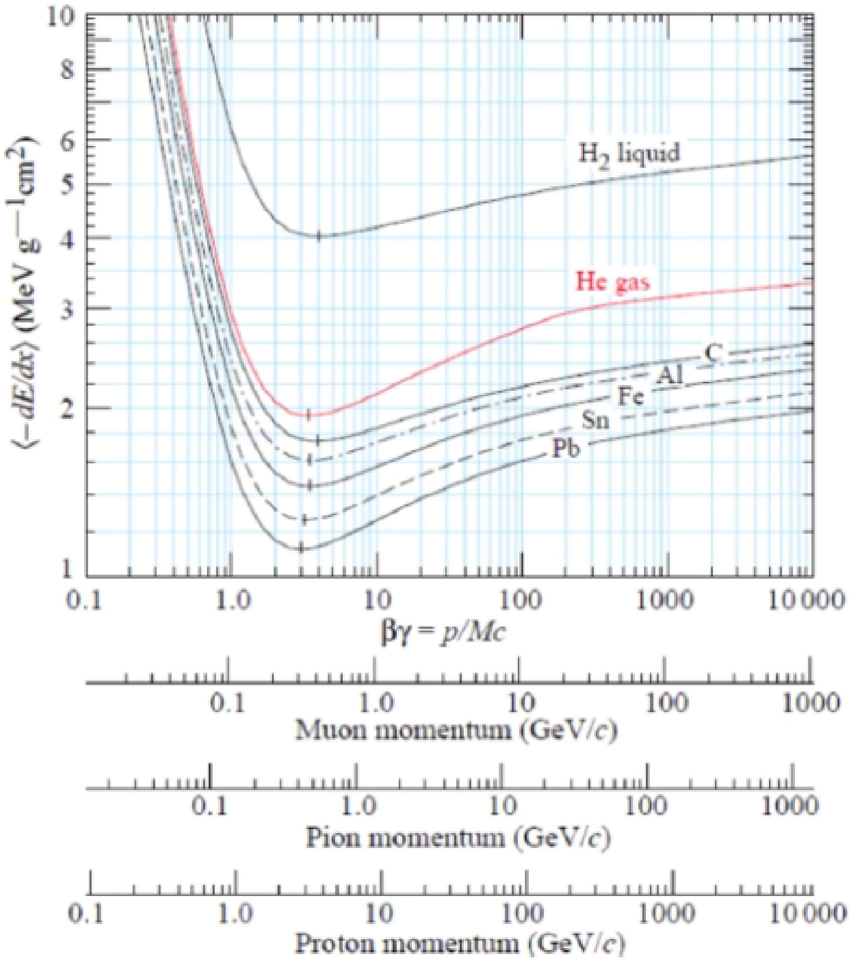


Figure 5.2

2 Calorimeters

The purpose of a calorimeter is to determine the total energy of an incident particle. It does so by absorbing the particle entirely, creating a measurable signal which is proportional to the energy of the incident particle. As it is destructive, it is always located after the non-destructive tracking system. It has no need for the B-field of the tracking detector, and may be called a "calo", "ECAL" or "HCAL".

There are two types of calorimeter:

- Electromagnetic Calorimeter: ECAL.
- Hadronic Calorimeter: HCAL.

We characterise the length of an ECAL in terms of radiation length X_0, typically $0.15X_0 - 0.3X_0$. It works by bremsstrahlung, and emitted bremsstrahlung photons causing pair production and more bremsstrahlung emission etc.

A HCAL works on nuclear hadronic interactions, so we instead define an interaction length λ_{int}, typically $5\lambda_{\text{int}} - 8\lambda_{\text{int}}$.

They don't measure the total energy of a particle if particles produced in the process pass straight through, so not all the energy in an ECAL will be captured, as some will pass straight through. Muons, neutrinos and sometimes pions pass straight through and escape, they are not (may not for a pion) be detected by the calorimeters.

An ideal calorimeter has an output signal "response" which is proportional to the energy of the input particle. We would ideally like a directly proportional linear relationship, but this may not always be realistic.

If we already know mass (from Cherenkov rings, to be discussed later in the course) and momentum from the tracking detector, why do we need a calo when we can just use $E^2 - p^2 = m^2$ to determine energy?

- Cherenkov rings and tracking systems only work with a charged particle. The calculation method is not sensitive enough, as the Cherenkov+tracker setup cannot detect γ, ν or neutral hadrons.
- Having a direct energy measurement adds another constraint to our system - the more information we can get the better. A direct energy measurement improves resolution and is more likely to be accurate than determining it from two other uncertain quantities.
- At very high momentum, (with little Coulomb scattering) the relative uncertainty of the transverse momentum of a tracking system is given by:

  $$\frac{\sigma_{p_\perp}}{p_\perp} \propto \frac{p_\perp}{BL^2}$$

  Therefore at a higher momentum, we have a higher uncertainty. At high momenta, there is a smaller deflection, so a high momentum track will just pass rapidly through the field with very little deflection and is therefore much more difficult to measure accurately - the tracking system degrades at higher momenta.

  For a calorimeter, we have nicer behaviour at high momenta:

  $$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{e}{C}$$

  Where:

  - a arises statistical fluctuations inherent to the measurement.
  - b arises from calibration effects (i.e. from non-linearity)
  - c arises from electronic noise.
  - $\oplus$ means to "add in quadrature", $(p \oplus q \oplus r) = \sqrt{p^2 + q^2 + r^2}$
- Trackers are relatively slow as they require ionisation clouds drifting towards a collection point and involve quite a computationally complex problem to recognise the patterns and reconstruct the track. A calorimeter relies on "scintillation" to convert energy to a response, which is much faster.

  This is important for triggering. Every 25ns, we need to decide whether or not to keep an event (as there is nowhere near enough storage/processing power) to perform detailed track reconstruction for every single event. We need a calorimeter to participate in this.

2.1 EM Calorimeters

There are two types of electromagnetic calorimeters, sampling or homogeneous. A sampling calorimeter is layered, with a layer of absorber producing a shower of particles (i.e. lead) in front of a collector layer with a scintillator that generates a signal when particles pass through.

HCALs are always sampling-type calorimeters, as it takes a lot more mass to slow down/break apart a hadron compared to a lighter charged particle.

Lets build a simple model of an electromagnetic shower that might be detected by a calorimeter:

We have a high energy photon (so we can mostly disregard scattering etc) enter the detector. This pair produces (i.e. an electron and a positron) which emits photons via bremsstrahlung emission. These emitted photons can then go on to pair produce themselves, generating (i.e.) an electron and a positron again, which emit more photons via bremsstrahlung, etc.

The maximum energy is deposited when the average particle energy (of particles developing in the shower) is the "critical energy".

Here, a photon has entered from the left, and the first few radiation lengths have left the material intact.

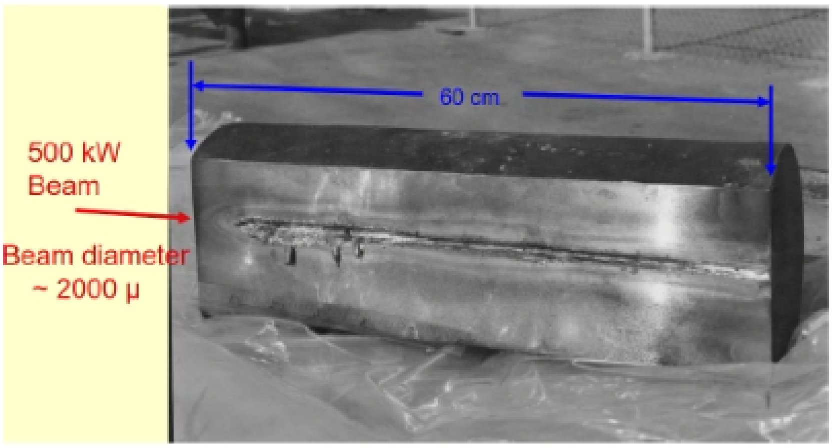


Figure 5.3: Damage to a copper block that has had energy dumped into it via this process.

After the first few radiation lengths, we have the "critical point", where a large amount of energy has been dumped. This point is defined as the point where the probability of bremsstrahlung and ionisation are equal.

The shower deposits a constant amount of energy per X_0 travelled (by definition of radiation length). Each step in path length, in terms of X_0 has a doubling number of particles and a halving individual particle energy. The angle of photon emission is fairly small, so the shower is narrow. This is unlike a hadronic calorimeter that has a wider shower. The width of the shower is given by the multiple scattering of the e^+ and e^-.

When the energy of the particle is less than the critical energy, ionisation dominates the interactions and the shower development stops rapidly. When the particle energy is $> E_c$, pair production and bremsstrahlung dominate.

After depth t (in units of X_0), the number of particles is 2^t, and the average particle energy is $E_0/2t$. The shower stops developing after $E = \frac{E_0}{2t} < E_c$. This happens when $E = 2t = E_0/E_c$. Hence:

$$t_{\max} \log 2 = \log \left(\frac{E_0}{E_c} \right)$$

$$t_{\max} = \frac{\log (E_0/E_c)}{\log 2}$$

This is what defines how deep our calorimeter needs to be in order to collect the energies we are expecting to require.

Mon 16 Feb 2026 16:00

Lecture 6 - End of Particle Physics: Calorimetry II and PID

1 EM Calorimetry: Shower Shape

The energy deposited vs the depth in material can be expressed as:

$$\frac{dE}{dt} = E_0 b \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)}$$

Where t is expressed as a length in units of radiation length $t = x/X_0$ and $\Gamma(a)$ is the standard gamma function $\Gamma(x) = \int_0^\infty t^{x-1} e^{-x}\, dx$. a, b are both constants.

For small t (early in the detector): t^{a-1} dominates as $e^{-bt} \approx 1$

For large t (later in the detector): e^{-bt} dominates.

Considering a plot of $\frac{dE}{dt}$ against t, we initially have polynomial growth, reaching a peak. We then from that peak decay exponentially with an infinite tail.

The maximum point of the shower is given at $t = t_{\max}$ and is achieved by further differentiating dE/dt and is given by:

$$t_{\max} = \frac{a-1}{b} = \ln\left(\frac{E_0}{E_c}\right) + C_{\gamma \text{ or } e^-}$$

For a photon, $C_\gamma = 0.5$ and for an electron, $C_e = -1$. Hence, the shower maximum is at a smaller value of t for an electron/positron compared to photon.

Crucially, we want to find the total energy deposited by the particle. We get that by integrating dE/dt:

$$\int_0^{\text{calo thickness}} \frac{dE}{dt}\, dt$$

While the tail is an infinitely long decaying exponential, we only care about the portion of the tail which is inside the calorimeter. Any energy that would be deposited past this point is lost:

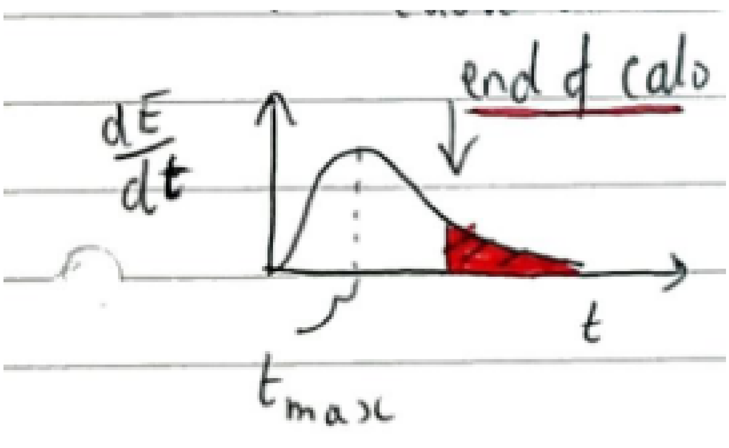


Figure 6.1

If the calorimeter is too thin, we need to make a large correction to adjust for this and we therefore have a bigger uncertainty. We want to ensure that the calorimeter is thick enough to capture the vast majority of deposited energy with minimal loss.

1.1 Resolution

The resolution for the calorimeter varies heavily based on the material used to construct it. Typically, however:

- ECAL resolution: $2 - 10\%$
- HCAL resolution: $50 - 100\%$

Different materials will have different characteristic radiation lengths,

For example:

- W has $X_0 = 0.35\text{cm}$, $\lambda_{\text{int}} = 9.9\text{cm}$, $R_M = 0.93\text{cm}$
- Pb has $X_0 = 0.56\text{cm}$, $\lambda_{\text{int}} = 17.6\text{cm}$, $R_M = 1.6\text{cm}$
- Fe has $X_0 = 1.8\text{cm}$, $\lambda_{\text{int}} = 16.7\text{cm}$, $R_M = 1.7\text{cm}$
- Cu has $X_0 = 1.4\text{cm}$, $\lambda_{\text{int}} = 15.3\text{cm}$, $R_M = 1.8\text{cm}$

R_M is the "Moliere Radius" and is the radial width in which 90% of the shower's energy deposit is confined. Note that we have different design considerations for an HCAL vs an ECAL, for example the size of an HCAL is generally much larger than that of an ECAL.

2 Particle Identification (PID)

We can now measure a particle's momentum and energy, and we now want to determine the mass of the particle. Surely we can do that with:

$$E^2 - p^2 = m^2$$

Unfortunately not…the resolution (especially for energy on a calo) is pretty poor, so we would end up with an unreasonably large uncertainty on our mass, too large to do reasonable particle identification with. Instead, we:

- Use information from the whole ensemble of detectors.
- Use hypothesis tests: proposing a specific particle type and then using all the information we have (i.e. momentum) see if this is a reasonably likely outcome.
- For example, we can use information on whether or not the particle reached the muon chambers to help narrow down what it could be. Alternatively, what is the pattern of energy deposit observed in the ECAL or HCAL?
- We can also use dedicated PID, such as:
  - A time of flight detector, determining the particle velocity (hence mass, with momentum measurements from a tracker).
  - Cherenkov detectors (not useful at high momenta, i.e >100GeV): As a recap, the particle emits a cone of photons at angle θ_c which depends on the refractive index of the material and β. Measuring this cone allows us to determine θ_c hence β hence v.

Here is an example of a detector setup:

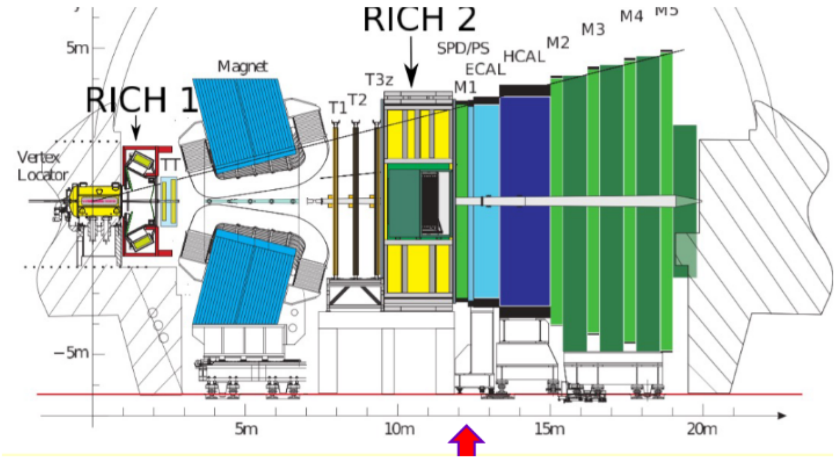
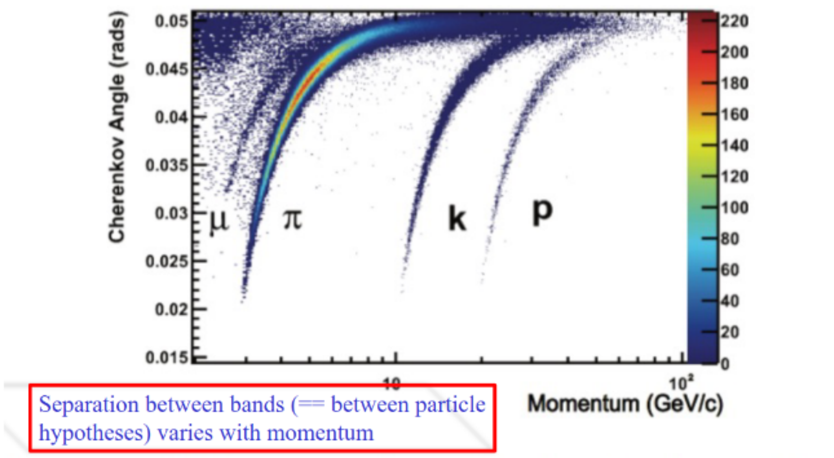


Figure 6.2

We have covered the vertex locator, magnet, tracking stations T1, T2, T3, the HCAL, ECAL and (while we haven't yet touched on them), we have 5 muon systems.

We are yet to discuss RICH1 and RICH2. These are called "Ring Imaging Cherenkov Detectors". These cones of Cherenkov emissions are focussed using a magnet onto a plane, where they form rings. The radii of these rings can be measured and the detector geometry can be used to determine the Cherenkov angle. The two detectors work similarly, but cover different ranges of measurable momenta.



Reconstructed Cherenkov angle as a function of track momentum

Figure 6.3

This Cherenkov angle can therefore be used to discriminate against particles, up to a limit of momentum, past which they become fairly useless.

NOTE: There was a practice exam question here that would be good to come back to during revision.

End of Particle Physics