
LC Temperature and Matter Lecture Notes

Year 1 Semester 2

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Tue 20 Jan 2026 12:00

Lecture 1

Mon 09 Feb 2026 11:00

Lecture 7 - Introduction to Temperature

We're starting the portion of the module on thermodynamics, looking at temperature, pressure etc and transitions involving these.

1 What is Temperature?

Particle for a single particle isn't well defined for the definition of temperature we want to use. There are two definitions:

- One is statistical, looking at average kinetic energy of particles.
- One is based on entropy and isn't looked at until second year.

We define temperature as *a statistical collection of energies for particles that make up the system for which we are measuring the temperature*. A material will have a range of kinetic energies, some very large, some very small, so we care about a statistical average. Thermalisation happens as a result of these high kinetic energy particles striking lower kinetic energy particles.

Temperature: A statistical collection of energies of the particles that comprise the system in question.

We can express temperature as using the Maxwell-Boltzmann distribution:

$$P(v) = 4\pi \left(\frac{M}{2\pi RT} \right)^{\frac{3}{2}} v^2 \exp\left(-\frac{Mv^2}{2RT} \right)$$

We look at this in more depth when we cover statistical physics later, but for now we care about the key features. If we plot $P(v)$ (the probability of finding a particle at certain velocity) against these velocities for a range of temperatures:

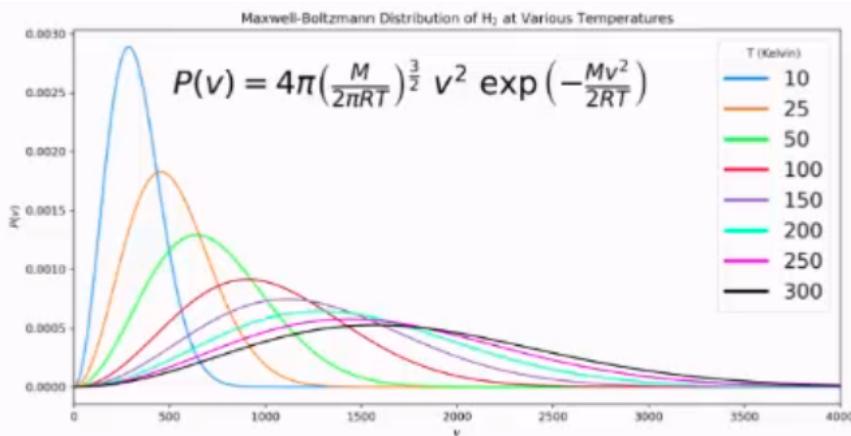


Figure 7.1

This has exponential decay, so theoretically we can find a particle at any velocity within a gas of any temperature. Particles with higher temperatures have a higher peak, so a higher average velocity.

2 Temperature Scales

We can use a number of different temperature scales:

- The Fahrenheit scale is set such that the freezing point of water is 32°F, and the boiling point of water at 212°F. It's designed to be based around the range of temperatures that a person could realistically experience in the core range from 0-200. It's not a useful unit and won't be used in exams.
- Celsius is the standard unit.
- Kelvin is the common unit of thermodynamics. It has the same graduations as degrees celsius but is shifted such that absolute zero is at 0K. Note that Kelvin has no degrees symbol. °C → K can be converted between using:

$$T(K) = T(^{\circ}C) + 273.15$$

3 Thermal Expansion

As a solid is heated, it expands in all directions. Consider a cuboid of height, width, depth W, L, D , we have new dimensions of $D + \Delta D, L + \Delta L, D + \Delta D$ after a small change in temperature. We can define the linear expansion coefficient (a material property) α_L based on these dimensions and the change in temperature:

$$\alpha_L \Delta T = \frac{\Delta L}{L} = \frac{\Delta W}{W} = \frac{\Delta D}{D}$$

If we consider the area instead, we have the area expansion coefficient:

$$\frac{\Delta A}{A} = \alpha_A \Delta T \approx 2\alpha_L \Delta T$$

The latter approximation can be derived as follows. Consider one dimension, i.e. L :

$$\begin{aligned} \frac{\Delta L}{L} &= \alpha_L \Delta T \\ \implies \Delta L &= L \alpha_L \Delta T \end{aligned}$$

So:

$$L_{\text{new}} = L + \Delta L = L(1 + \alpha_L \Delta T)$$

Pairing this with width to find an area (of a face):

$$\begin{aligned} W_{\text{new}} &= W + \Delta W = W(1 + \alpha_L \Delta T) \\ A_{\text{old}} &= L \times W \\ A_{\text{new}} &= L_{\text{new}} \times W_{\text{new}} \\ A_n &= LW(1 + \alpha_L \Delta T)^2 \\ &= LW(1 + 2\alpha_L \Delta T + \alpha_L^2 \Delta T^2) \end{aligned}$$

For a small change in temperature, the second order term is very small, hence:

$$\frac{\Delta(LW)}{LW} \approx 2\alpha_L \Delta T$$

The linear expansion coefficient has units of K⁻¹.

3.1 Thermal Stress

Consider a rod fixed between two immovable walls. The rod attempts to expand, but is fixed. This provides a thermal stress on the wall. If the rod has cross-sectional area A and Young Modulus Y , the thermal stress is given by:

$$\frac{F}{A} = Y \alpha_L \Delta T$$

This is why bridges etc need to have thermal expansion joints, otherwise they would buckle/break under this thermal stress.

3.2 Lennard Jones and Thermal Expansion

Thu 12 Feb 2026 13:00

Lecture 8 - Thermodynamics I

1 Temperature Scales II

In order to measure temperature, we need to define a quantity which directly depends on temperature - ideally linearly.

We can't measure temperature directly, we need this extra intermediary quantity. For example, thermometers use volume and measure the volume of a known quantity of a substance which behaves nicely at known temperatures.

This means that:

$$T = T_0 + k \frac{x - x_0}{x_0}$$

Where T_0 is a calibration point, at which point our quantity has value x_0 . For the celsius scale, we use $T_0 = 0^\circ\text{C}$ and we choose a value for the constant k such that when water boils, $T = 100^\circ\text{C}$.

However, $PV = nRT$ says that this is dependant on pressure too which isn't ideal, as for different pressures the relationship between volume and temperature differ and our scale is only consistent for a single pressure....

Instead of using the boiling/freezing point of water at 1ATM has our calibration point, we can use the "triple point" of water. This point only happens at a single pressure/volume, so is more consistent:

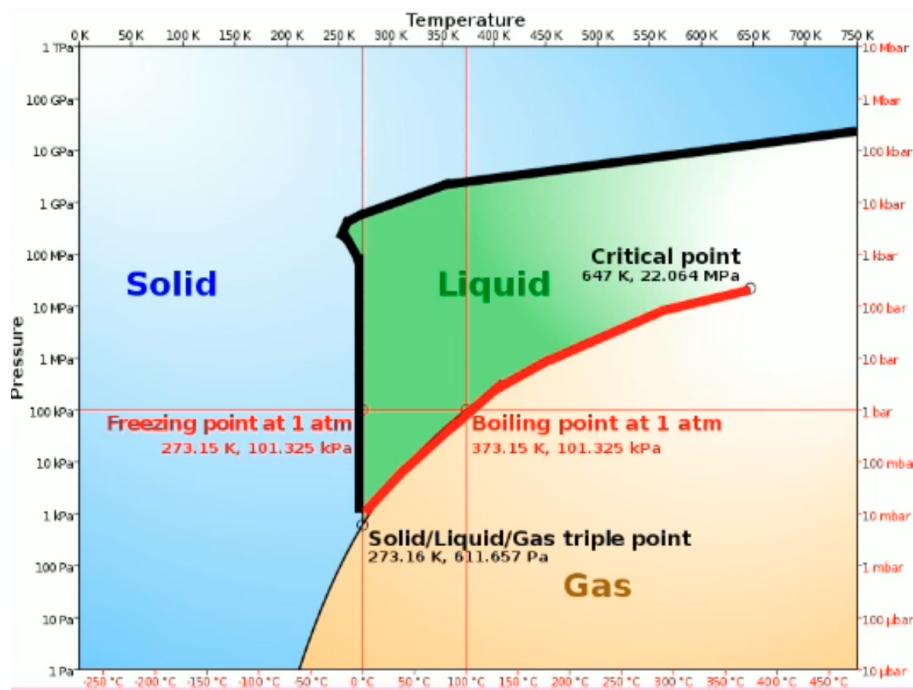


Figure 8.1

Alternatively, we could use the technique employed by the Kelvin scale and use absolute zero as our calibration point. Regardless of pressure/volume/amount of substance (provided its kept constant), they will all reach absolute zero at exactly the same theoretical temperature. We can therefore calibrate our scale off of this point.

2 Thermodynamics

Thermodynamics: The science of the relationship between heat, work, temperature and energy. Thermodynamics broadly deals with the transfer of energy from one place/form to another. Heat is a form of energy corresponding to a definite amount of mechanical work.

2.1 What is heat?

Heat is measure of thermal energy transfer between two systems. This shouldn't be confused with the colloquial understanding of heat i.e. being hot or cold, which is relative.

2.2 Systems

A system is a part of the universe we are investigating or discussing eg a room, bottle, galaxy etc. We often consider the ideal physics case of a closed system where the system doesn't exchange any heat with outside the system however this is not realistic.

We often discuss systems comprised of a container of some kind of fluid which either closed (with a lid) or open (no lid).

We can consider two types of walls:

- Adiabatic walls: No heat transfer possible (does not exist in practice).
- Diathermal walls: Heat transfer is possible through the wall.

2.3 Thermal Equilibrium

A system is in thermal equilibrium if it has a constant energy density throughout, so the system is in internal thermal equilibrium.

Two systems are in thermal equilibrium if there is no net transfer of heat energy between them. There will be some amount of heat transfer between both systems, and thermal equilibrium does not mean that there is no exchange of heat energy between the two. Some of the particles in one part of the system may statistically have higher energy and exchange energy, but there is no *net* transfer.

Consider some gas in a piston with volume, pressure and temperature V_I, P_I, T_I . As we pull out the piston, the volume of the gas changes and we have a new final temperature volume (and as pressure/volume are related to pressure), pressure and temperature V_F, P_F, T_F .

This does not happen instantly and there will be some time required to reach thermal equilibrium (a few seconds) after we have finished pulling out the system.

We can bring a system in contact with another system, i.e. placing a rubber duck (system B) in a bath (system A) or the ocean (system C). Which of these two pairs (A and B vs C and B) will thermalise first? While the duck will reach the temperature of the water faster when in the ocean than the bath, thermal equilibrium requires the entire system to thermalise. Therefore, as the ocean has much higher volume it takes longer for the ocean to entirely thermalise to the newer duck-adjusted temperature.

3 Definitions

- **Isothermal:** A change to a system which takes place at a **constant temperature**.
- **Isobaric:** A change to a system which takes place at a **constant pressure**.
- **Isochoric:** A change to a system which takes place at a **constant volume**.
- **Adiabatic:** A change to a system which takes place **without transfer of heat**. Temperature may change, but heat cannot enter or leave the system.

4 Heat Capacity

What is the relationship between heat energy ΔQ transferred to a system and the corresponding increase in system temperature ΔT ? This is given by:

$$\Delta Q = C\Delta T$$

Where C is the heat capacity in J/K, hence:

$$C = \frac{dQ}{dT}$$

C varies with the amount of material, so we additionally define specific and molar heat capacity:

- Specific Heat Capacity J/kgK : $C = mc$
- Molar Heat Capacity $J/molK$: $C = nc$

A higher heat capacity means a smaller increase in temperature for a given heat (i.e. more energy required to raise the temperature of the system by some amount).

To make things more difficult, heat capacity also varies with the system temperature. We also need to specify whether we are measuring isobaric or isochoric, as these will give different values:

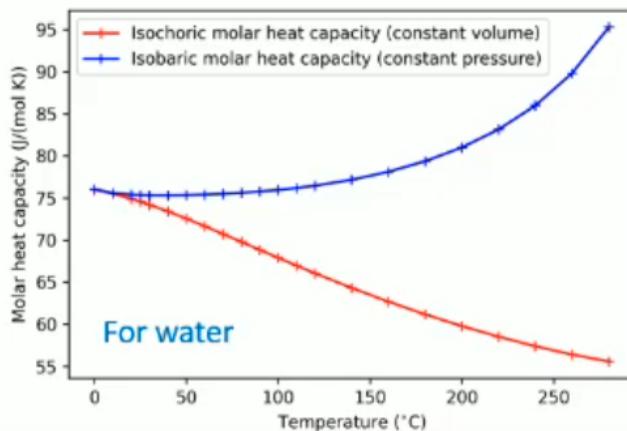


Figure 8.2

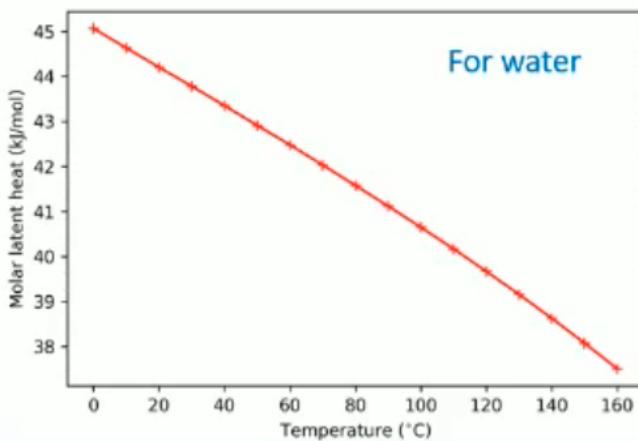


Figure 8.3

Notably, isobaric heat capacity is mostly constant around the standard values of liquid water at standard temperature:

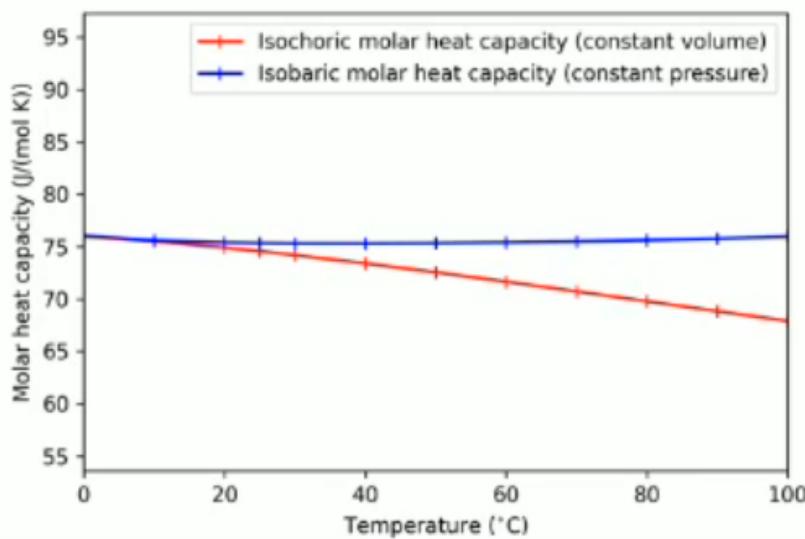


Figure 8.4

It is therefore not unreasonable to treat heat capacity as a constant in some circumstances, provided we state this as an assumption and justify it.

Tue 17 Feb 2026 12:00

Lecture 9 - Laws of Thermodynamics

1 0th Law of Thermodynamics

Consider two blocks next to each other, one with T_h and one with T_c . The blocks will thermalise to their mean $(T_h + T_c)/2$ in the trivial case where the blocks are identical.

Net heat will flow between the two systems until they have the same energy density and hence temperature.

If we have three systems: A, B, C. If we know that A and B are in thermal equilibrium, and A and C are in thermal equilibrium, then B and C are also in thermal equilibrium.

It's a fairly trivial axiom, so we denote it the zeroth law. Effectively, all objects in a system in equilibrium share the same temperature.

2 Ideal Gases

An ideal gas is defined as a collection of molecules (or atoms, if monatomic) that are non-interacting with each other (no interatomic forces) and collide elastically with each other. The internal energy of the gas is dependant on the velocities of the molecules, and hence on the temperature, and not on pressure or volume.

Boyle's Law: "The absolute pressure exerted by a given mass of an ideal gas is inversely proportional to the volume it occupies, if the temperature and amount of gas remain unchanged within a closed system."

Effectively:

$$P \propto \frac{1}{V}$$

Charles' Law: "When the pressure of a sample of an ideal gas is held constant, the Kelvin temperature and volume will be in direct proportion."

Effectively:

$$T \propto V$$

Ideal Gas Law: Since $P \propto \frac{1}{V}$ and $T \propto V$, we have: $PV = kT$, where k varies with context, for example when considering moles:

$$PV = nRT$$

Where n is the number of moles and $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$ is the gas constant.
Or for molecules:

$$PV = Nk_b T$$

Where $N = N_a n$ is the number of molecules, and $k_b = R/N_a$ is the Boltzmann Constant.

This is called an equation of state and allows us to describe the gas' state macroscopically. We generally express this on a P/V diagram, where each point on the plot represents a specific gas state. If we keep the amount of gas present constant, we can use any two of the variables to determine the third.

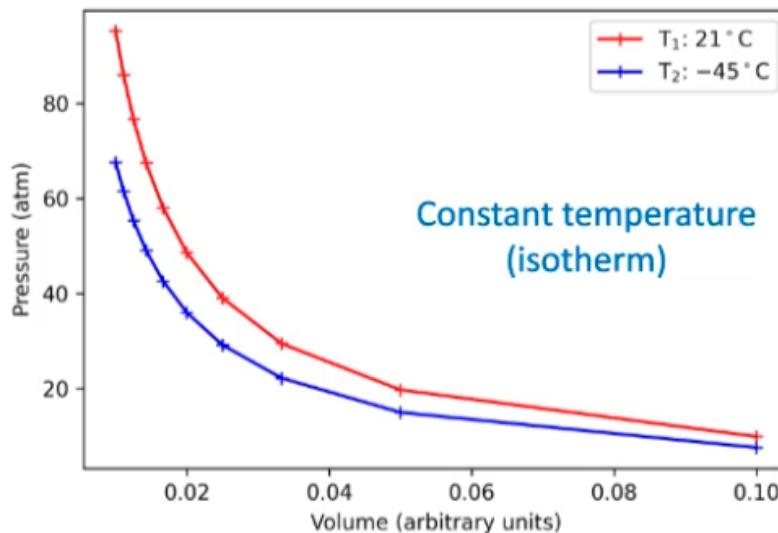


Figure 9.1

2.1 Joule's Second Law

Consider a system with a box divided in two. An ideal gas is contained in the leftmost half, and the rightmost half is a vacuum. We separate the two with a divider constituting an impermeable membrane.

We remove the membrane laterally, doing no work on the gas. Joule observed that the gas stayed at the same temperature as it diffused into the vacuum.

Consider the internal energy of the gas, denoted U , assuming that U is a function of two of the state variables. We know the temperature did not change, and the volume did, so $U(T, V)$.

We would expect:

$$dU = \frac{\partial U}{\partial T}dT + \frac{\partial U}{\partial V}dV$$

Temperature was observed experimentally to not change, hence $dT = 0$, so:

$$dU = 0 + \frac{\partial U}{\partial V}dV$$

Volume did change, so $dV \neq 0$. We did no work on the gas, as the divider was removed laterally, hence there was no change in internal energy, so:

$$dU = \frac{\partial U}{\partial V}dV = 0$$

And since $dU = 0$, we must have:

$$\frac{\partial U}{\partial V} \neq 0$$

This means that there is no dependence on volume for internal energy, hence U is dependent only on T , as found by Joule. This means that the gas Joule chose was well approximated by an ideal gas.

This is easier at higher temperatures, as a high temperature leads to high kinetic energies, so the interatomic forces becomes less significant and easier to disregard in reality.

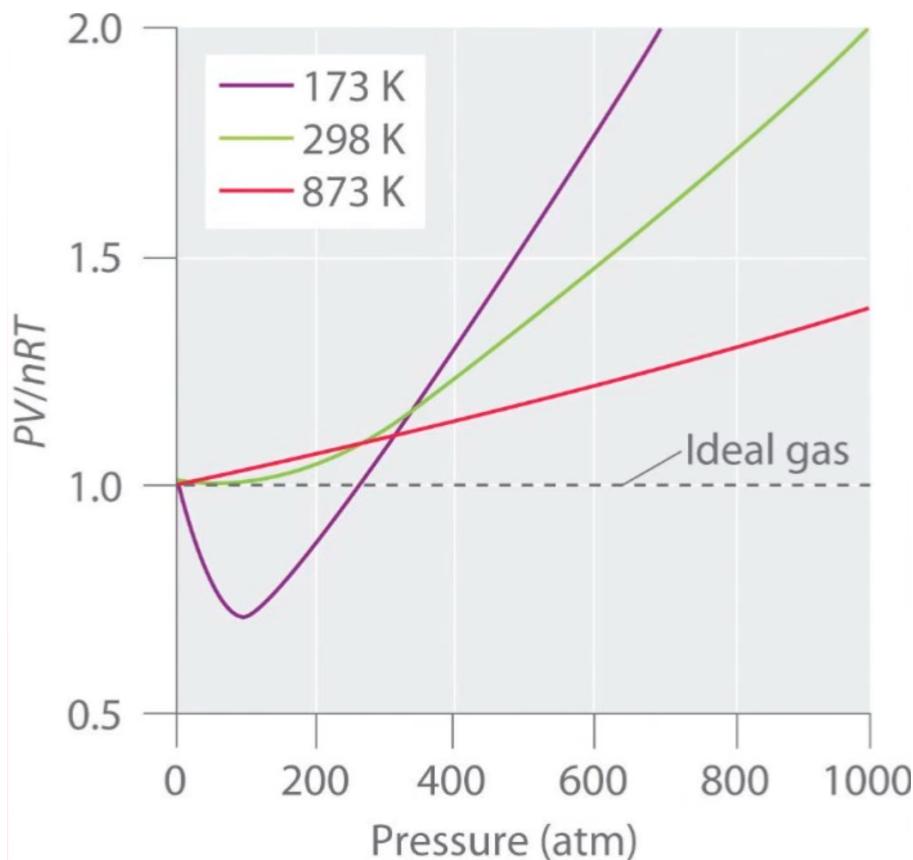


Figure 9.2

We see here that if the ideal gas law becomes a better approximation as we increase temperature, and a worse approximation as pressure increases.

From a LJ perspective, increasing the pressure decreases the average distance between gas molecules. This means that the potential between gas molecules is no longer negligible, and the assumption of zero potential no longer holds.

If we increase pressure even further, the force becomes repulsive and PV/nRT returns to being positive. At higher temperatures, this little dip isn't observed as a higher kinetic energy makes any interatomic forces relatively more negligible.

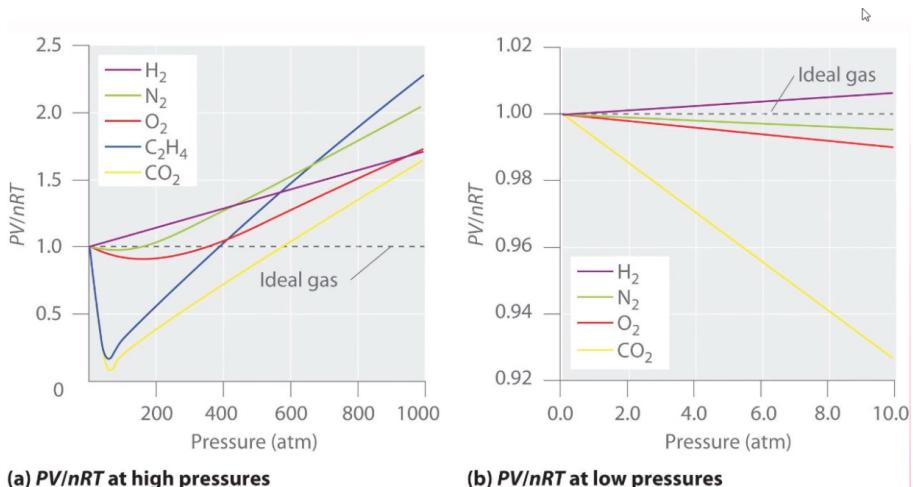


Figure 9.3

At low pressure or high temperature, the ideal gas law is a good assumption as:

- At low pressure, interatomic spacing is large enough to disregard interatomic forces.

- At high temperature, kinetic energy is large enough to comparatively disregard interatomic forces.

We deal solely with Maxwell-Boltzmann gases in this source that follow classical laws. We also have Fermi and Bose gases (made entirely of fermions and bosons respectively), but they're quantum mechanical, exotic and outside of this course.

2.2 Changing Energies

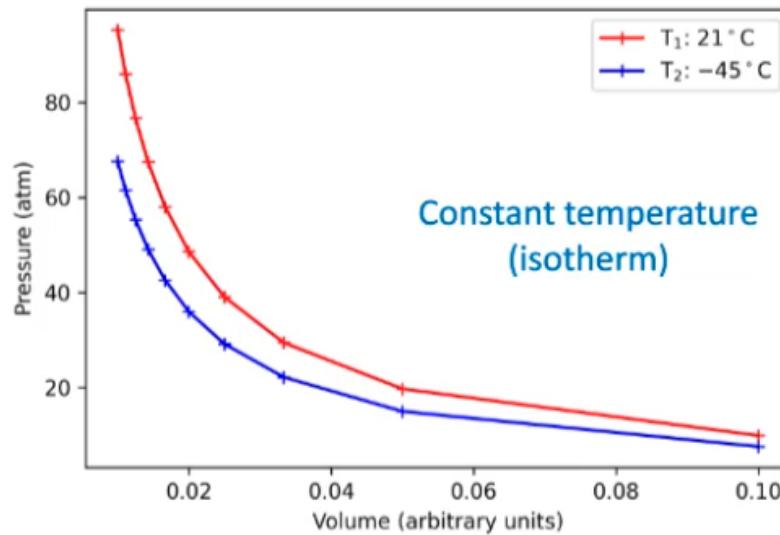


Figure 9.4

Say we want to go from T_1 to T_2 . We need to change the internal energy of the gas somehow. We have two general ways to change this:

- Add heat to the system by transferring heat to the gas at a constant volume.
- Do some work on the gas, i.e. by crushing it and reducing volume.

Consider a piston of area A , applying constant force F with extension ΔL . The pressure on the gas is $P = F/A$. The work done on the gas is:

$$\begin{aligned}\Delta W &= F\Delta L \\ \implies \Delta W &= P(A\Delta L) \\ \implies \Delta W &= P\Delta V\end{aligned}$$

Moving from infinitesimal Delta values:

$$W_{\text{on gas}} = \int dW = \int PdV$$

In the opposite case considering work done by the gas, conservation of energy says it must be oppositely signed:

$$W_{\text{by gas}} = - \int PdV$$

2.3 Mass Drop Experiment

The falling mass spins the paddles in the water, the work as potential energy is converted to kinetic energy raising the temperature of the water.

This tells us that mechanical work done and heat energy must be related in some manner.

3 1st Law of Thermodynamics

The change in internal energy of a system, ΔU is increased with increasing heat transfer into the system, Q_{in} and work done on the system, W_{on} :

$$\Delta U = Q_{\text{in}} + W_{\text{on}}$$

This is just conservation of energy.

Thu 19 Feb 2026 13:00

Lecture 10 - Thermodynamic Transitions

1 Thermodynamic Transitions

1.1 Isothermal (Fixed Temperature)

This may look like:

- A curve on a PV plot. As $PV = nRT$, a PV plot shows a $1/x$ relationship for constant T .
- A vertical straight line on a PT plot.
- A vertical straight line on a VT plot.

If the temperature change is 0, ΔU (which we derived last time only depends on temperature) follows:

$$\Delta T = \Delta U = 0$$

Hence:

$$0 = Q_{\text{in}} + W_{\text{on}}$$

1.2 Isochoric (Fixed Volume)

This may look like:

- A vertical straight line on a PV plot.
- A proportional relationship on a PT plot.
- A horizontal straight line on a VT plot.

For an isochoric transition, $\Delta U = Q_i n$, as:

$$W_{\text{on}} = (-) \int P dV$$

And as $dV = 0$, $W_{\text{on}} = 0$

1.3 Isobaric (Fixed Pressure)

This may look like:

- A horizontal straight line on a PV plot.
- A horizontal straight line on a PT plot.
- A proportional relationship on a VT plot.

For an isobaric transition, P is just a constant, so the integral:

$$W_{\text{on}} = \int_{V_1}^{V_2} P dV = [PV]_{V_1}^{V_2}$$

Where P is constant, which is nice and easy. Generally:

$$W_{\text{on}} = - \int P dV = (-)P\Delta V$$

$$\Delta U = \Delta Q - P\Delta V$$

2 Heat Capacities of Ideal Gases

Recall that:

$$\Delta Q = C\Delta T \implies C = \frac{dQ}{dT}$$

At a fixed volume:

$$C_V = \left(\frac{dQ_{in}}{dT} \right)_V$$

And at a fixed pressure:

$$C_P = \left(\frac{dQ_{in}}{dT} \right)_P$$

Mayer's Relation says that:

$$C_P = C_V + nR$$

This is an examinable derivation.

2.1 Deriving Mayer's Relation

Proof. Consider a gas piston. We have state variables V, P, T , and N is constant as the piston is sealed. The first law of thermodynamics says:

$$\Delta U = Q_{in} + W_{on}$$

We can consider this for both a constant volume or a constant pressure. For a constant volume:

$$\Delta U = \Delta Q_1$$

And for a constant pressure:

$$\Delta U = \Delta Q_2 - P\Delta V$$

Noting that the two Q s are different as we have two different transitions. We can create a slightly different expression for constant volume by multiplying by $\Delta T/\Delta T$:

$$\Delta U = \frac{\Delta Q_1}{\Delta T} \times \Delta T = C_V \Delta T$$

And for constant pressure:

$$\Delta Q_2 = \Delta U + P\Delta V = \frac{\Delta Q_2}{\Delta T} \Delta T = C_P \Delta T$$

Hence:

$$C_P \Delta T = \Delta U + P\Delta V$$

$$C_V \Delta T = \Delta U$$

Setting equal to each other based on ΔU :

$$\Delta U = C_V \Delta T = C_P \Delta T - P\Delta V$$

$$C_P \Delta T - C_V \Delta T = P\Delta V$$

$$\Delta T(C_P - C_V) = P\Delta V$$

$$\implies C_P - C_V = \frac{\Delta V}{\Delta T} \times P \quad \text{NB: } P \text{ is still constant.}$$

Using $PV = nRT$, for constant pressure we have $\frac{P\Delta V}{\Delta T} = nR$. Hence:

$$C_P - C_V = nR$$

$$C_P = C_V + nR$$

□

It is clear that $C_P > C_V$.

Note: We tend to use heat capacity at a constant volume unless specified.

3 Back to Thermodynamic Transitions

3.1 Adiabatic Transitions (No Heat Transfer)

These seem quite a bit like isothermal transitions, but look different on a PV plot:

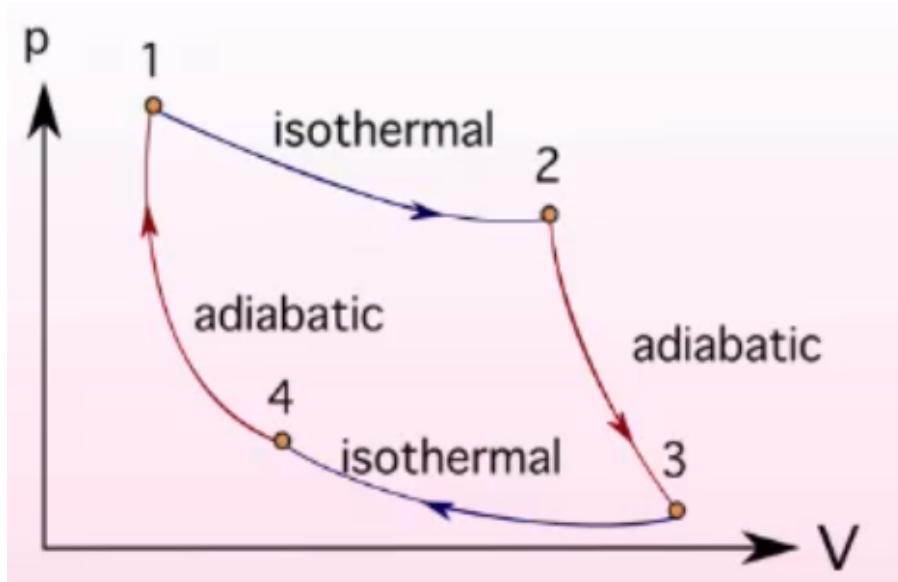


Figure 10.1: Differences in transitions on a PV Plot

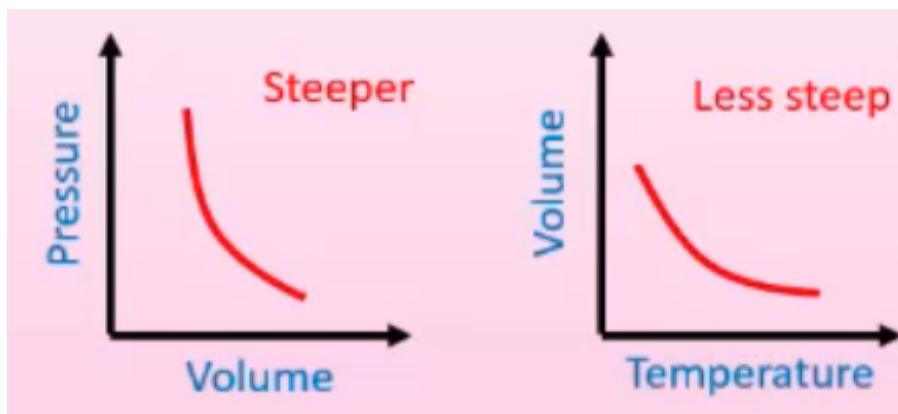


Figure 10.2: Adiabatic Transitions on a PV and VT Plot

For isothermal transitions, $PV = \text{const}$.

For adiabatic transitions, $PV^\gamma = \text{const}$ where $\gamma = \frac{C_P}{C_V}$

In a PV plane, adiabatic processes appear steeper than isothermal processes and adiabatic processes are steeper in the PV plane than the VT plane.

For an adiabatic transition, we can reduce the first law of thermodynamics like so:

$$\Delta U = Q_{\text{in}} + W_{\text{on}}$$

$Q_{\text{in}} = 0$ as there is no heat transfer, hence:

$$\Delta U = W_{\text{on}}$$

3.2 Derivations

Proof. For adiabatic transitions, where $Q = 0$.

The First Law becomes $\Delta U = Q_{\text{in}} + W_{\text{on}}$. Since $Q_{\text{in}} = 0$:

$$dU = 0 + dW_{\text{on}}$$

And assuming work is being done *on* the gas, the sign becomes negative:

$$\begin{aligned} dU &= -PdV \\ \frac{dUdT}{dT} &= -PdV \end{aligned}$$

We slightly abuse the definition of heat capacity here, and this will be elaborated on in a later lecture:

$$C_VdT = -PdV$$

Using $PV = nRT$, we have: $d(PV) = nRdT$:

$$\begin{aligned} nRdT &= PdV + VdP \\ dT &= \frac{PdV + VdP}{nR} \\ C_VdT &= -PdV \\ 0 &= C_V \frac{PdV + VdP}{nR} + PdV \\ 0 &= C_V(PdV + VdP) + nRpdV \\ 0 &= PdV(C_v + nR) + C_VVdP \\ 0 &= PdV(C_P) + VdP(C_V) \\ 0 &= \frac{C_P}{C_V}PdV + VdP \end{aligned}$$

Let $\gamma = \frac{C_P}{C_V}$, and dividing through by pressure and volume:

$$\begin{aligned} \gamma \frac{dV}{V} + \frac{dP}{P_s} \\ \int \gamma \frac{dV}{V} = - \int \frac{dP}{P} \\ \gamma \ln V + c_1 = - \ln(P) + c_2 \end{aligned}$$

Finally:

$$\gamma \ln(v) + \ln(p) = \text{const}$$

$$\ln(PV^\gamma) = \text{const}$$

$$PV^\gamma = e^{\text{const}} = \text{another constant}$$

Hence:

$$PV^\gamma = \text{const}$$

As required! □

Note that in a TV plane, we instead have:

$$TV^{\gamma-1} = \text{const}$$

Tue 24 Feb 2026 12:00

Lecture 11 - PV Planes and Transitions

Note RE: Last Lecture

To avoid confusion, the Physics convention is:

$$W_{\text{by}} = \int P dV$$

$$W_{\text{on}} = - \int P dV$$

1 PV Diagrams

We can move between points on a PV plane through processes which usually involve some kind of heat transfer and work. For example:

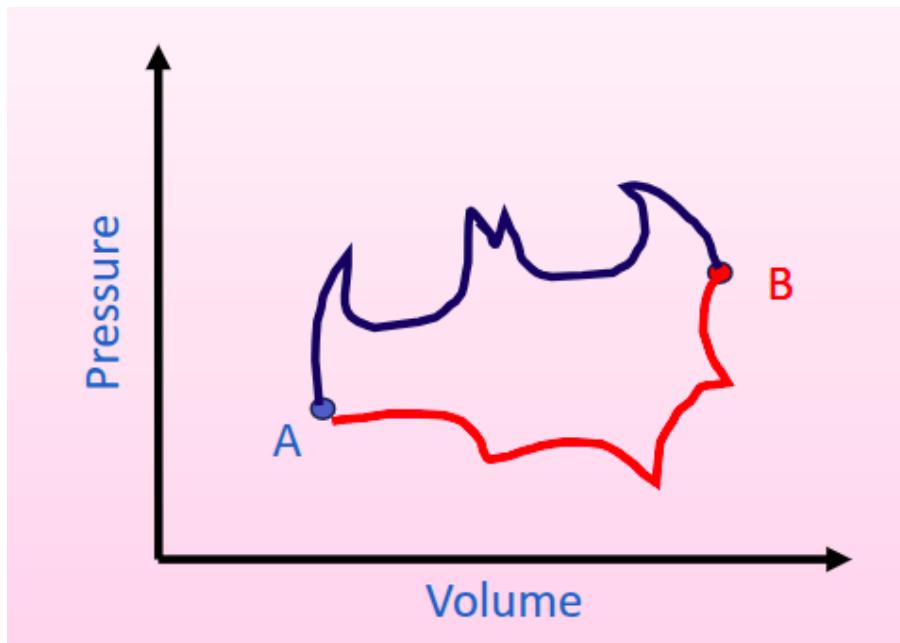


Figure 11.1

The two transitions from $A \rightarrow B$ are processes, and the nature of the process depends on the shape of the transition on the PV plot.

- Since the two possible transitions between A and B share the same start and end point, the change in internal energy U_{AB} is the same for both. Since we go from T_A to T_B in both cases (as U depends on T , which in turn depends on P, V so for the same P, V we have the same T), both paths have the same start/end internal energy, so the same change between them.
- The work done by the gas in the two paths is not the same, however. W_{by} is larger for the blue path as W is an integral of $\int P dV$, which represents the area under the curve. The blue line is further up the pressure axis, so the integral is larger and more work is done.

- What about heat input to the gas, Q_{in} ? Since the First Law says $\Delta U = Q_{in} + W_{on} \implies \Delta U = Q_{in} - W_{by}$. ΔU is constant, and $-W_{by}$ has a larger magnitude for blue, so must Q_{in} to keep ΔU constant.

These transitions are called a *PV Cycle* as we can start at *A*, go to *B* and end back up at *A*.

Consider a simple example with straight lines:

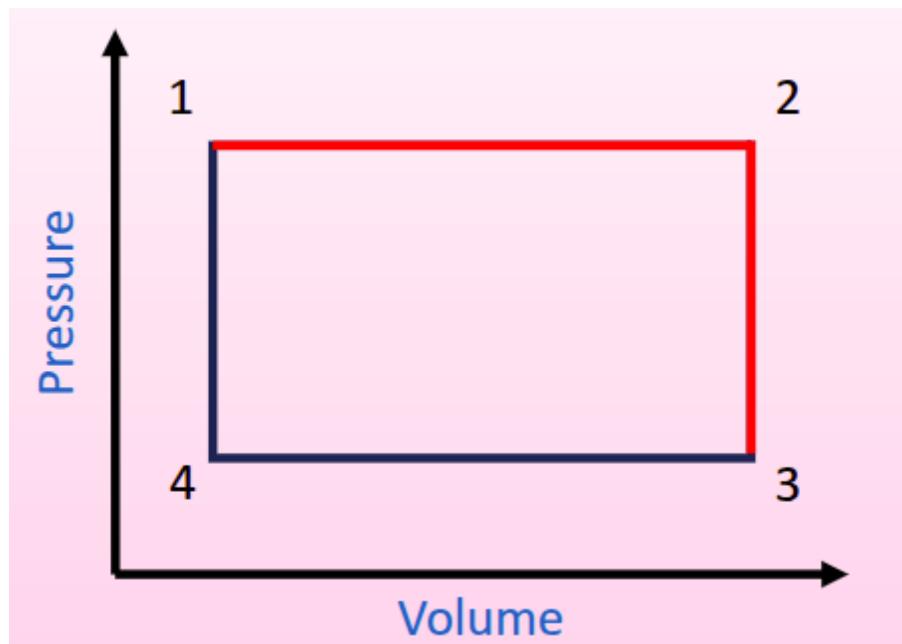


Figure 11.2

Where $1 \rightarrow 2$ and $3 \rightarrow 4$ are isochoric processes and $2 \rightarrow 3$ and $4 \rightarrow 1$ are isobaric. We can also look at isothermal and adiabatic processes:

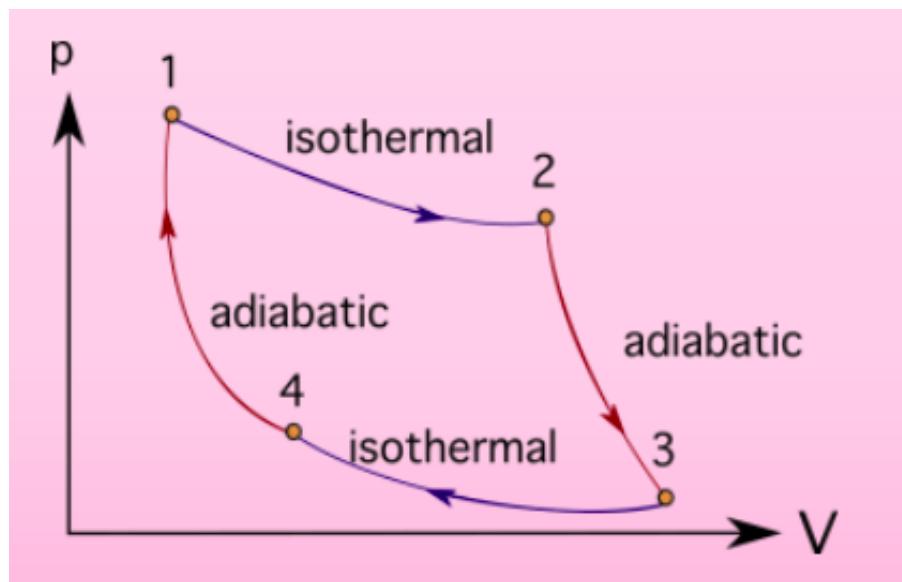


Figure 11.3

A cycle must follow these rules:

1. $\Delta U = 0$, for a full cycle.
2. Total work done is given by the area contained by the cycle.
3. Clockwise cycle for positive work, anticlockwise for negative work.

4. $Q_{in} + W_{on} = 0$ as $\Delta U = 0$.

2 Otto Cycle

This describes a petrol internal combustion engine:

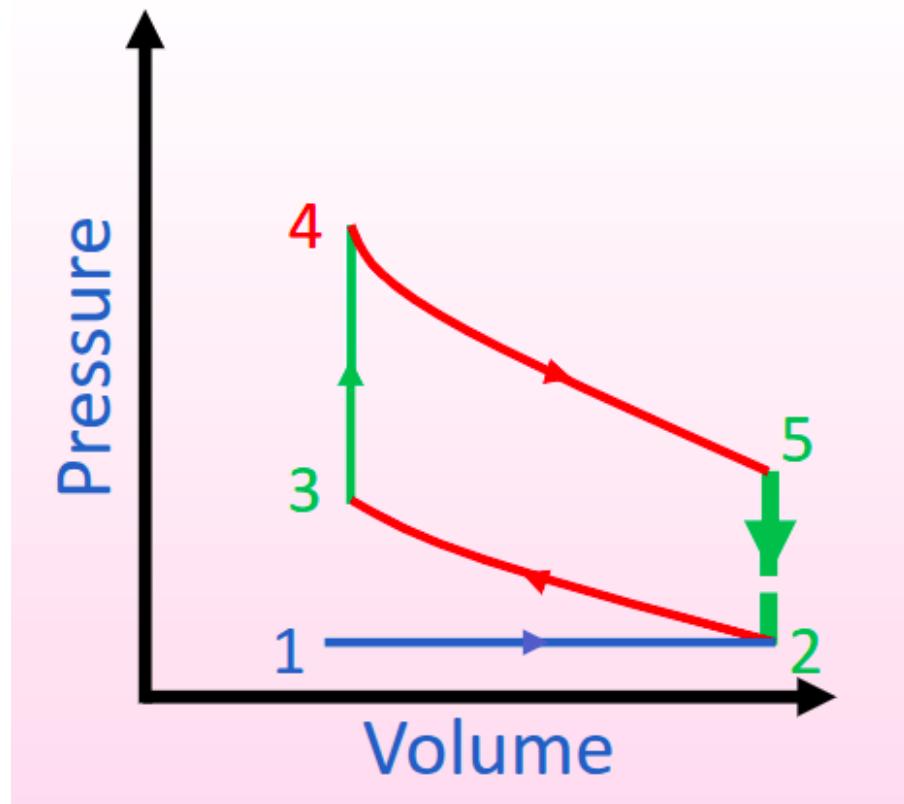


Figure 11.4: Blue: Isobaric, Green: Isochoric, Red: Adiabatic

This has the following components:

1. $1 \rightarrow 2$: Intake stroke: Isobaric introduction of air and fuel to the system.
2. $2 \rightarrow 3$: Adiabatic compression of the fluid as the piston compresses it.
3. $3 \rightarrow 4$: Isochoric introduction of heat energy to the system (ignition of the fluid mixture).
4. $4 \rightarrow 5$: Adiabatic expansion of the fluid mixture as the gas does work on the piston (pushing it down).
5. $5 \rightarrow 2$: Isochoric expulsion of heat from the system.
6. $2 \rightarrow 1$: Mass of air and exhaust gases released at a constant pressure.

3 Diesel Cycle

Here, the ignition and combustion process is performed at a constant pressure where work is done by the gas:

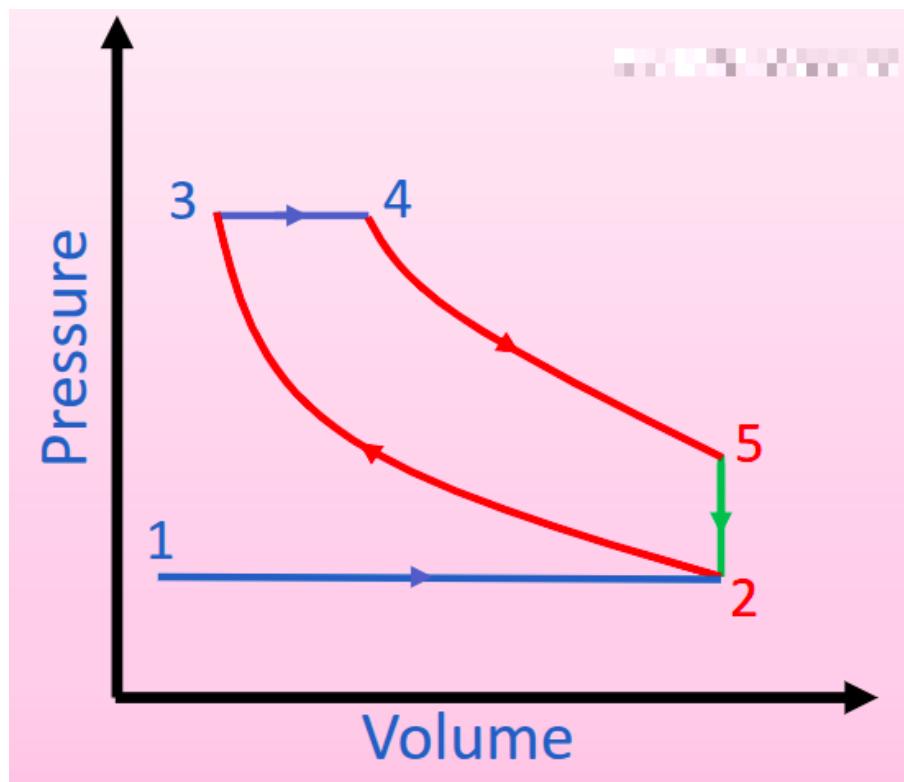


Figure 11.5: Blue: Isobaric, Green: Isochoric, Red: Adiabatic

In both cases, heat is introduced (by combustion) from 3 → 4, denoted Q_1 and heat is lost from 5 → 2 when heat is expelled from the system. The useful work done is $Q_1 - Q_2$ and is given by the area enclosed by the loop.

3.1 Example

Consider the adiabatic gas compression stroke in a diesel engine. (This is the adiabatic stroke where pressure is increasing/volume is decreasing, i.e. from 2 → 3.)

10^{-3}m^3 of nitrogen gas is initially at atmospheric pressure and 298K and is compressed to 1/15th of its original volume.

You may assume that nitrogen is an ideal diatomic gas with $\gamma = 1.4$ and $C_V = 20.85\text{J/Kmol}$.

Calculate:

1. The final pressure.
2. The final temperature.
3. The number of moles in the gas.
4. The change in internal energy.
5. The work done by the gas.

Part I

Atmospheric pressure is $101325 \approx 10^5\text{Pa}$. As this is an adiabatic transition, $PV^\gamma = \text{const}$. Hence:

$$\begin{aligned}
 P_1 V_1^\gamma &= P_2 V_2^\gamma \\
 \implies P_2 &= P_1 \left(\frac{V_1}{V_2} \right)^\gamma \\
 \implies P_2 &= P_1 \left(\frac{V_1}{(V_1/15)} \right)^\gamma \\
 \implies P_2 &= P_1 (15)^\gamma = 4.5\text{MPa}
 \end{aligned}$$

Part II

We can use either $PV = nRT$ twice, using the initial conditions to determine n or we can use:

$$T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$$

Using the same process gives:

$$T_2 = T_1(15)^{(\gamma-1)} = 880\text{K}$$

Part III

Since we know P_1, V_1 or P_2, V_2 we can use $PV = nRT$, as mentioned above. It would be a good sanity check to do both and compare the values.

Part IV

It's an adiabatic transition, we can relate $C_V = \frac{\partial U}{\partial T}$.

Integrating both sides gives:

$$\int dT = \int \frac{\partial U}{\partial T}$$

Thu 26 Feb 2026 13:00

Lecture 12 - Heat Transfer

1 Thermal Transport

Newton's Law of Cooling: “The rate of heat loss of a body is directly proportional to the difference in the temperatures between the body and the environment”

There are three means of thermal transport:

- **Conduction:** Heat is transmitted directly from one material to an adjoining material (or across parts of the same material) if there is a temperature difference between the two. There is no movement of the material.
- **Convection:** Movement of particles through a substances (often particles that comprise a fluid) that transport their heat energy along with them.
 - The Archimedes principle describes buoyancy based on the material’s density ρ , g and the volume of the the fluid displaced by the object, V :

$$F_b = \rho g V$$

- In convection, heating a particle causes an increase in bond size (by the LJP), hence the density decreases. This causes it to displace gas of a larger density, hence it rises.
- **Radiation:** Emission of electromagnetic radiation by all bodies which have heat (i.e. all bodies full stop), depending on their temperature.

1.1 Heat Baths

A heat bath is an object with a sufficiently large heat capacity, C such that a large change in its heat energy will result in a negligible change in temperature.

For example, a hot object placed in a very large bath of water (i.e. a whole swimming pool), or a building placed on a planet.

2 Conduction

Suppose we have a small uniform rod of length L and cross-sectional area A linking two heat baths. The rod is conducting and the heat baths have temperature T_1, T_2 .

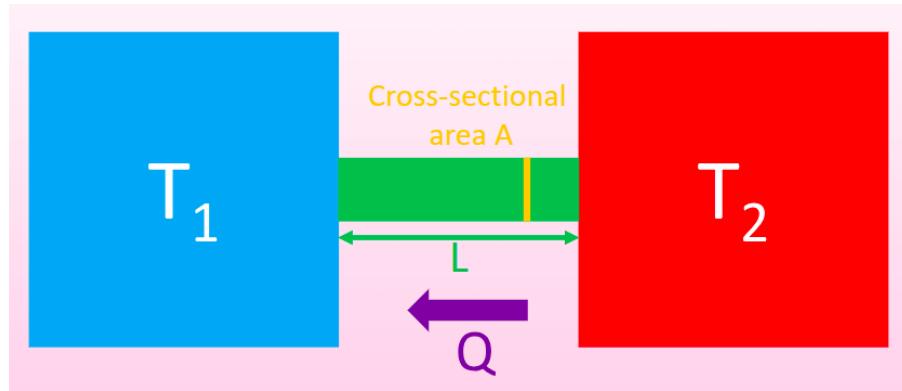


Figure 12.1

Eventually, we reach a “steady state” where all points on the road have a \dot{Q} which does not change with time.

$$\frac{d\dot{Q}}{dt} = \dot{Q} = \text{constant}, \quad \forall x \text{ along rod (in steady state)}$$

What does this value of \dot{Q} depend on? For a small section dx of the rod:

- Temperature change of the small section, dT .
- Cross-sectional area of the rod A .
- The “thermal conductivity” of the rod, κ^1 . It has units $\text{Wm}^{-1}\text{K}^{-1}$.

Putting these together gives us Fourier’s Law:

$$\dot{Q} = -\kappa A \frac{dT}{dx}$$

As T_2 is hotter, the flow of heat is from right to left. Therefore until thermalisation occurs, temperature increases from left to right across the rod ($\frac{dT}{dx} > 0$). However, the heat flow is in the opposite direction from right to left, hence the negative sign in the formula for \dot{Q} .

We want to show that $\frac{dT}{dx} = \frac{T_2 - T_1}{L}$:

$$\begin{aligned} \dot{Q} &= -\kappa A \frac{dT}{dx} \\ \frac{dT}{dx} &= -\frac{\dot{Q}}{\kappa A} \\ - \int \frac{\dot{Q}}{\kappa A} dx &= \int dT \\ - \int_0^L \frac{\dot{Q}}{\kappa A} dx &= \int_{T_1}^{T_2} dT \\ -\frac{\dot{Q}}{\kappa A} [x]_0^L &= T_2 - T_1 \\ \frac{dT}{dx} &= \frac{T_2 - T_1}{L} \end{aligned}$$

2.1 Thermal Conductivity

A higher value of κ means a material is more thermally conductive, for example:

- Diamond (natural): $\kappa = 2200 \text{ Wm}^{-1}\text{K}^{-1}$.
- Copper: $\kappa = 400 \text{ Wm}^{-1}\text{K}^{-1}$.
- Air: $\kappa = 0.02 \text{ Wm}^{-1}\text{K}^{-1}$.

¹We also have another very similar quantity labelled K which will be covered later and may be a source of confusion

- Aerogel: $\kappa = 0.003 \text{ W m}^{-1} \text{ K}^{-1}$.

Diamond is highly thermally conductive due to high frequency vibrations (“phonons”) being able to propagate through the material as a result of its rigidity.

There are two sources of heat conductivity, one being as a result of electrons and one as a result of phonons.

- Inside the material, we have two bands - valence and conduction. These bands are a thick collection of many energy levels very close together. As the Pauli exclusion principle says that no two electrons can share a quantum number, each level must shift infinitesimally to not overlap. The levels within each band are so close that we treat electron as being able to move freely between them.
- The valence band is where electrons are still bound to an atom, the conduction band is where they are bound to the material as a whole and are free to move through the material.
- For an insulator, the difference between the two bands is $\sim 10 \text{ eV}$. This represents a very small proportion of particles with sufficient energy and results in very little thermal excitation (hence little thermal conduction). As an electron is excited, it leaves a hole behind (that we treat as a particle h^+ for ease). We treat this hole as being able to move (as electrons themselves move).
- For an electron to fall back to the valence band, it must fall into a hole. In an insulator the conduction band is effectively empty. For a semiconductor, this gap is more like $\sim 1 \text{ eV}$, so a much higher number of electrons can jump, but this is still temperature dependant.
- In metals, the conduction and valence bands have little to no gap. Therefore, all electrons are free conduction electrons.

Diamonds sit on the boundary between being an insulator and a semiconductor. They work by phonons. As we heat a material, bonds in the material vibrate. As diamond is very rigid, vibrating one bond causes a ripple effect - effectively an wave of excitation (where one of these waves is called a phonon). This phonon propagation causes the transfer of vibrations hence the transfer of heat energy.

2.2 Thermal Properties

We can also define the:

- Thermal Resistivity: $\rho = 1/\kappa$.
- Thermal Conductance $K = \kappa A/L$.²
- Thermal Resistance: $R = L/(\kappa A)$.

This are the heat analogues of the equivalent terms for electrical conductance in a circuit.

Annoyingly, thermal conductivity also varies with temperature...It also does so in a really weird way depending on the material:

²This is the aforementioned annoying symbol overlap between κ and K

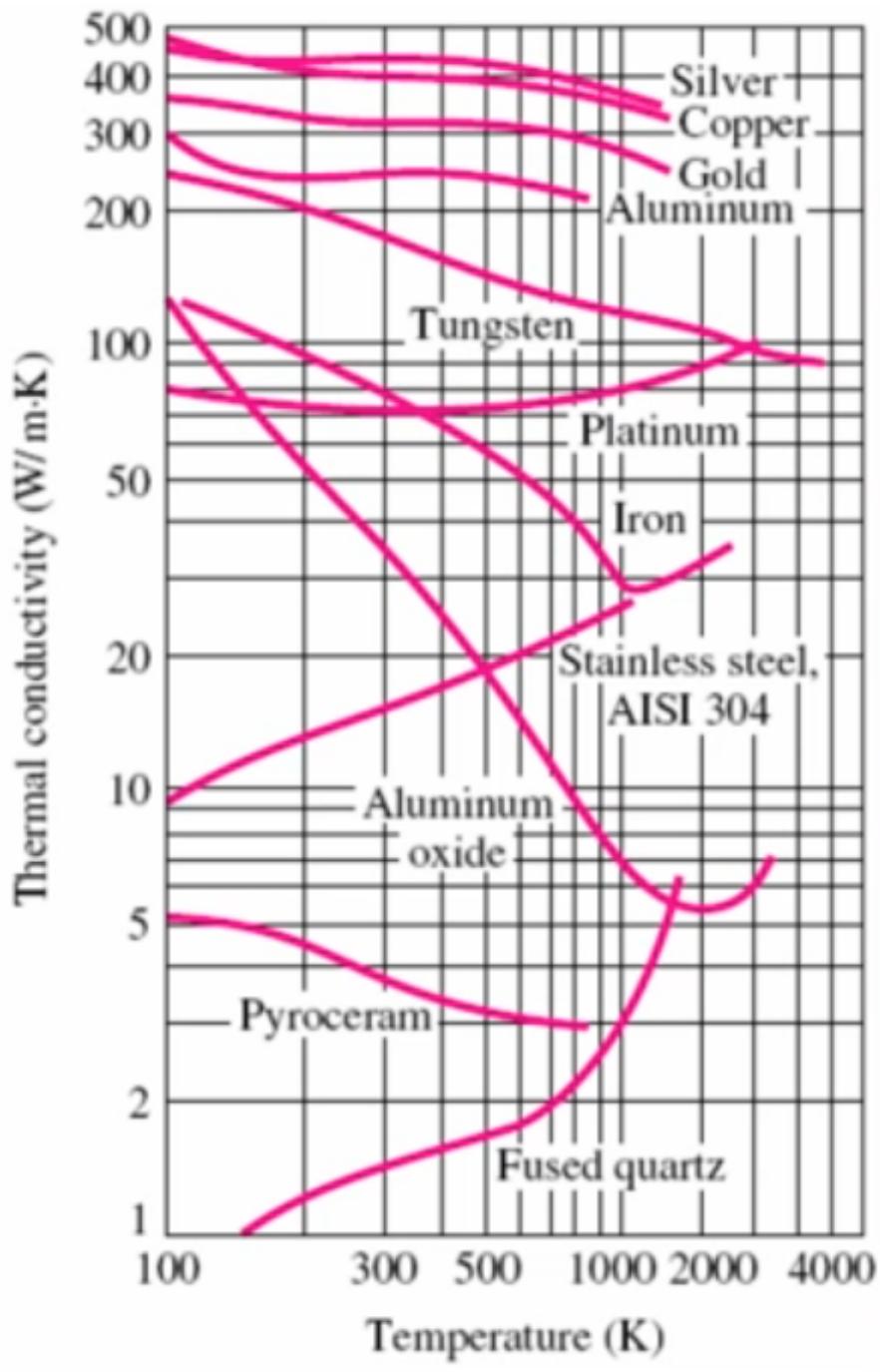


Figure 12.2: κ for various materials, as a function of temperature.

This comes from phonon excitation and electron excitation being two different competing process that both have different degrees of contribution at different temperatures.

This means that Newton's Law of Cooling no longer holds for $\kappa = f(T)$, as the proportionality no longer holds...