Note Title 2021-11-08

Derivation of momentum operator in position basis

$$|T(t)\rangle = \hat{p}\,|S(t)\rangle$$

$$\langle x|T(t)\rangle = \langle x|\hat{p}\,|S(t)\rangle =$$

expanding p

$$\Psi_T(x,t) = \langle x|\hat{p}\left(\int dp'\,|p'\rangle\langle p'|\right)|S(t)\rangle = \langle x|\int dp'\,p'\,|p'\rangle\langle p'|\int dx'\,|x'\rangle\underbrace{\langle x'|S(t)\rangle}_{=}$$

$$= \int dx'\int dp'\,p'\,\langle x|p'\rangle\langle p'|x'\rangle\,\Psi_S(x',t)$$

$$= \frac{1}{2\pi\hbar}\int dx'\int dp'\,p'\,\underbrace{e^{\frac{ip'(x-x')}{\hbar}}}_{\text{rewrite}}\,\Psi_S(x',t) \qquad p'' \triangleq \frac{p'}{\hbar}$$

$$= \frac{1}{2\pi\hbar}\int dx'\int \hbar\,dp''\,\hbar p''\,e^{ip''(x-x')}\,\Psi_S(x',t)$$

$$= \frac{i\hbar}{2\pi}\int dx'\left(\frac{d}{dx'}\int dp''\,e^{ip''(x-x')}\right)\Psi_S(x',t)$$

isn't this using pos. operator?

$$\Psi_T(x,t) = i\hbar \int dx' \left(\frac{d\,\delta(x-x')}{dx'}\right) \Psi_s(x',t) \quad *$$

$$u \Rightarrow du = \frac{d\,\Psi_s(x',t)}{dx'}\,dx'$$

$$v\,dv \Rightarrow v = \delta(x-x')$$

So

$$\Psi_T(x,t) = i\hbar \left\{ \Big(\Psi_s(x',t)\,\delta(x-x') \Big)_{-\infty}^{\infty} - \int \frac{d\,\Psi_s(x',t)}{dx'}\,\delta(x-x')\,dx' \right\}$$

$$= -i\hbar \frac{d\,\Psi_s(x,t)}{dx}$$

$\Rightarrow$ $\hat{p}$ in position basis $\equiv -i\hbar \dfrac{d}{dx}$

Note also from *

$$\langle x|\hat{p}|S(t)\rangle = \int dx'' \int dx' \, \langle x|x''\rangle \langle x''|\hat{p}|x'\rangle \langle x'|S(t)\rangle$$

$$= \int dx'' \int dx' \; \Psi_x^*(x'', t) \, \langle x''|\hat{p}|x'\rangle \, \Psi_S(x', t)$$

$$= \int dx' \; \langle x|\hat{p}|x'\rangle \, \Psi_S(x', t)$$

$$\Rightarrow \quad \langle x|\hat{p}|x'\rangle = i\hbar \frac{d}{dx'} \delta(x-x') = -i\hbar \frac{d\,\delta(x-x')}{dx}$$