Bra-Ket notation (Valid and invalid Combinations)

- Remember from last two classes that Ket vectors which ultimately become "states" in QM have Column Vector representations.

Now Consider three Ket vectors $|\phi\rangle, |\psi\rangle, |\chi\rangle$ that can be represented as (n x 1) column vector. Note that the Corresponding bra vectors $\langle\phi|, \langle\psi|, \langle\chi|$ Can be written as (1 x n) row vectors. Keeping this in mind and $\cdot$ implying regular matrix multiplication, round the valid expressions from the list below: (Also point out their own dimensions.)

1. $\cancel{|\phi\rangle} \cdot |\chi\rangle$
2. $|\chi\rangle \cdot \langle\psi|$
3. $\langle\phi| \cdot |\psi\rangle$
4. $\cancel{|\phi\rangle \cdot |\chi\rangle} \cdot \langle\psi|$
5. $|\phi\rangle \cdot \langle\chi| \cdot |\psi\rangle$
6. $\langle\phi| \cdot |\psi\rangle \cdot \langle\chi|$
7. $\langle\psi| \cdot |\phi\rangle \cdot |\chi\rangle$
8. $|\phi\rangle \cdot \langle\chi| \cdot |\psi\rangle$
9. $\langle\psi| \cdot \langle\phi| \cdot |\chi\rangle \cdot |\phi\rangle$
10. $\langle\chi| \cdot |\phi\rangle \cdot \langle\chi| \cdot |\psi\rangle$

A simple example of an operator and it's eigenbasis

- Remember from the last lecture that operators take Ket vectors to Ket vectors. Now consider a specific operator $\hat{\sigma}_y$ being represented by:

$$\hat{\sigma}_y \cong \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

(Also known as the Pauli-Y matrix).

(i) What is the dimension of the vector space of Ket vectors $\hat{\sigma}_y$ acts on?

2x1

(ii) What is the simpliest choice of a basis for the above vector space? (Denote the basis vectors as $|\phi_0\rangle, |\phi_1\rangle, |\phi_2\rangle \ldots\ldots$)

$$|\phi_0\rangle \cong \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\phi_1\rangle \cong \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|v\rangle = a|\phi_0\rangle + b|\phi_1\rangle$$

arbitrary

(iii) Represent $\hat{\sigma}_y$ as linear combination of all possible outer products between the basis vectors!

$$\hat{\sigma}_y = -i|\phi_0\rangle\langle\phi_1| + i|\phi_1\rangle\langle\phi_0|$$

(iv) Is $\hat{\sigma}_y$ a Hermitian operator?

$\Rightarrow$ Yes! $\hat{\sigma}_y^\dagger = \hat{\sigma}_y$

(V) What are the eigenvalues and eigenvectors of $\hat{\sigma}_y$?

$\Rightarrow$ Solve the eigenvalue problem.

Eigenvalue : $+1$: $\begin{pmatrix} 1 \\ i \end{pmatrix} \cong |+_y\rangle$

Eigenvalue: -1 : $\begin{pmatrix} 1 \\ -i \end{pmatrix} = |-_y\rangle$

(vi) Are they orthogonal? Can you make them orthonormal?

Yes! To normalise, we have

$$\frac{1}{\sqrt{|1|^2 + |i|^2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\frac{1}{\sqrt{|1|^2 + |-i|^2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

(vii) Do they form an ONB?

Yes, any two orthonormal vectors in 2D space form a basis.