Biostat 602 **Assignment 2** **Due: Thursday, January 19**

Turn in the homework problems only.

Homework Problems

1. Let $X_1, \cdots, X_n$ be *i.i.d.* random variables from the probability density function of the following form:

$$f_X(x|\mu,\sigma) = \frac{1}{\sigma}e^{-(x-\mu)/\sigma}, \quad \mu < x < \infty, 0 < \sigma < \infty.$$

   (a) Assuming that μ is known, find a one-dimensional sufficient statistic for σ.

   (b) Assuming that σ is known, find a one-dimensional sufficient statistic for μ.

   (c) Assuming that that both parameters are unknown, find a two-dimensional sufficient statistic for (μ,σ).

2. Let $X_1, \cdots, X_n$ be *i.i.d.* random variables from $N(0,\sigma^2)$ with the pdf

$$f_X(x|\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{x^2}{2\sigma^2}\right\}, \quad -\infty < x < \infty, \sigma^2 > 0$$

   (a) Apply the Factorization Theorem to show that $\sum_{i=1}^n X_i^2$ is a sufficient statistic for the parameter σ^2.

   (b) Is $\sum_{i=1}^n X_i^2$ also a minimal sufficient statistic for σ^2? Justify your answer.

3. Let $X_1, \cdots, X_n$ be *i.i.d.* random variables from a Poisson distribution whose probability mass function is given by

$$f_X(x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, \ x = 0, 1, 2, \ldots; \lambda > 0.$$

   (a) Find a one-dimensional sufficient statistic for parameter λ.

   (b) Show that your answer in (a) is also a minimal sufficient statistic.

4. Let $X_1, \cdots, X_n$ be a random sample from $Beta(\alpha,\beta)$. Find joint sufficient statistics for (α,β).

5. Let $X_1, \cdots, X_n$ be a random sample from $Cauchy(\theta, 1)$. Find a minimal sufficient statistic for θ.

6. Let $X_1, \cdots, X_n$ be a random sample from $Uniform(-\theta,\theta)$. Find a minimal sufficient statistic for θ.

Practice Problems

(a) C&B Exercise 6.1

(b) C&B Exercise 6.5

(c) C&B Exercise 6.6

(d) C&B Exercise 6.7

(e) C&B Exercise 6.16(b)