Biostat 602 **Assignment 3** **Due: Thursday, January 26**

Turn in the homework problems only.

Homework Problems

1. Let $X_1, \cdots, X_n$ be *i.i.d.* random variables from the probability density function of the following form:

$$f_X(x|\theta) = \begin{cases} \frac{2x}{\theta^2}, & 0 < x < \theta \\ 0, & \text{otherwise} \end{cases}$$

where $\theta > 0$. Find a minimal sufficient statistic for θ.

2. Suppose that $X_1, \cdots, X_n$ are *i.i.d.* random variables from pdf

$$f_X(x|\theta) = \theta x^{\theta-1} \exp\left(-x^\theta\right)$$

where $\theta > 0$, $x > 0$. Show that $(\log X_{(n)})/(\log X_{(1)})$ is an ancillary statistic.

3. Let $X_1, \cdots, X_n$ be *i.i.d.* random variables from a uniform distribution Uniform$(-\theta, \theta)$ with the pdf given by

$$f_X(x|\theta) = \frac{1}{2\theta} I\left(-\theta < x < \theta\right), \qquad \theta > 0$$

   (a) Is the two dimensional statistic $T_1(\mathbf{X}) = \left(X_{(1)}, X_{(n)}\right)$ a complete sufficient statistic? Justify your answer.

   (b) Is the one-dimensional statistic $T_2(\mathbf{X}) = \max_i \{|X_i|\}$ a complete sufficient statistic? Justify your answer.

4. Let $X_1, \cdots, X_n$ be *i.i.d.* random variables from $N(\mu, \sigma^2)$ population with <u>known μ</u>. Find a one-dimensional minimal sufficient statistic for σ^2.

5. Let $X_1, \cdots, X_n$ be *i.i.d.* observations uniformly drawn from $\{1, 2, \cdots, \theta\}$, where θ is a positive integer. This corresponds to a discrete uniform with pmf

$$f_X(x|\theta) = \begin{cases} 1/\theta & x = 1, 2, \cdots, \theta \\ 0 & \text{otherwise} \end{cases}$$

Show that $T(\mathbf{X}) = \max_i X_i$ is a complete, minimal sufficient statistic.

{ **Hint:** Showing completeness is somewhat involved. The direct way of doing that involves the followings two steps.

Step 1: Find the pmf of $T(\mathbf{X})$, i.e. find

$$\Pr(\max_i X_i = k) \quad \text{for } k = \{1, \cdots, \theta\}.$$

Step 2: Let g be any function. Set up the equation

$$E\left[g(T)\right] = \sum_{k=1}^{\theta} g(k) \Pr(\max_i X_i = k) = 0. \tag{1}$$

Since this identity is true for all θ, plug in (1) $\theta = 1, 2, \cdots$ etc. and establish that $g(k) = 0$ for all k. }

Practice Problems

(a) C&B Exercise 6.8

(b) C&B Exercise 6.9

(c) C&B Exercise 6.14

(d) C&B Exercise 6.17

(e) C&B Exercise 6.20

(f) C&B Exercise 6.22

(g) C&B Exercise 6.31