Biostat 602 **Assignment 4** **Due: Thursday, February 2**

Turn in the homework problems only.

Homework Problems

1. For each of the following statements, state whether it is *True* or *False*. If it is True, provide brief justification. If it is False, provide a counter-example.

   (a) Minimal sufficient statistics is always a function of the order statistics.

   (b) Any function of a minimal sufficient statistic is sufficient.

   (c) A complete statistic is always a one-to-one function of a minimal sufficient statistic.

   (d) One-to-one function of an ancillary statistic is ancillary.

   (e) If $X_1, \cdots, X_n$ is a random sample from a pdf conforming to a location family, then $\overline{X}$ also has a pdf that belongs to a location family.

2. Let X_1, X_2 be two *i.i.d.* random variables from the probability density function of the following form:

$$f_X(x|\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{x^2}{2\sigma^2}\right\}, \qquad x \in R, \sigma^2 > 0$$

   (a) Find a complete sufficient statistic for σ^2. Justify your answer.

   (b) Show that $\dfrac{X_1}{\sqrt{X_1^2 + X_2^2}}$ is an ancillary statistic.

   (c) Show that the expectation of the ancillary statistic in part (b) is zero, namely $\mathrm{E}\left[\dfrac{X_1}{\sqrt{X_1^2 + X_2^2}}\right] = 0$.

3. Let $X_1, \cdots, X_n$ be *i.i.d.* observations from a gamma distribution Gamma(α, β) with the pdf

$$f_X(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \qquad 0 \le x < \infty,\ \alpha > 0,\ \beta > 0$$

   (a) Show that the gamma distribution belongs to a full exponential family. Identify $h(\cdot), c(\cdot), w_j(\cdot), t_j(\cdot)$ terms in your proof.

(b) Find a complete sufficient statistic for $\boldsymbol{\theta} = (\alpha, \beta)$ and justify whether it is also a minimal sufficient statistic or not.

4. Let $X_1, \cdots, X_n$ be *i.i.d.* random variables from the two-parameter exponential distribution with probability density function of the following form:

$$f_X(x|\mu, \sigma) = \frac{1}{\sigma} e^{-(x-\mu)/\sigma}, \quad \mu < x < \infty, 0 < \sigma < \infty.$$

Consider the statistics $T_1 = X_{(1)}$ and $T_2 = X_{(1)} - \overline{X}$. Show that T_1 and T_2 are independently distributed.

5. Consider the previous problem again.

(a) Assuming σ is known, find a method of moments estimator of μ by using the lowest-order moment of X as possible.

(b) Assuming μ is known, find a method of moments estimator of σ by using the lowest-order moment of X as possible.

(c) Assuming both parameters are unknown, find a method of moments estimator of σ by using the lowest-order moments of X as possible.

(d) Assuming both parameters are unknown, find a method of moments estimator of the survival function $S(t) = \Pr(X > t)$ for a given $t > \mu$, assuming that X follows the pdf above. You may define a new random variable and use its moments.

Practice Problems

(a) C&B Exercise 6.20

(b) C&B Exercise 6.21

(c) C&B Exercise 6.27

(d) C&B Exercise 6.30