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FOR COMPUTATIONAL ENGINEERING & SCIENCES

FALL 2024

MEETING #4

Computational Modeling in Engineering and the Sciences
Computer Science Undergraduate Directed Reading Program

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AGENDA

- Discussion
- Physics review
- Formulating the 2-body problem
- Project structure
- Demo: creating project skeleton

Assignment: Skim the papers and articles posted in GitHub about Moriba Jah, an astrodynamicist and self-described “space environmentalist.” You may also want to take a look at his website and recent publications. What kinds of tools are used in the field of astrodynamics? What problems are we trying to solve? Who are the stakeholders beyond academia?

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Discussion: Find a paper where a system of Ordinary Differential Equations is used to solve a problem. Is it linear or nonlinear? What techniques were used to solve the system?

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PHYSICS REVIEW

- Kinematics
- Newton’s First Law
- Newton’s Law of Universal Gravitation
- Newton’s Third Law

PHYSICS REVIEW

Kinematics for Point Masses

$$\mathbf{r}$$

$$\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$$

$$\mathbf{a} = \frac{\mathrm{d}\mathbf{a}}{\mathrm{d}t} = \frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}$$

PHYSICS REVIEW

Newton's First Law

$$\sum \mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$$

PHYSICS REVIEW

Newton's First Law

$$\sum \mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$$
$$= \frac{\mathrm{d}(m\mathbf{v})}{\mathrm{d}t}$$

PHYSICS REVIEW

Newton's First Law

$$\sum \mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$$
$$= \frac{\mathrm{d}(m\mathbf{v})}{\mathrm{d}t}$$
$$= \frac{\mathrm{d}m}{\mathrm{d}t}\mathbf{v} + m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$$

PHYSICS REVIEW

Newton's First Law

$$\begin{aligned}\sum \mathbf{F} &= \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} \\ &= \frac{\mathrm{d}(m\mathbf{v})}{\mathrm{d}t} \\ &= \frac{\mathrm{d}m}{\mathrm{d}t}\mathbf{v} + m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} \\ &= m\mathbf{a}\end{aligned}$$

PHYSICS REVIEW

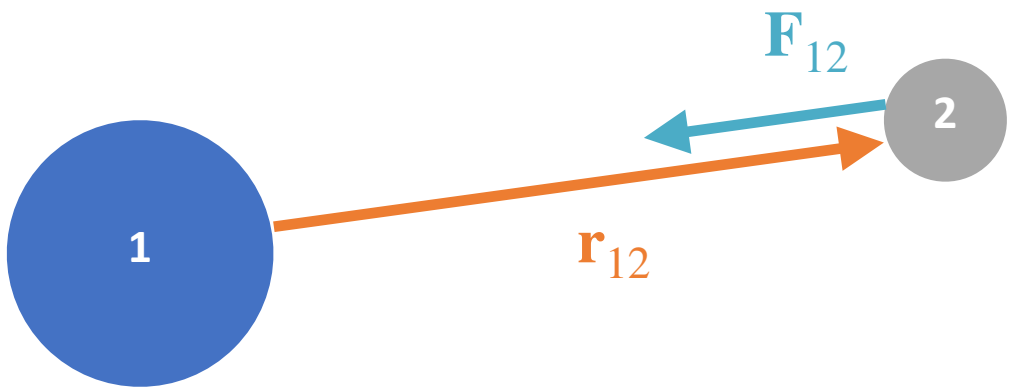
Newton's Law of Universal Gravitation

"Force by 1 on 2"

"Vector from 1 to 2"

$$\mathbf{F}_{12} = -\frac{Gm_1m_2}{\|\mathbf{r}_{12}\|^2}\hat{\mathbf{r}}_{12}$$

$$= -\frac{Gm_1m_2}{\|\mathbf{r}_{12}\|^3}\mathbf{r}_{12}$$



PHYSICS REVIEW

Newton's Third Law

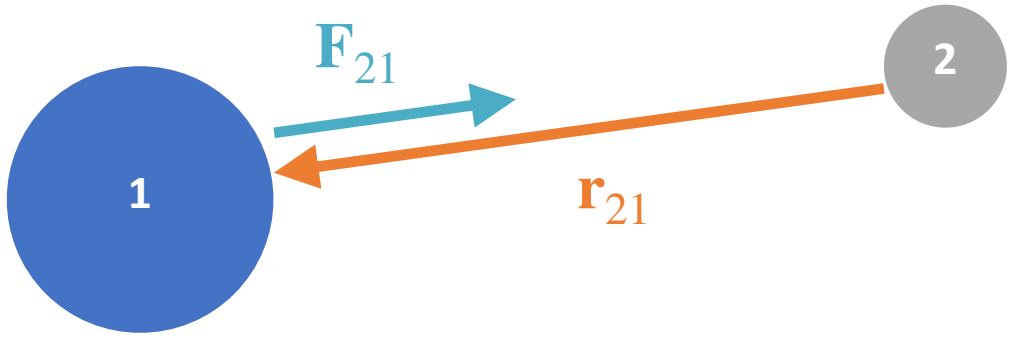
"Force by 2 on 1"

"Vector from 2 to 1"

What if $m_1 \gg m_2$?

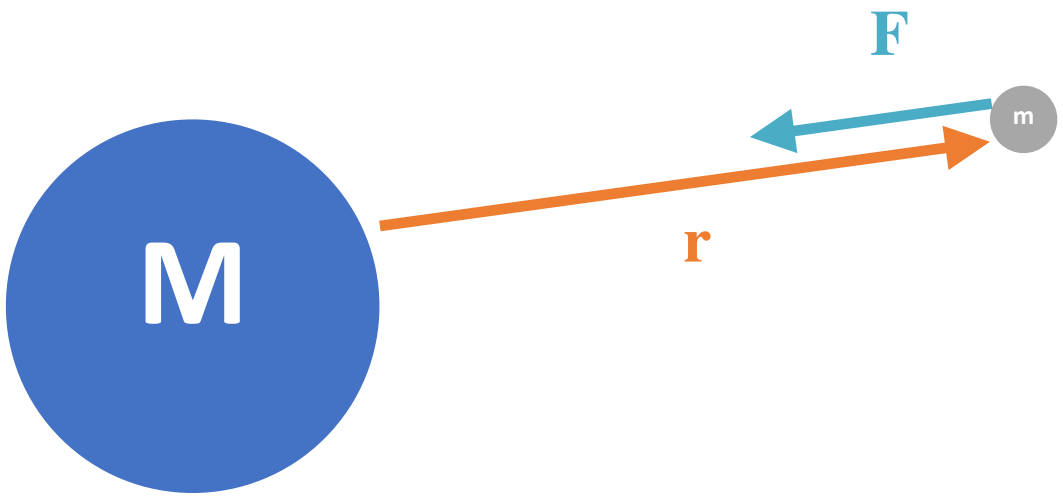
$$\mathbf{F}_{21} = -\frac{Gm_2m_1}{\|\mathbf{r}_{21}\|^2}\hat{\mathbf{r}}_{21}$$

$$= -\frac{Gm_2m_1}{\|\mathbf{r}_{21}\|^3}\mathbf{r}_{21}$$



FORMULATING THE 2-BODY PROBLEM

What if $m_1 \gg m_2$?

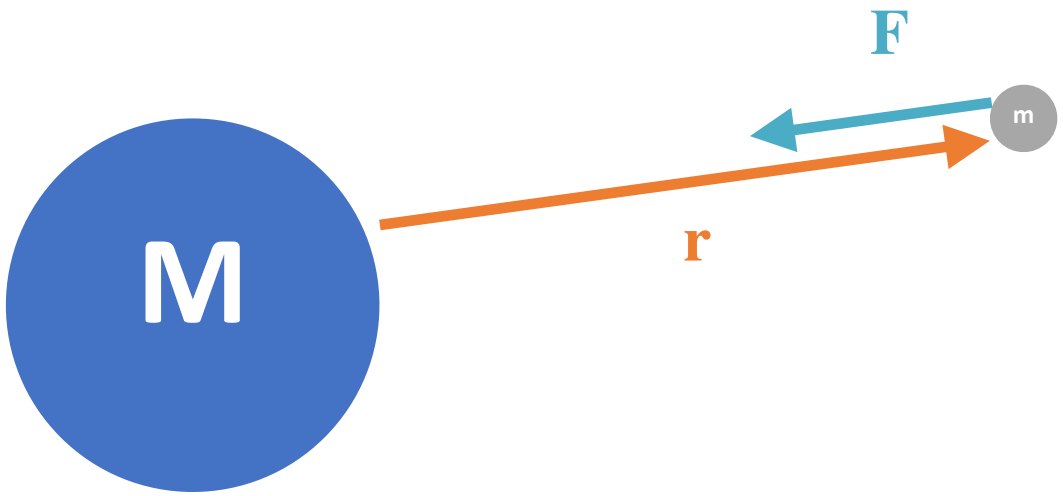


M is probably not going to move much, so let's fix its position and use it as the origin of our coordinate system.

Is this a reasonable assumption?

FORMULATING THE 2-BODY PROBLEM

What if $m_1 \gg m_2$?



M is probably not going to move much, so let's fix its position and use it as the origin of our coordinate system.

Is this a reasonable assumption?

Is it??

FORMULATING THE 2-BODY PROBLEM

Defining our differential equation

What is our “state vector?”

Derivation time!

$$\frac{d\mathbf{U}}{dt} = \mathbf{f}(\mathbf{U})$$

FORMULATING THE 2-BODY PROBLEM

Defining our differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} r_1 \\ r_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ -\frac{GM}{\|\mathbf{r}\|^3} r_1 \\ -\frac{GM}{\|\mathbf{r}\|^3} r_2 \end{bmatrix}$$

FORMULATING THE 2-BODY PROBLEM

Defining our differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} r_1 \\ r_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{GM}{\|\mathbf{r}\|^3} & 0 & 0 & 0 \\ 0 & -\frac{GM}{\|\mathbf{r}\|^3} & 0 & 0 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ v_1 \\ v_2 \end{bmatrix}$$

FORMULATING THE 2-BODY PROBLEM

Defining our differential equation

$$\frac{d\mathbf{U}}{dt} = \mathbf{f}(\mathbf{U})$$

FORMULATING THE 2-BODY PROBLEM

Defining our differential equation

$$\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}t} = \mathbf{A}(\mathbf{r})\mathbf{U}$$

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PROJECT STRUCTURE: SO YOU WANT TO BUILD A SIMULATION?

- Driver script: runs the show
- Problem setup
  - Parameter inputs (from file?)
  - Defining governing equations
  - Building your matrix
  - Applying constraints and boundary conditions
- Solution loop
- Output (to file?)
- Visualization and post-processing
- Error analysis and convergence studies
- Test scripts

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DEMO: CREATING PROJECT SKELETON

Demo time!

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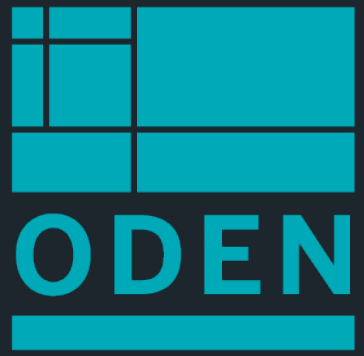

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