ODEN INSTITUTE
FOR COMPUTATIONAL ENGINEERING & SCIENCES

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MEETING #2

Computational Modeling in Engineering and the Sciences

ASHTON COLE
Graduate Student, Computational Hydraulics Group, The University of Texas at Austin

AGENDA

- Discussion of Konstantinovsky - Mastering the SEIR Model

- Introduction to ODEs

- Demo: ODEs in 1D

Assignment: Read first chapter of Baigent - Lotka-Volterra Dynamics. We'll discuss next week.

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Discussion of Konstantinovsky

Introduction to ODEs

Introducing ODEs

- **ordinary differential equation:** a differential equation with only one **independent** variable
- Introducing the IVP:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = f(u, t)$$
$$u(t_0) = u_0$$
$$u(t) = ?$$

Introducing ODEs

Reintroducing a thing from Calculus you probably thought you would never see again in your life

- **Taylor Series** are very helpful to derive and evaluate derivative approximations.
  - E.g., a polynomial approximating $x(t)$ centered at $t = \tau$
- The nth derivative of this polynomial is exact at τ and is usually a reasonable guesstimate in the neighborhood.

$$u(t) \approx p_\tau(t) = u(\tau) + \frac{du}{dt}\bigg|_\tau (t - \tau) + \frac{1}{2}\frac{d^2u}{dt^2}\bigg|_\tau (t - \tau)^2 + \frac{1}{3!}\frac{d^3u}{dt^3}\bigg|_\tau (t - \tau)^3$$

$$+ \dots + \frac{1}{n!}\frac{d^nu}{dt^n}\bigg|_\tau (t - \tau)^n + \dots$$

Introducing ODEs

Deriving integration methods from finite differences
- Limit definitions of derivative are good place to start
  - What if h is small?
- Elaborate schemes are less intuitive, but when in doubt…

$$\frac{\mathrm{d}u}{\mathrm{d}t}\bigg|_{\tau} := \lim_{h \to 0} \frac{u(\tau + h) - u(\tau)}{h}$$

$$= \lim_{h \to 0} \frac{u(\tau) - u(\tau - h)}{h}$$

$$= \lim_{h \to 0} \frac{u(\tau + h) - u(\tau - h)}{2h}$$

Introducing ODEs

Deriving integration methods from finite differences

- Limit definitions of derivative are good place to start
  - What if h is small?
- Elaborate schemes are less intuitive, but when in doubt…

$$\frac{\mathrm{d}u}{\mathrm{d}t}\bigg|_{\tau} := \lim_{h \to 0} \frac{u(\tau + h) - u(\tau)}{h} \qquad\qquad = \frac{u(\tau + h) - u(\tau)}{h} + \mathcal{O}(h)$$

$$= \lim_{h \to 0} \frac{u(\tau) - u(\tau - h)}{h} \qquad\qquad = \frac{u(\tau) - u(\tau - h)}{h} + \mathcal{O}(h)$$

$$= \lim_{h \to 0} \frac{u(\tau + h) - u(\tau - h)}{2h} \qquad\qquad = \frac{u(\tau + h) - u(\tau - h)}{2h} + \mathcal{O}(h^2)$$

Introducing ODEs

Deriving integration methods from finite differences

- Forward Euler method

$$\frac{\mathrm{d}x}{\mathrm{d}t}\bigg|_t \approx \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

$$x(t + \Delta t) = x(t) + \Delta t f(x(t), t)$$

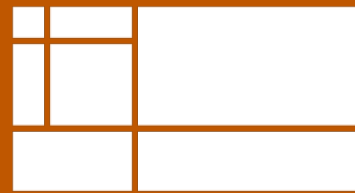$$x_{n+1} = x_n + \Delta t f(x_n, t_n)$$

Introducing ODEs

Deriving integration methods from finite differences

- Backward Euler method

$$\frac{\mathrm{d}x}{\mathrm{d}t}\bigg|_t \approx \frac{x(t) - x(t - \Delta t)}{\Delta t}$$

$$x(t) = x(t - \Delta t) + \Delta t f(x(t), t)$$

$$x_{n+1} = x_n + \Delta t f(x_{n+1}, t_{n+1})$$

Demo: ODEs in 1D

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\alpha u, \quad \alpha > 0$$

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