ODEN INSTITUTE
FOR COMPUTATIONAL ENGINEERING & SCIENCES

FALL 2024

MEETING #3

Computational Modeling in Engineering and the Sciences
Computer Science Undergraduate Directed Reading Program

ASHTON COLE
Graduate Student Fellow, The University of Texas at Austin

AGENDA

- Discussion
- Results from last week
- Outline moving forwards
- Introduction to ODEs in ND
- Demo: programming explicit Euler for ODE's

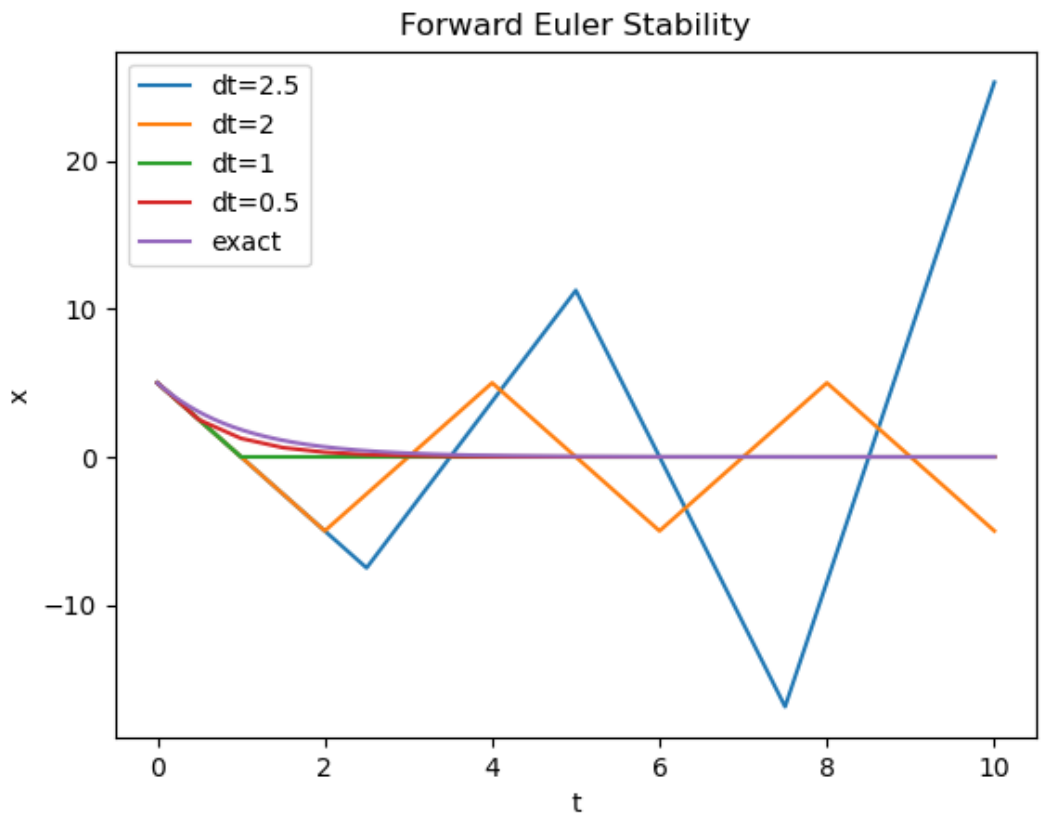
Assignment: Find a paper where a system of Ordinary Differential Equations is used to solve a problem. Is it linear or nonlinear? What techniques were used to solve the system?

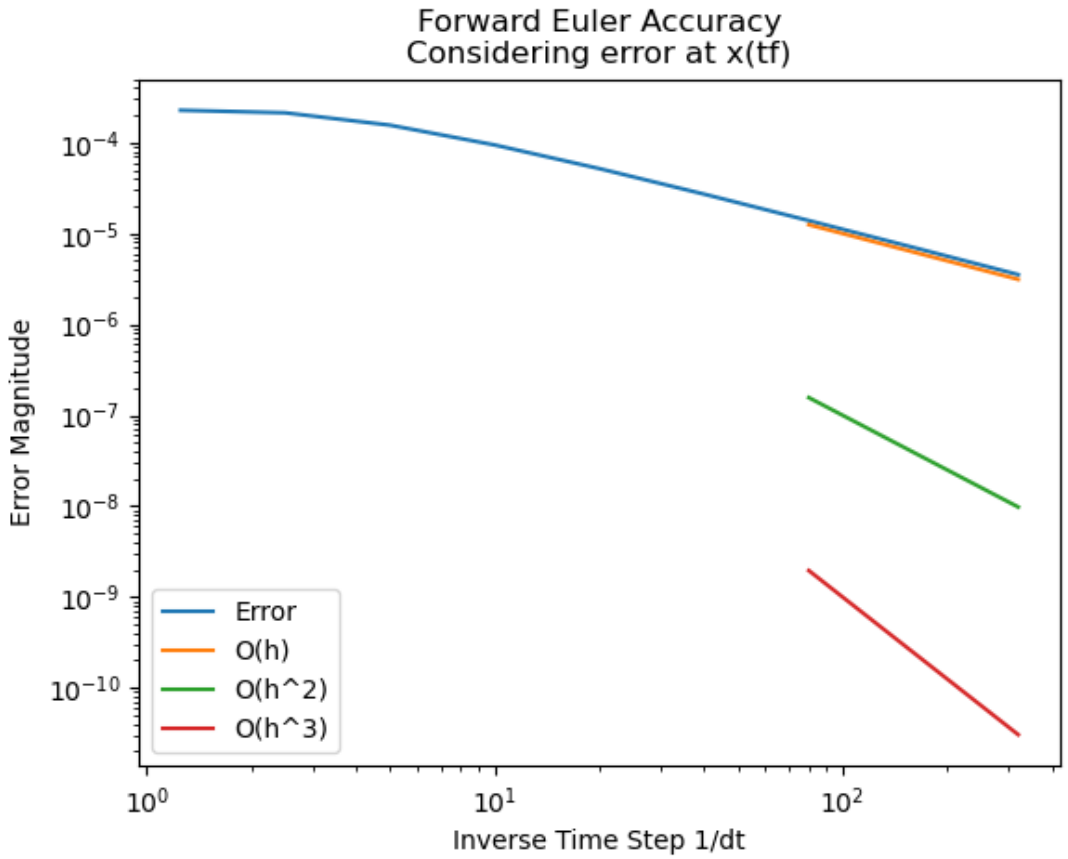
ODEN INSTITUTE
FOR COMPUTATIONAL ENGINEERING & SCIENCES

Discussion: Find a paper discussing high-order integration methods for ordinary differential equations, e.g., Runge-Kutta, Adams-Bashforth, &c. Look for the motivation, advantages, disadvantages, and broader context of the method.

ODEN INSTITUTE
FOR COMPUTATIONAL ENGINEERING & SCIENCES

RESULTS FROM LAST WEEK





ODEN INSTITUTE
FOR COMPUTATIONAL ENGINEERING & SCIENCES

OUTLINE MOVING FORWARDS

- Develop some simple integrators
- Discuss simulation structure
- Astrodynamics
- 2 body problem
- N body problem
- Next project?

ORDINARY DIFFERENTIAL EQUATION(S)

ordinary differential equation
a differential equation with only one **independent** variable

Introducing the IVP:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x, t)$$
$$x(t_0) = x_0$$
$$x(t) = ?$$

ORDINARY DIFFERENTIAL EQUATION(S)

system of
ordinary differential equations

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{f}(\mathbf{x}, t)$$
$$\mathbf{x}(t_0) = \mathbf{x}_0$$
$$\mathbf{x}(t) = ?$$

ORDINARY DIFFERENTIAL EQUATION(S)

linear system of ordinary differential equations

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{A}\mathbf{x}$$
$$\mathbf{x}(t_0) = \mathbf{x}_0$$
$$\mathbf{x}(t) = ?$$

ORDINARY DIFFERENTIAL EQUATION(S)

Forward Euler method

$$\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_t \approx \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$x(t+\Delta t) = x(t) + \Delta t f(x(t), t)$$

$$x_{n+1} = x_n + \Delta t f(x_n, t_n)$$

ORDINARY DIFFERENTIAL EQUATION(S)

Forward Euler method

$$\left.\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}\right|_t \approx \frac{\mathbf{x}(t+\Delta t)-\mathbf{x}(t)}{\Delta t}$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t\,\mathbf{f}(\mathbf{x}(t), t)$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \Delta t\,\mathbf{f}(\mathbf{x}_n, t_n)$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \Delta t\,\mathbf{A}\mathbf{x}_n$$

ORDINARY DIFFERENTIAL EQUATION(S)

Backward Euler method

$$\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_t \approx \frac{x(t) - x(t - \Delta t)}{\Delta t}$$

$$x(t) = x(t - \Delta t) + \Delta t f(x(t), t)$$

$$x_{n+1} = x_n + \Delta t f(x_{n+1}, t_{n+1})$$

ORDINARY DIFFERENTIAL EQUATION(S)

Backward Euler method

$$\left.\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}\right|_t \approx \frac{\mathbf{x}(t) - \mathbf{x}(t - \Delta t)}{\Delta t}$$

$$\mathbf{x}(t) = \mathbf{x}(t - \Delta t) + \Delta t \mathbf{f}(\mathbf{x}(t), t)$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \Delta t \mathbf{f}(\mathbf{x}_{n+1}, t_{n+1})$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \Delta t \mathbf{A} \mathbf{x}_{n+1}$$

$$\mathbf{x}_{n+1} = (\mathbf{I} - \Delta t \mathbf{A})^{-1} \mathbf{x}_n$$

ODEN INSTITUTE
FOR COMPUTATIONAL ENGINEERING & SCIENCES

DEMO: PROGRAMMING ODE TIME INTEGRATORS

Demo time!

Today, let’s do Explicit Euler and Explicit RK2 for general ODE’s.

ODEN INSTITUTE
FOR COMPUTATIONAL ENGINEERING & SCIENCES

Assignment: Find a paper where a system of Ordinary Differential Equations is used to solve a problem. Is it linear or nonlinear? What techniques were used to solve the system?

ODEN INSTITUTE

FOR COMPUTATIONAL ENGINEERING & SCIENCES