PUBLICATIONS OF THE ASTRONOMICAL SOCIETY OF THE PACIFIC
88:374–376, August 1976

TIDAL EFFECTS INDUCED IN A SPHERICAL GALAXY BY THE PASSAGE OF A COMPANION

WILLIAM J. GUTOWSKI, JR.* AND RICHARD B. LARSON
Yale University Observatory

Received 1976 March 10

A gravitational encounter between a spherical galaxy and a smaller companion is simulated by computing a large number of stellar orbits in the restricted three-body approximation. For the case considered, i.e., galaxies of $10^{11}\,\mathfrak{M}_\odot$ and $5 \times 10^{10}\mathfrak{M}_\odot$ passing in parabolic orbits at a closest distance of 20 kpc, the larger galaxy loses about 8% of its mass, and its outer regions swell up by $\sim$ 50% in radius. These effects may be important in interpreting the measured luminosities and light distributions of elliptical galaxies in cosmological studies.

Key words: galaxies — gravitational encounter — evolution of galaxies

I. Introduction

A number of authors have recently studied the effect of galaxy collisions on the structure of spherical galaxies. Most investigations, such as those of Sastry and Alladin (1970), Gallagher and Ostriker (1972), and Richstone (1975) have considered conditions comparable to those existing in dense clusters, where the relative orbits of interacting galaxies are highly hyperbolic. In this paper we consider a model which is more appropriate for a galaxy which is perturbed by the passage of a smaller companion, for example in a binary or small multiple system where the velocities are more nearly parabolic. Lauberts (1974) has previously done N-body calculations for slow interactions betweeen spherical systems, but only for the case of equal masses.

Here we adopt the restricted three-body approximation to simulate the dynamics of a pair of interacting galaxies. This means that all of the mass of the system is effectively assigned to two point masses which follow Keplerian orbits that can be treated analytically, thus simplifying the calculations. A large number of massless test particles are initially placed in orbit around the larger point mass to simulate the distribution of stellar orbits in a spherical galaxy, and their motions are followed numerically throughout the course of the encounter. After the encounter, the distribution of orbits is again examined to see how the structure and kinematics of the galaxy have been altered.

II. The Model

The parent galaxy is represented by a gravitating core surrounded by a halo of massless test particles, which are initially placed on a series of concentric shells whose radii range from 5 kpc to 35 kpc in steps of 5 kpc. Each shell is populated with 168 stars, which are distributed as follows. On an equatorial ring defined by the intersection of each shell with the plane of the perturber's orbit, eight points are chosen at intervals of $\pi/4$ radians. At each point six stars are placed, and are given velocity vectors pointing in each direction along three mutually perpendicular axes, one normal to the equatorial plane and the other two in the plane and inclined at 45° to the direction to the center; in this way an isotropic velocity distribution is crudely simulated. All velocity vectors are of equal magnitude, taken to be equal to the velocity of a circular orbit. Two additional rings of stars at latitudes $\pm 45°$ from the equatorial plane are then constructed by rotating the velocity configuration from each point in the equatorial plane by $\pm\pi/4$ radians toward the poles. Finally, twelve stars are located at each pole by rotating through $\pm\pi/2$ radians the velocity configurations at two points spaced a quarter of a circle apart on the equatorial ring. Because of symmetry with respect to the equatorial plane, only 108 orbits need actually be calculated for each spherical shell. It is hoped that this procedure gives a sufficiently large and representative sample of stellar orbits to allow a reliable estimation of statistical quantities, such as the fractional mass lost from each shell.

The core of the parent galaxy and the perturbing galaxy are each the source of a softened potential

$$V(r) = -G\mathfrak{M}/(r + a) \quad ,$$

where the core size a is taken to be 1 kpc. A softened potential is used because a few of the stars acquire highly elongated orbits which carry them close to either center of mass; for these cases, a softened potential is more realistic than a point mass and avoids the difficulty of calculating a very close passage near a

*Present address: Princeton University Observatory.

point mass. The mass $\mathfrak{M}$ is taken to be $10^{11}\,\mathfrak{M}_\odot$ for the main galaxy and half this value for the perturber. The perturber is assumed to have a parabolic orbit with a perigalactic distance of 20 kpc; this might represent, for example, the limit of a highly elliptical orbit for a binary pair.

A fourth-order Runge-Kutta method with variable step size is used to integrate the equations of motion for each stellar orbit. Taking $t = 0$ to be the moment of perigalactic passage, the orbits are integrated between starting and stopping times of -7.5×10^8 yr and 7.5×10^8 yr; at these times the perturber is far enough from the main galaxy that any spurious effects caused by the sudden appearance or disappearance of the second galaxy can be neglected, as is verified experimentally.

III. Results

It is of interest to calculate the average change in energy and radius for stars from each spherical shell, and the fractional mass from each shell which escapes from the galaxy. Since these quantities involve averages over each shell and since the stars are not distributed uniformly over the shells, weighting factors have been applied for each ring which are proportional to the fractional area of the sphere which lies closest to each ring.

Figure 1 shows the mean change ΔE in the energy per unit mass

$$E = \frac{1}{2}\,v^2 - \frac{G\mathfrak{M}}{r + a}$$

of stars from each shell as a function of initial shell radius. The crosses and solid curve give the mean change in energy for all stars, and the circles and dashed curve refer only to those stars which remain bound to the main galaxy. Figure 2 shows the fractional mass from each shell which is lost from the galaxy. As might be expected, the effects of the encounter are small for shells of radius smaller than the distance of closest passage (20 kpc), but become quite important for shells at larger radii where the orbits are slower and the stars are relatively weakly bound.

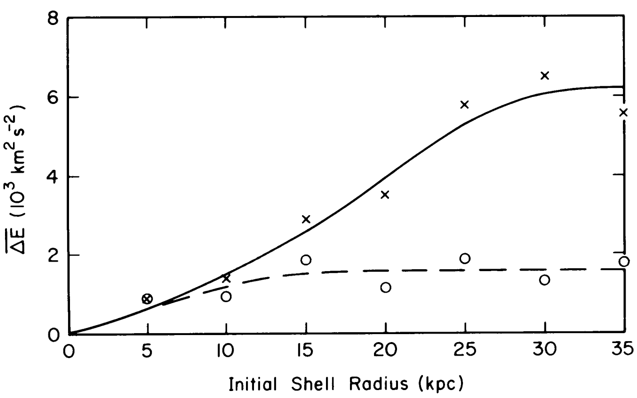

FIG. 1 — The average increase in energy per unit mass $\langle \Delta E \rangle$ for stars from each spherical shell. The crosses and solid curve give the average energy change for all stars from each shell, and the circles and dashed curve refer only to those stars which remain bound to the parent galaxy.

For those stars which remain bound, the increase in mean energy corresponds to an increase in average distance from the center of the galaxy, i.e., to a swelling up of the galaxy. If we define a mean final radius r for the stars from each shell by assuming that the mean final energy E_f of these stars is that of a circular orbit of radius r, as assumed for the initial condition, we have

$$E_f = \frac{G\mathfrak{M}r}{2(r + a)^2} - \frac{G\mathfrak{M}}{r + a} \quad ,$$

from which we can evaluate r by solving a quadratic equation. The results obtained in this way for the fractional increase in mean radius for stars from each shell are shown in Figure 3 (crosses and dashed curve). The fractional change in mean radius is negligible (less than the numerical accuracy) for stars from the shell at $r = 5$ kpc, and increases strongly with increasing radius; again, the largest effect occurs in the outermost part of the system.

Since the above method does not take into account that a star in an elongated orbit is more likely to be found on the slow outermost part of its orbit than elsewhere, a second estimate of the expansion effect has been made by simply calculating the average radius of the stars from each shell at the end of the integration, assuming that their positions at this time provide an adequate statistical sampling of the time-averaged distribution of stars in the galaxy. In this respect the initial configuration does not represent a "most probable" view of the galaxy, so the final average radii cannot be compared directly with the initial shell radii; we have therefore normalized the increase in shell radii by assuming that there is no change for the

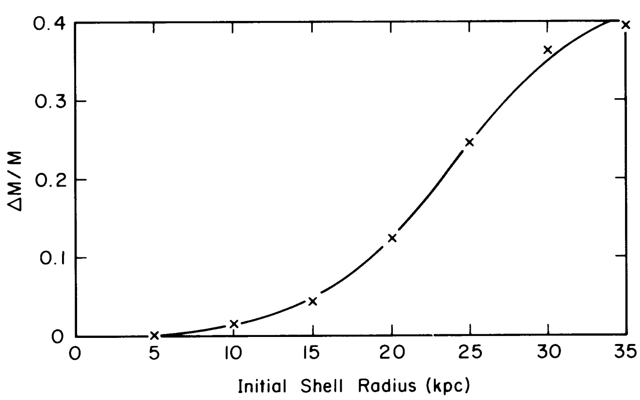

FIG. 2 — The fractional mass loss $\Delta\mathfrak{M}/\mathfrak{M}$ for each initial shell.

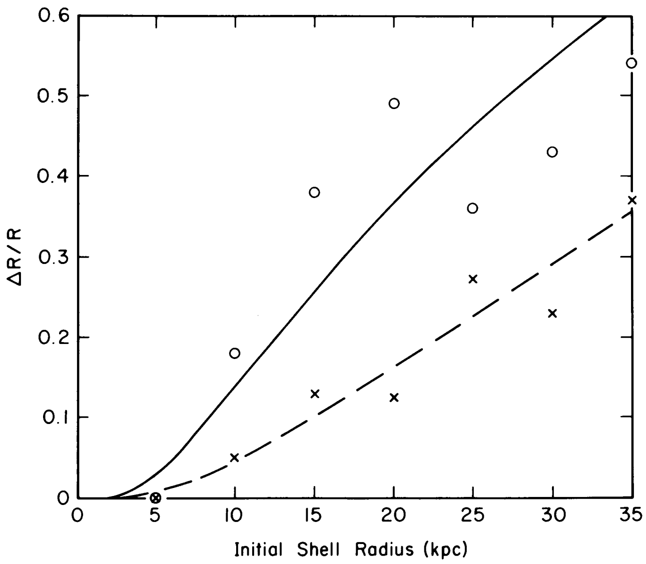

FIG. 3—The fractional increase in mean radius $\Delta R/R$ for those stars from each initial shell which remain bound. The crosses and dashed curve give the result obtained from the mean change in energy, and the circles and solid curve give the result obtained by averaging the final radii.

shell at $r = 5$ kpc, as was found by the first method. The resulting fractional increases in radius are indicated by the circles and solid curve in Figure 3. The second method leads to a somewhat larger estimate of the expansion factor, apparently because the stellar orbits tend to become more elongated as a result of the encounter.

In order to evaluate integrated quantities such as the total fractional mass loss from the galaxy, it is necessary to assume a radial density distribution throughout the halo of the galaxy and assign a mass to each shell. If we assume a density law of the form

$$\rho(r) = \rho_0 r^{-3} \quad ,$$

which is approximately valid for the envelopes of elliptical galaxies, we obtain a total fractional mass loss of 8% from the system of spherical shells. Since the bulk of the mass of the galaxy is probably in the region represented by the shells ($r > 5$ kpc), this is also an approximation for the total mass loss from the galaxy. The total fractional increase in energy is 0.08 when all stars are considered, or 0.04 when only those stars which remain bound are considered.

The energy gained by the stars is extracted from the orbit of the perturber, changing it from a parabolic to an elliptical orbit. If, for purposes of estimating the energy interchange, we assign a mass of $10^{11}\,\mathfrak{M}_\odot$ to the system of shells and assume that the perturber's mass remains constant at $5 \times 10^{10}\,\mathfrak{M}_\odot$, we find that the perturber acquires a negative energy sufficient to put it in an elliptical orbit of semimajor axis 45 kpc. Successive encounters will continue to reduce this quantity until, after a few orbits, the two galaxies spiral together and merge.

IV. Discussion

The results obtained here are approximately consistent with those of Lauberts (1974), considering that he studied collisions between equal-mass galaxies. Lauberts found a fractional mass loss of roughly 10%–20% and a fractional change in energy of up to 25%. A difference from the present results is that most of the mass lost from one galaxy is captured by the other, whereas in the present case most of the lost mass escapes from the system altogether. This difference is possibly attributable to the use of equal masses by Lauberts, although distortions in the mass distribution which are allowed by the N-body model of Lauberts may also play an important role.

In any case, the possibility that some mass will escape from the system is important for cosmological studies which consider the evolution in luminosity of galaxies. Ostriker and Tremaine (1975) have considered the effect of dynamical friction in causing galaxies to accrete their smaller companions and grow in luminosity, at a rate which may be cosmologically significant. The present work demonstrates two mechanisms which tend to counteract this effect: mass may be lost from the system through the effects of repeated encounters, and the mass which does not escape is dispersed over a larger volume, again possibly decreasing the detected luminosity. More detailed calculations will be needed to decide to what extent the increase in luminosity due to accretion is negated by mass loss, but the present result suggests that this effect could be important; the mass lost from the main galaxy is 16% of the mass of the companion, and considering that the companion must also lose mass and that several orbital passages must occur before the companion is accreted, a major fraction of the mass of the accreted companion may in fact be lost.

REFERENCES

Gallagher, J. S., and Ostriker, J. P. 1972, *A.J.* **77**, 288.
Lauberts, A. 1974, *Astr. and Ap.* **33**, 231.
Ostriker, J. P., and Tremaine, S. D. 1975, *Ap. J.* (*Letters*) **202**, L113.
Richstone, D. O. 1975, *Ap. J.* **200**, 535.
Sastry, G. N., and Alladin, S. M. 1970, *Ap. and Space Sci.* **7**, 261.