

**FALL 2024**

# MEETING #2

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Computational Modeling in Engineering and the Sciences  
Computer Science Undergraduate Directed Reading Program

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# AGENDA

- Discussion
- Introduction to ODEs in 1D
- Demo: programming explicit Euler

**Assignment:** Find a paper discussing high-order integration methods for ordinary differential equations, e.g., Runge-Kutta, Adams-Bashforth, &c. Look for the motivation, advantages, disadvantages, and broader context of the method.

**Discussion: Research applications of “computational modeling” and find an interesting example to tell us about.**

# ORDINARY DIFFERENTIAL EQUATION(S)

## ordinary differential equation

a differential equation with only one **independent** variable

Introducing the IVP:

$$\frac{dx}{dt} = f(x, t)$$

$$x(t_0) = x_0$$

$$x(t) = ?$$

## ORDINARY DIFFERENTIAL EQUATION(S)

**Reintroducing a thing from Calculus that you probably never thought you would use again in your life**

**Taylor Series** are very helpful to derive and evaluate derivative approximations.

E.g., a polynomial approximating  $x(t)$  centered at  $t = \tau$

The  $n$ th derivative of this polynomial is exact at  $\tau$  and is usually a reasonable guesstimate in the neighborhood.

$$x(t) \approx p_{\tau}(t) = x(\tau) + \left. \frac{dx}{dt} \right|_{\tau} (t - \tau) + \frac{1}{2} \left. \frac{d^2x}{dt^2} \right|_{\tau} (t - \tau)^2 + \frac{1}{3!} \left. \frac{d^3x}{dt^3} \right|_{\tau} (t - \tau)^3 \\ + \dots + \frac{1}{n!} \left. \frac{d^n x}{dt^n} \right|_{\tau} (t - \tau)^n + \dots$$

# ORDINARY DIFFERENTIAL EQUATION(S)

## Deriving integration methods from finite differences

We can use the definition of a derivative to make some fudgy approximations.

$$\begin{aligned}\frac{dx}{dt} \Big|_t &:= \lim_{h \rightarrow 0} \frac{x(t+h) - x(t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x(t) - x(t-h)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x(t+h) - x(t-h)}{2h}\end{aligned}$$

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$$\begin{aligned}\frac{dx}{dt} \Big|_t &:= \lim_{h \rightarrow 0} \frac{x(t+h) - x(t)}{h} &= \frac{x(t+h) - x(t)}{h} + \mathcal{O}(h) \\ &= \lim_{h \rightarrow 0} \frac{x(t) - x(t-h)}{h} &= \frac{x(t) - x(t-h)}{h} + \mathcal{O}(h) \\ &= \lim_{h \rightarrow 0} \frac{x(t+h) - x(t-h)}{2h} &= \frac{x(t+h) - x(t-h)}{2h} + \mathcal{O}(h^2)\end{aligned}$$

## Forward Euler method

### ORDINARY DIFFERENTIAL EQUATION(S)

$$\left. \frac{dx}{dt} \right|_t \approx \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

$$x(t + \Delta t) = x(t) + \Delta t f(x(t), t)$$

$$x_{n+1} = x_n + \Delta t f(x_n, t_n)$$



## Backward Euler method

### ORDINARY DIFFERENTIAL EQUATION(S)

$$\left. \frac{dx}{dt} \right|_t \approx \frac{x(t) - x(t - \Delta t)}{\Delta t}$$

$$x(t) = x(t - \Delta t) + \Delta t f(x(t), t)$$

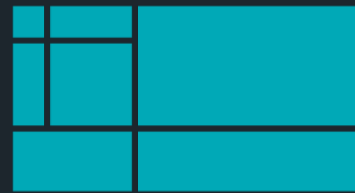
$$x_{n+1} = x_n + \Delta t f(x_{n+1}, t_{n+1})$$

## DEMO: PROGRAMMING EXPLICIT EULER

Demo time!

$$\frac{dx}{dt} = -\alpha x, \quad \alpha > 0$$

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