

FALL 2025

MEETING #2

Computational Modeling in Engineering and the Sciences

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AGENDA

- Discussion of Konstantinovsky - Mastering the SEIR Model
- Introduction to ODEs
- Demo: ODEs in 1D

Assignment: Read first chapter of Baigent - Lotka-Volterra Dynamics. We'll discuss next week.

Discussion of Konstantinovsky

Introduction to ODEs

Introducing ODEs

- **ordinary differential equation:** a differential equation with only one **independent** variable
- Introducing the IVP:

$$\frac{du}{dt} = f(x, t)$$
$$u(t_0) = u_0$$
$$u(t) = ?$$

Introducing ODEs

Reintroducing a thing from Calculus you probably thought you would never see again in your life

- **Taylor Series** are very helpful to derive and evaluate derivative approximations.
 - E.g., a polynomial approximating $x(t)$ centered at $t = \tau$
- The n th derivative of this polynomial is exact at τ and is usually a reasonable guesstimate in the neighborhood.

$$u(t) \approx p_\tau(t) = u(\tau) + \frac{du}{dt} \Big|_\tau (t - \tau) + \frac{1}{2} \frac{d^2u}{dt^2} \Big|_\tau (t - \tau)^2 + \frac{1}{3!} \frac{d^3u}{dt^3} \Big|_\tau (t - \tau)^3 \\ + \dots + \frac{1}{n!} \frac{d^nu}{dt^n} \Big|_\tau (t - \tau)^n + \dots$$

Introducing ODEs

Deriving integration methods from finite differences

- Limit definitions of derivative are good place to start
 - What if h is small?
- Elaborate schemes are less intuitive, but when in doubt...

$$\begin{aligned}\frac{du}{dt} \Big|_{\tau} &:= \lim_{h \rightarrow 0} \frac{u(\tau + h) - u(\tau)}{h} \\ &= \lim_{h \rightarrow 0} \frac{u(\tau) - u(\tau - h)}{h} \\ &= \lim_{h \rightarrow 0} \frac{u(\tau + h) - u(\tau - h)}{2h}\end{aligned}$$

Introducing ODEs

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$$\begin{aligned}\frac{du}{dt} \Big|_{\tau} &:= \lim_{h \rightarrow 0} \frac{u(\tau + h) - u(\tau)}{h} &= \frac{u(\tau + h) - u(\tau)}{h} + \mathcal{O}(h) \\ &= \lim_{h \rightarrow 0} \frac{u(\tau) - u(\tau - h)}{h} &= \frac{u(\tau) - u(\tau - h)}{h} + \mathcal{O}(h) \\ &= \lim_{h \rightarrow 0} \frac{u(\tau + h) - u(\tau - h)}{2h} &= \frac{u(\tau + h) - u(\tau - h)}{2h} + \mathcal{O}(h^2)\end{aligned}$$

Introducing ODEs

Deriving integration methods from finite differences

- Forward Euler method

$$\left. \frac{dx}{dt} \right|_t \approx \frac{x(t + \Delta t) - x(t)}{\Delta t}$$
$$x(t + \Delta t) = x(t) + \Delta t f(x(t), t)$$
$$x_{n+1} = x_n + \Delta t f(x_n, t_n)$$

Introducing ODEs

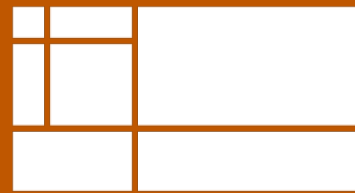
Deriving integration methods from finite differences

- Backward Euler method

$$\left. \frac{dx}{dt} \right|_t \approx \frac{x(t) - x(t - \Delta t)}{\Delta t}$$
$$x(t) = x(t - \Delta t) + \Delta t f(x(t), t)$$
$$x_{n+1} = x_n + \Delta t f(x_{n+1}, t_{n+1})$$

Demo: ODEs in 1D

$$\frac{du}{dt} = -\alpha u, \quad \alpha > 0$$



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