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Homework Assignment 3
CS 663: Digital Image Processing

2021-10-06

Exercise 6

Let $\mathcal{F}$ denote the continuous Fourier transform operator. The Fourier transform of a function $f(t)$ is then denoted as $\mathcal{F}(f(t))$ and can be computed as follows:

$$\mathcal{F}(f(t))(\mu) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}dt$$

Lets define $\mathcal{G}(t') = \mathcal{F}(\mathcal{F}(f(t)))(t')$, which can be computed as follows:

$$\mathcal{G}(t') = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}dt \right) e^{-j2\pi t'\mu}d\mu$$

Rearranging the integrals to first compute the integral w.r.t μ and then compute the outer integral w.r.t t, we have the following:

$$\mathcal{G}(t') = \int_{-\infty}^{\infty} f(t) \left(\int_{-\infty}^{\infty} e^{-j2\pi\mu(t+t')}d\mu \right) dt$$

Now, we know that the expression in the integral w.r.t μ is a dirac delta function given by:

$$\int_{-\infty}^{\infty} e^{-j2\pi\mu(t+t')}d\mu = \delta(t+t')$$

Thereby, plugging this into the expression for $\mathcal{G}(t')$, we have the following:

$$\mathcal{G}(t') = \int_{-\infty}^{\infty} f(t)\delta(t+t')dt$$

Clearly, the above expression evaluates to:

$$\mathcal{G}(t') = f(-t')$$

Since t' is just a dummy variable, we can equivalently write:

$$\mathcal{G}(t) = \mathcal{F}(\mathcal{F}(f(t))) = f(-t)$$

Thus, we have:

$$\mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t))))) = \mathcal{F}(\mathcal{F}(f(-t))) = f(-(-t)) = f(t)$$

Hence, proved.