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Homework Assignment 2
CS 663: Digital Image Processing

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Exercise-3

Let $p_I(i)$ denote the probability density function for image $I(x,y)$. Let $J(x,y)$ be the image constructed by randomly sampling a zero-mean gaussian distribution with standard deviation σ. Therefore $p_J(j) = \mathcal{N}(0,\sigma)$. Now, in order to corrupt the original image $I(x,y)$, we add $J(x,y)$ to it. Hence, the motive is to compute the probability distribution function of $K(x,y) = I(x,y) + J(x,y)$. Let $p_{IJ}(i,j)$ denote the joint probability of $I = i$ and $J = j$.

The probability density function $p_K(k)$ can be computed as follows:

$$p_K(k) = \int_{-\infty}^{\infty} p_{IJ}(i, k-i) di$$

Since the two random variables, I and J are independent we get:

$$p_K(k) = \int_{-\infty}^{\infty} p_I(i) p_J(k-i) di$$

Therefore, the PDF of K is simply a convolution of the PDFs of I and J. Substituting $p_J(k-i)$ using the definition of gaussian distribution,

$$p_J(k-i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(k-i)^2}{2\sigma^2}}$$

We finally have the following expression for the PDF of the resulting noisy image K:

$$p_K(k) = (p_I * p_J)(k) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} p_I(i) e^{-\frac{(k-i)^2}{2\sigma^2}} di$$