Ashutosh Sharma (190100027)
Kritti Sharma (180100060)
Shreyas Chandgothia (180260037)

Homework Assignment 2
CS 663: Digital Image Processing

2021-08-31

Exercise-4

(a)
Filters which can be expressed in the form of an outer product are called separable filters. Let us assume that a (3×3) filter f can be expressed as an outer product of a (3×1) vector u and a (1×3) vector v, i.e. $f = u \bigotimes v$ as follows:

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \bigotimes \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} u_1v_1 & u_1v_2 & u_1v_3 \\ u_2v_1 & u_2v_2 & u_2v_3 \\ u_3v_1 & u_3v_2 & u_3v_3 \end{bmatrix} = \begin{bmatrix} u_1 \times \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \\ u_2 \times \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \\ u_3 \times \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \end{bmatrix}$$

Therefore, we see that a special property of separable filters is that each of its rows is a scalar multiple of any of the other rows (similarly for columns).
Now, the Laplacian mask with a -4 in the center is expressed as follows:

$$f = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Clearly, the second row is not a scalar multiple of the first row or the third row. Hence, the Laplacian mask f is not a separable filter.

(b)
We want to explore if we can implement the Laplacian mask f entirely using two 1D convolutional masks. We know that applying two convolutional masks to an image one by one, is same as first generating the convolution of the two masks and then applying the resultant mask as a convolution on the image. Thus, in effect, we want to find out if the Laplacian mask f can be expressed as a convolution of two 1D masks.

Now, if both of these 1D convolutional masks are either horizontal or vertical, then we will end up getting their convolution also as a 1D mask, i.e. only horizontal or vertical, respectively. Hence, clearly, one of these two 1D masks will have to be horizontal and the other one has to be vertical, since the Laplacian f is a 2D mask. Further we know that the convolution of a horizontal 1D mask with a vertical 1D one, is same as their outer product, calculated by effectively treating both of them as vectors.

Thus, in essence, we want that the outer product of the two 1D masks has to be equal to f. However, in part (a), we have seen that the Laplacian mask is not a separable filter, which means that f cannot be factorised into an outer product form of two 1D convolutions. Hence, we cannot implement f entirely using only 1D convolutions.