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Homework Assignment 1
CS 663: Digital Image Processing

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Exercise-2

Consider a random variable R with probability density function $p_R(r)$ and a transformed random variable S with probability density function $p_S(s)$ where the transformation function $T(r)$ to perform histogram equalization is defined as follows:

$$s = T(r) = (L-1)\int_0^r p_R(w)dw \tag{1}$$

The probability density function of S can be computed as:

$$p_S(s) = \frac{p_R(r)}{|T'(r)|} \tag{2}$$

$$T'(r) = (L-1)\frac{d}{dr}(\int_0^r p_R(w)dw) = (L-1)p_R(r) \tag{3}$$

Substituting $T'(r)$ from eqn 3 in eqn 2,

$$p_S(s) = \frac{p_R(r)}{(L-1)p_R(r)} = \frac{1}{L-1} \tag{4}$$

Let Q be the transformed random variable by applying histogram equalization to S with transformation function $U(s)$, then, the probability density function of Q, $p_Q(q)$ can be computed as:

$$q = U(s) = (L-1)\int_0^s p_S(w)dw \tag{5}$$

$$p_Q(q) = \frac{p_S(s)}{|U'(s)|} \tag{6}$$

$$U'(s) = (L-1)\frac{d}{dr}(\int_0^s p_S(w)dw) = (L-1)p_S(s) \tag{7}$$

Substituting $U'(r)$ from eqn 7 in eqn 6,

$$p_Q(q) = \frac{p_S(s)}{(L-1)p_S(s)} = \frac{1}{L-1} \tag{8}$$

The result produced is the same as that of the first round. Both, $p_Q(q)$ and $p_S(s)$, are uniform distributions.