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Homework Assignment 3
CS 663: Digital Image Processing

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Exercise 4

For a function $f(x)$, from the Differentiation Theorem, we know that:

$$\mathcal{F}(\frac{d^n f(x)}{dx^n}) = (j2\pi\mu)^n F(\mu)$$

Therefore, expanding this concept to a 2D image $f(x, y)$, we know that:

$$\mathcal{F}(\frac{\partial f(x,y)}{\partial x}) = j2\pi\mu F(\mu,\nu) = \mathcal{G}_x$$

where $\mathcal{G}_x$ is the Fourier transform of the gradient of the function $f(x, y)$ in the x direction. Similarly, $\mathcal{G}_y$ is the Fourier transform of the gradient of the function $f(x, y)$ in the y direction and is given as:

$$\mathcal{F}(\frac{\partial f(x,y)}{\partial y}) = j2\pi\nu F(\mu,\nu) = \mathcal{G}_y$$

The gradients in x and y direction can be computed by taking inverse Fourier transforms as follows:

$$f_x = \mathcal{F}^{-1}(\mathcal{G}_x) = \mathcal{F}^{-1}(j2\pi\mu F(\mu,\nu)))$$

and

$$f_y = \mathcal{F}^{-1}(\mathcal{G}_y) = \mathcal{F}^{-1}(j2\pi\nu F(\mu,\nu))$$

The magnitude of the gradient can then be computed as $\sqrt{f_x^2 + f_y^2}$. Thus we are successfully able to calculate the gradient magintude at all spatial pixel locations from the Fourier Transform of the image $F(\mu, \nu)$.