

## Exercise-3

We are given two images  $I$  and  $J$  whose intensity values (in each location) are randomly drawn from the known probability mass functions (PMFs)  $p_I(i)$  and  $p_J(j)$  respectively. Let image  $K$  be defined as  $I + J$ . Assuming that intensity values lie in the set  $\{0, 1, 2, \dots, L-1\}$ , intensity values in image  $K$  will lie in the set  $\{0, 1, 2, \dots, 2L-2\}$ .

Let  $(x_0, y_0)$  denote a pixel in each image:  $K, I, J$ . The probability that  $K(x_0, y_0) = k$  will be equal to the sum of the probability for  $[(I(x_0, y_0) = 0 \text{ and } J(x_0, y_0) = k) \text{ or } (I(x_0, y_0) = 1 \text{ and } J(x_0, y_0) = k-1) \text{ or } (I(x_0, y_0) = 2 \text{ and } J(x_0, y_0) = k-2) \dots \text{ or } (I(x_0, y_0) = k \text{ and } J(x_0, y_0) = 0)]$ . Mathematically, this can be computed as:

$$\begin{aligned} p_K(k) &= (p_I(0) \times p_J(k)) + (p_I(1) \times p_J(k-1)) + (p_I(2) \times p_J(k-2)) + \dots + (p_I(k) \times p_J(0)) \\ &= \sum_{m=0}^k p_I(m) p_J(k-m) \\ &= (p_I * p_J)(k) \end{aligned} \tag{1}$$

The expression resembles the **convolution** operation!