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**Homework Assignment 3**
CS 663: Digital Image Processing

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# Exercise 5

## Part-a

We are given that the function $f(x,y)$ is real. Let $\mathcal{F}(\mu,\nu)$ denote its Discrete Fourier Transform, which is given as:

$$\mathcal{F}(\mu,\nu) = \frac{1}{\sqrt{W_1W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)exp(-j\frac{2\pi\mu}{W_1}x)exp(-j\frac{2\pi\nu}{W_2}y)$$

Taking complex conjugate on both sides of the above equation, we get:

$$\mathcal{F}(\mu,\nu) = \frac{1}{\sqrt{W_1W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f^*(x,y)exp(j\frac{2\pi\mu}{W_1}x)exp(j\frac{2\pi\nu}{W_2}y)$$

The above equation can be rewritten by writing the positive coefficient of $x$, $y$ in the fraction of the exponential as double negative and then rearranging one negative sign. Since $f(x,y)$ is real, we have $f^*(x,y) = f(x,y)$, so the equation can be written as follows:

$$\mathcal{F}^*(\mu,\nu) = \frac{1}{\sqrt{W_1W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)exp(-j\frac{2\pi(-\mu)}{W_1}x)exp(-j\frac{2\pi(-\nu)}{W_2}y)$$

Observe that he R.H.S. of the equation is exactly same as $\mathcal{F}(-\mu,-\nu)$, given by:

$$\mathcal{F}(-\mu,-\nu) = \frac{1}{\sqrt{W_1W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)exp(-j\frac{2\pi(-\mu)}{W_1}x)exp(-j\frac{2\pi(-\nu)}{W_2}y)$$

Therefore,

$$\mathcal{F}^*(\mu,\nu) = \mathcal{F}(-\mu,-\nu)$$

Hence, proved.

## Part-b

For the second part, it is given that $f(x,y)$ is real and even. From the previous result, since $f(x,y)$ is real, we have $\mathcal{F}^*(\mu,\nu) = \mathcal{F}(-\mu,-\nu)$. Zero-padding the original function $f(x,y)$ to infinite extent in both directions, we can write the DFT of $f(x,y)$ as:

$$\mathcal{F}(\mu,\nu) = \frac{1}{\sqrt{W_1W_2}} \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x,y)exp(-j\frac{2\pi\mu}{W_1}x)exp(-j\frac{2\pi\nu}{W_2}y)$$

Thus $\mathcal{F}(-\mu,-\nu)$ is computed as follows:

$$\mathcal{F}(-\mu,-\nu) = \frac{1}{\sqrt{W_1W_2}} \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x,y)exp(-j\frac{2\pi(-\mu)}{W_1}x)exp(-j\frac{2\pi(-\nu)}{W_2}y)$$

Now, replacing by $x = -x'$ and $y = -y'$, we get:

$$\mathcal{F}(-\mu,-\nu) = \frac{1}{\sqrt{W_1W_2}} \sum_{x'=-\infty}^{\infty} \sum_{y'=-\infty}^{\infty} f(-x',-y')exp(-j\frac{2\pi(-\mu)}{W_1}(-x'))exp(-j\frac{2\pi(-\nu)}{W_2}(-y'))$$

Since $f(x,y)$ is even as well, we have $f(-x,-y) = f(x,y)$. Therefore, we can rewrite this equation by cancelling the double negatives in the exponentials as:

$$\mathcal{F}(-\mu,-\nu) = \frac{1}{\sqrt{W_1W_2}} \sum_{x'=-\infty}^{\infty} \sum_{y'=-\infty}^{\infty} f(x',y')exp(-j\frac{2\pi\mu}{W_1}x')exp(-j\frac{2\pi\nu}{W_2}y')$$

On looking carefully, we can observe that the R.H.S. is nothing but $\mathcal{F}(\mu, \nu)$, since $x'$ and $y'$ are just dummy variables. So, we have:

$$\mathcal{F}(-\mu, -\nu) = \mathcal{F}(\mu, \nu)$$

From Part-a, we have the following result:

$$\mathcal{F}^*(\mu, \nu) = \mathcal{F}(-\mu, -\nu)$$

Using the above two equations, we get:

$$\mathcal{F}^*(\mu, \nu) = \mathcal{F}(-\mu, -\nu) = \mathcal{F}(\mu, \nu)$$

Hence, proved.