Ashutosh Sharma (190100027)
Kritti Sharma (180100060)
Shreyas Chandgothia (180260037)

Homework Assignment 1
CS 663: Digital Image Processing

2021-08-17

Exercise-4

Part (a)

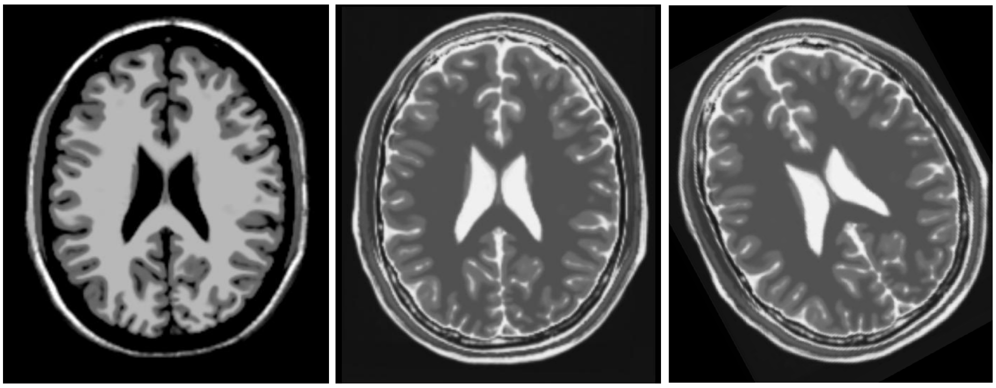
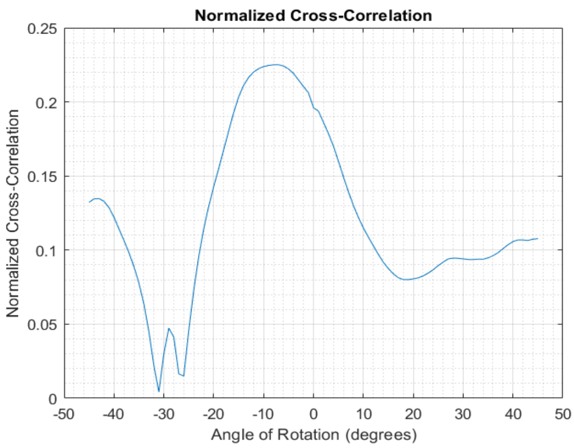
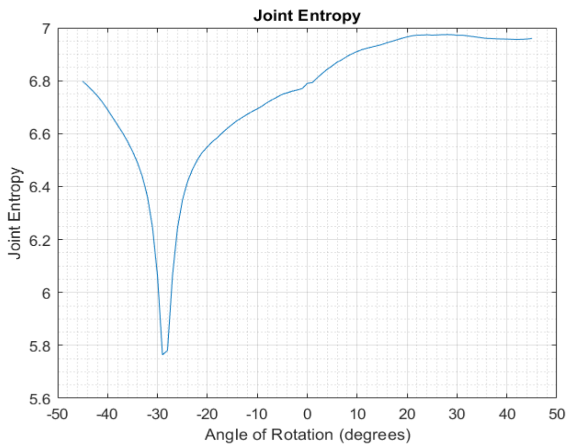
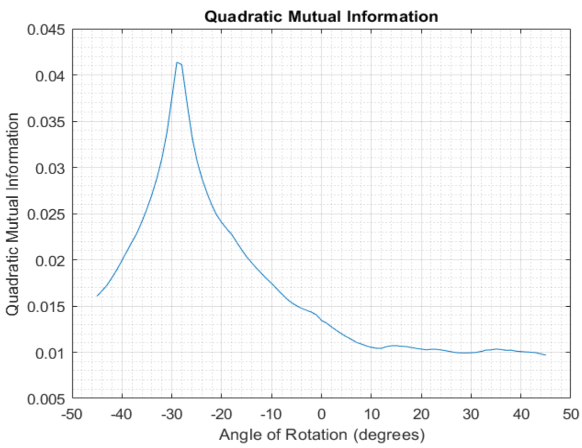


Figure 1: Magnetic Resonance Images of human brain. Left (J1) and middle (J2) images have physical correspondence at all pixels. Right (J3) image is a $28.5°$ rotation of J2 in anticlockwise direction.

Part (b), (c), (d)







The functions to compute normalized cross-correlation (NCC), joint entropy (JE) and quadratic mutual information (QMI) have been implemented in MATLAB and the corresponding codes have been submitted along with this assignment submission.

Observations from Plots:

Normalized cross-correlation should maximize at the optimal angle of rotation. We observe a sharp local maxima at $-29°$(which is the optimal angle) and a smooth global maxima at $-8°$. The rationale for this observation is uncertain, but a possible reason might be that the two images are not perfect negatives of each other. This can be seen by closely observing the two images at the boundary region of the brain, where they both seem to be identical rather than negatives of each other.

In the joint entropy plot, we observe a sharp dip at $-29°$, which correctly signifies that this the optimal angle of rotation according to the joint entropy metric.

In the quadratic mutual information plot, we again observe a peak in the metric when the angle of rotation is $-29°$. This is along expected lines and further explained in §(f).

Part (e)

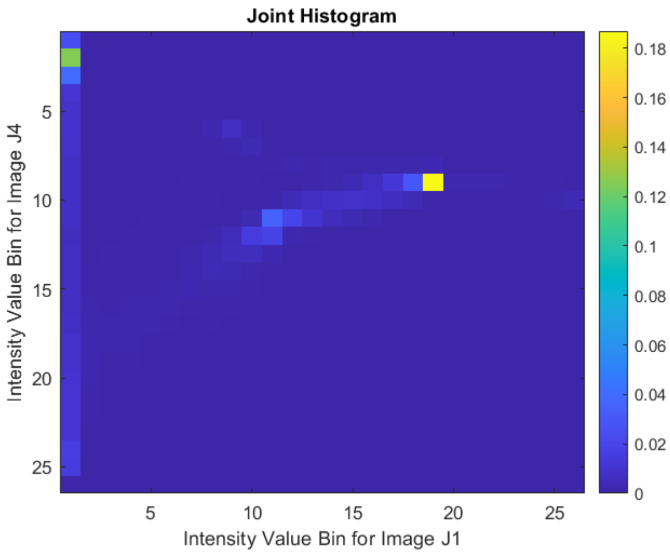


Figure 2: Joint histogram between J1 and J4.

Observations from Joint Histogram:

The images are approximately negatives of each other, hence we observe that the joint histogram is bright along approximately $-45°$ line. Its not exactly along the $-45°$ line because the optimal angle from our analysis is $29°$ whereas the true rotation angle is $28.5°$. Also, we observe that image J_1 has a lot of counts in the $0 - 10$ bin which can be attributed to the fact that the J_1 image has many dark pixels.

Part (f)

The Quadratic Mutual Information metric depicts the degree of mutual dependence between the two random variables I_1 and I_2. As the two images begin to align, the dependence between the two variables will go on increasing and will be maximum when they perfectly align, meaning that the amount of information known to us about I_1 will be the same as that when observing I_2.

Mathematically, if the two random variables are independent, then the joint probability $p_{I_1 I_2}(i_1, i_2)$ will be approximately equal to the product of $p_{I_1}(i_1)$ and $p_{I_2}(i_2)$, hence giving a QMI of roughly zero. As the two variables become more and more dependent, the QMI value will go on increasing, giving a maximum value when the two variable are completely dependent.