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Homework Assignment 3
CS 663: Digital Image Processing

2021-10-06

Exercise 7

The images in the second sub-figure of Figure 1 given in the question, visually depict the spatial representation of the Fourier-space filters described in the first sub-figure.
Let the high-pass filters in the Fourier space by denoted by $\mathcal{H}(\mu, \nu)$. The spatial representation of the filters, denoted by $h(x, y)$ can be obtained by taking the discrete inverse Fourier transform as follows:

$$h(x,y) = \frac{1}{\sqrt{W_1W_2}} \sum_{\mu=0}^{W_1-1} \sum_{\nu=0}^{W_2-1} \mathcal{H}(\mu,\nu)exp(j\frac{2\pi x}{W_1}\mu)exp(j\frac{2\pi y}{W_2}\nu)$$

where (W_1, W_2) correspond to the finite size of the filters in the Fourier space.

The central region in the spatial domain corresponds to $x \approx 0, y \approx 0$. Thus, near the center, the exponential factors are close to unity and we have:

$$h(x \approx 0, y \approx 0) \approx h(0,0) = \frac{1}{\sqrt{W_1W_2}} \sum_{\mu=0}^{W_1-1} \sum_{\nu=0}^{W_2-1} \mathcal{H}(\mu,\nu)$$

Now, as can be seen from the images of the filters, the value of the high-pass filters $\mathcal{H}(\mu, \nu)$ in the Fourier Space are always positive and close to 1 for the majority of the frequency domain. Thus, the above sum will become very large as it corresponds to the volume under the curve which would be obtained if the high-pass filter were to be plotted in 3-dimensions in the Fourier Space. Thus strong spikes in the center are observed.