Error Control in Relay Networks

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***Abstract*—The report deals with the study of multi-hop relay network architecture that has been introduced in the next generation wireless cellular networks such as LTE advanced and WiMax systems as a promising technique to improve the coverage and the throughput of the network. The report illustrates both theoretical as well as analytic study of performance of different error control schemes for relay networks.**

I. Introduction

THIS report deals with a multi-hop system as described in [1]. To fully understand this report it is required to first go through [1]. Following report has two major sections, first section analyzes relay systems with infinite retransmissions and no packet loss, where as second part deals with a little more realistic system constituting limited number of retransmission along with packet loss. First section illustrates detailed theoretical and analytic study of three types of relay networks: Amplify-and-Forward (AF) relaying with end-to-end ARQ, Decode-and-Forward (DF) relaying with end-to-end ARQ, and DF relaying with hop-by-hop ARQ. The second section briefly analyses AF relaying with end-to-end ARQ and limited retransmissions.

II. Relay Network with Infinite Retransmissions

The queuing systems in consideration in this section are shown in Fig 1. Each of the nodes in the systems have one server and the associated queues are independent. They have infinite length buffer and arrival and departure process are Poisson. There is no bound on the number of retransmissions.

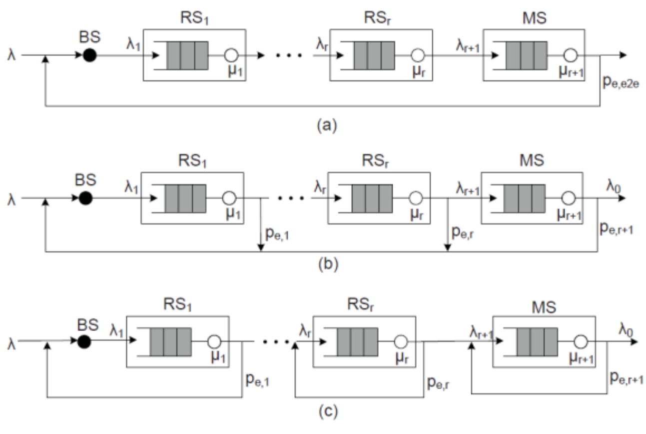

Fig. 1. Queuing networks for (a) AF relaying with end-to-end ARQ, (b) DF relaying with end-to-end ARQ, and (c) DF relaying with hop-by-hop ARQ

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TABLE I
List of Parameters

Number of relays	r
Packet arrival rate	λ
Packet processing time	$\sim exp(\mu)$, $\mu = \mu_{AF}$ for AF-RS, $\mu = \mu_{DF}$ for DF-RS, $\mu = \mu_{MS}$ for MS
Packet error probability of j$^{\text{th}}$ hop	$p_{e,j} \in [0,1]$ with $j = \{1,2,...,r+1\}$

A. Theoretic Analysis

Parameters for the system are illustrated in table I.

1) AF relaying with end-to-end ARQ: As it can be seen in Fig 1(a), it's a series of $M/M/1$ queuing systems with an end-to-end feedback(λ_{e2e}).Except for one end to end feedback, there is one entry and one exit point in each node, thus by observing the movement of packets in the system we can write following equations.

$$\lambda_1 = \lambda_2 .... = \lambda_{r+1} \tag{1}$$
$$\lambda_1 = \lambda + \lambda_{e2e} \tag{2}$$
$$\lambda_{e2e} = p_{e,e2e}\lambda_{r+1} \tag{3}$$
$$\lambda = \lambda_{r+1}(1 - p_{e,e2e}) \tag{4}$$

using equations $(1) - (4)$,

$$\lambda_j = \frac{\lambda}{1 - p_{e,e2e}},\ j \in \{1, 2..., r+1\} \tag{5}$$
$$\lambda_{e,e2e} = \frac{p_{e,e2e}}{1 - p_{e,e2e}}\lambda \tag{6}$$

The packets received at MS would be in error if there has been error at any of the nodes. So, we can write equation (7).

$$\begin{aligned} p_{e,e2e} &= 1 - p_{no\ error} \\ p_{e,e2e} &= 1 - \prod_{j=1}^{r+1}(1 - p_{e,j}) \\ \Rightarrow \lambda_j &= \frac{\lambda}{\prod_{i=1}^{r+1}(1 - p_{e,i})} \end{aligned} \tag{7}$$

As each sub-system is an $M/M/1$ queue, average number of packets in waiting ($N_{q,j}$), under service ($N_{s,j}$) or in total (N_j) can be written written as in equations $(8) - (11)$. we can directly use the formula derived in the lectures for average number of packets in an M/M/1 system

$$\overline{N}_j = \overline{N}_{q,j} + \overline{N}_{s,j} \tag{8}$$

$$\overline{N}_j = \frac{\rho_j}{1-\rho_j} = \frac{\lambda_j}{\mu_j - \lambda_j} \tag{9}$$

$$\overline{N}_{s,j} = \rho_j \tag{10}$$

$$\overline{N}_{q,j} = \frac{\rho_j^2}{1-\rho_j} \tag{11}$$

$$where,\ j \in \{1, 2..., r+1\},\ and\ \rho_j = \frac{\lambda_j}{\mu_j}$$

Now in order to find the average queuing delay, Little's theorem can be applied on each of the sub-system..

$$\overline{W}_j = \frac{\overline{N}_{q,j}}{\lambda_j},\ j \in \{1, 2...r+1\} \tag{12}$$

using equations $(11) - (12)$,

$$\begin{aligned} \overline{W}_j &= \frac{1}{\lambda_j}\left(\frac{\rho_j^2}{1-\rho_j}\right) \\ \Rightarrow \overline{W}_j &= \frac{1}{\mu_j}\left(\frac{\lambda_j}{\mu_j - \lambda_j}\right),\ j \in \{1, 2...r+1\} \end{aligned} \tag{13}$$

Average end-to-end delay ($\overline{T}$) can be found by using recursion. Let T be the sum of time delay in the nodes 1 to $r+1$. If N packets enter the system, then all of them will take on an average T time to come out of $r+1^{\text{th}}$ node. On an average, $p_{e,e2e}N$ packets would be in error and need to be retransmitted which will take on average $\overline{T}$ time.

$$\begin{aligned} N\overline{T} &= NT + p_{e,e2e}N\overline{T} \\ \Rightarrow NT &= N\overline{T}(1 - p_{e,e2e}) \\ \Rightarrow \overline{T} &= \frac{T}{1 - p_{e,e2e}} \end{aligned} \tag{14}$$

T is the sum of time delays in all the nodes, thus it can be written as in equation (15).

$$\begin{aligned} T &= \sum_{j=1}^{r+1} T_j \\ \Rightarrow T &= \sum_{j=1}^{r+1} \frac{N_j}{\lambda_j},\ (Little's\ Th.) \\ \Rightarrow T &= \sum_{j=1}^{r+1} \frac{1}{\lambda_j}\frac{\rho_j}{1-\rho_j},\ (from\ eq(9)) \\ \Rightarrow T &= \sum_{j=1}^{r+1} \frac{1}{\mu_j - \lambda_j} \end{aligned} \tag{15}$$

Using equations $(14) - (15)$, average end-to-end delay can written as equation (16).

$$\overline{T} = \frac{1}{1 - p_{e,e2e}} \sum_{j=1}^{r+1} \frac{1}{\mu_j - \lambda_j} \tag{16}$$

List of formulae derived for AF relaying with end-to-end ARQ are compiled in table II.

TABLE II
AF RELAYING WITH END-TO-END ARQ

$p_{e,e2e}$	$1 - \prod_{j=1}^{r+1}(1 - p_{e,j})$
λ_1	$\frac{\lambda}{\prod_{i=1}^{r+1}(1-p_{e,i})}$
$\overline{W}_j$	$\frac{1}{\mu_j}\left(\frac{\lambda_j}{\mu_j - \lambda_j}\right)$
$\overline{N}_j$	$\frac{\lambda_j}{\mu_j - \lambda_j}$
$\overline{T}$	$\frac{1}{1-p_{e,e2e}} \sum_{j=1}^{r+1} \frac{1}{\mu_j - \lambda_j}$

2) DF relaying with end-to-end ARQ: As it can be seen in Fig 1(b), it's a series of $M/M/1$ queuing systems with an end-to-end ARQ. However, this time as the system is DF, each node has a feedback branch connected to the BS. After each hop, packets with $\lambda_{j,e}$ intensity are in error and are dropped and thus total feedback intensity would be the sum of each of these($\lambda_{e,fb}$). By observing the movement of packets in the system we can write following equations.

$$\lambda_1 = \lambda + \lambda_{e,fb} \tag{17}$$

$$\lambda_{e,fb} = \sum_{i=1}^{r+1} \lambda_{e,i} \tag{18}$$

$$\lambda_{e,j} = \lambda_j p_{e,j},\ j \in \{1, 2...r+1\} \tag{19}$$

Using equations $(17) - (19)$,

$$\begin{aligned} \lambda_j &= \lambda_{j-1} - \lambda_{j-1,e} \\ \Rightarrow \lambda_j &= \lambda_{j-1}(1 - p_{e,j}) \\ \Rightarrow \lambda_j &= \lambda_1 \prod_{i=1}^{j-1}(1 - p_{e,i}),\ j \in \{2, 3...r+1\} \end{aligned} \tag{20}$$

From equation (20), we have the expression for arrival intensity for all the nodes in terms of λ_1. Now, since the overall system arrival intensity should be conserved since no packets are blocked, thus we can write equation (21).

$$\begin{aligned} \lambda &= \lambda_{r+1}(1 - p_{e,r+1}) \\ \Rightarrow \lambda &= \lambda_1 \prod_{i=1}^{r+1}(1 - p_{e,i}) \\ \Rightarrow \lambda_1 &= \frac{\lambda}{\prod_{i=1}^{r+1}(1 - p_{e,i})} \end{aligned} \tag{21}$$

Using equations $(20) - (21)$,

$$\begin{aligned} \lambda_j &= \frac{\lambda}{\prod_{i=1}^{r+1}(1 - p_{e,i})} \prod_{k=1}^{j-1}(1 - p_{e,k}) \\ \lambda_j &= \frac{\lambda}{\prod_{i=j}^{r+1}(1 - p_{e,i})},\ j \in \{1, 2..., r+1\} \end{aligned} \tag{22}$$

As each sub-system is an $M/M/1$ queue, average number of packets in waiting ($N_{q,j}$), under service ($N_{s,j}$) or in total

(N_j) can be written written as in equations $(8)-(11)$.we can directly use the formula derived in the lectures for average number of packets in an M/M/1 system Using equations (9) and (22),

$$\overline{N}_j = \frac{\lambda}{(\mu_j \prod_{i=j}^{r+1}(1-p_{e,i}))-\lambda} \tag{23}$$

Now in order to find the average queuing delay, Little's theorem can be applied on each of the sub-system..

$$\overline{W}_j = \frac{\overline{N}_{q,j}}{\lambda_j},\ j \in \{1, 2...r+1\} \tag{24}$$

using equations $(11), (22)$ and (24),

$$\begin{aligned} \overline{W}_j &= \frac{1}{\lambda_j}(\frac{\rho_j^2}{1-\rho_j}) \\ \Rightarrow \overline{W}_j &= \frac{1}{\mu_j}(\frac{\lambda_j}{\mu_j-\lambda_j}) \\ \Rightarrow \overline{W}_j &= \frac{1}{\mu_j}(\frac{\lambda}{(\mu_j \prod_{i=j}^{r+1}(1-p_{e,i}))-\lambda}),\ j \in \{1...,r+1\} \end{aligned} \tag{25}$$

Average end-to-end delay ($\overline{T}$) can be found using Little's theorem over the entire system.

$$\overline{T} = \frac{\overline{N}}{\lambda} \tag{26}$$

N is the average total of the packets in the system.

$$\begin{aligned} \overline{N} &= \sum_{j=1}^{r+1} \overline{N}_j \\ \overline{N} &= \sum_{j=1}^{r+1} \frac{\lambda}{(\mu_j \prod_{i=j}^{r+1}(1-p_{e,i}))-\lambda}\ (from\ eq(23)) \end{aligned} \tag{27}$$

Using equations $(26)-(27)$,

$$\overline{T} = \sum_{j=1}^{r+1} \frac{1}{(\mu_j \prod_{i=j}^{r+1}(1-p_{e,i}))-\lambda} \tag{28}$$

List of formulae derived for DF relaying with end-to-end ARQ are compiled in table III.

3) DF relaying with hop-by-hop ARQ: As it can be seen in Fig 1(c), it's a series of $M/M/1$ queuing systems with each having an ARQ. After each hop, packets with $\lambda_{j,e}$ intensity are in error and have to retransmitted. By observing the movement of packets in the system we can observe that each sub-system can be treated in an isolated manner and thus following equations can be written.

$$\lambda_j = \lambda + \lambda_{e,j} \tag{29}$$

$$\lambda_{e,j} = \lambda_j p_{e,j},\ j \in \{1, 2, ...r+1\} \tag{30}$$

Using equations $(29)-(30)$,

$$\lambda_j = \frac{\lambda}{1-p_{e,j}},\ j \in \{1, 2, ...r+1\} \tag{31}$$

As each sub-system is an $M/M/1$ queue, average number of packets in waiting ($N_{q,j}$), under service ($N_{s,j}$) or in total (N_j) can be written written as in equations $(8)-(11)$. we can directly use the formula derived in the lectures for average number of packets in an M/M/1 system Using equations (9) and (31),

$$\overline{N}_j = \frac{\lambda}{\mu_j(1-p_{e,j})-\lambda} \tag{32}$$

Now in order to find the average queuing delay, Little's theorem can be applied on each of the sub-system..

$$\overline{W}_j = \frac{\overline{N}_{q,j}}{\lambda_j},\ j \in \{1, 2...r+1\} \tag{33}$$

using equations $(11), (31)$ and (33),

$$\begin{aligned} \overline{W}_j &= \frac{1}{\lambda_j}(\frac{\rho_j^2}{1-\rho_j}) \\ \Rightarrow \overline{W}_j &= \frac{1}{\mu_j}(\frac{\lambda_j}{\mu_j-\lambda_j}) \\ \Rightarrow \overline{W}_j &= \frac{1}{\mu_j}(\frac{\lambda}{\mu_j(1-p_{e,j})-\lambda}),\ j \in \{1...,r+1\} \end{aligned} \tag{34}$$

Average end-to-end delay ($\overline{T}$) can be found using Little's theorem over the entire system.

$$\overline{T} = \frac{\overline{N}}{\lambda} \tag{35}$$

N is the average total of the packets in the system.

$$\begin{aligned} \overline{N} &= \sum_{j=1}^{r+1} \overline{N}_j \\ \overline{N} &= \sum_{j=1}^{r+1} \frac{\lambda}{\mu_j(1-p_{e,j})-\lambda}\ (from\ eq(32)) \end{aligned} \tag{36}$$

Using equations $(35)-(36)$,

$$\overline{T} = \sum_{j=1}^{r+1}, \frac{1}{\mu_j(1-p_{e,i})-\lambda} \tag{37}$$

List of formulae derived for DF relaying with hop-by-hop ARQ are compiled in table III.

TABLE III
DF RELAYING

	End-to-End ARQ	Hop-By-Hop ARQ
λ_j	$\frac{\lambda}{\prod_{i=j}^{r+1}(1-p_{e,i})}$	$\frac{\lambda}{1-p_{e,j}}$
$\overline{W}_j$	$\frac{1}{\mu_j}(\frac{\lambda}{(\mu_j \prod_{i=j}^{r+1}(1-p_{e,i}))-\lambda})$	$\frac{1}{\mu_j}(\frac{\lambda}{\mu_j(1-p_{e,j})-\lambda})$
$\overline{N}_j$	$\frac{\lambda}{(\mu_j \prod_{i=j}^{r+1}(1-p_{e,i}))-\lambda}$	$\frac{\lambda}{\mu_j(1-p_{e,j})-\lambda}$
$\overline{T}$	$\sum_{j=1}^{r+1} \frac{1}{(\mu_j \prod_{i=j}^{r+1}(1-p_{e,i}))-\lambda}$	$\sum_{j=1}^{r+1} \frac{1}{\mu_j(1-p_{e,i})-\lambda}$

B. Numerical Analysis

In this section, we will numerically compare performance of the three types of relay networks which have been theoretically studied in the previous section.

1) Stability region: A relay network is considered stable if all the queues in it are stable. For a queue to be stable, arrival rate should be less than the service rate (ie. $\rho < 1$). In AF relay network and DF with Hop-by-Hop ARQ relay network, all the queues have identical arrival rates. However, in DF with End-to-End ARQ relay network, arrival rate decreases with increasing distance from BS. So, we can take the first node as the critical node for analyzing maximum allowed arrival rate (λ_{max}). Using $p_{e,j} = 0.1$ for $j \in \{1, 2, ..., r+1\}$, $\mu_{DF} = 1$ and $\mu_{AF} = \mu_{MS} = 2\mu_{DF}$, and the formulae in tables II and III we can write following numerical relations between number of relays(r) and λ_{max}.

$$\lambda_{max,AF} = 2 * (0.9)^{r+1} \tag{38}$$

$$\lambda_{max,DF,E2E} = 0.9^{r+1} \tag{39}$$

$$\lambda_{max,DF,H2H} = 0.9 \tag{40}$$

Fig 2 shows the plot of λ_{max} for the three systems for $r \in \{1, 2...8\}$.

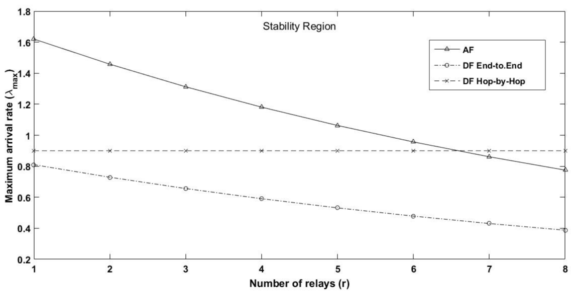

Fig. 2. Stability region for AF, DF with End-to-End ARQ and DF with Hop-by-Hop ARQ relay networks

In AF relay network, as the number of relay nodes increase, probability of packets reaching MS being in error ($p_{e,e2e}$) also increases. So, the intensity of retransmissions would also increase, which in turn would increase packet arrival rate at each of the sub-system. Thus, as r goes to infinity, λ_{max} would tend to zero.

Similarly in DF relay networks with End-to-End ARQ, as the number of relays increase, probability of a packet incurring error on any one of the node increases, thus increasing the intensity of retransmission. As r tends to infinity, λ_{max} would tend to zero.

However, in DF relay networks with End-to-End ARQ λ_{max} is completely unaffected by number of relay nodes. Each sub-system can be treated in complete isolation. Thus even if r tends to infinity this system would not be affected and λ_{max} would still be $1 - p_{e,j}$.

2) Arrival rate-delay characteristics: In this section, we would analyze the relationship between Arrival rate and delay of the system. Assuming number of relays to be equal to four ($r = 4$) and other parameters to be the same as before, we can write equations $(41) - (43)$ and sketch the plot as shown in fig. 3 using formulae in tables II and III. Using values of λ_{max} from equations $(38) - (40)$, in equations $(41) - (43)$ range of λ is $(0, \lambda_{max})$.

$$\overline{T}_{AF} = \frac{5}{2(0.9)^5 - \lambda} \tag{41}$$

$$\overline{T}_{DF,E2E} = \frac{4}{(0.9)^5 - \lambda} + \frac{1}{2(0.9)^5 - \lambda} \tag{42}$$

$$\overline{T}_{AF} = \frac{5}{2(0.9)^5 - \lambda} \tag{43}$$

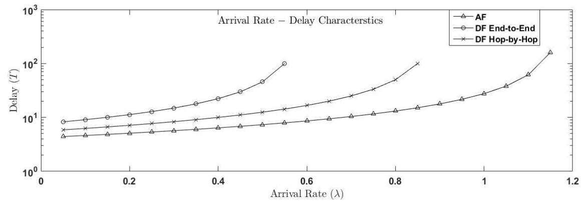

Fig. 3. Arrival rate-delay characteristics of AF, DF with End-to-End ARQ and DF with Hop-by-Hop ARQ relay networks

The graph shows that AF has less delay than DF networks, and Hop-by-Hop ARQ has lesser delay than End-to-End ARQ.

As the probability of packets in error ($p_{e,j}$) increases, there would be more retransmissions and the average delay is bound to increase. Thus, the curves in the plot (shown in fig. 3) would shift upwards.

3) AF vs. DF: Taking $r = 4$, $\mu_{DF} = 1$ and $\mu_{AF} = \mu_{MS} = 2\mu_{DF}$, $\lambda = 0.5\lambda_{max}(r)$, we can write the relation between time delay and packet error probability, and plot the same (fig. 4) by varying $p_{e,j}$ from 0.05 to 0.95 using formulae in tables II and III.

$$p_e = p_{e,1} = p_{e,2} .... = p_{e,5} \tag{44}$$

$$\overline{T}_{AF} = \frac{5}{(1 - p_e)^5} \tag{45}$$

$$\overline{T}_{DF,E2E} = \frac{26}{3(1 - p_e)^5} \tag{46}$$

$$\overline{T}_{DF,H2H} = \frac{26}{3(1 - p_e)} \tag{47}$$

Form the graph we can observe that for p_e above 0.13, DF

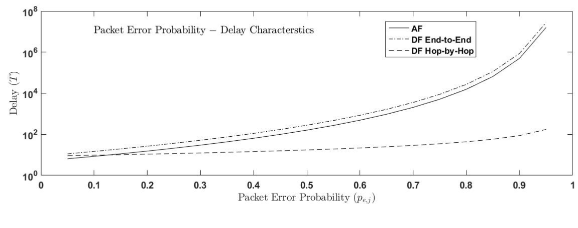

Fig. 4. Packet error probability-delay characteristics of AF, DF with End-to-End ARQ and DF with Hop-by-Hop ARQ relay networks

relaying with Hop-by-Hop ARQ clearly has least system delay. Where as for p_e above 0.13, AF relaying has least system delay. Thus, we can conclude that based on the metric of system delay, for systems with less packet error probability AF relaying can be used, and for systems with higher packet error probability DF relaying with Hop-by-Hop ARQ should be used. Besides system delay, we can consider other factors as well like complexity and cost of the relaying nodes. In AF relay networks relay nodes are rather simple and would cost less as compared to DF networks. Also, in the case of Hop-by-Hop ARQ each node would be even more complex as compared to End-to-End ARQ. This factor might shift the odds towards choosing AF relaying networks with End-to-End ARQ. However, if packet error probability is in considerable limits, DF relay with Hop-by-Hop ARQ is highly recommended.

Now, for a fixed value of $p_e = 0.1$ and $\mu_{AF} = k\mu_{DF},\ k \in [1,4]$ while rest of the parameters being same, we can compare the system delay of AF for different values of k with that DF networks as shown in table IV. By observing system delay

TABLE IV
SYSTEM DELAY

Packet Processing Time (μ)	System Delay
μ_{DF} End-to-End	14.68
μ_{DF} Hop-by-Hop	9.63
μ_{AF} for $k = 1$	16.93
μ_{AF} for $k = 2$	8.47
μ_{AF} for $k = 3$	5.64
μ_{AF} for $k = 4$	4.23

values from the table IV, for k=2 and above AF relay network is better, while, for k=1, DF network with Hop-by-Hop ARQ has performance.

III. RELAY NETWORK WITH FINITE RETRANSMISSIONS

The queuing system in consideration in this section are shown in Fig 5. Each of the nodes have one server and the associated queues are independent. They have infinite buffers and arrival and departure process are Poisson. The number of retransmissions is limited in this system.

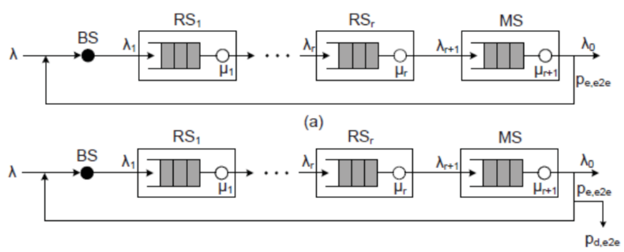

Fig. 5. Queuing networks for AF relaying with end-to-end ARQ in the cases that (a) the maximum number of retransmission $= \infty$ and (b) the maximum number of retransmission $R < \infty$.

A. Theoretic Analysis

If a packet need to be retransmitted, then it would be dropped if it remains in error in the next R-1 retransmissions as well. Thus, the packet drop probability given that the packet needs to be retransmitted ($p_{d,e2e}$) can be written in terms of probability of packet being in error while being transmitted from from BS to MS ($p_{e,e2e}$) as shown in equations $(48) - (50)$.

$$p_{e,e2e} = 1 - \prod_{j=1}^{r+1}(1 - p_{e,j}) \tag{48}$$

$$p_{d,e2e} = (p_{e,e2e})^{R-1} \tag{49}$$

Using equations $(48) - (49)$,

$$p_{d,e2e} = (1 - \prod_{j=1}^{r+1}(1 - p_{e,j}))^{R-1} \tag{50}$$

Now, in order to find the packet loss ratio, we need to find the intensity of packets being dropped. By understanding the movement of packets in the system we can formulate relations between arrival intensity (λ), packet drop intensity (λ_{dr}), packet retransmission intensity (λ_r) and throughput (λ_o).

$$\lambda_1 = \lambda_2 ..... = \lambda_{r+1} \tag{51}$$

$$\lambda_1 = \lambda + \lambda_r \tag{52}$$

$$\lambda_o = \lambda - \lambda_{dr} \tag{53}$$

$$\lambda_{dr} = (p_{e,e2e} \cdot p_{d,e2e})\lambda_1 \tag{54}$$

$$\lambda_r = p_{e,e2e}(1 - p_{d,e2e})\lambda_1 \tag{55}$$

Using equations $(51) - (55)$, we can express all the intensities in terms of λ.

$$\lambda_1 = \frac{\lambda}{1 - p_{e,e2e}(1 - p_{d,e2e})} \tag{56}$$

$$\lambda_{dr} = \frac{p_{e,e2e} \cdot p_{d,e2e}}{1 - p_{e,e2e}(1 - p_{d,e2e})}\lambda \tag{57}$$

$$\lambda_r = \frac{p_{e,e2e}(1 - p_{d,e2e})}{1 - p_{e,e2e}(1 - p_{d,e2e})}\lambda \tag{58}$$

$$\lambda_o = \frac{1 - p_{e,e2e}}{1 - p_{e,e2e}(1 - p_{d,e2e})}\lambda \tag{59}$$

Packet loss ratio (P_L) would be the ratio of intensity of packets dropped to the intensity of packet arrival.

$$\begin{aligned} P_L &= \frac{\lambda_{dr}}{\lambda} \\ \Rightarrow P_L &= \frac{p_{e,e2e} \cdot p_{d,e2e}}{1 - p_{e,e2e}(1 - p_{d,e2e})} \\ \Rightarrow P_L &= \frac{(p_{e,e2e})^R}{1 - p_{e,e2e} + (p_{e,e2e})^R} \end{aligned} \tag{60}$$

The system would be stable if all queues are stable (ie. $\rho_j < 1,\ for\ j \in \{1, 2...r+1\}$), thus we can find λ_{max} by equating arrival intensity in the sub-system to the packet processing intensity.

$$\begin{aligned} \lambda_1 &= \mu_{AF} \\ \Rightarrow \mu_{AF} &= \lambda_{max}(r)\frac{1}{1 - p_{e,e2e}(1 - p_{d,e2e})} \\ \Rightarrow \lambda_{max}(r) &= \mu_{AF}(1 - p_{e,e2e}(1 - p_{d,e2e})) \\ \Rightarrow \lambda_{max}(r) &= \mu_{AF}(1 - p_{e,e2e} + (p_{e,e2e})^R) \end{aligned} \tag{61}$$

Throughput(λ_o) of the system can be given by equation (59). To find the end to end delay, we can use Little's theorem on the system. Each sub-system is an M/M/1 system, thus we can directly use the formula derived in the lectures for average number of packets in an M/M/1 system.

$$\begin{aligned}
\overline{T} &= \frac{\overline{N}}{\lambda_{eff}} \\
\Rightarrow \overline{T} &= \frac{\sum_{j=1}^{r+1} \overline{N}_j}{\lambda - \lambda_{dr}} \\
\Rightarrow \overline{T} &= \frac{\sum_{j=1}^{r+1} \frac{\lambda_j}{\mu_j - \lambda_j}}{\lambda_o} \\
\Rightarrow \overline{T} &= (r+1)\frac{\lambda_1}{\lambda_o(\mu_{AF} - \lambda_1)} \\
\Rightarrow \overline{T} &= \frac{r+1}{(1-p_{e,e2e})(\mu_{AF} - \frac{\lambda}{1-p_{e,e2e}(1-p_{d,e2e})})}
\end{aligned} \tag{62}$$

B. Numerical Analysis

We are taking $r = 4$, $\mu_{AF} = \mu_{MS} = 2$, and $p_{e,j} = 0.1,\ for\ j \in \{1, 2, ..., r+1\}$. Using these, we can plot the packet loss ratio when R increases from 3 to 8 for $\lambda = 0.5\lambda_{max}(r)$.

$$\begin{aligned}
p_{e,e2e} &= 1 - (0.9)^5 \\
\Rightarrow P_L(R) &= \frac{(1-(0.9)^5)^R}{(0.9)^5 + (1-(0.9)^5)^R}
\end{aligned} \tag{63}$$

Form fig. 6 we can observe that packet loss ratio decreases

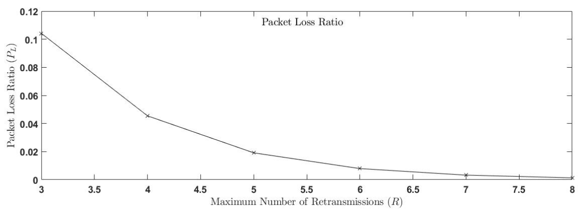

Fig. 6. Packet Loss Ratio(P_L) versus maximum number of retransmissions(R)

rapidly by increasing maximum number of retransmissions. Now, we will try to express delay in terms of throughput.

$$\begin{aligned}
\lambda_1 &= \frac{\lambda_o}{1-p_{e,e2e}} \\
\overline{T} &= (r+1)\frac{\lambda_1}{\lambda_o(\mu_{AF} - \lambda_1)} \\
\Rightarrow \overline{T} &= \frac{(r+1)}{(\mu_{AF}(1-p_{e,e2e}) - \lambda_o)} \\
\Rightarrow \overline{T} &= \frac{5}{2(0.9)^5 - \lambda_o}
\end{aligned} \tag{64}$$

We need to find the range of λ_o for $\lambda < \lambda_{max}(r)$ for $R = 4$.

$$\begin{aligned}
\lambda_{o,max} &= \frac{1-p_{e,e2e}}{1-p_{e,e2e}(1-p_{d,e2e})}\lambda_{max} \\
\Rightarrow \lambda_{o,max} &= \frac{(0.9)^5}{(0.9)^5 + (1-(0.9)^5)^4}\lambda_{max} \\
\lambda_{max} &= 2((0.9)^5 + (1-(0.9)^5)^4) \\
\Rightarrow \lambda_{o,max} &= 2(0.9)^5
\end{aligned} \tag{65}$$

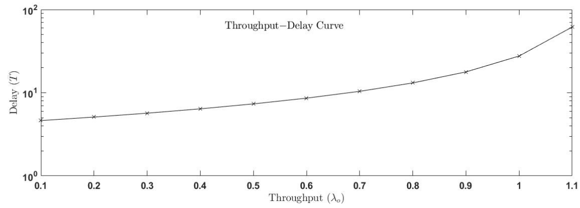

Fig. 7. Variation in delay ($\overline{T}$) with increment of throughput (λ_o)

If we compare maximum throughput of finite retransmission system with that of AF relay network with infinite retransmission, then we can see that they are equal.

$$\lambda_{o,max} = \lambda_{AF,max} = 2(0.9)^{r+1}$$

Expression for average time delay is also same for both the cases.

$$\overline{T}_{finite} = \overline{T}_{AF} = \frac{5}{\lambda_{o,max} - \lambda_o}$$

Thus, we can conclude that AF infinite retransmission model is good enough for analyzing throughput and average time delay for a corresponding real world finite retransmission system.

IV. Conclusion

This project elaborately analyzed different types of error control multi-hop relay network architecture and derived essential parameters for each of the type. The project also dealt with practical relay networks with finite retransmissions, and deduced that they can be modeled as infinite retransmission relay systems for throughput and average delay analysis.

References

[1] *EP2200 Course Project*, KTH Royal Institute of Technology, Mar. 2016.