Microeconomic Theory for Applications 2023
Problem Set 2: Consumer Theory

Please upload your solutions to e-learning by 4pm on September 27 (reminder: late problem sets will not be accepted). Show your work and make an effort to argue as clearly as possible in each of the exercises.

1. Consider the expenditure function

$$e(p_x, p_y, \bar{u}) = \frac{\bar{u}}{\frac{1}{p_x} + \alpha \frac{1}{p_y}}, \text{ where } \alpha > 1.$$

   Suppose the initial situation is $(p_x, p_y, I) = (1, 1, 1)$ and that things change to $(p'_x, p'_y, I') = (1/2, p'_y, 1)$. For which values of p'_y is the consumer worse off after the change?

2. A consumer has utility function

$$u(L, F) = L^A F^{1-A},$$

   where L represents hours of leisure, F is food, and A is a parameter that satisfies $0 < A < 1$. The consumer has 24 hours that she can divide between leisure and work. For each hour that she works, she earns a salary $s > 0$, and each unit of food costs p. She has initial wealth w (i.e., this wealth is independent of how much she works).

   (a) Write down the consumer's budget constraint and make a picture of it in (L, F) space.

   (b) Solve the consumer's utility maximization problem and obtain the demand for leisure $L(p, s, w)$ and food $F(p, s, w)$.

   (c) Find a condition on the parameters such that the consumer does not work (Hint: examine the inequality $L(p, s, w) \geq 24$).

   (d) How does the consumer's demand for leisure respond to a change in initial wealth w? How does the consumer's demand for leisure respond to a change in the salary s? Use partial derivatives to answer both questions.

3. Consider utility function $u(x, y) = \min\{ax, y\}$, where $a > 0$.

   (a) Calculate the consumer's Marshallian demand functions and indirect utility function.

   (b) Calculate the consumer's compensated demand functions and expenditure function.

   (c) How does the Marshallian and Compensated demands for good 1 respond to a change in the price of good 2? Answer by using derivatives.

   (d) Suppose $a = 3$, $p_y = 1$ and $I = 10$. In the same diagram, graph

      - the consumer's Marshallian demand curve for good 1, $x^*(p_x, 1, 10)$ and
      - the consumer's compensated demand curve for good 1, $x^C(p_x, 1, v(1, 1, 10))$.

In the diagram, you should have x on the horizontal axis and p_x on the vertical axis. Notice that the utility level for the compensated demand curve equals the indirect utility for $p_x = 1$, $p_y = 1$ and $I = 10$. While the graphs do not have to be exact, the demand curves should have the correct shape and relate to each other in the correct way.

(e) Suppose $p_x = 1$, $p_y = 1$ and $I = 10$, and that the price of good 1 changes to $p'_x = 2$. Calculate the income and substitution effects for good 1.

4. Evaluate whether the following claims are true or false (do not forget to motivate your answer, just answering "true" or "false" gives zero credit.

   (a) Consider a utility maximizing consumer with a differentiable utility function $u(x, y)$ that satisfies $u_1 > 0$ and $u_2 > 0$.

   Claim: The consumer's demand functions could be given by $x(p_x, p_y, I) = \frac{I}{p_x}$ and $y(p_x, p_y, I) = \frac{I}{p_y}$.

   (b) Suppose that as the price of apples doubles, Sten's demand for apples declines by 10 units.

   Claim: If the substitution effect is -8, then we can conclude that apples must be an inferior good for Sten.

   (c) Gunilla consumption set consists of corn tortillas (good 1) and wheat tortillas (good 2). The price of a pound of corn tortillas is \$2 and the price of a pound of wheat tortillas is \$1. She purchases a bundle such that her MRS equals 4.

   Claim: Since Gunilla's MRS is not equal to the price ratio, her chosen bundle cannot maximize her utility.

   (d) Suppose the consumer's utility function is $u(x, y) = x^2 + y^2$ and that $p_x = p_y = 1$.

   Claim: Given the specified prices, the income consumption curve of the consumer will be given by the line $y = x$.