# PROJECT 1: BRAKE SYSTEM MODELLING AND CONTROL

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## Objective:

To determine the transfer function of air brake system consisting of electro-pneumatic regulator (EPR) and design a negative feedback P, PI, PD and PID controller given certain performance.

## Dominant frequency and magnitude:

| Frequency | Input dominant frequency | Output dominant frequency | Maximum input | Maximum output |
|---|---|---|---|---|
| 0.1 | 2.7176 | 2.4085 | 0.10681 | 0.10681 |
| 0.2 | 2.969 | 2.5804 | 0.19836 | 0.19836 |
| 0.3 | 2.7007 | 2.2066 | 0.30518 | 0.30518 |
| 0.4 | 2.9587 | 2.2205 | 0.39673 | 0.39673 |
| 0.5 | 2.4634 | 1.6486 | 0.48828 | 0.48828 |
| 0.6 | 2.6665 | 1.6096 | 0.61035 | 0.61035 |
| 0.7 | 2.9601 | 1.4435 | 0.7019 | 0.7019 |
| 0.8 | 2.957 | 1.1919 | 0.79346 | 0.79346 |
| 0.9 | 3.0363 | 1.1118 | 0.90027 | 0.90027 |
| 1.0 | 2.9275 | 1.027 | 1.0071 | 1.0071 |
| 1.1 | 2.9853 | 0.86415 | 1.0986 | 1.0986 |
| 1.2 | 2.5158 | 0.69169 | 1.2207 | 1.2207 |

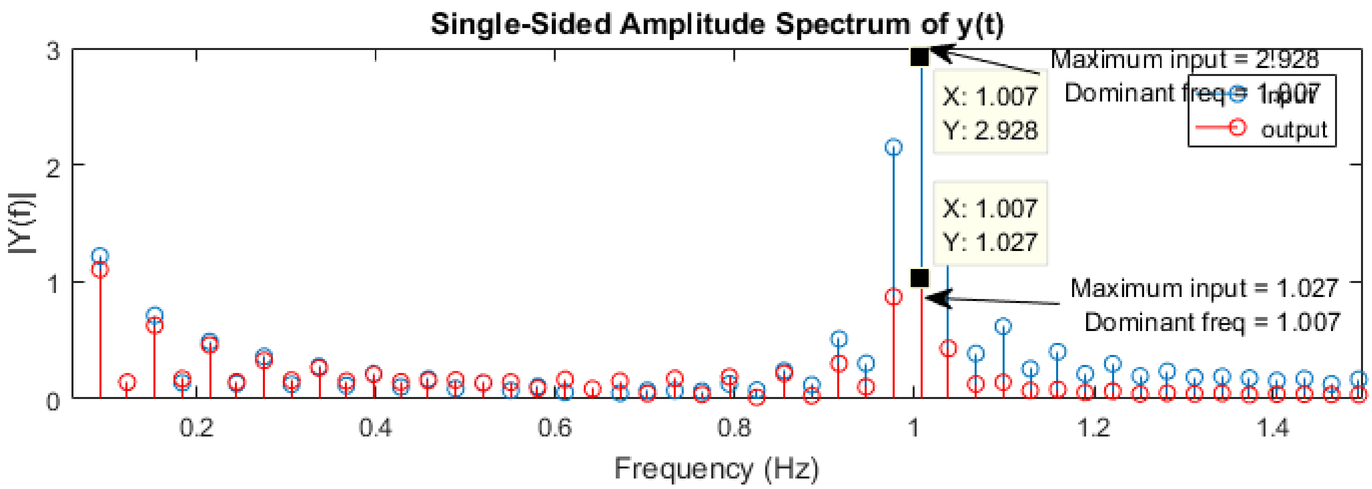



*Figure 1 Spectrum for frequency of 1.0 Hz*

As it can be observed in figure1, the max output peak is at 0.06 Hz frequency which is at the lower end. But we are ignoring that value as it is in low frequency region and it could be due to some sort of noise in the system.

# Bode Plot and plant function:

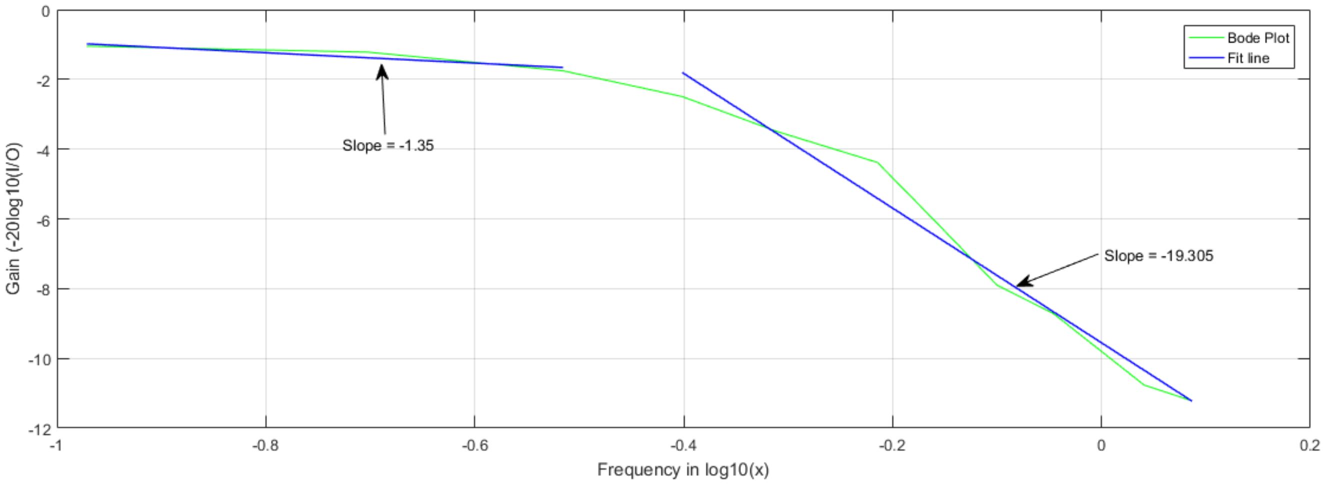


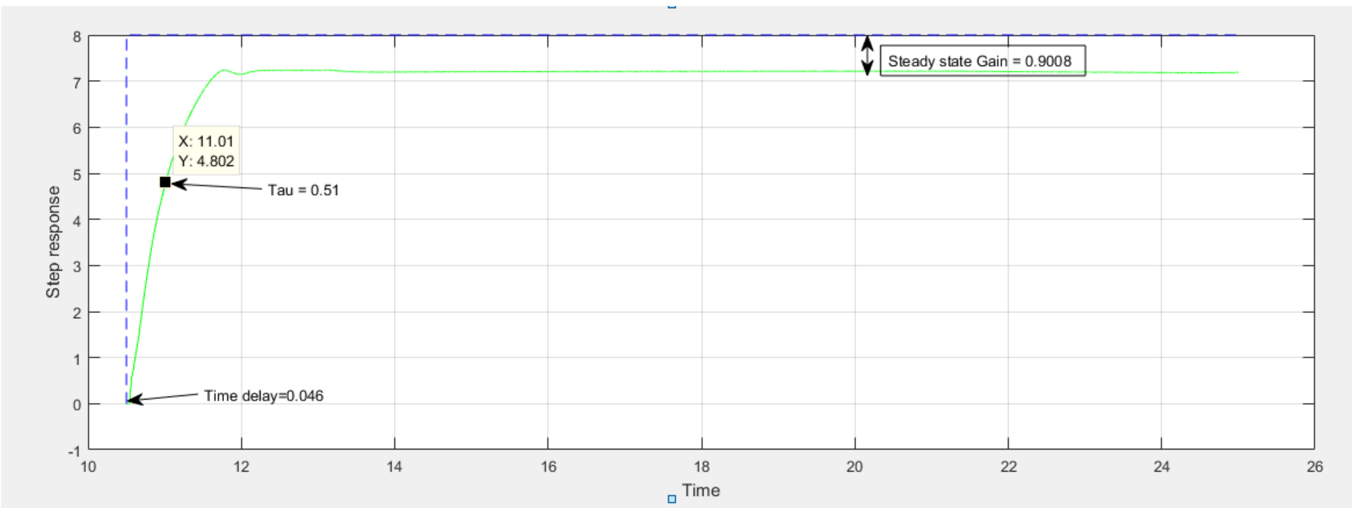

*Figure 2: Bode plot and curve fitting a straight line for low frequency range and high frequency*

As it can be observed above the slope is close to -20 which means it can be taken as a second order system, represented by:

$$G(s) = \frac{K}{1+\tau s}\, e^{-Ts}$$

After using PADE's approximation, analysing the step response of the system by plotting voltage and pressure with time, we can calculate time delay, gain and time constant.

$$G(s) = \frac{K\,(2+Ts)}{(2-Ts)(1+\tau s)}$$




$$\text{Mean Gain} = 0.9001 \qquad\qquad \text{Average time delay} = 0.053$$

For time constant of system, we have to find MAPE for each tau in the range of tau calculated from the experimental data. The tau which given the minimum MAPE is the system time constant.

| Time constant value | Step = 5V | Step = 6V | Step = 7V | Step = 8V | Average MEAP |
|---|---|---|---|---|---|
| 0.448 | 2.433004 | 1.833455 | 1.74271 | 1.621707 | 1.907719 |
| 0.468 | 2.672494 | 1.929929 | 1.496579 | 1.967454 | 2.016614 |
| 0.488 | 2.951729 | 2.123724 | 1.343769 | 2.374608 | 2.198457 |
| 0.508 | 3.261444 | 2.396872 | 1.331888 | 2.801256 | 2.447865 |
| 0.528 | 3.618369 | 2.716745 | 1.488686 | 3.220199 | 2.765461 |
| 0.548 | 4.00446 | 3.067889 | 1.777665 | 3.631845 | 3.120465 |
| 0.568 | 4.393517 | 3.450167 | 2.126571 | 4.038013 | 3.502067 |
| 0.588 | 4.778874 | 3.855409 | 2.504922 | 4.438933 | 3.894535 |

Looking at the above table for minimum MEAP, time constant is 0.448.

Now, our system plant function is fully known

$$G(s) = \frac{0.9001}{1 + 0.448s}\, e^{-0.053s}$$

# PID Controller design:

Controller function for PID is given by :

$$C(s) = Kp + \frac{Ki}{s} + Kd * s$$

For proportional controller Kp can be obtained by plotting root locus and performance constraints. With the constrained region in root locus, the range of Kp can be found.

For PI controller, integral gain is taken as $Ki = \frac{Kp}{\tau}$ , where $\tau$ is time constant. This relation is taken from a paper where it has been stated this approximation of Ki works for LTI system very effectively.

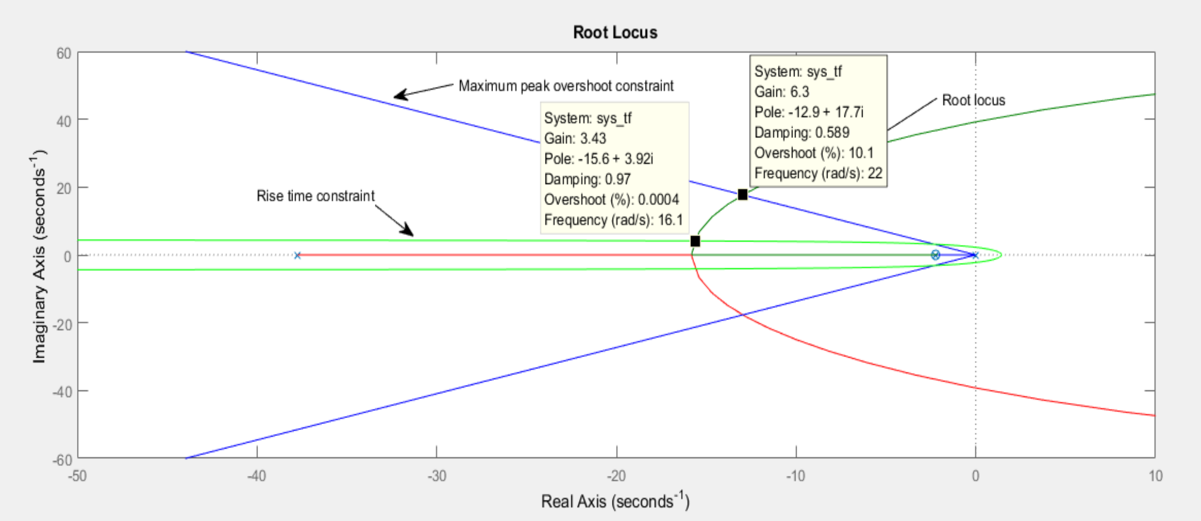



*Figure 3 Root locus plot for PID controller*

Differential term of PID controller is very sensitive to noise, so a very small ratio of Kd to Kp is chosen to minimise the effect of noise in system. as $Kd = 0.001 * Kp$

Effectively, controller function for PID becomes:

$$C(s) = Kp(1 + \frac{1}{0.448 * s} + 0.001 * s)$$

| Type | Kp | | Ki | | Kd | |
|---|---|---|---|---|---|---|
| | min | max | min | max | min | max |
| PID | 3.43 | 6.3 | 7.65625 | 14.0625 | 0.00343 | 0.0063 |
| P | 2.94 | 6.14 | - | - | - | - |
| PI | 3.53 | 6.15 | 7.879464 | 13.72768 | - | - |
| PD | 3.02 | 6.28 | - | - | 0.00302 | 0.00628 |

*Figure 4 Controller gains for each of the system*

For optimising the controller for steady state error and performance criteria, we observe that for max values of Kp, the steady state error is minimum. This result makes sense because as Kp increases the system responds more aggressive to error values and hence the steady state error decreases, but also peak overshoot and oscillations of the system increases.