AOA | Homework - 8

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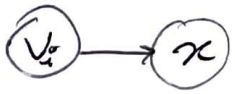
35.2-3> Consider the closest-point heuristic for building an approximate traveling salesperson tour whose cost function satisfies the triangle inequality. Prove that this heuristic returns a tour whose total cost is not more than twice the cost of an optimal tour.

⇒ Closest point heuristic: Begin with a trivial cycle consisting of a single arbitrarily chosen vertex. At each step, identify the vertex u that is not on the cycle but whose distance to any vertex on the cycle is minimum. If that vertex on the cycle ~~(nearest u) is~~ that is nearest to u is vertex v. Then extend the cycle to include u by inserting u just after v. Repeat until all vertices are on the cycle. Using this heuristic lets take a use case to understand the cost:-

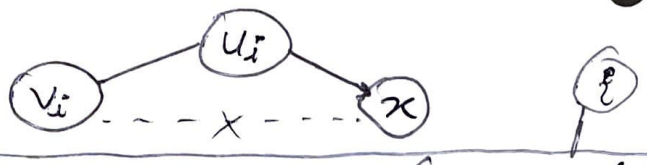
· Let us assume in iteration i that u_i be the vertex that is not on the cycle and we want to extend to cycle and let say v_i was connected to some x. u_i will lead to removal of edge $v_i \to x$ and addition of edge $v_i \to u_i$ & $u_i \to x$.

· Initially.



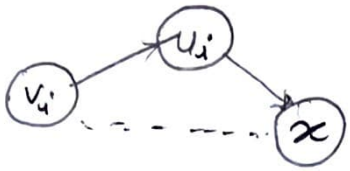
Δ Cost = $c(v_i, x)$

⇒ after i^{th} iteration.



$\Delta Cost = c(v_i, u_i) + c(u_i, x) - c(v_i, x)$

Also, using triangle property we can say;



$$c(u_i, x) \leq c(v_i, x) + c(v_i, u_i)$$

$$\boxed{c(u_i, x) - c(v_i, x) \leq c(v_i, u_i)} \quad \text{(ii)}$$

Substituting eqn(ii) in eqn (i), we will get.

~~cost = c(v_i, u_i)~~

Increase in cost in iteration $i \leq 2c(v_i, u_i)$

$\therefore$ total cost of output tour $\leq \boxed{2\sum_{i=2}^{n} c(v_i, u_i)}$

$\Rightarrow$ Prim algorithm: In Prim's MST all vertices would have been added in same order

$\therefore$ total cost of output tour = $\boxed{\sum_{i=2}^{n} c(v_i, u_i)}$

we know from class that prim's algo MST cost is lower bound of optimal cost.

i.e $\boxed{c(\text{Prim's MST}) \leq c(\text{Optimal}).}$

and from above eqn we could say that.

$$\boxed{c(\text{closest point Heu}) \leq 2c(\text{Prim's MST})}$$

Hence, $\boxed{c(\text{Closet point Heuristic}) \leq 2c(\text{Optimal})}$

i.e given heuristic returns a tour whose total cost is not more than twice the cost of an optimal tour.