Homework - 9 | AOA | Name: ASHUTOSH CHAUHAN
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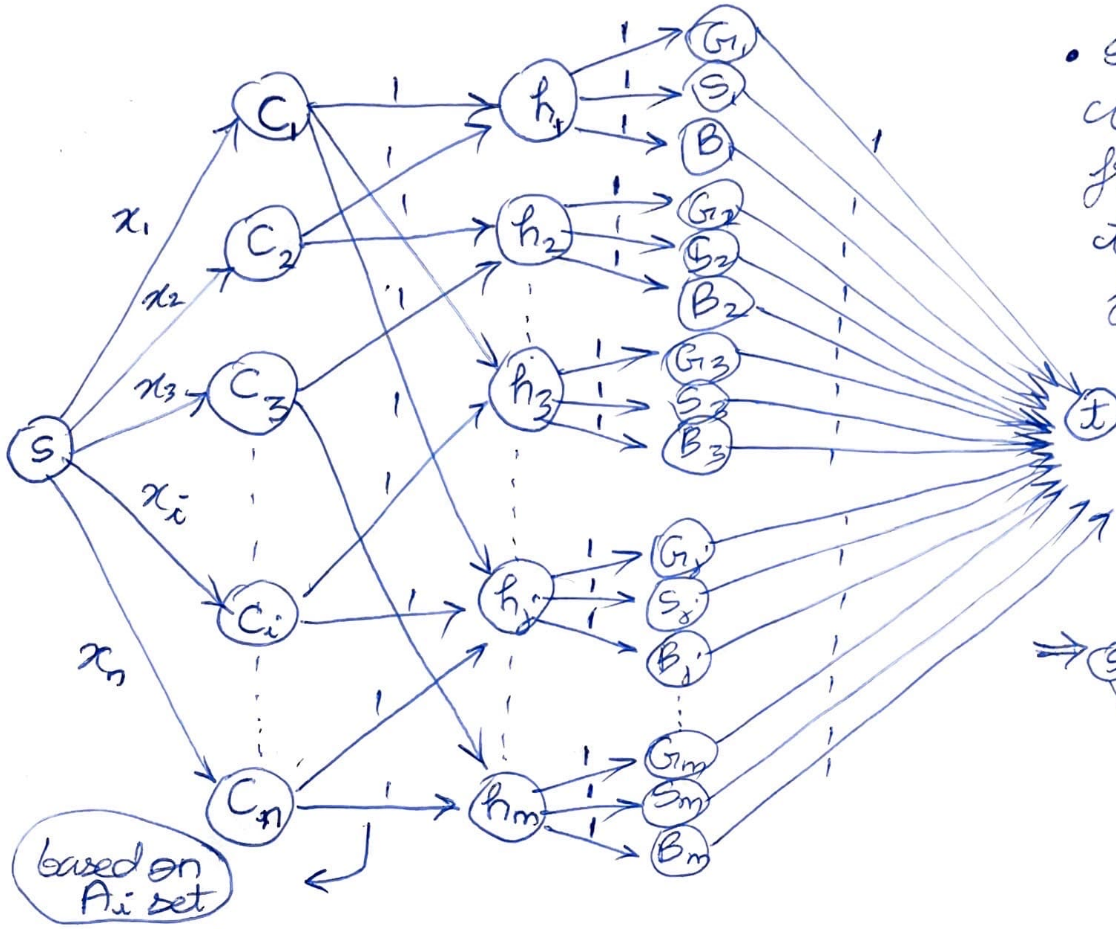
Q.) Olympics 2020 problem. Given any vector $X = (x_1, x_2, \ldots, x_n)$, is X a possible outcome of Olympics 2020? Design an efficient algorithm to determine if the answer if "yes" or "no", prove its correctness and analyze its time complexity.

Ans- **Main Idea**:
In the given problem number of medals are constant so we could think this problem as a network flow problem and maximum medal no. as max flow. Then we will use Edmonds-Karp algorithm to verify if the max flow of formed graph is equal to the maximum medals. If equal we will return 'Yes' else 'No'.

Algorithm:

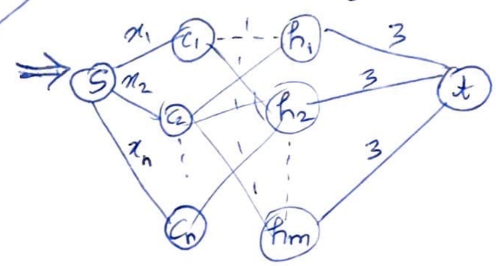
- The graph will contain a source 's', a sink 't', nodes 'C_i' representing i^{th} country and node 'h_j' to represent j^{th} game.
- For each country 'C_i' and game 'h_j' such that $C_i \subseteq A_i$ (i.e set of countries who sent athletes to partiicite in game h_j), we add an edge from node 'C_i' to 'h_j' with capacity 1.
- For each game 'h_j', we add three nodes to represent the gold medal, silver medal and bronze medal, denoted as: 'G_j', 'S_j' and 'B_j' respectively.
- We add edges from node 'h_j' to: 'G_j', 'S_j' and 'B_j' with capa 1.

→ We ~~gave~~ set the capacity of edges between a country and game as 1, since in question it is given that a country can ~~parti~~ send atmost 1 player to any particular game.



• In this graph we want to send flow from the source node to sink node where flow represent the medals.

• We also could simplify this graph by removing G_j, S_j & B_j



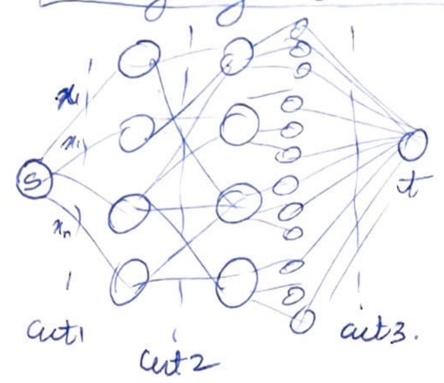
• Each edge from country → game, represent medal that country could win in this game.

• Each edge from game → medal node, represent what position is won by that country.

• If maxflow $= 3m \Rightarrow$ X is valid possible outcome
$\neq 3m \Rightarrow$ X is not a possible outcome
, where m is no. of games.

~~Proof of~~

• **Proof of Correctness**



In given graph we can see that the capacity of each cut is equal to the size of flow i.e 3m

~~If X vector will be a valid possible outcome then and if we receive 3m flow at sink.~~

If we are getting 3m flow at sink, we could say that at every cut the flow will be equal to the capacity of cut i.e 3m. This is only possible if source is giving 3m flow as sum of flow received by sink is equal to sum of flow sent by source. The above point proves that $\sum(x_1, x_2 \ldots x_n) = 3m$. And since we set the capacity of edge from country to game as 1, it will ensure that only one participant from a country participate in one game. The above two points proves the correctness of our algorithm.

• **Time Complexity** :-

• Time to do flow augmentation in Edmonds-Karp algorithm $= O(VE)$.

• Time to search shortest path every time $= O(V+E)$
$= O(E) \quad (\because |E| \geq |V|-1)$

$\therefore$ Total time complexity $= O(VE) * (E) = O(VE^2)$

; where, $V = n+m+2$
$E = n + mn + m$

$\therefore \ TE = O((n+m+2)(n+mn+m)^2)$
$\cong O((m+n)(n+mn+m)^2)$