Algorithms

Lecture Topic: NP Completeness (Part 5)

Anxiao (Andrew) Jiang

Roadmap of this lecture:

1. NP Completeness

    1.1 Prove the "Traveling Salesman Problem" is NPC.

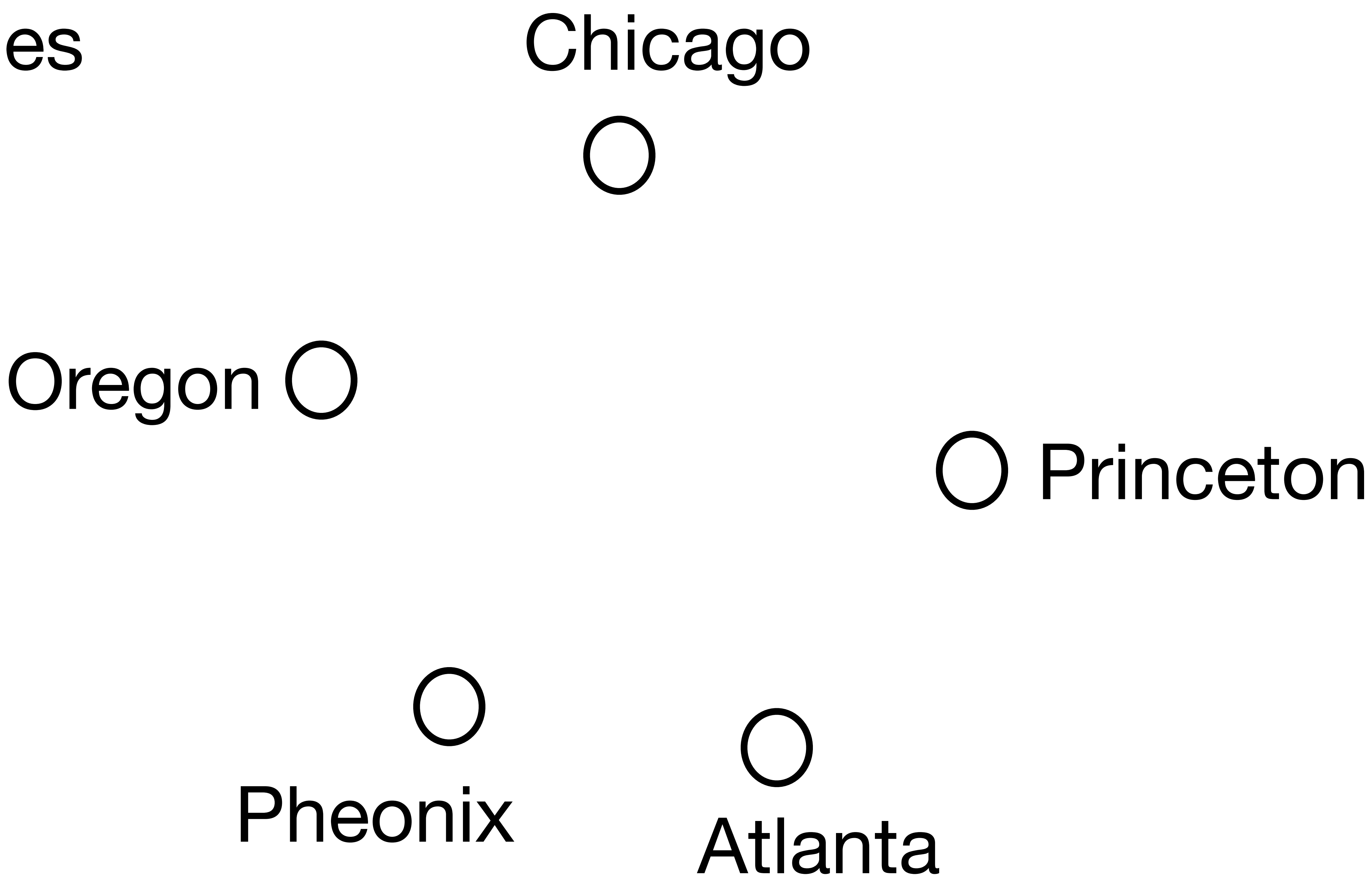
    1.2 Prove the "Subset Sum Problem" is NPC.

Traveling Salesman Problem (TSP)

Known NPC Problems:

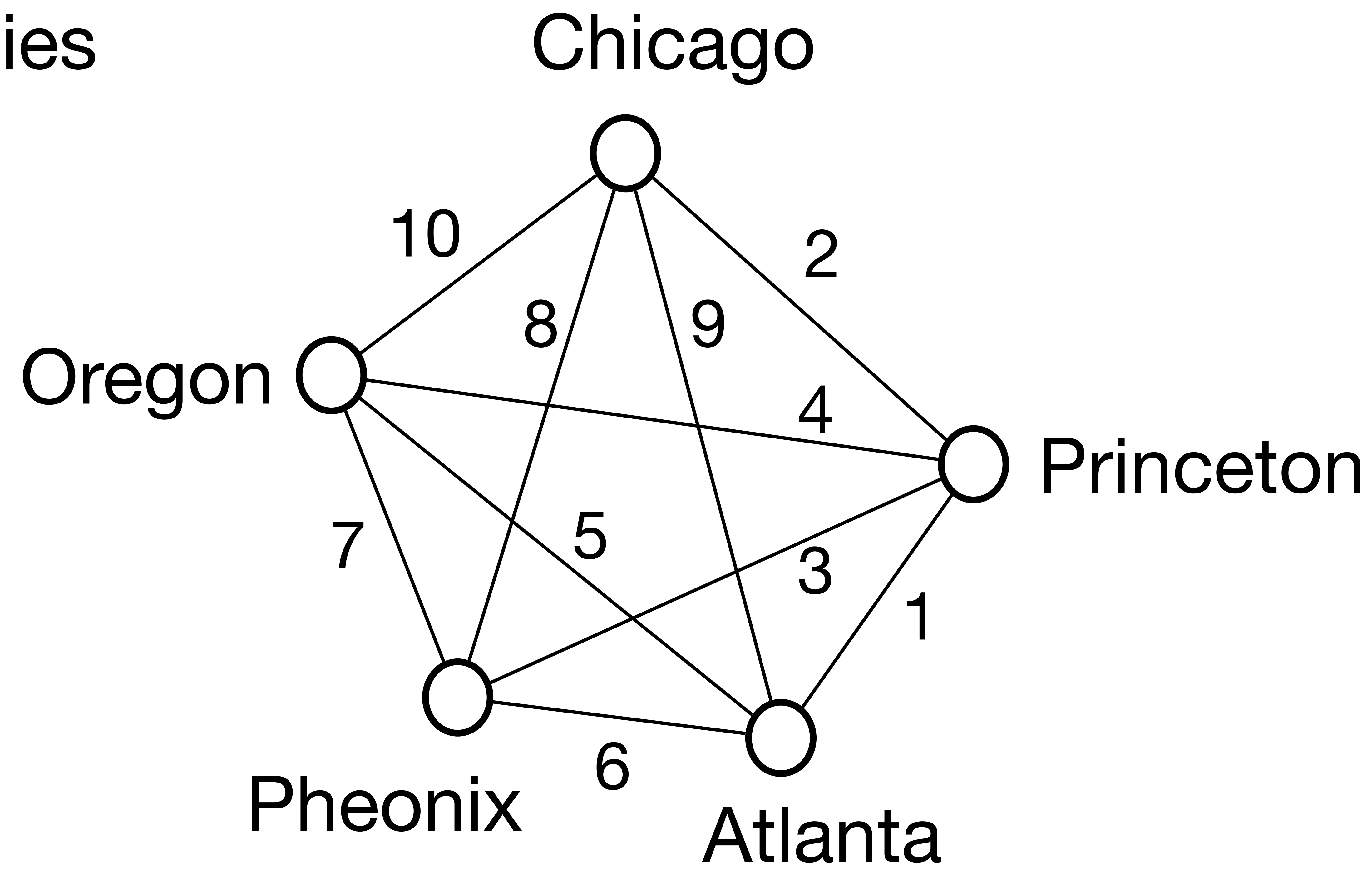
1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem

n cities



Traveling Salesman Problem (TSP)

n cities



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Traveling Salesman Problem (TSP)

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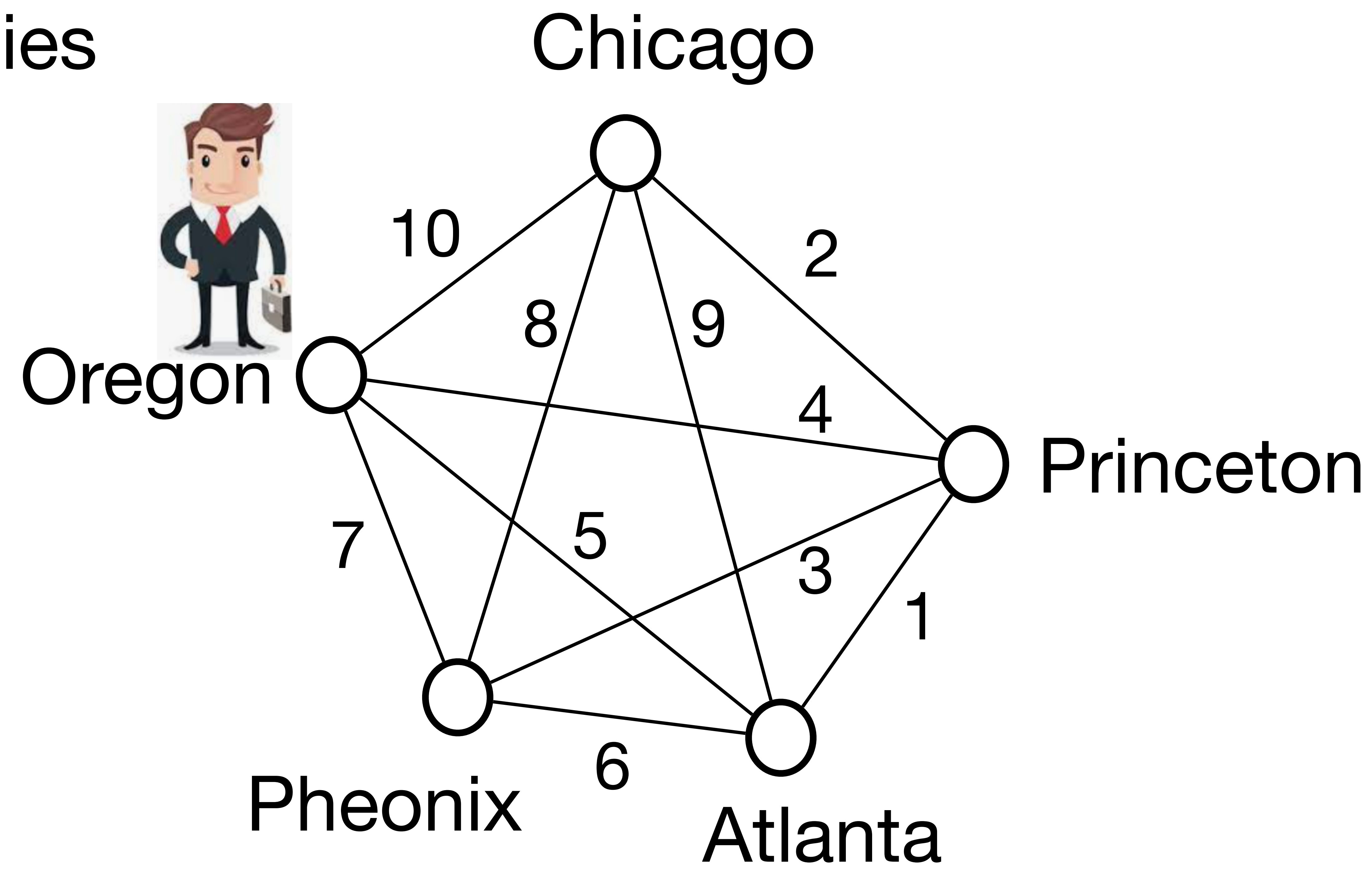

Known NPC Problems:

1) 3-CNF SAT Problem
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4) Hamiltonian Cycle Problem

How can the salesman visit each city exactly once, and return home via the shortest journey?

Traveling Salesman Problem (TSP)

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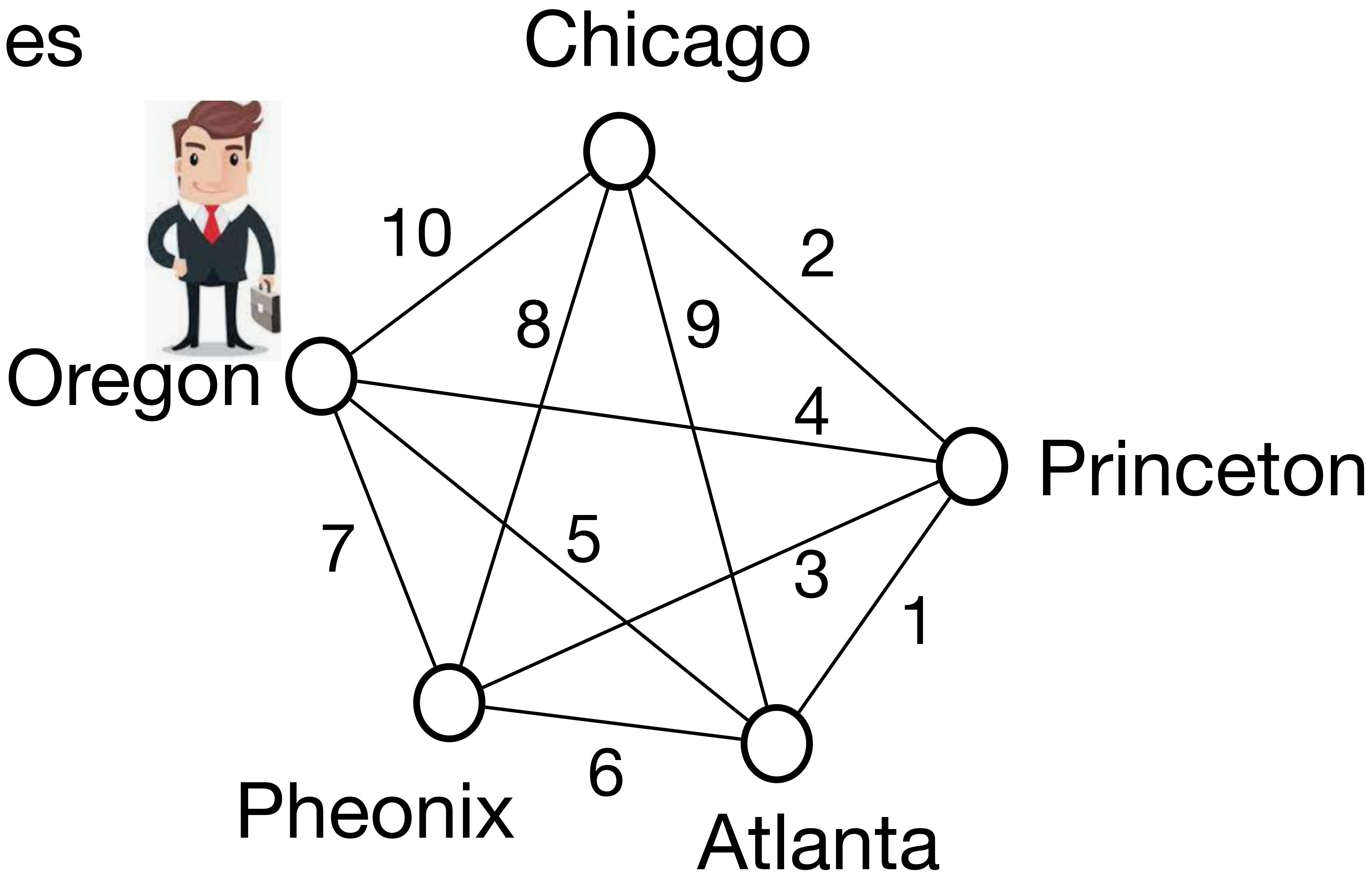

Hamiltonian cycle

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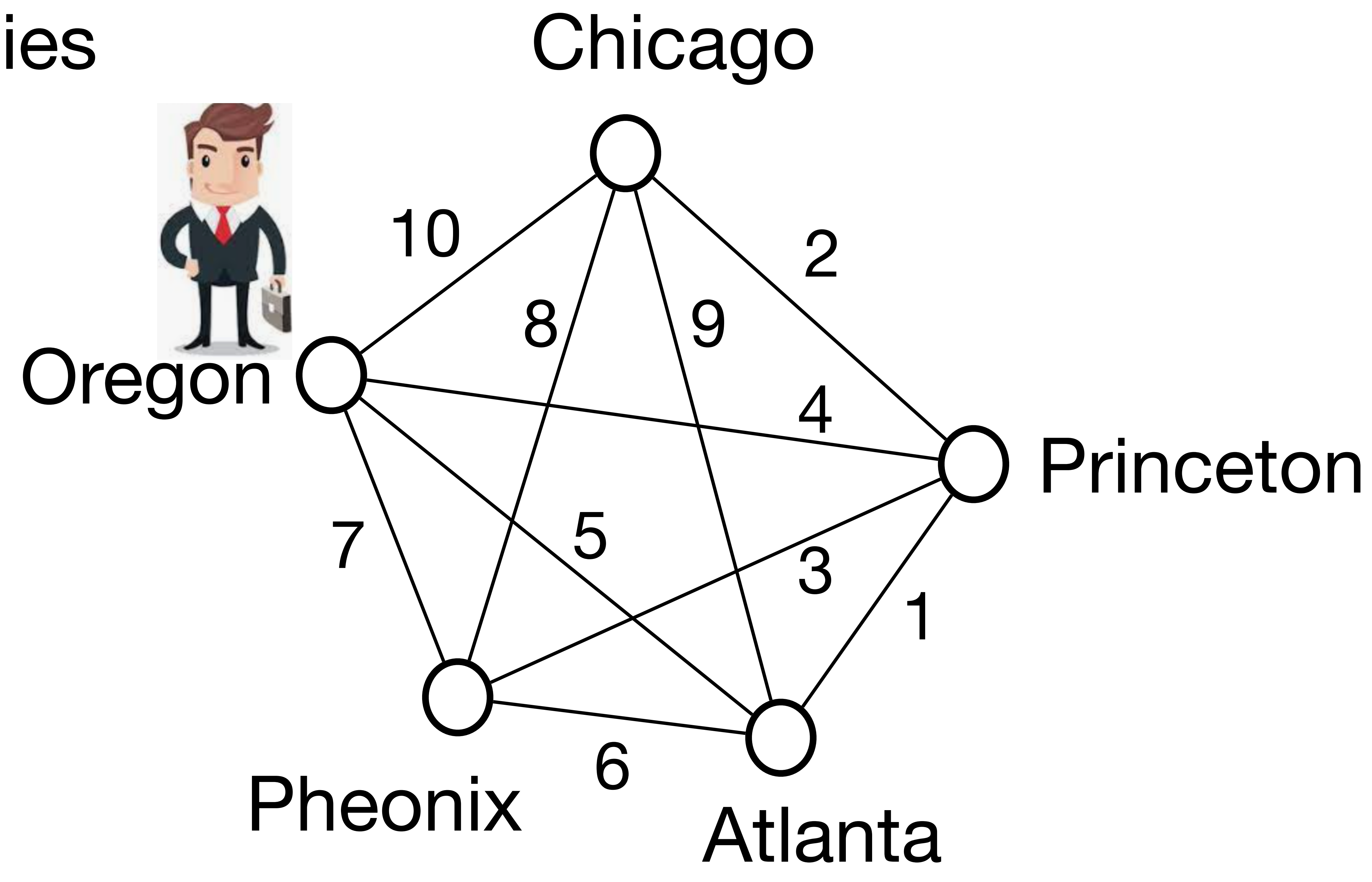
Traveling Salesman Problem (TSP)

n cities



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Shortest

Hamiltonian cycle

Traveling Salesman Problem (TSP)

Input: An undirected complete graph G=(V,E), where every edge $(u, v) \in E$ has a non-negative integer weight $w(u, v)$. An integer $k \geq 0$.

Question: Does G have a Hamiltonian cycle of weight $\leq k$?

Known NPC Problems:

1) 3-CNF SAT Problem
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4) Hamiltonian Cycle Problem

Theorem: $TSP \in NPC$.

Traveling Salesman Problem (TSP)

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Known NPC Problems:

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3) Vertex Cover Problem
4) Hamiltonian Cycle Problem

Theorem: $TSP \in NPC$.

Proof: 1) $TSP \in NP$.

Certificate: a Hamiltonian cycle of weight $\leq k$.

Polynomial-time verification.

2) Which known NPC problem do we want to reduce to TSP?

We want to prove: Hamiltonian Cycle Problem $\leq_p$ TSP

Hamiltonian Cycle Problem:

Input: An undirected graph $G = (V, E)$.

Question: Does G have a Hamiltonian cycle?

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Input: An undirected complete graph G=(V,E), where every edge $(u, v) \in E$ has a non-negative integer weight $w(u, v)$. An integer $k \geq 0$.

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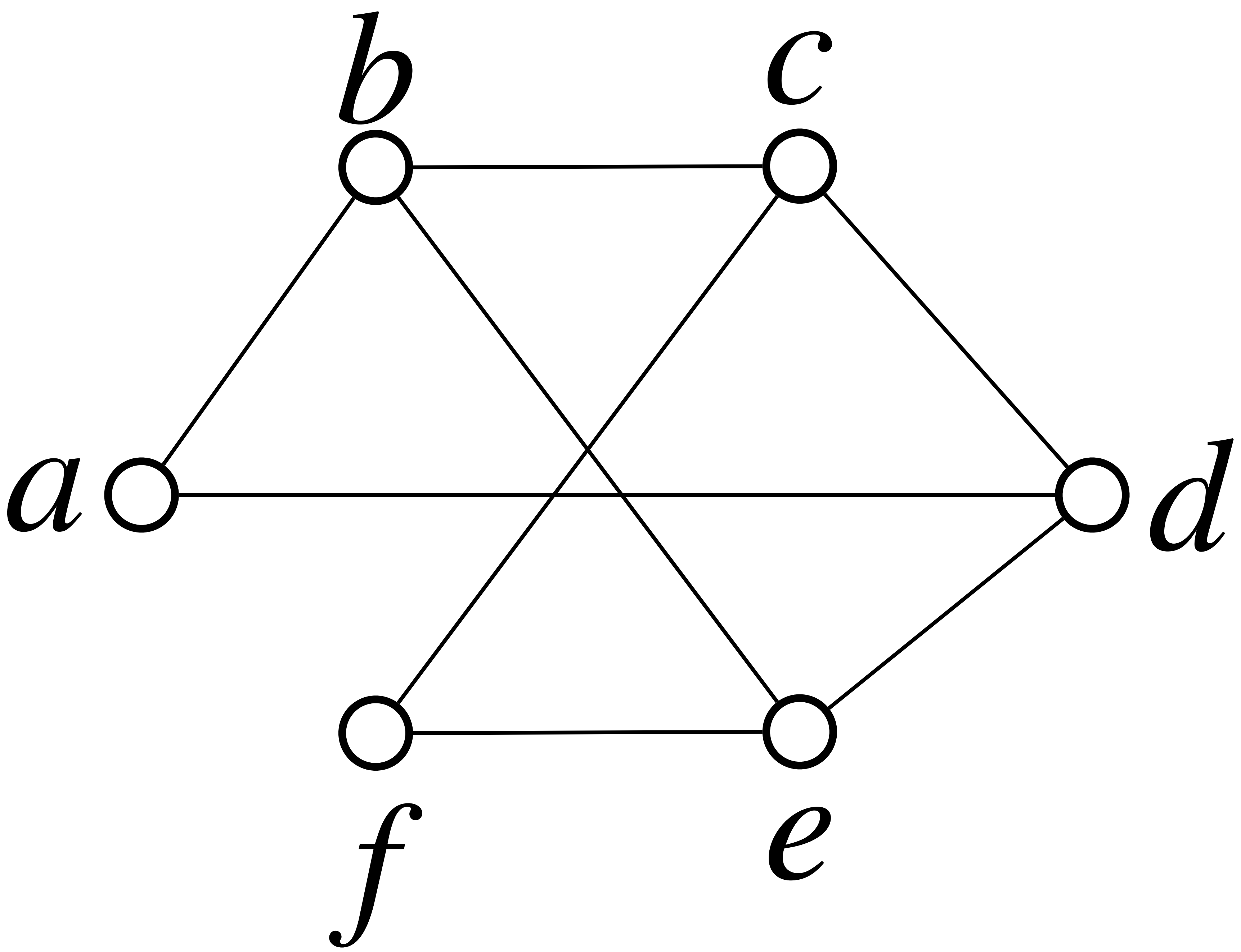
Example instance:



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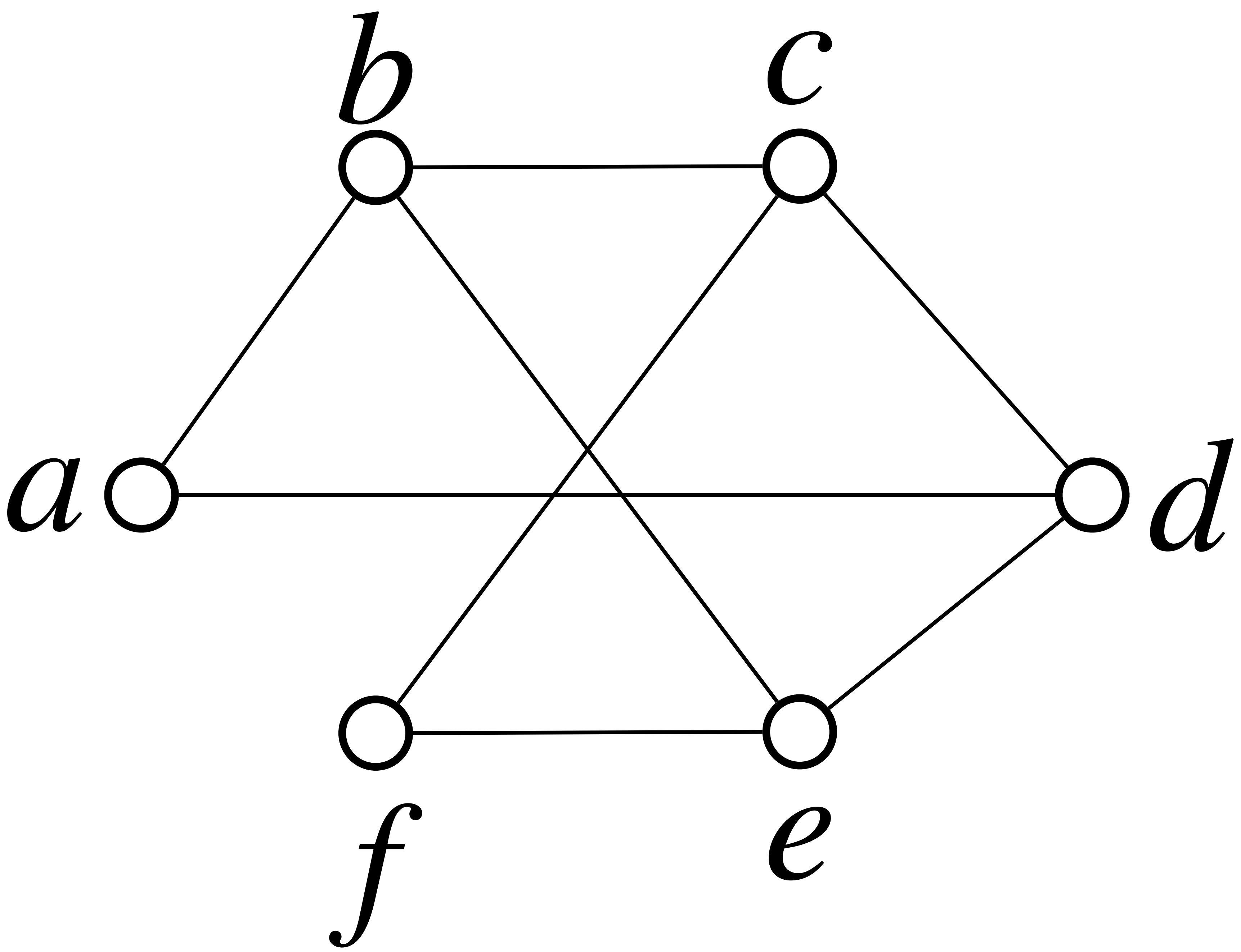

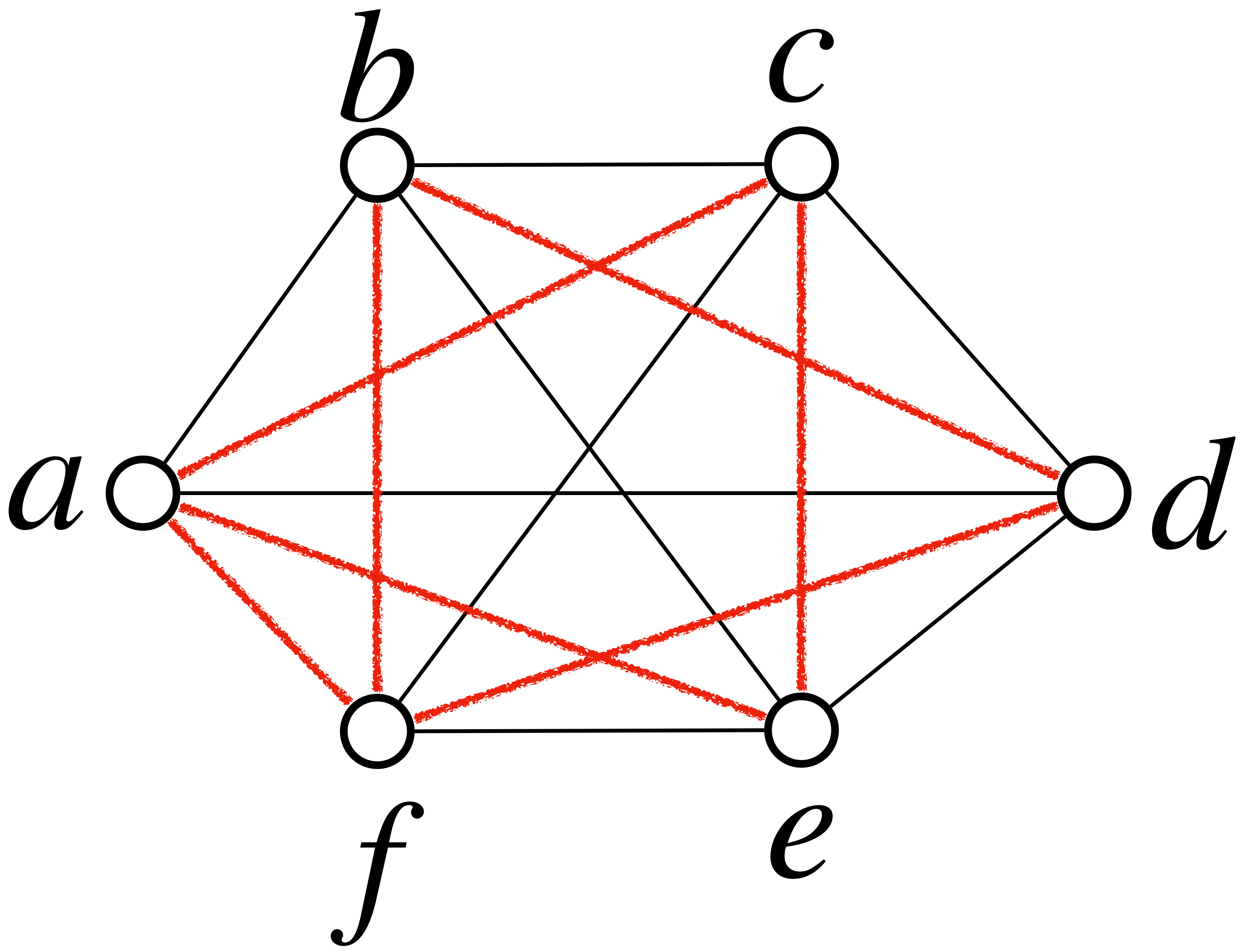

Black edges: weight 0 Red edges: weight 1

k=0

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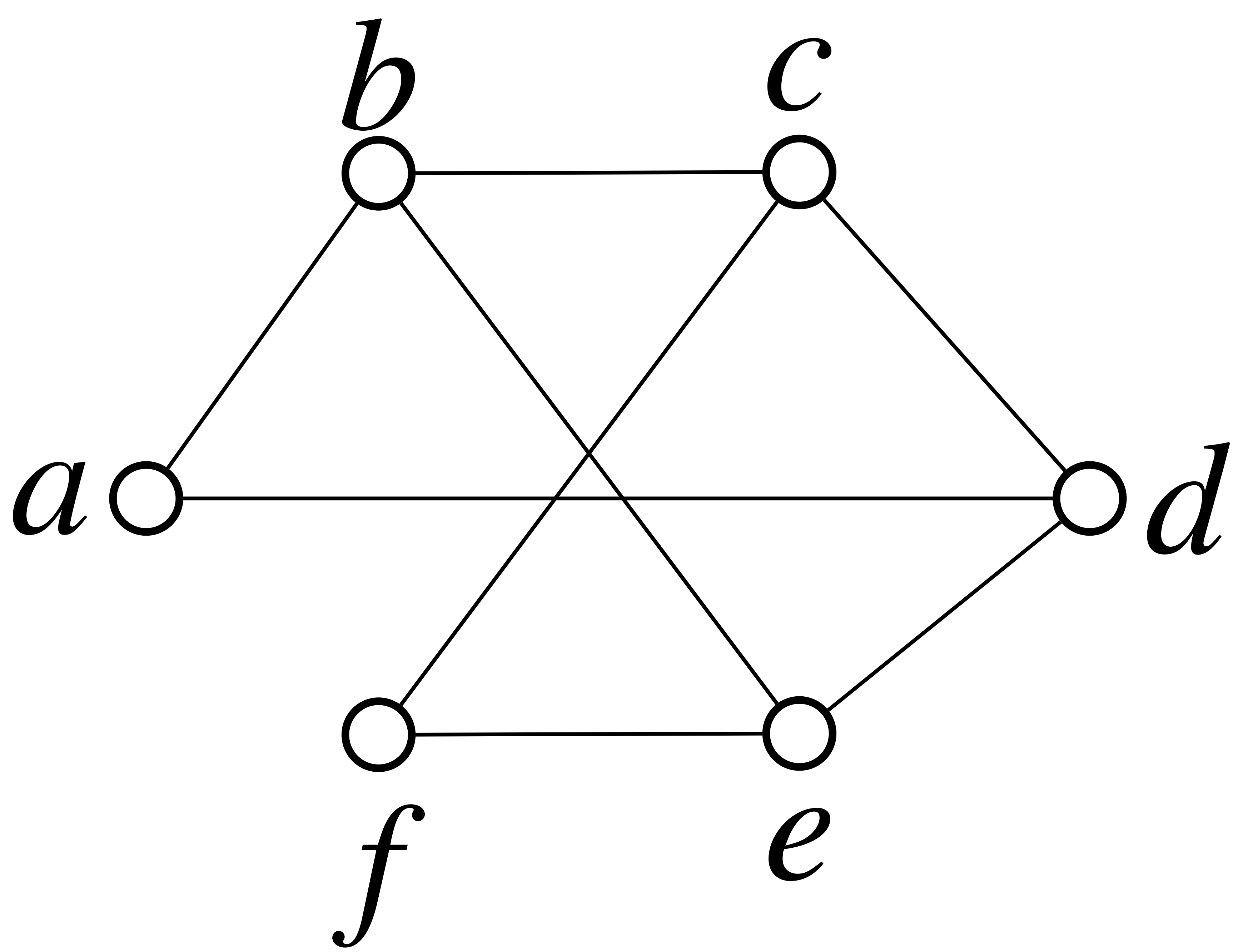
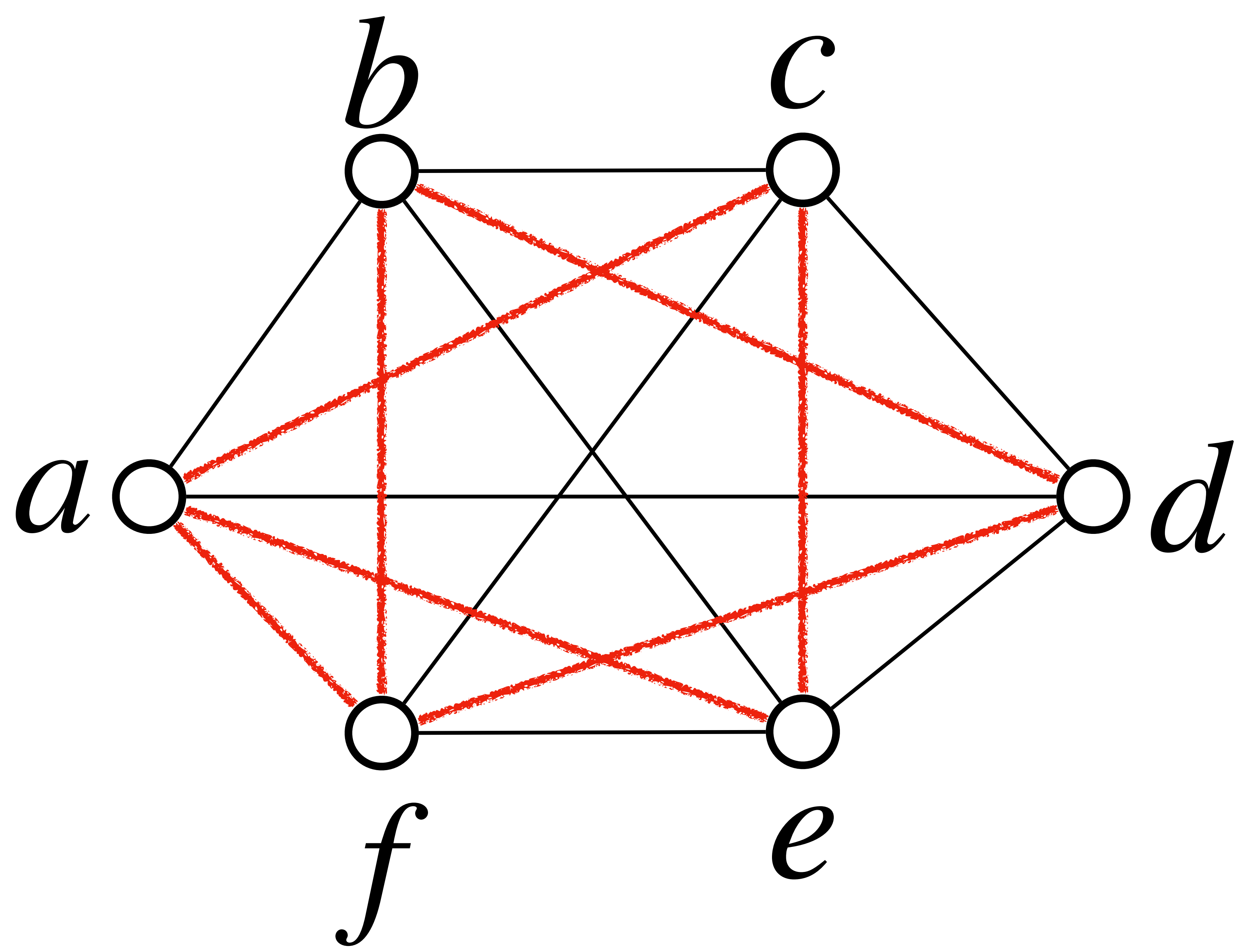
Question: Does G have a Hamiltonian cycle of weight $\leq k$?

Example instance:

Polynomial-time mapping. Does it preserve YES/NO answer?





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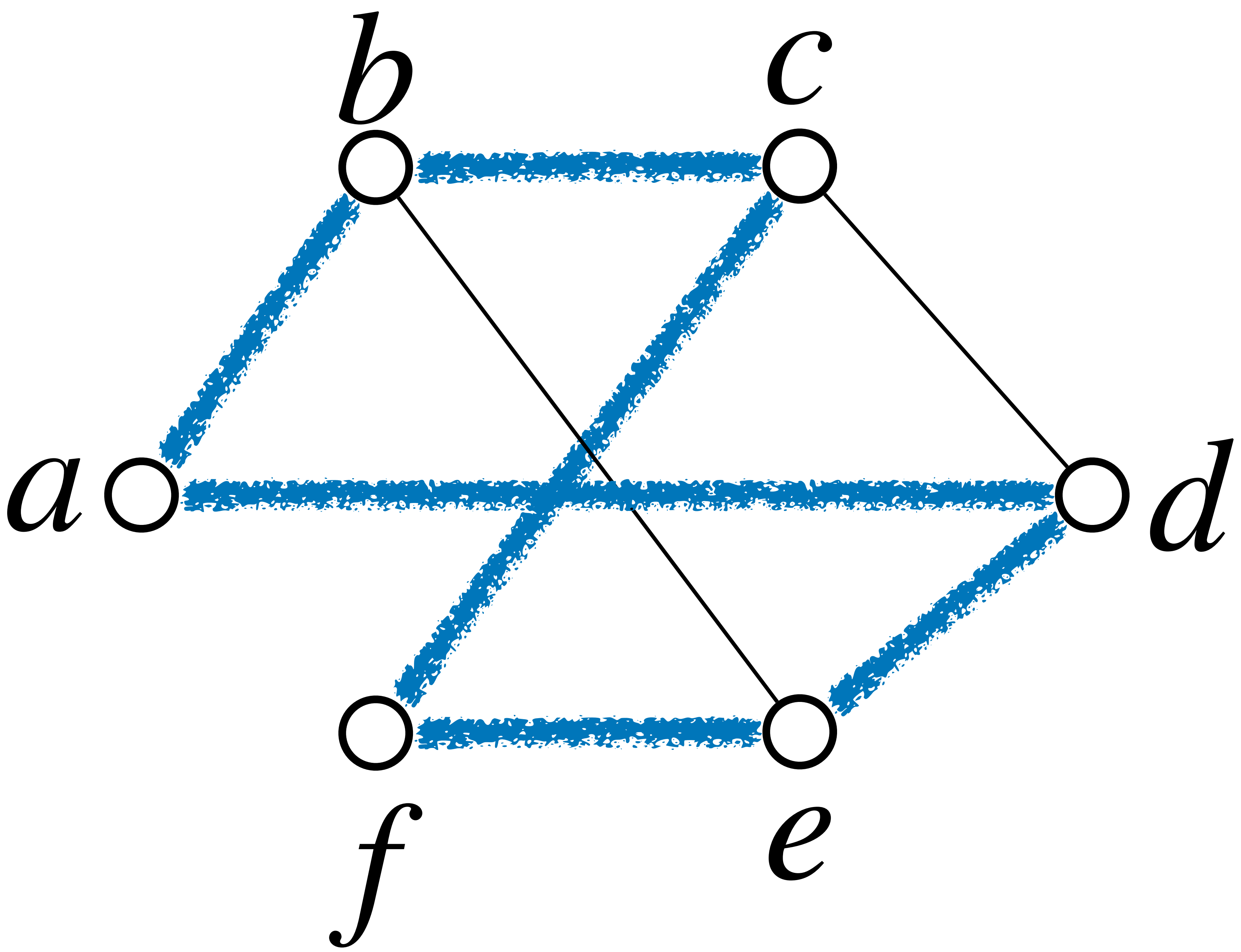
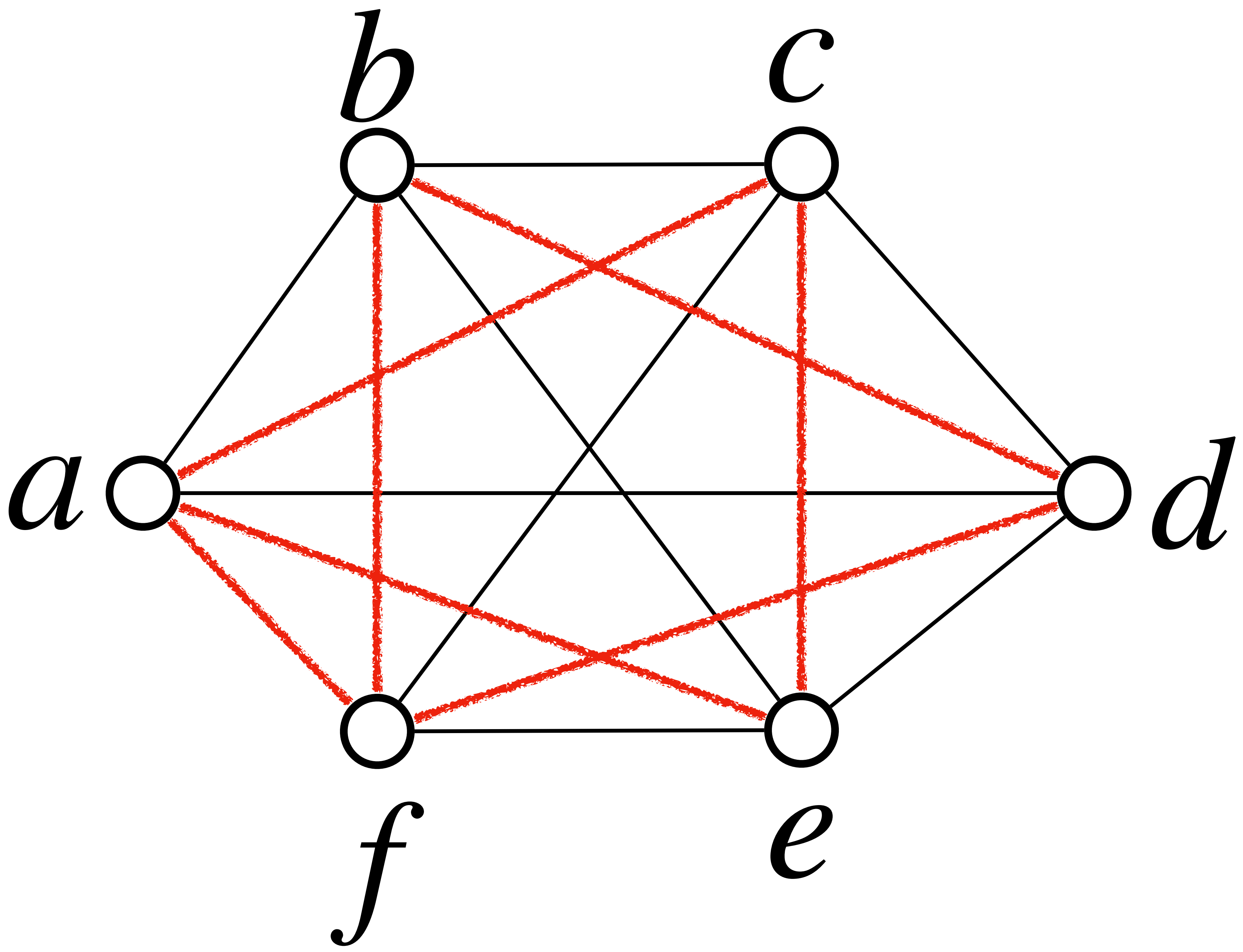
Assume answer "YES"





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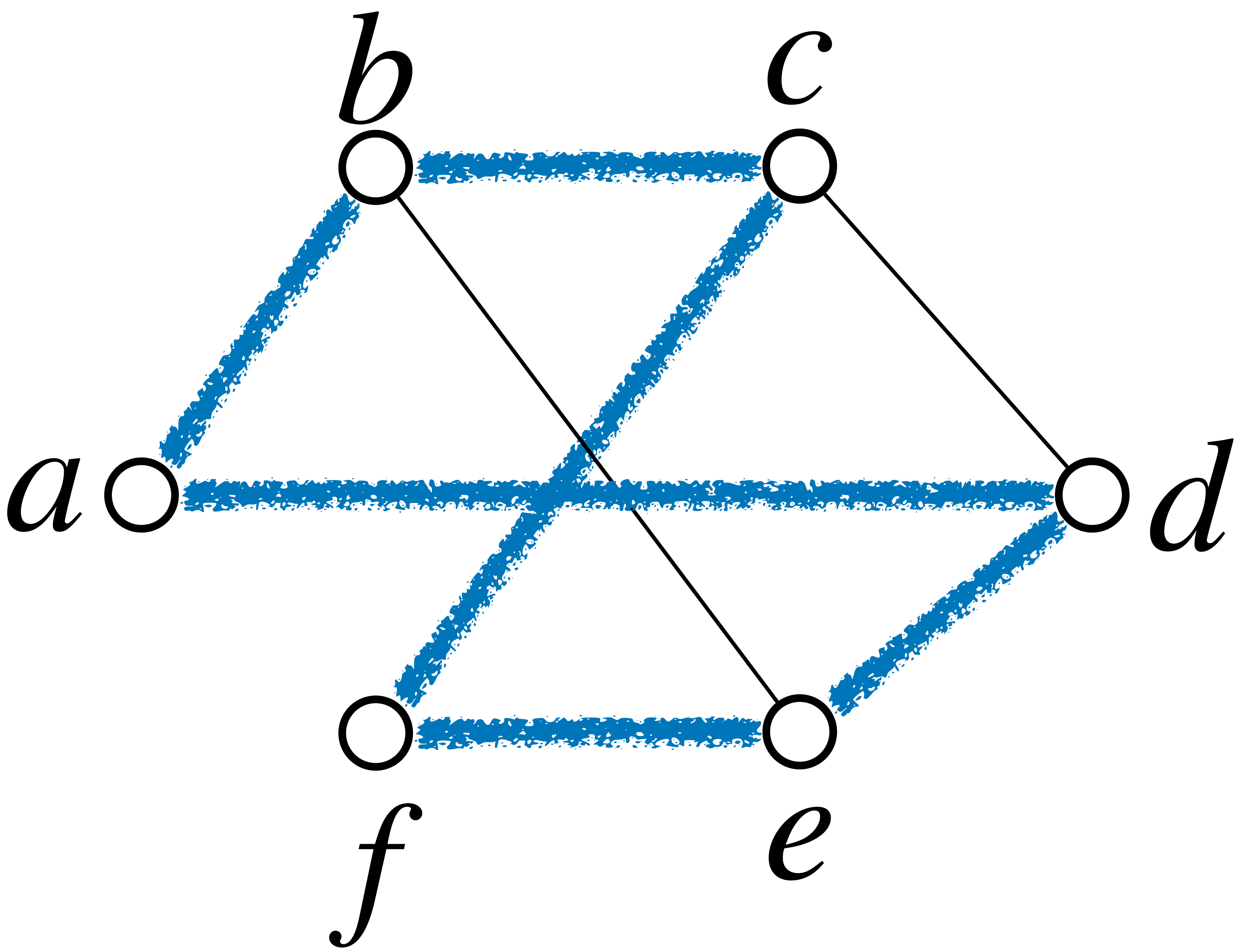
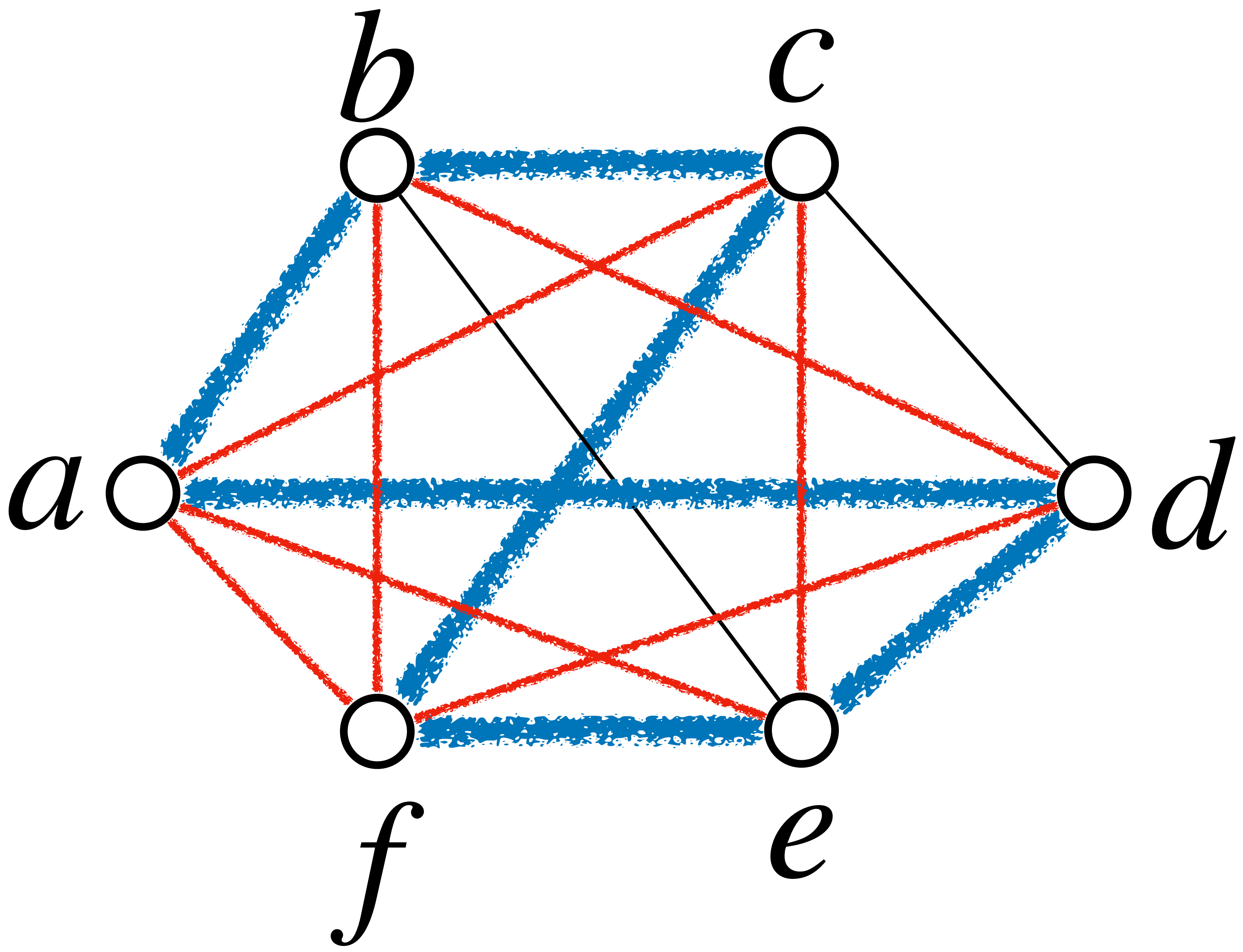
Question: Does G have a Hamiltonian cycle of weight $\leq k$?

Assume answer "YES"



Answer "YES"



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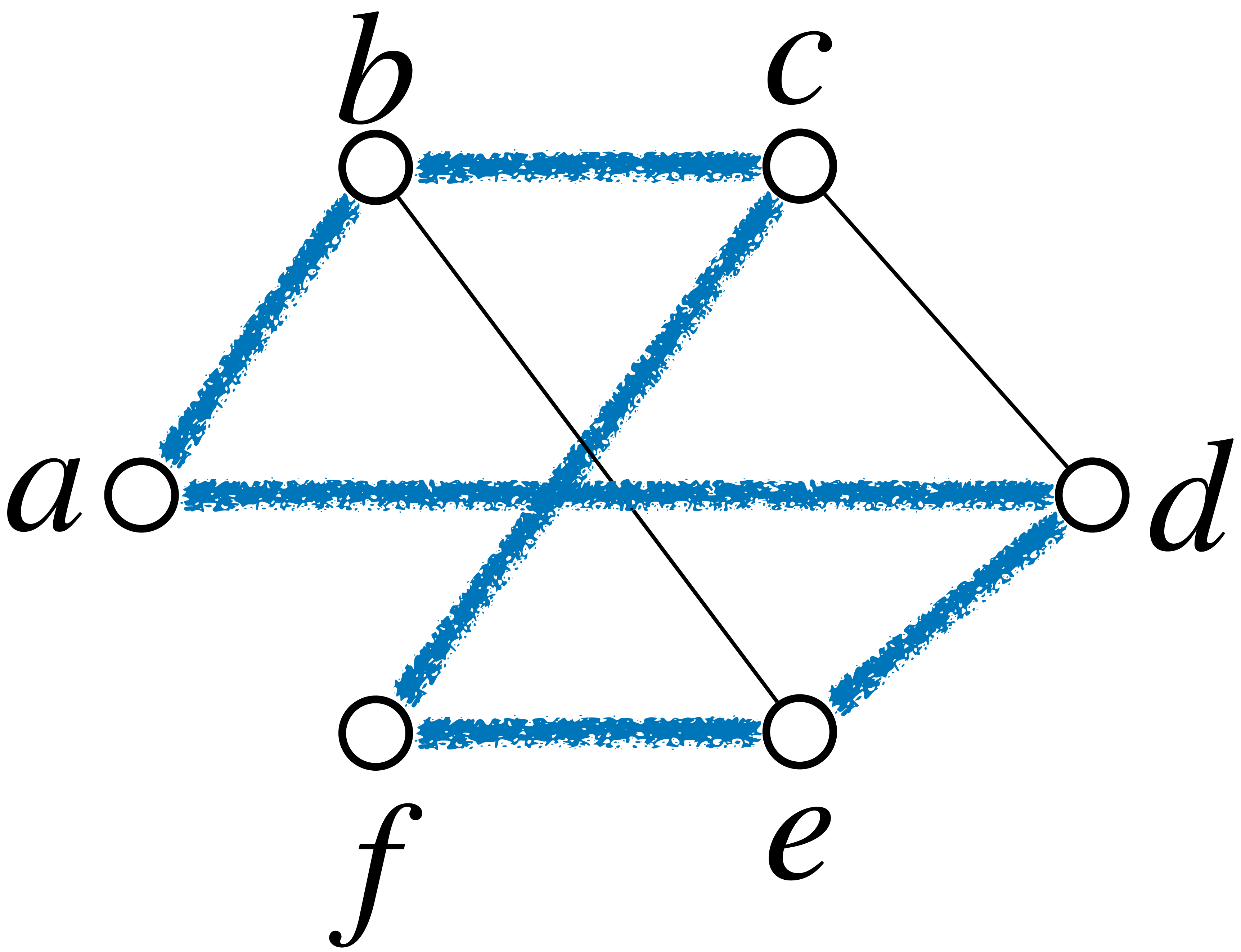
Answer "YES"



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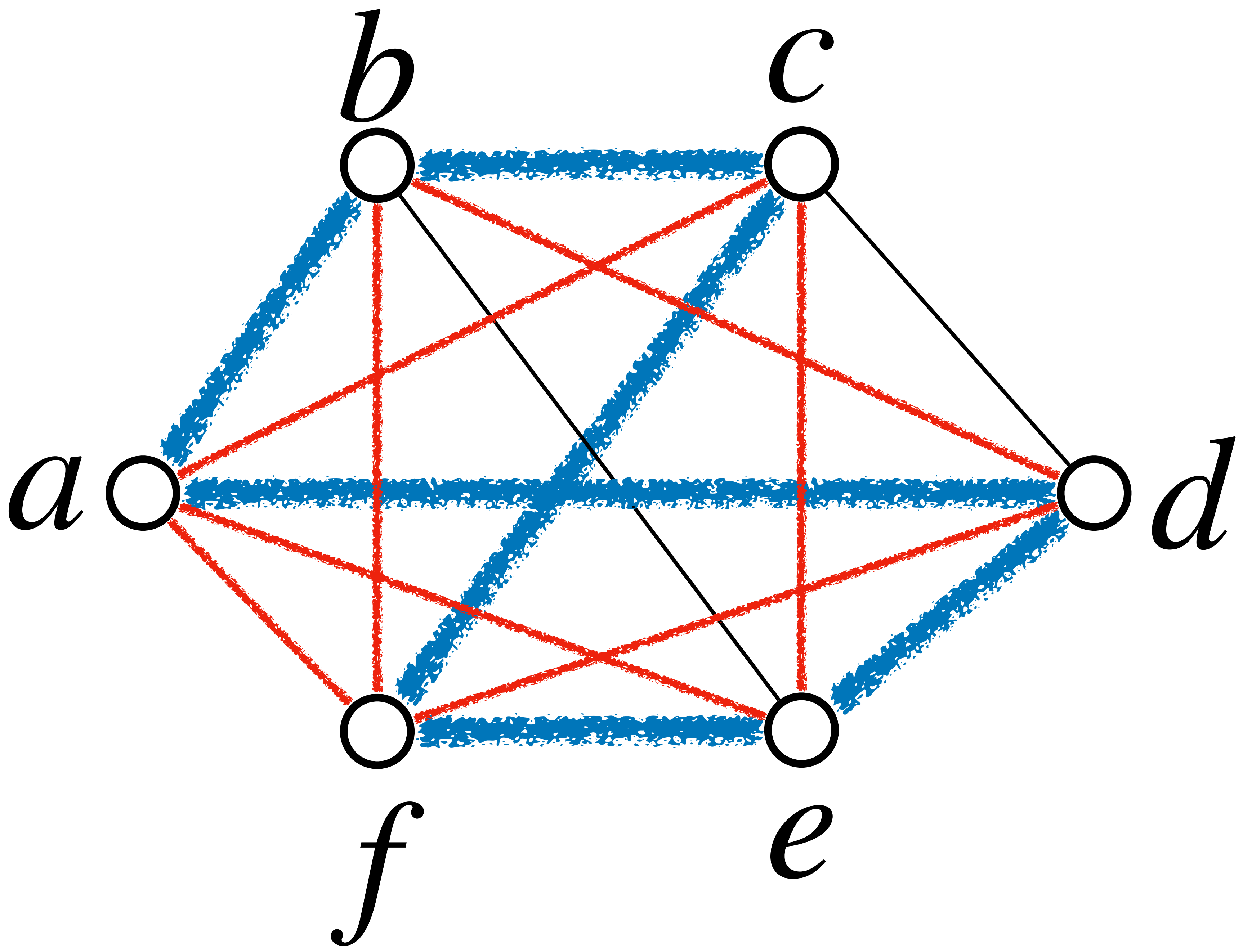

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k=0

We have proved: Hamiltonian Cycle Problem $\leq_p$ TSP

$$TSP \in NPC$$

Quiz questions:

1. What is the main idea for proving the NP-completeness of TSP?
2. Is TSP on a general graph (instead of a complete graph) also NP-complete?

Roadmap of this lecture:

1. NP Completeness

    1.1 Prove the "Traveling Salesman Problem" is NPC.

    1.2 Prove the "Subset Sum Problem" is NPC.

Subset Sum Problem

Input: A set S of positive integers.

A target integer t > 0.

Question: Does S have a subset S' whose sum equals t ?

Known NPC Problems:

1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem
5) Traveling Salesman Problem

Subset Sum Problem

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Example: $S = \{1,2,7\},\ t = 8.$

Subset Sum Problem

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Example: $S = \{1,2,7\},\ t = 8.$

Answer: YES.

$S' = \{1,7\}$.

Subset Sum Problem

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Question: Does S have a subset S' whose sum equals t ?

Known NPC Problems:

1) 3-CNF SAT Problem
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Example: $S = \{1,2,7\},\ t = 6.$

Subset Sum Problem

Input: A set S of positive integers.

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Known NPC Problems:

1) 3-CNF SAT Problem
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Example: $S = \{1,2,7\}$, $t = 6$.

Answer: NO.

Subset Sum Problem

Input: A set S of positive integers.

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Question: Does S have a subset S' whose sum equals t ?

Known NPC Problems:

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Example: $S = \{1,2,7,14,49,98,343,686,2409,2793,16808,17206,117705,117993\}$

$t = 138457$

Subset Sum Problem

Input: A set S of positive integers.

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Question: Does S have a subset S' whose sum equals t ?

Known NPC Problems:

1) 3-CNF SAT Problem
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Example:

$$S = \{1,2,7,14,49,98,343,686,2409,2793,16808,17206,117705,117993\}$$

$$t = 138457$$

Answer: Yes.

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Known NPC Problems:

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Theorem: Subset Sum Problem $\in NPC$.

Subset Sum Problem

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Theorem: Subset Sum Problem $\in NPC$.

Proof: 1) Subset Sum Problem $\in NP$.

Certificate: A subset S' whose sum equals t.

Polynomial-time verification.

2) Which known NPC problem do we want to reduce to the "Subset Sum Problem"?

We want to prove 3-CNF SAT Problem $\leq_p$ Subset Sum Problem .

3-CNF SAT Problem:

Input: A CNF formula with n variables and k clauses, where each clause is the "OR" of 3 literals.

Question: Does there exist a solution to the variables that make the formula be true?

Subset Sum Problem

Input: A set S of positive integers. A target integer t > 0.

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Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

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$$n = 3,\ k = 4$$

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$$n = 3,\ k = 4$$

All numbers have n+k digits

	x_1	x_2	x_3	C_1	C_2	C_3	C_4

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$$n = 3,\ k = 4$$

All numbers have n+k digits

x_1	x_2	x_3	C_1	C_2	C_3	C_4
1	0	0				
1	0	0				
0	1	0				
0	1	0				
0	0	1				
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	x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	1	0	0	1			
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x_2	0	1	0	0			
$\bar{x}_2$	0	1	0	1			
x_3	0	0	1	0			
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	x_1	x_2	x_3	C_1	C_2	C_3	C_4
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3-CNF SAT Problem: → Subset Sum Problem

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$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

All numbers have n+k digits

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	1	0	0	1	0	0	1
	$\bar{x}_1$	1	0	0	0	1	1	0
	x_2	0	1	0	0	0	0	1
	$\bar{x}_2$	0	1	0	1	1	1	0
	x_3	0	0	1	0	0	1	1
	$\bar{x}_3$	0	0	1	1	1	0	0
$2k$ rows		0	0	0				
		0	0	0				
		0	0	0				
		0	0	0				
		0	0	0				
		0	0	0				
		0	0	0				
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3-CNF SAT Problem:

Subset Sum Problem

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$$n = 3,\ k = 4$$

All numbers have n+k digits

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
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	x_3	0	0	1	0	0	1	1
	$\bar{x}_3$	0	0	1	1	1	0	0
$2k$ rows		0	0	0	1	0	0	0
		0	0	0	2	0	0	0
		0	0	0	0	1	0	0
		0	0	0	0	2	0	0
		0	0	0	0	0	1	0
		0	0	0	0	0	2	0
		0	0	0	0	0	0	1
		0	0	0	0	0	0	2

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$$n = 3,\ k = 4$$

S has 2(n+k) numbers,
all of which have n+k digits

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2

3-CNF SAT Problem:

Subset Sum Problem

S has 2(n+k) numbers,
all of which have n+k digits

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Polynomial-time mapping.

Does it preserve YES/NO answers?

Subset Sum Problem

S has 2(n+k) numbers,
all of which have n+k digits

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

S has 2(n+k) numbers,
all of which have n+k digits

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume "YES".

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

S has 2(n+k) numbers, all of which have n+k digits

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

S has 2(n+k) numbers,
all of which have n+k digits

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume "YES".

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

First n bits are already matched.

Why?

Subset Sum Problem

S has 2(n+k) numbers,
all of which have n+k digits

$2n$ rows; $2k$ rows

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume "YES".

Subset Sum Problem

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Sum is between 1 and 3 here. Why?

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

$2n$ rows

$2k$ rows

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$
$x_2 = 0$
$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Subset Sum Problem

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Sum is between 1 and 3 here. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Subset Sum Problem

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x_2} \vee \bar{x_3}$

$C_2 = \bar{x_1} \vee \bar{x_2} \vee \bar{x_3}$

$C_3 = \bar{x_1} \vee \bar{x_2} \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Sum is between 1 and 3 here. Why?

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows: x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x_1}$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x_2}$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x_3}$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume "YES".

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows: x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume "YES".

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$
$x_2 = 0$
$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1 $\quad v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$ $\quad v_1' =$	1	0	0	0	1	1	0
	x_2 $\quad v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$ $\quad v_2' =$	0	1	0	1	1	1	0
	x_3 $\quad v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$ $\quad v_3' =$	0	0	1	1	1	0	0
$2k$ rows	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Subset Sum Problem

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Sum is between 1 and 3 here. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem: → Subset Sum Problem

Assume "YES". → Answer is "YES".

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$$x_1 = 0$$
$$x_2 = 0$$
$$x_3 = 1$$

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Subset Sum Problem

Assume “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = ?$

$x_2 = ?$

$x_3 = ?$

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Subset Sum Problem

Assume “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$\boxed{x_1 = 0}$

$x_2 = ?$

$x_3 = ?$

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	$\boxed{1}$	1	1	4	4	4	4

3-CNF SAT Problem:

Subset Sum Problem

Assume “YES”.

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$\boxed{x_2 = 0}$

$x_3 = ?$

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	$\boxed{1}$	1	4	4	4	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$\boxed{x_3 = 1}$

Subset Sum Problem

Assume “YES”.

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
2n rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
2k rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	$\boxed{1}$	4	4	4	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Assume "YES".

Sum is at least 1. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	$\boxed{4}$	4	4	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Assume “YES”.

Sum is at least 1. Why?

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows: x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	$\boxed{4}$	4	4	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Assume "YES".

Sum is at least 1.

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows: x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v'_1 =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v'_2 =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v'_3 =$	0	0	1	1	1	0	0
$2k$ rows	$s_1 =$	0	0	0	1	0	0	0
	$s'_1 =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s'_2 =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s'_3 =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s'_4 =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	$\boxed{4}$	4	4

3-CNF SAT Problem:

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$$x_1 = 0$$
$$x_2 = 0$$
$$x_3 = 1$$

Subset Sum Problem

Assume "YES".

Sum is at least 1.

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows: x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	$\boxed{4}$	4

3-CNF SAT Problem:

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Assume “YES”.

Sum is at least 1.

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	$\boxed{4}$

3-CNF SAT Problem:

Answer is “YES”.

Subset Sum Problem

Answer is “YES”.

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$$x_1 = 0$$

$$x_2 = 0$$

$$x_3 = 1$$

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

"YES" for 3-CNF SAT Problem $\Leftrightarrow$ "YES" for Subset Sum Problem

3-CNF SAT Problem $\leq_p$ Subset Sum Problem

Subset Sum Problem $\in NPC$

Quiz questions:

1. What is the main idea for proving the NP-completeness of the “Subset Sum Problem”?

2. What are applications of the “Subset Sum Problem”?