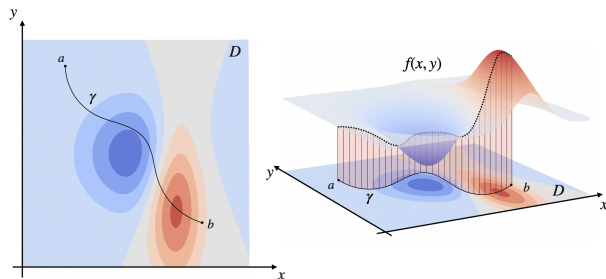# 221110   Line Integral - Curve integral

The line integral is a tool for calculating th
area mapped   by a   curve 'γ' that lies
on the (x_1, x_2) plane, and who's other
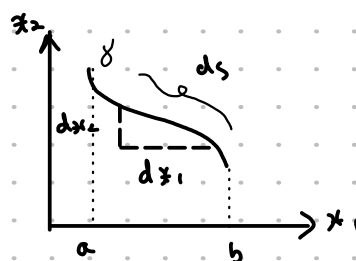end intersects The surface created by a
multivariate function $u(x_1, x_2)$

If 'ds' is an Infinitesimally small
length along 'γ'.

$$ds = \sqrt{dx_1^2 + dx_2^2}$$

$$= \frac{dx_1}{dx_1} \sqrt{dx_1^2 + dx_2^2}$$

$$= \sqrt{1 + \frac{dx_2^2}{dx_1^2}} \cdot dx_1$$

$$= \sqrt{1 + f'(x_1)} \; dx_1 \quad (\because y = x_2 = f(x_1))$$

$\therefore$ The integral along 'γ' intersecting $u(x_1, x_2)$

$$\int_\gamma u(x_1, x_2) \, ds = \int_a^b u(x_1, f(x_1)) \sqrt{1 + f'(x_1)} \cdot dx_1$$

$$\int_\gamma u(\underline{x}) \, ds = \int_a^b u(x_1, f(x_1)) \sqrt{1 + f'(x_1)} \; dx_1$$