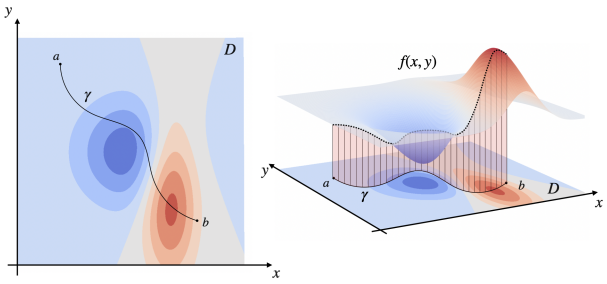
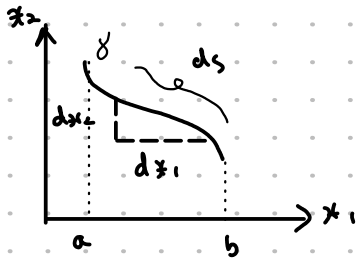
221110 Line Integral - Curve integral

The line integral is a tool for calculating th area mapped by a curve 'γ' that lies on the (x_1, x_2) plane, and who's other end intersects the surface created by a multivariate function $u(x_1, x_2)$



If 'ds' is an Infinitesimally small length along 'γ'.



$$ds = \sqrt{dx_1^2 + dx_2^2}$$

$$= \frac{dx_1}{dx_1}\sqrt{dx_1^2 + dx_2^2}$$

$$= \sqrt{1 + \frac{dx_2^2}{dx_1^2}}\, dx_1$$

$$= \sqrt{1 + (f'(x_1))^2}\, dx_1 \quad (\because y = x_2 = f(x_1))$$

∴ The integral along 'γ' intersecting $u(x_1, x_2)$

$$\int_\gamma u(x_1, x_2)\, ds = \int_a^b u(x_1, f(x_1))\, \sqrt{1 + (f'(x_1))^2}\, dx_1$$

$$\int_\gamma u(\underline{x})\, ds = \int_a^b u(x_1, f(x_1)) \sqrt{1 + (f'(x_1))^2}\, dx_1$$