Topological Sort (BFS)

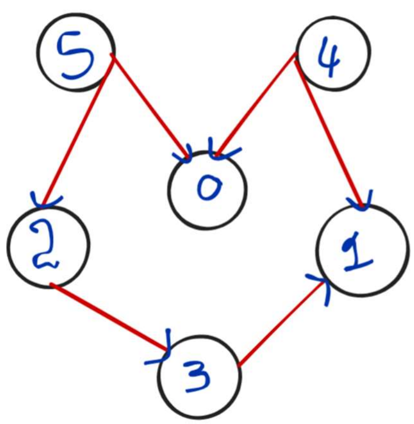
Problem statement: Given a graph, find the topological order for the given graph.

Topological sort: The linear ordering of nodes/vertices such that if there exists an edge between 2 nodes u,v then ‘u’ appears before ‘v’.

Example:

Example:

Input:



Output: 4 5 2 0 3 1

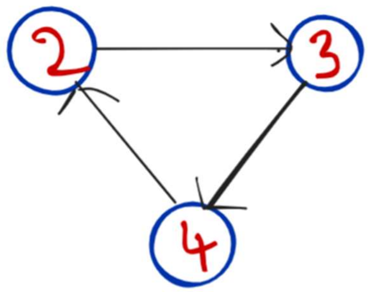
Explanation: 4 is appearing before its neighbours (1,0)

5 is appearing before its neighbours (0,2)

2 is appearing before its neighbours (3)

3 is appearing before its neighbours (1)

Note: This can be possible only for DAG (Directed acyclic graph) because in an undirected graph we can’t decide which node will come first because there will be no direction, and if there is a cycle topological order will not be possible (See below figure to understand why it is not possible for graphs containing cycle).



2 ->3 ->4 ->2 here if you can observe

2 is appearing before 3

3 is appearing before 4

4 is appearing before 2

But 2 has already appeared before 4, So topological ordering will not be possible

Intuition:

The question states that if there is an edge between u and v then u should appear before v, Which means we have to start this question from a node that doesn't have any previous edges. But how to find that node that has no edge before if? Here, we use the concept of in-degree (Number of edges pointing toward a node). We find an in-degree which has indegree=0 and we start from this. We use the Indegree concept to find topological sorting. Let's see how.

Approach:

1. In order to maintain the In-degree of each and every node, we take an array of size v(where v is the number of nodes).
2. Find in-degree of all nodes and fill them in an array
3. Now take Queue data structure and add nodes that have in-degree is 0 (as we discussed in the intuition), and simply apply bfs on queue with some condition.

4. Take the top/peek node from Queue (let the popped node be x), add this x to our answer. (If you can observe clearly the above step is nothing but Normal BFS traversal).

   We'll apply some conditions to the BFS:

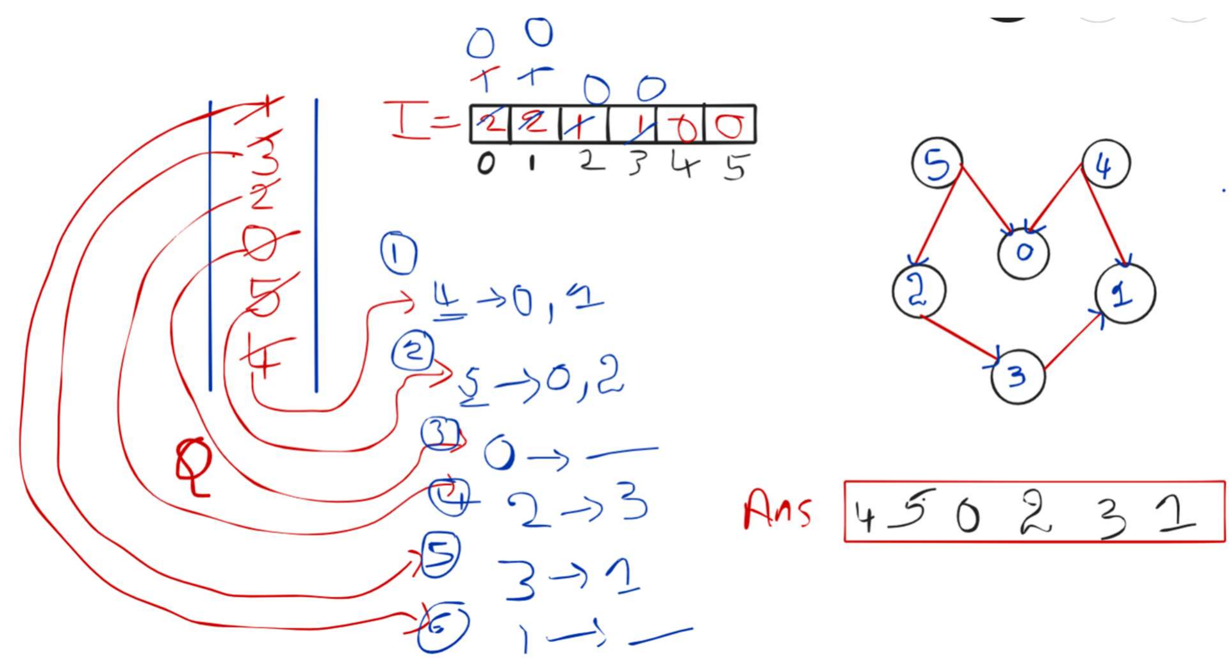
      1. Now take neighbour nodes of popped nodes and reduce their in-degree by 1.
      2. Check if any of the popped element nodes in degree becomes 0, after reducing in-degree by 1 if it happens then add this neighbour element to the queue for which the in-degree became zero.
5. Repeat step 4 till the queue becomes empty.

We'll apply some conditions to the BFS:

a) Now take neighbour nodes of popped nodes and reduce their indegree by 1.

b) Check if any of popped element nodes in degree become 0, after reducing in-degree by 1, if it happens then add this neighbour element which in-degree became 0 to the queue.

Let's see how it works pictorially:



Code:

- C++ Code
- Java Code

```
#include<bits/stdc++.h>
using namespace std;

class Solution {
public:
        vector<int> topo(int N, vector<int> adj[]) {
        queue<int> q;
            vector<int> indegree(N, 0);
            for(int i = 0;i<N;i++) {
                for(auto it: adj[i]) {
                    indegree[it]++;
                }
            }

            for(int i = 0;i<N;i++) {
                if(indegree[i] == 0) {
                    q.push(i);
                }
            }
            vector<int> topo;
            while(!q.empty()) {
                int node = q.front();
                q.pop();
                topo.push_back(node);
                for(auto it : adj[node]) {
                    indegree[it]--;
                    if(indegree[it] == 0) {
                        q.push(it);
                    }
                }
```

```
            }
            return topo;
        }
};



int main()
{

        vector<int> adj[6];
        adj[5].push_back(2);
        adj[5].push_back(0);
        adj[4].push_back(0);
        adj[4].push_back(1);
        adj[3].push_back(1);
        adj[2].push_back(3);

        Solution obj;
        vector<int> v=obj.topo(6, adj);
        for(auto it:v)
        cout<<it<<" ";


    return 0;
}
```

Output:

4 5 2 0 3 1

Time Complexity: O(N+E)

Space complexity: O(N)+O(N)