

**Natural Numbers** - Common counting numbers.

**Prime Number** - A natural number greater than 1 which has only 1 and itself as factors.

**Composite Number** - A natural number greater than 1 which has more factors than 1 and itself.

**Whole Numbers** - The set of Natural Numbers with the number 0 adjoined.

**Integers** - Whole Numbers with their opposites (negative numbers) adjoined.

**Rational Numbers** - All numbers which can be written as fractions.

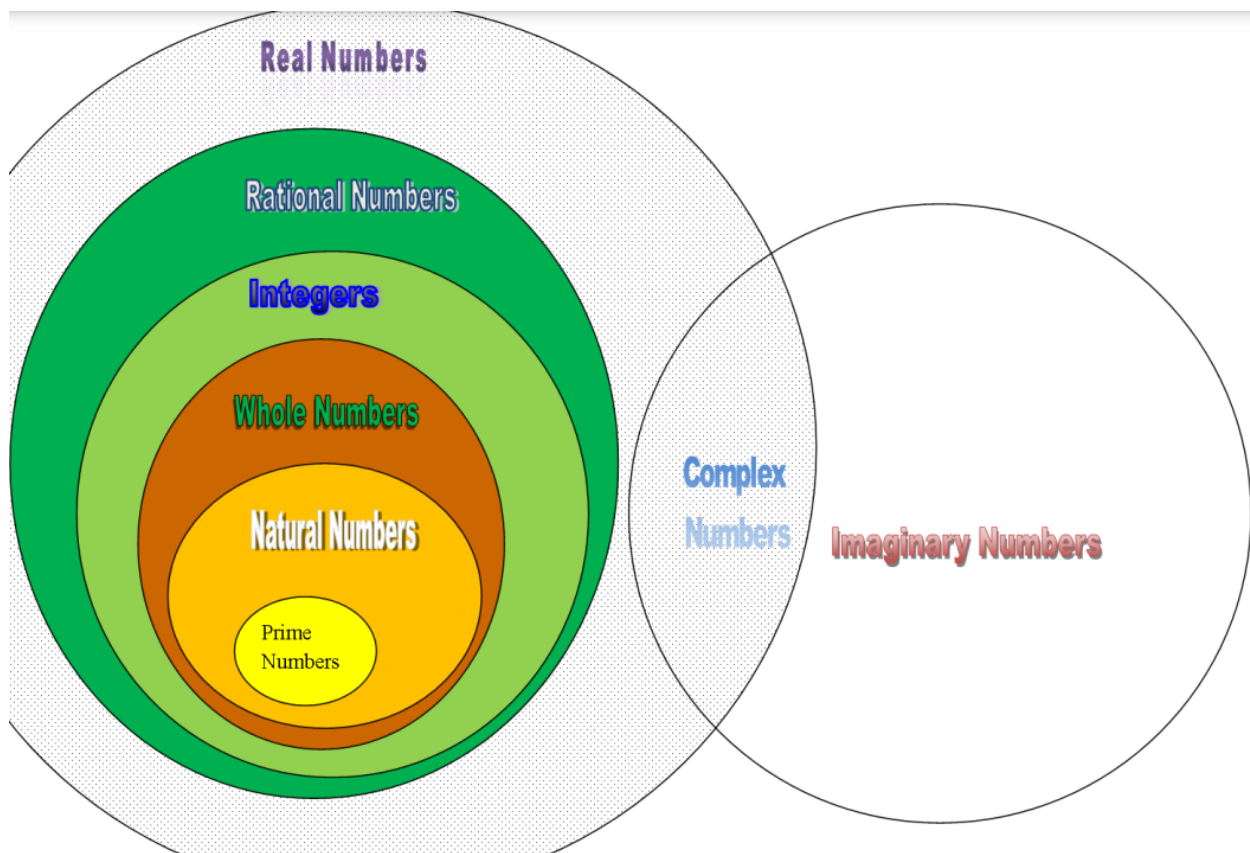
**Irrational Numbers** - All numbers which cannot be written as fractions.

**Real Numbers** - The set of Rational Numbers with the set of Irrational Numbers adjoined.

**Complex Number** - A number which can be written in the form  $a + bi$  where  $a$  and  $b$  are real numbers and  $i$  is the square root of  $-1$ .

Type of Number	Example
Natural Numbers	$N=1,2,3,4,\dots$
Prime Number	$P=2,3,5,7,11,13,17,\dots$
Composite Number	$4,6,8,9,10,12,\dots$
Whole Numbers	$W=0,1,2,3,4,\dots$
Integers	$Z=\dots,-3,-2,-1,0,1,2,3,\dots$
Rational Numbers	$Q=-12,0.33333\dots,52,1110,\dots,\dots$
Irrational Numbers	$F=\dots,\pi,2-\sqrt{\phantom{x}},0.121221222\dots$

Real Numbers	$R = \dots, -3, -1, 0, 15, 1.1, 2 - \sqrt{2}, 3, \pi, \dots$
Complex Number	$C = \dots, -3 + 2i, 0, 1 + 3i, \dots$



**Permutation:** A permutation is an arrangement in a definite order of several objects taken some or all at a time. Let us take 10 numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The number of different 4-digit-PIN which can be formed using these 10 numbers is 5040.  $P(10,4) = 5040$ .

**Formula:**  $nPr = \frac{n!}{(n-r)!}$

**Combination:** A combination is all about grouping. The number of different groups which can be formed from the available things can be calculated using combinations. Let us try to understand this with a

simple example. A team of 2 is formed from 5 students(William, James, Noah, Logan, and Oliver). This the combination of 'r' persons from the available 'n' persons is given as  $nCr = \frac{n!}{r!(n-r)!}$ . The combinations can happen in the following 10 ways by which the team of 2 could be formed.

- William James
- William Noah
- William Logan
- William Oliver
- James Noah
- James Logan
- James Oliver
- Logan Noah
- Logan Oliver
- Oliver Noah

This is a simple example of combinations.  $C(5,2) = 10$ .

Use combination when order does not matter.

**Recursion:**

$${}^nC_1 = \frac{n!}{(n-1)! * 1!} \quad \left\{ \begin{array}{l} {}^nC_r = \frac{n!}{r! * (n-r)!} \end{array} \right.$$

**Simplification:**

$${}^nC_1 = \frac{n!}{(n-1)! * 1!}$$

$${}^nC_1 = \frac{n * \cancel{(n-1)!}}{\cancel{(n-1)!} * 1}$$

$${}^nC_1 = n$$

We can write generic function for this (recurrence relation to find the factorial)

$${}^nC_1 = \frac{n!}{(n-1)! * 1!}$$

$${}^nC_1 = \frac{n * \cancel{(n-1)!}}{\cancel{(n-1)!} * 1} \quad \left. \vphantom{\frac{n * \cancel{(n-1)!}}{\cancel{(n-1)!} * 1}} \right\} n! = n * (n-1)! \quad \rightarrow \quad f(n) = n * f(n-1)$$

$${}^nC_1 = n \quad \quad \quad f(n-1) = (n-1) * f(n-2)$$

$$\quad \quad \quad f(n-2) = (n-2) * f(n-3)$$

$$\quad \quad \quad \vdots$$

If we keep doing this, will go infinite loop. To break this, we should have Base case.

$$f(1) = 1$$

Mathematical Def:

$$f(n) = \begin{cases} n * f(n-1) & \text{If } n > 1 \\ 1 & \text{If } n = 1 \end{cases}$$

$${}^nC_1 = \frac{n!}{(n-1)! * 1!}$$

$${}^nC_1 = \frac{n * \cancel{(n-1)!}}{\cancel{(n-1)!} * 1}$$

$${}^nC_1 = n$$

$$n! = n * (n-1)! \quad \Rightarrow \quad f(n) = n * f(n-1)$$

$$f(1) = 1$$

Recursive case

$$f(n) = \begin{cases} n * f(n-1) & \text{If } n > 1 \\ 1 & \text{If } n = 1 \end{cases}$$

When a function calls itself directly or indirectly it is called recursive function and the process is called Recursion.

OR

It is a powerful, problem solving technique where solution of a larger problem defined in terms of smaller instances of itself.

$$\Sigma(n) = 1+2+3+4+\dots+n$$

$$\Sigma(n-1) = 1+2+3+4+\dots+(n-2)+(n-1)$$

$$\Sigma(n) = (1+2+3+4+\dots+n-1) + n$$

$$\Sigma(n) = \Sigma(n-1) + n$$

$$\Sigma(n) = \begin{cases} \Sigma(n-1) + n & \text{If } n > 1 \\ 1 & \text{If } n = 1 \end{cases}$$

OR

$$\text{sum}(n) = \begin{cases} \text{sum}(n-1) + n & \text{If } n > 1 \\ 1 & \text{If } n = 1 \end{cases}$$