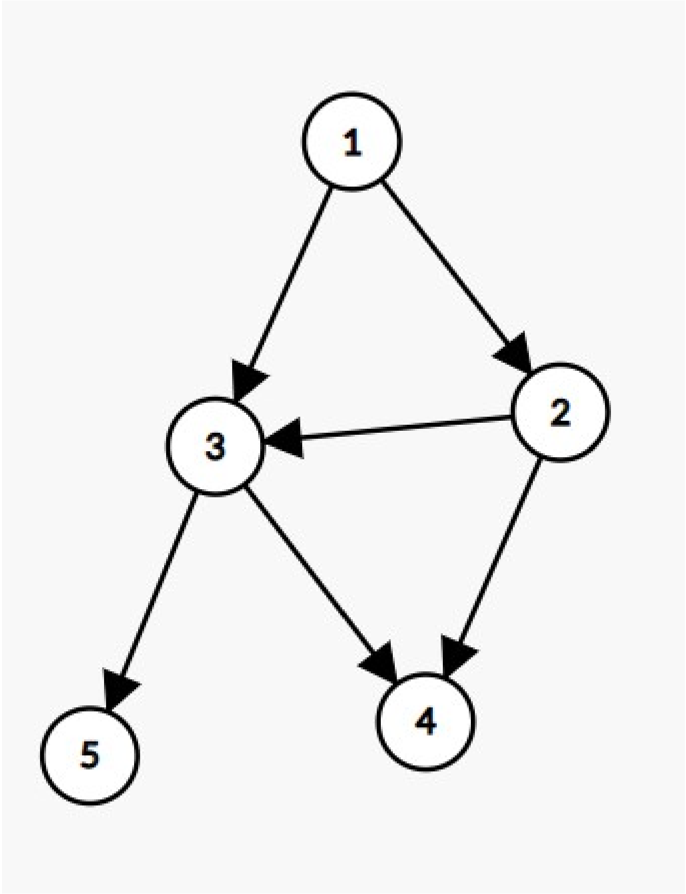
Topological Sort Using DFS
Problem Statement: Given a DAG(Directed Acyclic Graph), print all the vertex of the graph in a topologically sorted order. If there are multiple solutions, print any.
Pre-req: DFS traversal, Graphs, Stack data structure.
Examples:

Example 1:

Input:

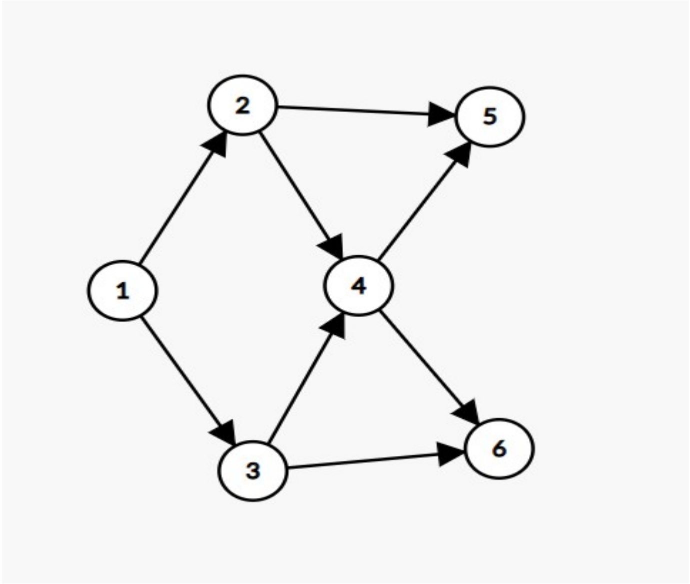


Output:

One of the solutions is 1,2,3,5,4

Example 2:

Input:

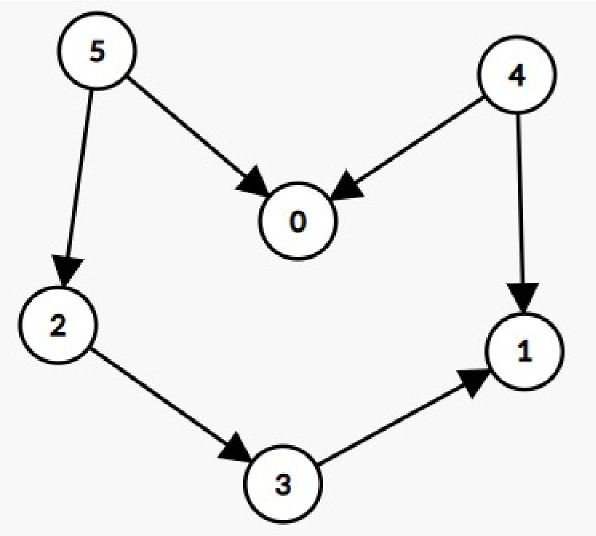


Output: One of the solution is 1,2,3,4,5,6

Solution

***Disclaimer**: Don't jump directly to the solution, try it out yourself first.*

Intuition:
-> First of all let's understand Topological Sorting. It means linear ordering of vertices such that there is an edge u—-> v**,** u appears before v in the ordering.
Suppose for a given graph,



Some of the possible Topological orders can be:

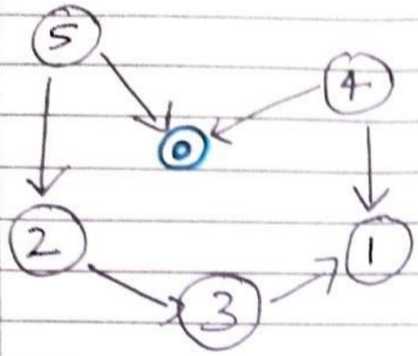
1. 5,4,2,3,1,0
2. 4,5,2,3,1,0

-> In both cases we can see, that
4->0 (4 appears before 0) , 5>3 (5 appears before 3), …
-> Similarly there can be **multiple toposorts order for the given graph** but the condition should be if there is an edge u->v then **u should always appear before v.**
-> Topological Sorting is applicable only for **DAG(Directed Acyclic Graph)**. Why is it so?
Because of the following reasons:

1. For Undirected graphs ,only u->v is not applicable . It cannot be sure whether the edge is between u to v or v to u (u-v) .
2. In a cyclic graph there will always be a dependency factor . You cannot make sure that you can have linear ordering of vertices.

-> Finally, now you have a clear understanding of what Topological Sorting is. We will be using the **DFS(Depth First Search)** method to solve the problem. What we will be doing is for each vertex we will explore its adjacent vertex. After exploring, we will store the current vertex in a data structure to maintain Topo Sort.
Approach:
We will be using the following data structure to get Topo sort:

1. Visited Vector – To store visit of each vertex
2. Stack – To maintain the topo sort order.



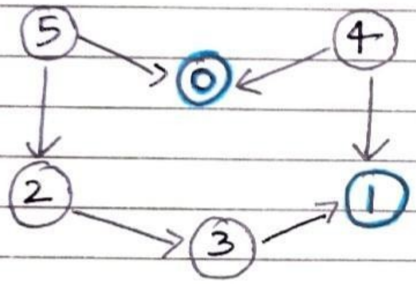
0

stack

1					
~~0~~	0	0	0	0	0
0	1	2	3	4	5

dfs(0)

x x



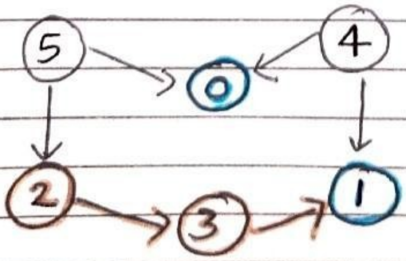
1
0

stack

1	1				
~~0~~	~~0~~	0	0	0	0
0	1	2	3	4	5

dfs(1)

x x



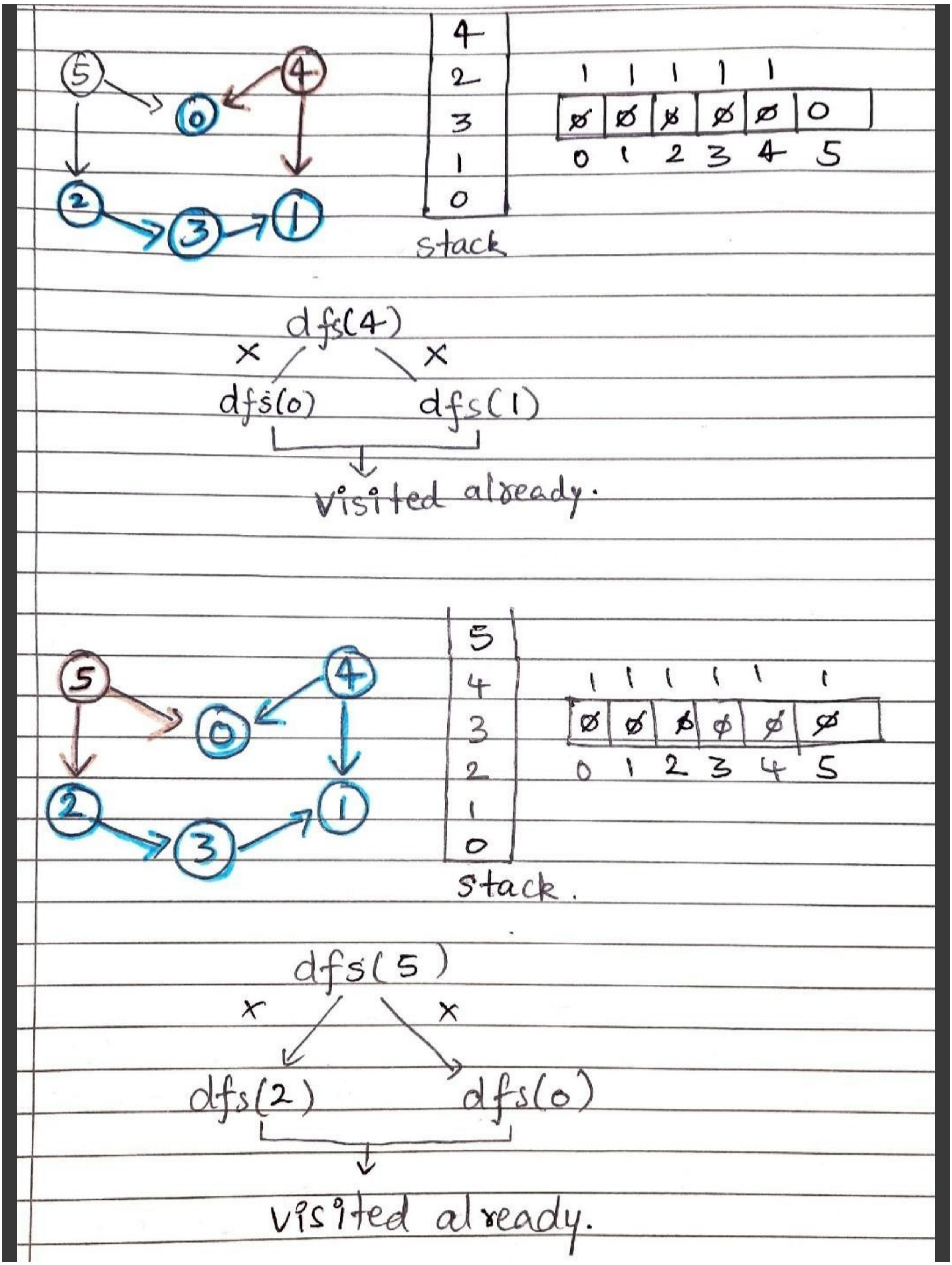
2
3
1
0

stack

1	1	1	1		
~~0~~	~~0~~	~~0~~	~~0~~	0	0
0	1	2	3	4	5

dfs(2)

$\rightarrow$ dfs(3) $\xrightarrow{X}$ dfs(1) [visited]



Did you notice something while using Stack? Just because there was an edge from u to v. Dfs call will go from u to v. The 1st dfs (v) will get over first and then dfs(u). Here we are making sure that if u->v, then we will **first push v into the stack and then u will be pushed**. This is how Topological order is maintained in the Stack.

Code:

- C++ Code
- Java Code

```
#include <bits/stdc++.h>
```

```cpp
using namespace std;

class Solution {
  void findTopoSort(int node, vector < int > & vis, stack < int > & st, vector < int > adj[]) {
    vis[node] = 1;

    for (auto it: adj[node]) {
      if (!vis[it]) {
        findTopoSort(it, vis, st, adj);
      }
    }
    st.push(node);
  }
  public:
    vector < int > topoSort(int N, vector < int > adj[]) {
      stack < int > st;
      vector < int > vis(N, 0);
      for (int i = 0; i < N; i++) {
        if (vis[i] == 0) {
          findTopoSort(i, vis, st, adj);
        }
      }
      vector < int > topo;
      while (!st.empty()) {
        topo.push_back(st.top());
        st.pop();
      }
      return topo;

    }
};

// { Driver Code Starts.
int main() {

  int N = 6;

  vector < int > adj[5 + 1];

  adj[5].push_back(2);
  adj[5].push_back(0);
  adj[4].push_back(0);
  adj[4].push_back(1);
  adj[2].push_back(3);
  adj[3].push_back(1);

  Solution obj;
  vector < int > res = obj.topoSort(6, adj);

  cout << "Toposort of the given graph is:" << endl;
  for (int i = 0; i < res.size(); i++) {
    cout << res[i] << " ";
  }

  return 0;
}
```

Output:

Toposort of the given graph is:

5 4 2 3 1 0

Time Complexity: O(N+E)

N = Number of node , E = Number of Edges

Space Complexity: O(N) + O(N)

Visited Array and Stack data structure. Both will be using O(N).

Auxiliary Space Complexity: O(N)

Recursion call of DFS