

Time complexity:

1) $O(n)$

①
$$\begin{aligned} & \text{for}(i=0; i < n; i++) \text{---} n+1 \\ & \{ \\ & \quad \text{stmt; ---} n \\ & \} \end{aligned}$$

$$\underline{\quad\quad\quad}$$

$$O(n)$$

$$\begin{aligned} & \text{for}(i=1; i < n; i=i+2) \\ & \{ \\ & \quad \text{stmt; ---} \frac{n}{2} \\ & \} \end{aligned}$$

$$\underline{\quad\quad\quad}$$

$$O(n)$$

①
$$\begin{aligned} & \text{for}(i=n; i > 0; i--) \\ & \{ \\ & \quad \text{stmt; ---} n \\ & \} \end{aligned}$$

$$\underline{\quad\quad\quad}$$

$$O(n)$$

$$\begin{aligned} & \text{for}(i=1; i < n; i=i+20) \\ & \{ \\ & \quad \text{stmt; ---} \frac{n}{20} \\ & \} \end{aligned}$$

$$\underline{\quad\quad\quad}$$

$$O(n)$$

1 — 10 $f(n) = \frac{n}{20}$

2) $O(n^2)$

$$\begin{aligned} & \text{for}(i=0; i < n; i++) \text{---} n+1 \\ & \{ \\ & \quad \text{for}(j=0; j < n; j++) \text{---} n \times (n+1) \\ & \quad \{ \\ & \quad \quad \text{stmt; ---} n \times n \\ & \quad \} \\ & \} \end{aligned}$$

$$\underline{\quad\quad\quad}$$

$$O(n^2)$$

Time Complexity #1

③
$$\begin{aligned} & \text{for}(i=0; i < n; i++) \\ & \{ \\ & \quad \text{for}(j=0; j < i; j++) \\ & \quad \{ \\ & \quad \quad \text{stmt; ---} \\ & \quad \} \\ & \} \end{aligned}$$

i	j	no. of time
0	0x	0
1	0x 1x	1
2	0 1 2x	2
3	0 1 2 3	3
...
n	n	n

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$f(n) = \frac{n^2+1}{2}$$

$$O(n^2)$$

3) $O(n^{1/2})$

Complexity #1

$$\begin{aligned} & p=0; \\ & \text{for}(i=1; p \leq n; i++) \\ & \{ \\ & \quad p = p + i; \\ & \} \end{aligned}$$

Assume $p > n$

$$\therefore p = \frac{k(k+1)}{2}$$

$$\frac{k(k+1)}{2} > n$$

$$k^2 > n$$

$$k > \sqrt{n}$$

$$\begin{aligned} & i & p \\ & 1 & 0+1=1 \\ & 2 & 1+2=3 \\ & 3 & 1+2+3 \\ & 4 & 1+2+3+4 \\ & \vdots & \vdots \\ & k & 1+2+3+4+\dots+k \end{aligned}$$

$$O(\sqrt{n})$$

4) $O(\log n)$

⑤

```

for (i=1; i < n; i = i * 2)
{
    stmt;
}

```

Assume $i \geq n$
 $\therefore i = 2^k$
 $\therefore 2^k \geq n$
 $2^k = n$
 $k = \log_2 n$

$\frac{i}{1}$
 $1 \times 2 = 2$
 $2 \times 2 = 2^2$
 $2^2 \times 2 = 2^3$
 \vdots
 2^k

⑥

```

for (i = n; i >= 1; i = i / 2)
{
    stmt;
}

```

Assume $i < 1$
 $\therefore \frac{n}{2^k} < 1$
 $\frac{n}{2^k} = 1$
 $n = 2^k$
 $k = \log_2 n$

$O(\log_2 n)$

$\frac{i}{n}$
 $\frac{n}{2}$
 $\frac{n}{2^2}$
 $\frac{n}{2^3}$
 \vdots
 $\frac{n}{2^k}$

⑧

```

for (i=0; i < n; i++)
{
    stmt;
}
for (j=0; j < n; j++)
{
    stmt;
}

```

n
 n
 $2n$
 $O(n)$

⑨

```

p = 0
{
    for (i=1; i < n; i = i * 2)
    {
        p++;
    }
    for (j=1; j < p; j = j * 2)
    {
        stmt;
    }
}

```

$p = \log n$
 $\log p$
 $O(\log \log n)$

```
for(i=0; i*i<n; i++)  
{  
    stmt;  
}
```

$$i*i < n$$

$$i*i \geq n$$

$$i^2 = n$$

$$i = \sqrt{n}$$