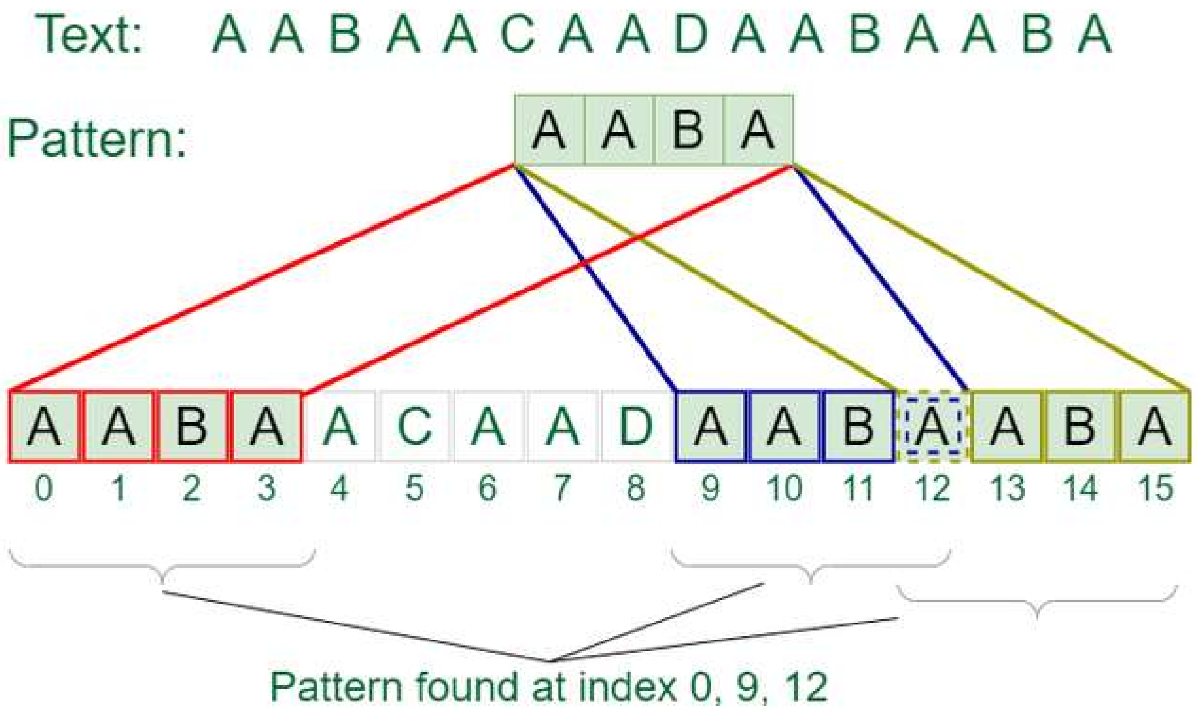
KMP Algorithm for Pattern Searching

Given a text **txt[0 . . . N-1]** and a pattern **pat[0 . . . M-1]**, write a function search(char pat[], char txt[]) that prints all occurrences of pat[] in txt[]. You may assume that **N > M**.

Examples:

Input: *txt[] = "THIS IS A TEST TEXT", pat[] = "TEST"*
Output: *Pattern found at index 10*
Input: *txt[] = "AABAACAADAABAABA"*
pat[] = "AABA"
Output: *Pattern found at index 0, Pattern found at index 9, Pattern found at index 12*


Text: A A B A A C A A D A A B A A B A
Pattern:
A A B A
A A B A A C A A D A A B A A B A
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Pattern found at index 0, 9, 12

Pattern searching is an important problem in computer science. When we do search for a string in a notepad/word file or browser or database, pattern-searching algorithms are used to show the search results.
We have discussed the Naive pattern-searching algorithm in the previous post. The worst case complexity of the Naive algorithm is O(m(n-m+1)). The time complexity of the KMP algorithm is O(n+m) in the worst case.
KMP (Knuth Morris Pratt) Pattern Searching:

Examples:
1) *txt[] = "AAAAAAAAAAAAAAAAAB", pat[] = "AAAAB"*
2) *txt[] = "ABABABCABABABCABABABC", pat[] = "ABABAC" (not a worst case, but a bad case for Naive)*
The KMP matching algorithm uses degenerating property (pattern having the same sub-patterns appearing more than once in the pattern) of the pattern and improves the worst-case complexity to **O(n+m)**.
The basic idea behind KMP's algorithm is: whenever we detect a mismatch (after some matches), we already know some of the characters in the text of the next window. We take advantage of this information to avoid matching the characters that we know will anyway match.
Matching Overview
txt = "AAAAABAAABA"
pat = "AAAA"
*We compare first window of **txt** with **pat***
*txt = "**AAAA**ABAAABA"*
*pat = "**AAAA**" [Initial position]*
*We find a match. This is same as [Naive String Matching](). *
*In the next step, we compare next window of **txt** with **pat**.*
*txt = "**AAAAA**BAAABA"*
*pat = "**AAAA**" [Pattern shifted one position]*
This is where KMP does optimization over Naive. In this second window, we only compare fourth A of pattern
with fourth character of current window of text to decide whether current window matches or not. Since we know
first three characters will anyway match, we skipped matching first three characters.
Need of Preprocessing?
An important question arises from the above explanation, how to know how many characters to be skipped. To know this,
we pre-process pattern and prepare an integer array lps[] that tells us the count of characters to be skipped

Preprocessing Overview:

- KMP algorithm preprocesses pat[] and constructs an auxiliary **lps[]** of size **m** (same as the size of the pattern) which is used to skip characters while matching.

- Name **lps** indicates the longest proper prefix which is also a suffix. A proper prefix is a prefix with a whole string not allowed. For example, prefixes of "ABC" are "", "A", "AB" and "ABC". Proper prefixes are "", "A" and "AB". Suffixes of the string are "", "C", "BC", and "ABC".
- We search for lps in subpatterns. More clearly we focus on sub-strings of patterns that are both prefix and suffix.
- For each sub-pattern pat[0..i] where i = 0 to m-1, lps[i] stores the length of the maximum matching proper prefix which is also a suffix of the sub-pattern pat[0..i].

lps[i] = the longest proper prefix of pat[0..i] which is also a suffix of pat[0..i].

Note: lps[i] could also be defined as the longest prefix which is also a proper suffix. We need to use it properly in one place to make sure that the whole substring is not considered.

Examples of lps[] construction:

For the pattern "AAAA", lps[] is [0, 1, 2, 3]
For the pattern "ABCDE", lps[] is [0, 0, 0, 0, 0]
For the pattern "AABAACAABAA", lps[] is [0, 1, 0, 1, 2, 0, 1, 2, 3, 4, 5]
For the pattern "AAACAAAAAC", lps[] is [0, 1, 2, 0, 1, 2, 3, 3, 3, 4]
For the pattern "AAABAAA", lps[] is [0, 1, 2, 0, 1, 2, 3]

Preprocessing Algorithm:

In the preprocessing part,

- We calculate values in **lps[]**. To do that, we keep track of the length of the longest prefix suffix value (we use **len** variable for this purpose) for the previous index
- We initialize **lps[0]** and **len** as 0.
- If **pat[len]** and **pat[i]** match, we increment **len** by 1 and assign the incremented value to lps[i].
- If pat[i] and pat[len] do not match and len is not 0, we update len to lps[len-1]
- See **computeLPSArray()** in the below code for details

Illustration of preprocessing (or construction of lps[]):

pat[] = "AAACAAAA"

=> len = 0, i = 0:

- *lps[0] is always 0, we move to i = 1*

=> len = 0, i = 1:

- *Since pat[len] and pat[i] match, do len++,*

- *store it in lps[i] and do i++.*
- *Set len = 1, lps[1] = 1, i = 2*

=> len = 1, i = 2:

- *Since pat[len] and pat[i] match, do len++,*
- *store it in lps[i] and do i++.*
- *Set len = 2, lps[2] = 2, i = 3*

=> len = 2, i = 3:

- *Since pat[len] and pat[i] do not match, and len > 0,*
- *Set len = lps[len-1] = lps[1] = 1*

=> len = 1, i = 3:

- *Since pat[len] and pat[i] do not match and len > 0,*
- *len = lps[len-1] = lps[0] = 0*

=> len = 0, i = 3:

- *Since pat[len] and pat[i] do not match and len = 0,*
- *Set lps[3] = 0 and i = 4*

=> len = 0, i = 4:

- *Since pat[len] and pat[i] match, do len++,*
- *Store it in lps[i] and do i++.*
- *Set len = 1, lps[4] = 1, i = 5*

=> len = 1, i = 5:

- *Since pat[len] and pat[i] match, do len++,*
- *Store it in lps[i] and do i++.*
- *Set len = 2, lps[5] = 2, i = 6*

=> len = 2, i = 6:

- *Since pat[len] and pat[i] match, do len++,*
- *Store it in lps[i] and do i++.*
- *len = 3, lps[6] = 3, i = 7*

=> len = 3, i = 7:

- *Since pat[len] and pat[i] do not match and len > 0,*
- *Set len = lps[len-1] = lps[2] = 2*

=> len = 2, i = 7:

- *Since pat[len] and pat[i] match, do len++,*
- *Store it in lps[i] and do i++.*
- *len = 3, lps[7] = 3, i = 8*

We stop here as we have constructed the whole lps[].

Implementation of KMP algorithm:

Unlike the Naive algorithm, where we slide the pattern by one and compare all characters at each shift, we use a value from lps[] to decide the next characters to be matched. The idea is to not match a character that we know will anyway match.

How to use lps[] to decide the next positions (or to know the number of characters to be skipped)?

- We start the comparison of pat[j] with j = 0 with characters of the current window of text.
- We keep matching characters txt[i] and pat[j] and keep incrementing i and j while pat[j] and txt[i] keep **matching**.
- When we see a **mismatch**
    - We know that characters pat[0..j-1] match with txt[i-j…i-1] (Note that j starts with 0 and increments it only when there is a match).
    - We also know (from the above definition) that lps[j-1] is the count of characters of pat[0…j-1] that are both proper prefix and suffix.
    - From the above two points, we can conclude that we do not need to match these lps[j-1] characters with txt[i-j…i-1] because we know that these characters will anyway match. Let us consider the above example to understand this.

Below is the illustration of the above algorithm:

*Consider txt[] = "**AAAAABAAABA**", pat[] = "**AAAA**"*

*If we follow the above LPS building process then **lps[] = {0, 1, 2, 3}***

***-> i = 0, j = 0:** txt[i] and pat[j] match, do i++, j++*

***-> i = 1, j = 1:** txt[i] and pat[j] match, do i++, j++*

***-> i = 2, j = 2:** txt[i] and pat[j] match, do i++, j++*

***-> i = 3, j = 3:** txt[i] and pat[j] match, do i++, j++*

-> i = 4, j = 4:** Since j = M, print pattern found and reset j, **j** = lps[j-1] = lps[3] = **3

Here unlike Naive algorithm, we do not match first three characters of this window. Value of lps[j-1] (in above step) gave us index of next character to match.

***-> i = 4, j = 3:** txt[i] and pat[j] match, do i++, j++*

-> *i = 5, j = 4:* *Since j == M, print pattern found and reset j,* ***j*** *= lps[j-1] = lps[3] =* ***3***
Again unlike Naive algorithm, we do not match first three characters of this window. Value of lps[j-1] (in above step) gave us index of next character to match.
-> *i = 5, j = 3:* *txt[i] and pat[j] do NOT match and j > 0, change only j.* ***j*** *= lps[j-1] = lps[2] =* ***2***
-> *i = 5, j = 2:* *txt[i] and pat[j] do NOT match and j > 0, change only j.* ***j*** *= lps[j-1] = lps[1] =* ***1***
-> *i = 5, j = 1:* *txt[i] and pat[j] do NOT match and j > 0, change only j.* ***j*** *= lps[j-1] = lps[0] =* ***0***
-> *i = 5, j = 0:* *txt[i] and pat[j] do NOT match and j is 0, we do i++.*
-> *i = 6, j = 0:* *txt[i] and pat[j] match, do i++ and j++*
-> *i = 7, j = 1:* *txt[i] and pat[j] match, do i++ and j++*
We continue this way till there are sufficient characters in the text to be compared with the characters in the pattern…
Below is the implementation of the above approach:

- C++
- Java
- Python3
- C#
- Javascript
- PHP

```
// C++ program for implementation of KMP pattern searching

// algorithm


#include <bits/stdc++.h>


void computeLPSArray(char* pat, int M, int* lps);


// Prints occurrences of pat[] in txt[]
```

```c
void KMPSearch(char* pat, char* txt)
{
    int M = strlen(pat);
    int N = strlen(txt);

    // create lps[] that will hold the longest prefix suffix
    // values for pattern
    int lps[M];

    // Preprocess the pattern (calculate lps[] array)
    computeLPSArray(pat, M, lps);

    int i = 0; // index for txt[]
    int j = 0; // index for pat[]
    while ((N - i) >= (M - j)) {
        if (pat[j] == txt[i]) {
            j++;
            i++;
        }

        if (j == M) {
            printf("Found pattern at index %d ", i - j);
            j = lps[j - 1];
        }

        // mismatch after j matches
        else if (i < N && pat[j] != txt[i]) {
            // Do not match lps[0..lps[j-1]] characters,
            // they will match anyway
            if (j != 0)
                j = lps[j - 1];
            else
                i = i + 1;
        }
    }
}

// Fills lps[] for given pattern pat[0..M-1]
void computeLPSArray(char* pat, int M, int* lps)
{
    // length of the previous longest prefix suffix
    int len = 0;

    lps[0] = 0; // lps[0] is always 0

    // the loop calculates lps[i] for i = 1 to M-1
    int i = 1;
    while (i < M) {
        if (pat[i] == pat[len]) {
            len++;
            lps[i] = len;
            i++;
        }
        else // (pat[i] != pat[len])
        {
            // This is tricky. Consider the example.
            // AAACAAAA and i = 7. The idea is similar
            // to search step.
            if (len != 0) {
                len = lps[len - 1];

                // Also, note that we do not increment
                // i here
            }
            else // if (len == 0)
            {
                lps[i] = 0;
                i++;
            }
        }
    }
}

// Driver code
int main()
{
    char txt[] = "ABABDABACDABABCABAB";
    char pat[] = "ABABCABAB";
    KMPSearch(pat, txt);
    return 0;
}
```
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Output

```
Found pattern at index 10
```

Time Complexity: O(N+M) where N is the length of the text and M is the length of the pattern to be found.
Auxiliary Space: O(M)