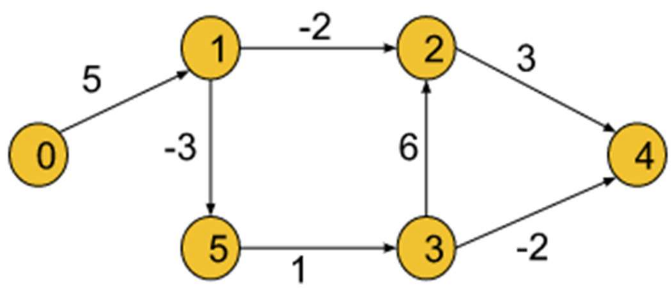
Bellman Ford Algorithm: G-41

Problem Statement: Given a weighted, directed and connected graph of V vertices and E edges, Find the shortest distance of all the vertices from the source vertex S.

Note: If the Graph contains a negative cycle then return an array consisting of only -1.

Example 1:
`Input Format:`

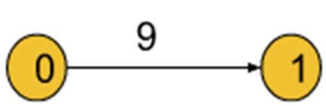
```
V = 6,

E = [[3, 2, 6], [5, 3, 1], [0, 1, 5], [1, 5, -3], [1, 2, -2], [3, 4, -2], [2,
4, 3]],

S = 0
```



`Result:` `0 5 3 3 1 2`

`Explanation:` `Shortest distance of all nodes from the source node is returned.`

Example 2:
`Input Format:` `V = 2, E = [[0,1,9]],  S = 0`



`Result:` `0 9`

`Explanation:` `Shortest distance of all nodes from the source node is returned.`

Solution

Disclaimer: *Don't jump directly to the solution, try it out yourself first. [Problem link](#).*

Solution:

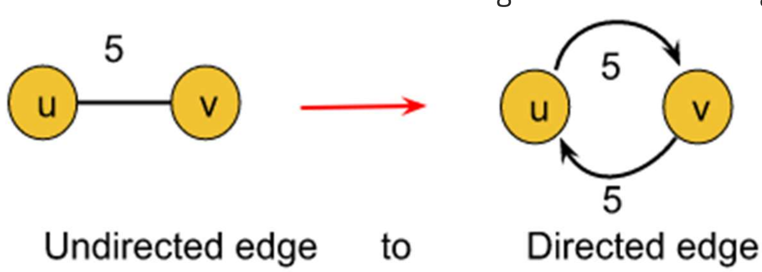
The bellman-Ford algorithm helps to find the shortest distance from the source node to all other nodes. But, we have already learned ***Dijkstra's algorithm (***Dijkstra's algorithm article link***)*** to fulfill the same purpose. Now, the question is ***how this algorithm is different from Dijkstra's algorithm***.

While learning Dijkstra's algorithm, we came across the following two situations, where Dijkstra's algorithm failed:

- ***If the graph contains negative edges.***
- ***If the graph has a negative cycle (In this case Dijkstra's algorithm fails to minimize the distance, keeps on running, and goes into an infinite loop. As a result it gives TLE error).***

Negative Cycle: *A cycle is called a negative cycle if the sum of all its weights becomes negative. The following illustration is an example of a negative cycle:*

Bellman-Ford's algorithm successfully solves these problems. ***It works fine with negative edges*** as well as ***it is able to detect if the graph contains a negative cycle***. But this algorithm is only applicable for ***directed graphs***. In order to apply this algorithm to an undirected graph, we just need to convert the undirected edges into directed edges like the following:



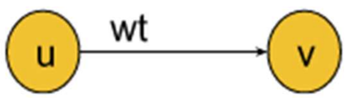
Explanation: An undirected edge between nodes u and v necessarily means that there are two opposite-directed edges, one towards node u and the other towards node v. So the above conversion is valid.

After converting the undirected graph into a directed graph following the above method, we can use the Bellman-Ford algorithm as it is.

Intuition:

In this algorithm, the edges can be given in any order. The intuition is to relax all the edges for N-1(N = no. of nodes) times sequentially. After N-1 iterations, we should have minimized the distance to every node.

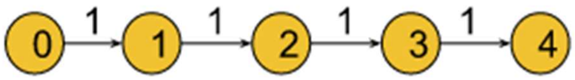
Let's understand what the relaxation of edges means using an example.



Let's consider the above graph with dist[u], dist[v], and wt. Here, wt is the weight of the edge and dist[u] signifies the shortest distance to reach node u found until now. Similarly, dist[v](maybe infinite) signifies the shortest distance to reach node v found until now. If the distance to reach v through u(i.e. dist[u] + wt) is smaller than dist[v], we will update the value of dist[v] with (dist[u] + wt). This process of updating the distance is called the relaxation of edges.

We will apply the above process(i.e. minimizing the distance to reach every node) N-1 times in the Bellman-Ford algorithm.

Two follow-up questions about the algorithm: Why do we need exact N-1 iterations?
Let's try to first understand this using an example:



Checking in each iteration	Given order of the edges: u	v	wt
dist[3] + 1 < dist[4]	3	4	1
dist[2] + 1 < dist[3]	2	3	1
dist[1] + 1 < dist[2]	1	2	1
dist[0] + 1 < dist[1]	0	1	1

- In the above graph, the algorithm will minimize the distance of the i^{th} node in the i^{th} iteration like dist[1] will be updated in the 1st iteration, dist[2] will be updated in the 2nd iteration, and so on. So we will need a total of 4 iterations(i.e. N-1 iterations) to minimize all the distances as dist[0] is already set to 0.
  Note: *Points to remember since, in a graph of N nodes we will take at most N-1 edges to reach from the first to the last node, we need exact N-1 iterations. It is impossible to draw a graph that takes more than N-1 edges to reach any node.*
- ***How to detect a negative cycle in the graph?***

    - We know if we keep on rotating inside a negative cycle, the path weight will be decreased in every iteration. But according to our intuition, we should have minimized all the distances within N-1 iterations(that means, after N-1 iterations no relaxation of edges is possible).
    - In order to check the existence of a negative cycle, we will relax the edges one more time after the completion of N-1 iterations. And if in that N^{th} iteration, it is found that further relaxation of any edge is possible, we can conclude that the graph has a negative cycle. Thus, the Bellman-Ford algorithm detects negative cycles.

Approach:

Initial Configuration:

distance array(dist[ ]): The dist[] array will be initialized with infinity, except for the source node as dist[src] will be initialized to 0.

The algorithm steps will be the following:

1. First, we will initialize the source node in the distance array to 0 and the rest of the nodes to infinity.
2. Then we will run a loop for N-1 times.
3. Inside that loop, we will try to relax every given edge.
   For example, one of the given edge information is like (u, v, wt), where u = starting node of the edge, v = ending node, and wt = edge weight. For all edges like this we will be checking if node u is reachable and if the distance to reach v through u is less than the distance to v found until now(i.e. **dist[u] and dist[u]+ wt < dist[v]**).
4. After repeating the 3rd step for N-1 times, we will apply the same step one more time to check if the negative cycle exists. If we found further relaxation is possible, we will conclude the graph has a negative cycle and from this step, we will return a distance array of -1(i.e. minimization of distances is not possible).
5. Otherwise, we will return the distance array which contains all the minimized distances.

Note: *If you wish to see the dry run of the above approach, you can watch the video attached to this article.*

Code:

- C++ Code
- Java Code

```
#include <bits/stdc++.h>
using namespace std;

class Solution {
public:
	/*  Function to implement Bellman Ford

	*   edges: vector of vectors which represents the graph

	*   S: source vertex to start traversing graph with

	*   V: number of vertices

	*/
	vector<int> bellman_ford(int V, vector<vector<int>>& edges, int S) {
		vector<int> dist(V, 1e8);
		dist[S] = 0;
		for (int i = 0; i < V - 1; i++) {
			for (auto it : edges) {
```

```
                                int u = it[0];
                                int v = it[1];
                                int wt = it[2];
                                if (dist[u] != 1e8 && dist[u] + wt < dist[v]) {
                                        dist[v] = dist[u] + wt;
                                }
                        }
                }
                // Nth relaxation to check negative cycle
                for (auto it : edges) {
                        int u = it[0];
                        int v = it[1];
                        int wt = it[2];
                        if (dist[u] != 1e8 && dist[u] + wt < dist[v]) {
                                return { -1};
                        }
                }


                return dist;
        }
};


int main() {

        int V = 6;
        vector<vector<int>> edges(7, vector<int>(3));
        edges[0] = {3, 2, 6};
        edges[1] = {5, 3, 1};
        edges[2] = {0, 1, 5};
        edges[3] = {1, 5, -3};
        edges[4] = {1, 2, -2};
        edges[5] = {3, 4, -2};
        edges[6] = {2, 4, 3};

        int S = 0;
        Solution obj;
        vector<int> dist = obj.bellman_ford(V, edges, S);
        for (auto d : dist) {
                cout << d << " ";
        }
        cout << endl;

        return 0;
}
```

Output: 0 5 3 3 1 2

Time Complexity: O(V*E), where V = no. of vertices and E = no. of Edges.

Space Complexity: O(V) for the distance array which stores the minimized distances.