Homography-Based Projective and Affine Rectification

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I. Introduction

Images of buildings and other planar surfaces often appear distorted due to perspective projection. As a result, parallel lines may converge and angles may no longer be preserved. These effects make it difficult to directly measure real-world dimensions from a single image.

This project addresses this problem by applying geometric rectification techniques to a single view of a planar scene. First, projective distortion is removed by estimating vanishing points and the vanishing line of the plane. This step restores parallelism in the image. Next, affine distortion is eliminated by enforcing orthogonality constraints between selected line pairs, producing a metric-rectified image where angles are preserved.

Finally, a homography is computed between image points and known world coordinates on the planar surface. This transformation allows image points to be mapped back into the world coordinate system, enabling accurate distance measurements. The methods are applied to multiple images of planar structures, demonstrating that reliable metric information can be recovered from a single image without explicit camera calibration.

II. Methodology

The proposed approach follows a structured three-stage pipeline to eliminate geometric distortions and recover metric information from a single planar image. Each stage removes a specific type of distortion and builds upon the previous result.

A. Image Acquisition and Preprocessing

A single image containing a planar surface is selected as input. Due to perspective projection, the image exhibits projective distortion. The image is loaded into MATLAB and converted to grayscale if necessary to simplify processing while preserving geometric structure.

B. Projective to Affine Rectification

The first stage removes projective distortion so that parallel lines in the real world remain parallel in the rectified image.

1) Point Selection on a Planar Rectangle: Four points corresponding to the corners of a rectangular structure on the plane are manually selected. Although the rectangle appears distorted in the image, its opposite sides are known to be parallel in the real world. The selected points are represented in homogeneous coordinates.

2) Line and Vanishing Point Computation: Lines are computed from pairs of points using the cross product. Each pair of parallel lines intersects at a vanishing point in the image. Two vanishing points are obtained by intersecting opposite sides of the rectangle and are normalized for numerical stability.

3) Vanishing Line Estimation: The vanishing line of the plane is computed by taking the cross product of the two vanishing points. This line represents the image of the line at infinity of the planar surface.

4) Affine Rectification: A projective transformation (homography) is constructed to map the vanishing line to the line at infinity. Applying this homography removes projective distortion, producing an affine-rectified image in which parallelism is preserved.

C. Affine to Sim Rectification

Although affine rectification preserves parallelism, angles and lengths remain distorted. The second stage removes affine distortion using orthogonality constraints.

1) Selection of Orthogonal Line Pairs: Two pairs of lines that are known to be perpendicular in the real world are selected from the affine-rectified image. Each line is defined using two manually selected points.

2) Metric Rectification: The constraint system is solved using Singular Value Decomposition (SVD). Cholesky decomposition is then applied to recover the affine correction matrix. A rectification homography is constructed and applied to the image, producing a metric-rectified image where angles and shapes are preserved up to a scale factor.

D. Real-World Measurement Using Homography

The final stage introduces scale to enable real-world distance measurement.

1) Point Correspondence Selection: Four points are selected from the metric-rectified image, and their corresponding real-world coordinates are defined using known physical dimensions of the scene.

2) Homography Estimation: A projective transformation between image coordinates and world coordinates is computed using the Direct Linear Transform (DLT) algorithm. The resulting linear system is solved using Singular Value Decomposition.

3) Distance Measurement: Image points are mapped to world coordinates using the inverse homography. After normalization, Euclidean distances are computed directly in real-world units.

sectionPart 1: Projective to Affine Rectification

E. *Objective*

The objective of this part is to remove projective (perspective) distortion from a single image of a planar surface. This is achieved by estimating the vanishing line of the plane and mapping it to infinity. After rectification, parallel lines in the real world appear parallel in the image.

F. *Mathematical Concept*

In homogeneous coordinates, the line at infinity of the Euclidean plane is represented as:

$$\ell_\infty = [0\ \ 0\ \ 1]^T$$

Due to perspective projection, this line is mapped to a finite vanishing line in the image:

$$L_\infty = [l_1\ \ l_2\ \ l_3]^T$$

A projective homography that maps the vanishing line to infinity is defined as:

$$H_p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}$$

Applying this homography removes projective distortion while preserving affine properties.

G. *Algorithm*

1) Point Selection: Four points corresponding to the corners of a real-world rectangle are manually selected in clockwise order:

$$p_1 : \text{bottom-left},\ p_2 : \text{bottom-right},\ p_3 : \text{top-right},\ p_4 : \text{top-left}$$

Each point is represented in homogeneous coordinates:

$$p_i = [x_i\ \ y_i\ \ 1]^T$$

2) Line Computation: Lines corresponding to rectangle edges are computed using cross products:

$$\ell_{12} = p_1 \times p_2, \quad \ell_{34} = p_3 \times p_4$$

$$\ell_{23} = p_2 \times p_3, \quad \ell_{14} = p_1 \times p_4$$

3) Vanishing Points and Vanishing Line: Vanishing points are obtained by intersecting pairs of parallel lines:

$$v_1 = \ell_{12} \times \ell_{34}, \quad v_2 = \ell_{23} \times \ell_{14}$$

Each vanishing point is normalized:

$$v_i \leftarrow \frac{v_i}{v_{i,3}}$$

The vanishing line of the plane is:

$$L_\infty = v_1 \times v_2 \quad \text{with} \quad L_{\infty,3} = 1$$

Applying H_p produces the affine-rectified image.

III. Part 2: Affine to Similarity (Metric) Rectification

A. *Introduction*

After affine rectification, parallelism is preserved but angles and lengths remain distorted. This part removes affine distortion by enforcing orthogonality constraints.

B. *Affine Transformation Model*

An affine transformation is defined as:

$$\mathbf{x}' = A\mathbf{x} + \mathbf{t}$$

In homogeneous form:

$$\begin{bmatrix} \mathbf{x}' \\ 1 \end{bmatrix} = \begin{bmatrix} A & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix}$$

C. *Orthogonality Constraints*

For two orthogonal directions in the world:

$$\mathbf{d}_1^T \mathbf{d}_2 = 0$$

Under affine transformation:

$$\mathbf{u}_1 = A\mathbf{d}_1, \quad \mathbf{u}_2 = A\mathbf{d}_2$$

Substituting:

$$\mathbf{u}_1^T (A^{-T} A^{-1}) \mathbf{u}_2 = 0$$

Define:

$$S = A^{-T} A^{-1} = \begin{bmatrix} s_{11} & s_{12} \\ s_{12} & s_{22} \end{bmatrix}$$

Thus:

$$\mathbf{u}_1^T S \mathbf{u}_2 = 0$$

D. *Metric Rectification*

Using two orthogonal line pairs, a homogeneous system:

$$M\mathbf{s} = 0$$

is formed and solved using SVD.

From:

$$S^{-1} = AA^T$$

a Cholesky decomposition gives:

$$S^{-1} = LL^T$$

The metric rectification homography is:

$$H_m = \begin{bmatrix} L^{-1} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}$$

IV. Part 3: Real-World Measurement Using Homography

A. *Objective*

The goal of this part is to recover real-world distances by estimating a homography between image and world coordinates.

B. Homography Model

Let:

$$p_i = [x_i \;\; y_i \;\; 1]^T, \quad P_i = [X_i \;\; Y_i \;\; 1]^T$$

The mapping is:

$$p = HP$$

C. Direct Linear Transform

Each correspondence provides:

$$p \times (HP) = 0$$

For four points:

$$A\mathbf{h} = 0$$

which is solved using SVD to obtain H.

D. Distance Measurement

World coordinates are recovered by:

$$P = H^{-1}p$$

After normalization:

$$(X, Y) = \left(\frac{P_1}{P_3}, \frac{P_2}{P_3}\right)$$

The Euclidean distance is:

$$d = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}$$

V. Result: Homography Matrix Estimation

A planar homography was estimated to map points from the world plane ($Z = 0$) to image coordinates. Four corresponding point pairs were selected interactively between the reference image and a known world coordinate grid. The homography was computed using the Direct Linear Transform (DLT), which solves a linear system derived from the projective mapping constraint.

1) Estimated Homography: The resulting homography matrix is

$$H_1 = \begin{bmatrix} 2878.0764 & -8085.2967 & 4219.5707 \\ 1298.4275 & -3757.7002 & 2064.1627 \\ 1.5696 & -3.2491 & 1.0000 \end{bmatrix} \tag{1}$$

$$H_2 = \begin{bmatrix} 194.2654 & 495.9643 & 1410.2287 \\ 404.4570 & 144.0774 & 375.2287 \\ 0.1374 & 0.3183 & 1.0000 \end{bmatrix} \tag{2}$$

$$H_3 = \begin{bmatrix} 2878.0764 & -8085.2967 & 4219.5707 \\ 1298.4275 & -3757.7002 & 2064.1627 \\ 1.5696 & -3.2491 & 1.0000 \end{bmatrix} \tag{3}$$

$$p = HP, \tag{4}$$

where $p = [u, v, 1]^T$ represents image coordinates and $P = [X, Y, 1]^T$ represents world plane coordinates. The homography encodes rotation, scale, translation, and perspective distortion in a single 3×3 matrix defined up to scale.

2) Geometric Interpretation: The first two rows primarily capture rotation and anisotropic scaling, the third row models perspective effects, and the last column corresponds to translation. This structure is consistent with the standard decomposition of planar projective mappings.

3) Inverse Mapping for Measurement: World coordinates are recovered by applying the inverse transformation

$$P = H^{-1}p. \tag{5}$$

After normalization of homogeneous coordinates, the recovered (X, Y) values provide real-world measurements. A reference length of 55.5 cm was used to establish the metric scale, enabling direct distance computation from image data.

VI. Conclusion

This work presented a homography-based method for removing geometric distortions from a single image of a planar surface and enabling accurate real-world measurement without camera calibration. The process first removed projective and affine distortions to recover metric structure, and then estimated a homography to map image points to world coordinates. The results show that the method successfully restores planar geometry and allows reliable measurement, with only small errors caused by manual point selection. This demonstrates that projective geometry provides a practical and effective solution for planar reconstruction from a single image. The approach can be applied in areas such as document analysis, architectural imaging, robotics, augmented reality, and photogrammetry. Future improvements may focus on automating feature detection and using more robust estimation techniques to increase accuracy and reduce user effort.

References

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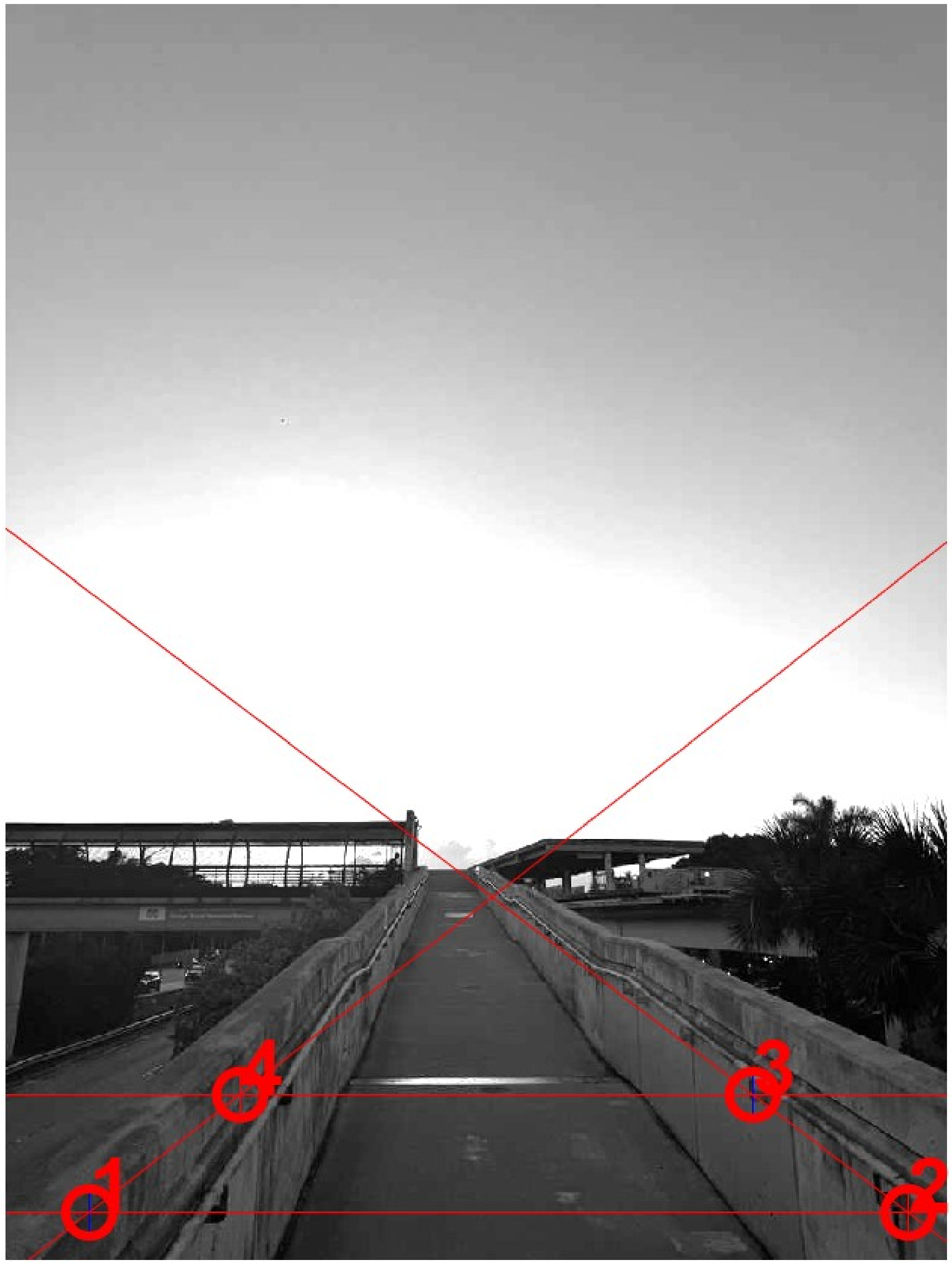

Fig. 1. Projective-to-affine rectification result (Example 1).

Fig. 2. Projective-to-affine rectification result (Example 2).

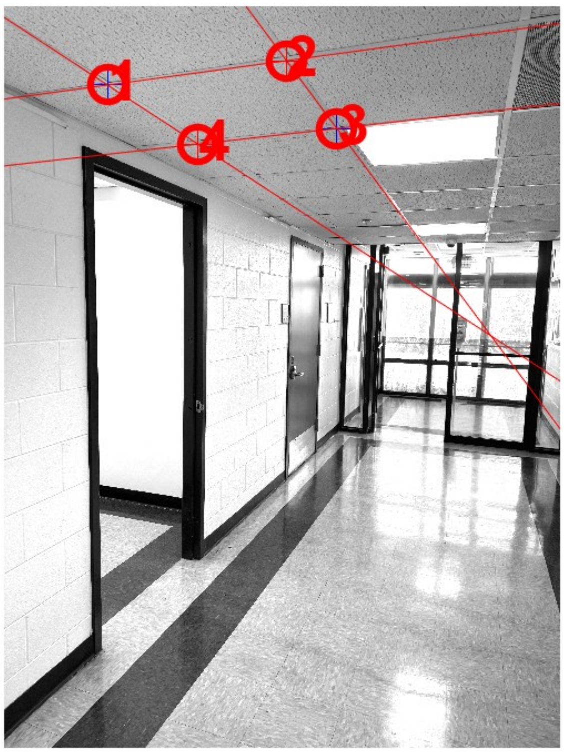

Fig. 3. Projective-to-affine rectification result (Example 3).

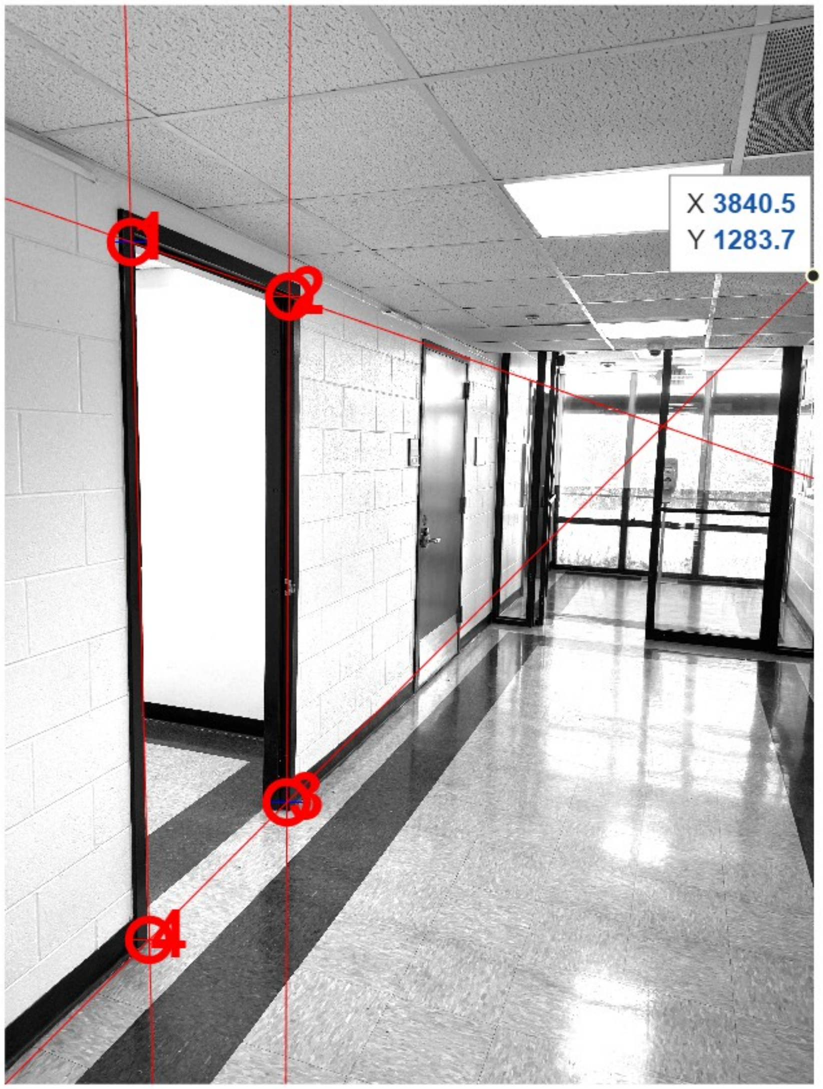

Fig. 4. Projective-to-affine rectification result (Example 4).

Fig. 5. Metric-rectified result05

Fig. 6. Metric-rectified result06

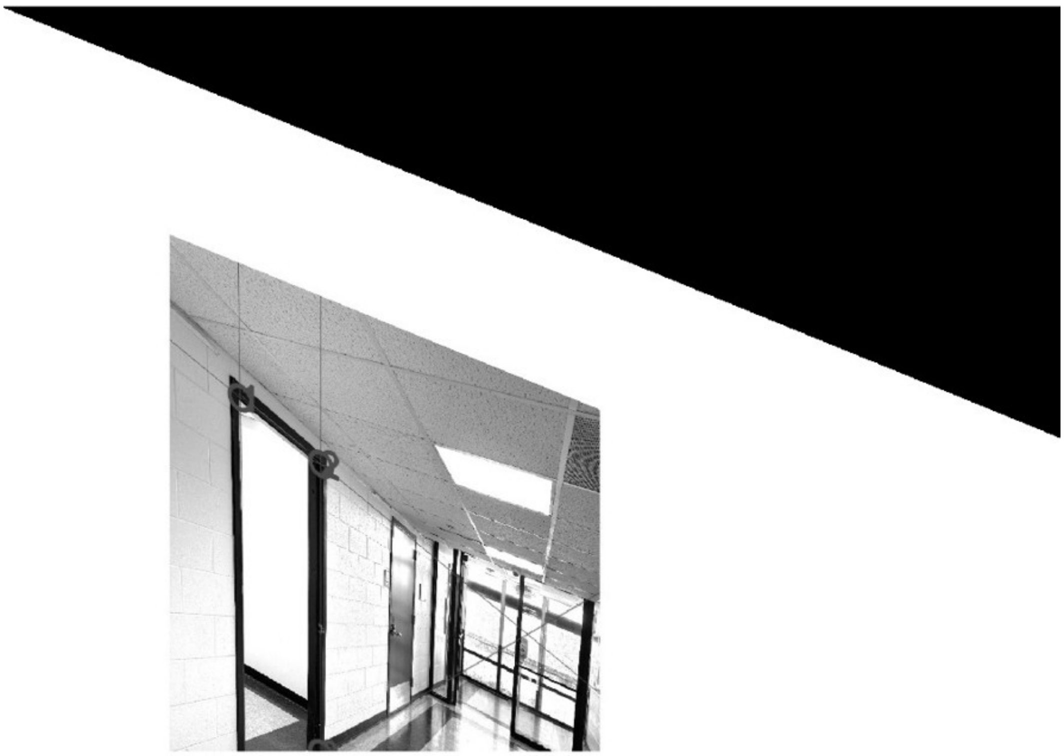

Fig. 7. Metric-rectified result07

Fig. 8. Metric-rectified result08