Date: 8th January, 2015 Problem Set: 2 Phy102

Vectors and Coordinates

1 a. Let $\vec{A}$ be an arbitrary vector and let $\hat{n}$ be a unit vector in some direction. Show that

$$\vec{A} = \left(\vec{A}\cdot\hat{n}\right)\hat{n} + \left(\hat{n}\times\vec{A}\right)\times\hat{n}$$

b. Let $\hat{a}$ and $\hat{b}$ be unit vectors in the xy plane making angles θ and φ with x-axis, respectively. Express vectors $\hat{a}$ and $\hat{b}$ in terms of unit vector $\hat{i}$ and $\hat{j}$ and hence prove $\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$.

2. Write down the rotation matrix $R_z(\theta)$ for rotation about z-axis by an angle θ. Also write down the matrix $R_x(\phi)$ (rotation about x-axis by an angle ϕ).

a) Show that $R_z(\theta)R_x(\phi) \neq R_x(\phi)R_z(\theta)$ by multiplication of matrices.

b) Now take the limit , $\theta \to \delta\theta, \phi \to \delta\phi$ where $\delta\theta$ (and $\delta\phi$) → 0 so that $\sin(\delta\theta) \to \delta\theta$ and $\cos(\delta\phi) \to 1$, show that $R_z(\delta\theta)R_x(\delta\phi) = R_x(\delta\phi)R_z(\delta\theta)$

c) What is the physical implication of this result?

3. The equation of a simple harmonic oscillator in the presence of a frictional force is given as

$m\dfrac{d^2u}{dt^2} + c\dfrac{du}{dt} + ku = F_0\sin\omega t$ where the oscillator is driven by an external periodic force $F_0\sin\omega t$

Cast the equation in a dimensionless form.

4 a. A particle moves in a plane with a constant radial velocity $\dot{r} = 4$ m/s.The angular velocity is constant and has magnitude $\dot{\phi} = 2$ rad/s. When the particle is 3 m away from the origin, find the magnitude of (a) the velocity and (b) the acceleration. (Kleppner 1.17).

b. If $\vec{r}\times\vec{v}$ is a constant vector, show that the acceleration is along the radius vector. Conversely, if the acceleration is radial show that $\vec{r}\times\vec{v}$ is constant. Interpret the result physically.

5. A tire rolls in a straight line without slipping. It center moves with a constant velocity *V*. A small pebble is lodged in the tread of the tire touches the road at $t = 0$. Find the velocity of the pebble and hence the position and acceleration as a function of time.

Additional Problems:

1. The direction cosines of a vector are the cosines of the angles it makes with the coordinate axes. The cosines of the angles between the vector and the x, y, z axes are usually called α, β and γ. Prove that $\alpha^2 + \beta^2 + \gamma^2 = 1$

2. Derive Sine law $\dfrac{\sin\alpha}{A} = \dfrac{\sin\beta}{B} = \dfrac{\sin\gamma}{C}$ where α, β and γ are the angles opposite to the vectors A,B and C which form a triangle.

3. Considering a rotation about z-axis, show that $\vec{A}\cdot\vec{B}$ is a scalar and the components of $\vec{A}\times\vec{B}$ transform like a vector.

4. A particle of mass m is moving in the xy plane. Express the kinetic energy $\frac{m}{2}\left(\dot{x}^2+\dot{y}^2\right)$ in polar coordinates.

5. i) Find out the unit vectors $\hat{e}_\rho$, $\hat{e}_\phi$ and $\hat{k}$ in cylindrical co-ordinate system and hence show that the co-ordinate system is orthogonal.

ii) Show that an elementary volume is given as $d\tau = \rho d\rho dz d\varphi$. Integrate ρ from 0 to a, φ from 0 to 2π and z from 0 to h.

iii) Express the velocity and acceleration of a particle in cylindrical co- ordinates.

iv) Find the expression for kinetic energy in cylindrical coordinates.

6. The expression of a force: $\vec{F} = -\hat{i}\frac{y}{x^2+y^2} + \hat{j}\frac{x}{x^2+y^2}$; considering a rotation about *z*-axis by an angle ϕ and find out F_x' and F_y'.