

Practice-sheet 1

Date: 8 August, 2014

1. What is the time complexity of each of the following subroutine as a function of N ? Give proper reasons.
 - (a)

```
int sum = 0;
for (int i = 1; i < N; i *= 2)
    for(int j = 0; j < i; j++)
        sum++;
```
 - (b)

```
int sum = 0;
for (int i = 1; i < N; i *= 2)
    for (int j = 0; j < N; j++)
        sum++;
```
2. Given an array of size n storing a sequence of 0's followed by all 1's. Design an efficient algorithm to compute the smallest index in A containing a 1.
3. You are given 2 sorted arrays A and B each of size n . Design an $O(n)$ algorithm to merge them to make another sorted array C of size $2n$.
4. Give an algorithm to find the smallest and second smallest elements from a list of N items using the minimum number of comparisons. Note that here we are interested in the exact number and not an upper bound in terms of big "O" notation.
5. Given two arrays storing n numbers, design an algorithm to print out all elements that appear in both arrays. The output should be in sorted order.
6. Given a positive integer n and a list containing $n - 1$ distinct integers in the range $[1, n]$, design an $O(n)$ time algorithm to find the missing number. You are not allowed to modify the list even temporarily. Your algorithm is allowed to use only $O(1)$ extra space. However, you may assume that every arithmetic operation takes $O(1)$ time.
7. Given a $n \times n$ matrix of 0s and 1s such that in each row no 0 comes before a 1, design an algorithm to find the row with the most 0s. The algorithm should run in $O(n)$ time.

8. Analyze the following algorithm called Euclid's algorithm for GCD of two numbers. Mention its time complexity as a function of input size (number of bits of **a** and number of bits in **b**).

```
GCD(a,b)    // here a is greater than or equal to b.
{
    while b <> 0
    {
        t <- b
        b <- a mod b
        a <- t
    }
    return a
}
```