

Problem 1. Consider n boys and n girls in a school seeking to be paired. Each boy has a preference list that ranks all girls and each girl has a preference list that ranks all boys. The set of n boys is divided into two categories: *normal* and *weird* and similarly for the girls. Suppose that there is a number k between 1 and n (included) such that there are k weird boys and k weird girls: thus there are $n - k$ normal boys and $n - k$ normal girls.

Suppose that the preference lists satisfies the following property. A weird person prefers a weird partner to a normal partner, and, a normal person prefers a normal partner to a weird partner.

Show that in every stable matching, every weird boy is paired with a weird girl.

Problem 2. *True or False.* Prove your answer or give a counterexample.

1. In every instance of the Stable Matching problem (done in the class), there is a stable matching containing a pair (b, g) such that g is ranked first on the preference list of b and b is ranked first on the preference list of g .
2. Consider an instance of the stable matching problem where there is a boy b and a girl g such that g is ranked first in the preference list of b and b is ranked first in the preference list of g . Then in every stable matching S for this instance, b is paired with g .

Problem 3. *College Counselling.* Consider a scenario where there n students and m colleges, with the i th college having k_i seats. Each student has his/her own ranking for the m colleges. Each college has a ranking of the students in order of preference. Assume that the total number of seats in the m colleges is less than the number of students.

You have to design an algorithm that assigns students to colleges such that all available positions are filled and the assignment is *stable*. The assignment of students to colleges is stable if neither of the following two kinds of situations (or instability) arises.

- *Instability 1.* There are students s and s' and a college c such that
 - s is assigned to c .
 - s' is not assigned to any college.
 - c prefers s' to s .
- *Instability 2.* There are students s and s' and colleges c and c' such that
 - s is assigned to c .
 - s' is assigned to c' .
 - s prefers c' to c .
 - c' prefers s to s' .

An assignment of students to colleges is said to be *stable* if neither kind of instability arises.

Show that there is always an assignment of students to colleges that is stable and give an algorithm for finding such an assignment.