

Hunter-Rabbit Game

Difficulty level: *high*

This exercise will illustrate the fact that algorithm design (even inefficient ones) for a problem is not a trivial exercise. Some of you will remember this exercise for years to come.

There is an infinite line. There is a rabbit and a hunter. The hunter is very lazy but very clever. The rabbit is standing at some integer location on the line in the beginning. The hunter is also standing at some point on this line, but he is not aware of the location of the rabbit. The game starts as follows. The rabbit makes a crucial decision for the rest of its life. It either chooses to stay at its initial position for the rest of its life or it chooses to run continuously for the rest of its life. In case it chooses to run, it chooses one fixed direction along the line (towards $+\infty$ or $-\infty$) and then continuously runs in that direction for the rest of its life. Note that hunter is totally unaware of the choice(s) made by the rabbit. The rabbit runs at fixed speed and covers unit distance in one minute. So at the end of each minute it is at an integer location. For example, if it is running towards $+\infty$ and at the end of t minutes it is at point i , then at the end of $t + 1$ minutes, it is at point $i + 1$. Let us now see what does the hunter do ? The hunter is very lazy and continues to stay at the same initial point. However, he, at the end of each minute, can select a point on the line as a target and shoot. He is also aware that the rabbit is always at some integer position at the end of each minute. If at the end of some minute, the hunter shoots the position which is the same where the rabbit is, the rabbit dies and the game ends.

Design an effective procedure (algorithm) which hunter can follow so that the game ends irrespective of the starting position of the rabbit and its initial choice. Note that the number of minutes for which the game runs has to be bounded by a *function* of the initial distance between the rabbit and the hunter. However, it would be great if your procedure can ensure that the duration of the game (number of minutes) is of the order of quadratic or linear in terms of the initial distance between the rabbit and the hunter.



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