PHY 103: General Physics 2 (2014 – 2015, Semester – I)

Department of Physics
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Assignment-1

Note: The questions marked with circles are to be solved by the students as Home Work. These will not be solved in the tutorials. The students are encouraged to clear any doubts on these questions during the office hours of tutors.

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1. The plot of a function looks like a hill on a flat plane.

$$h(x,y) = \exp[(2xy - 3x^2 - 4y^2 - 18x + 28y - 5)/60]$$

(a) Where is the top of the hill located?
(b) How high is the hill?
(c) How steep is the slope at a point (1,1)?. In what direction is the slope steepest, at that point?.

2. Given a scalar function

$$V = \left(\sin\frac{\pi}{2}x\right)\left(\sin\frac{\pi}{3}y\right)e^{-z},$$

Determine,
(a) the magnitude and direction of the maximum rate of increase of V at the point P(1,2,3)
(b) the rate of increase of V at P in the direction of the origin.

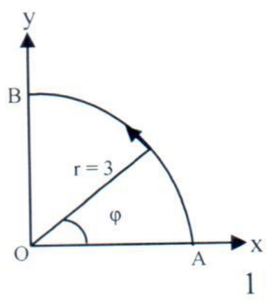
3. (a) The vector $\vec{A}$ is given in spherical polar coordinates as $\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$. Find A in cylindrical coordinates. Write down the transformation matrix.

4. Given $\vec{F} = \hat{\mathbf{x}}\,(\mathbf{xy}) - \hat{y}\,(\mathbf{2x})$

(a) Evaluate the scalar line integral

$$\int_A^B \overrightarrow{F} \cdot d\vec{\ell}$$

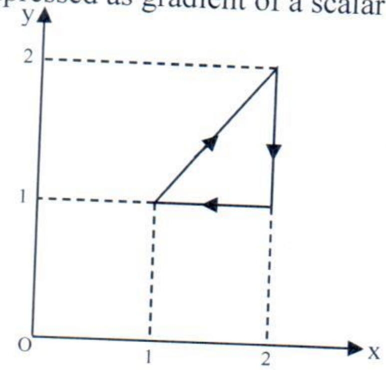
along the quarter circle shown in the figure below, using both cartesian and cylindrical coordinates.



5. Assume the vector function $\vec{A} = \hat{x}\,(3x^2y^3) - \hat{y}\,(x^3y^2)$

a) Evaluate $\int \left(\vec{\nabla} \times \vec{A}\right).d\vec{s}$ over the triangular area (Do not use Stokes' theorem).

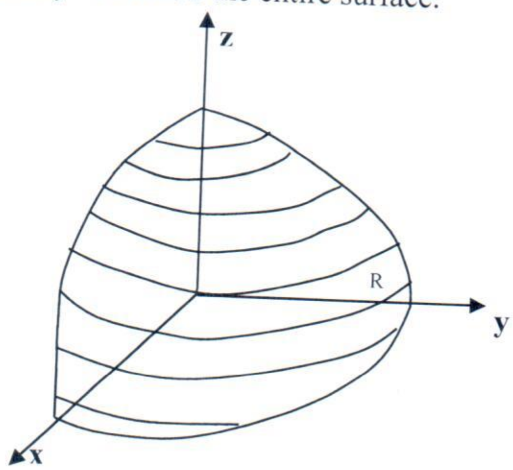
b) Can $\vec{A}$ be expressed as gradient of a scalar? Explain.



6. Check the divergence theorem for the function

$$\vec{v} = (\ r^2\cos\theta\ )\hat{r} + (\ r^2\cos\phi\ )\hat{\theta} - (\ r^2\cos\theta\sin\phi\ )\hat{\phi}$$

using the volume of one octant of the sphere of radius R (shown below). Make sure you include the entire surface.



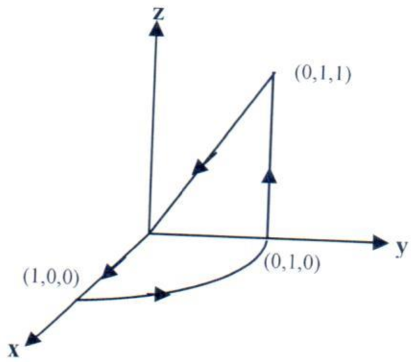


Figure for Question# 8

7. Evaluate the integral

$$J = \int_V e^{-r}\left(\vec{\nabla}.\frac{\hat{r}}{r^2}\right)d\tau$$

where V is a sphere of radius R, centered at the origin .

a) using the divergence theorem and performing the resulting surface integral.

b) using Dirac Delta function

8. Compute the line integral of

$$\vec{v} = \left(r\cos^2\theta\right)\hat{r} - \left(r\cos\theta\sin\theta\right)\hat{\theta} + 3r\,\hat{\phi}$$

around the path shown in figure above (the points are labeled by their Cartesian coordinates). **Do it in spherical coordinates**. Check your answer, using Stokes' theorem.