Gems from the world of data structures and algorithms

A matter of life and death

Difficulty level: *difficult*

On his way to meet Mhysa, Peter meets Jon and they set off across the narrow sea. But as fate would have it, on their way they are attacked by the Stone Men who destroy their ship and only a fraction of their crew barely manage to survive. Now in the middle of nowhere, they find themselves standing on a thin plank of wood surrounded by water as far as the horizon stretches. The plank is just enough for two people to stand side by side and **N** people to stand from end to end and as luck would have it exactly **2N** sailors including Peter and Jon have survived. To top it all, the plank is going down slowly but steadily.

Jon, being the knight that he is, is prompt in suggesting that they throw half the sailors overboard so that the rest can survive. Peter mutters a curse under his breath but stops Jon. He has figured out that they can all survive if they stand in a particular order on the plank. If the heaviest of them stand on one end (end **A**) of the plank, and the lightest on the other side (end **B**, they can all survive for a few more hours. They quickly look around only to find out that they are standing in the exact opposite order. Now they need to shuffle their arrangement so as to reach the optimal ordering and they need to do it quick. They only have $\frac{2}{3}N^2$ seconds until the plans sinks and each swap takes **1 second** The plank is so thin that only two people standing consecutive positions can swap places. Also as sailors and knights cannot be trusted, only one swap can be made in one second so that Peter can oversee it. Is this even possible? Can it be done in $\frac{2}{3}N^2$ seconds?

TLDR;

There are **N** positions on a plank and at each position $k = 2$ people are standing such that at position i, k people numbered $n - i + 1$ are standing. In each step only 2 people standing adjacent to each other can swap positions. What is the minimum number of steps required to completely reverse the ordering such that at position i, k people numbered i should be standing.

For example in the case $k = 1$, $\frac{n(n-1)}{2}$ steps would be required to achieve this. So naturally one would think that $n(n-1)$ steps should be required for $k = 2$, but can you do better than this? Well Peter has to. He owes Jon and he always pays his debt.

every art is beautiful and so is the art of algorithm design ...