Indian Institute of Technology Kanpur

Mini Project -1

MTH 308A – Principles of Numerical Analysis

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FINDING THE MINIMUM DISTANCE BETWEEN GIVEN POINTS AND CURVE.

We have a given simple closed curve C in $\mathbb{R}^2$ given by C : [0, 1] → $\mathbb{R}^2$ such that
$C(t) = (X(t), Y(t))$, $0 \leq t \leq 1$, where X, Y : $\mathbb{R} \to \mathbb{R}$ are infinitely differentiable periodic functions with period 1. We have also been given a point $(xo\ ,\ yo) \in \mathbb{R}^2$
We have to find the closest point (x_c, y_c) on the curve C.

I am using Newton's Method to solve this problem.

We know that Newton's Method for a function is

$$\boldsymbol{x(m+1) = x(m) - \frac{f(x(m))}{f'(x(m))}}$$

For this problem, we define a distance function which we have to minimize eventually.

$$\boldsymbol{Distance\ Function} = \sqrt{(\boldsymbol{x(t) - xo})^2 + (\boldsymbol{y(t) - yo})^2}$$

Minimizing the distance function, we get the same t minimizing the square of the distance function.

$$\boldsymbol{f(t) = (x(t) - xo)^2 + (y(t) - yo)^2}$$

For minima, $f'(t) = 0$.

We can find the roots of $f'(t) = 0$ by Newton's Method.
Applying Newton's Method on $f'(t)$.

For this, we have to initialize the value of $t(0)$.

I am initializing this value by randomly picking a value in (0,1) from a uniform distribution. Applying Newton's Method using $t(0) = rand;$ on $f'(t)$, we get $t's$ such that my $f'(t) < eps$

eps is already given in the *main.m* file.

This will give approximated values of $'t'$ such that $f'(t) = 0$, or we may say $f(t)$ is optimum.

But this does not ensure that for given set of $t's$, only minima will occur. For this, I have taken 1000 values of $t's$ which may or may not be same.

After finding such 1000 values of $t's$ (may or may not be unique), I'm storing the value of $'t'$ at which the square of the distance function is minimum, and hence the distance function is min.

One reason of choosing the number 1000 is to increase the probability of ensuring that my reported result is for the global minima. It will be very rare when the 1000 values of $'t'$ will account only for the maxima of the square of the distance function. In such cases, when a given curve will have a large number of local maximums present, we will have to increase the number 1000 to sufficiently required number say 10000 or 100000 or more.