

Grade Table (for checker use only)

| Question | Points | Score |
|----------|--------|-------|
| 1        | 6      |       |
| 2        | 6      |       |
| 3        | 8      |       |
| 4        | 10     |       |
| 5        | 12     |       |
| 6        | 10     |       |
| 7        | 12     |       |
| 8        | 12     |       |
| 9        | 12     |       |
| 10       | 12     |       |
| Total:   | 100    |       |

Team Members:

- .....
- .....
- .....

**Write your team name on top of each page.**

This test will be very easy if you solve it with a relaxed mind.

If you have any queries, contact the invigilator. However, no questions on the validity/correctness of a question will be entertained.

1. (6 points) Eleven gears are placed on a plane, arranged in a chain, as shown below. Can all the gears rotate simultaneously? Explain your answer.



(4 points) What if we have a chain of 572 gears?

2. (6 points) Prove that if a number has an odd number of divisors, then it is a perfect square.

3. (8 points) If  $p$ ,  $4p^2 + 1$ , and  $6p^2 + 1$  are prime numbers, find  $p$ .


4. (10 points) How many ways are there to place: a) two bishops, b) two knights, c) two queens on a chessboard so that they do not attack each other?

5. (12 points) Prove that all the numbers in the series

$10001, 100010001, 1000100010001, \dots$

are composite.

6. (10 points) How can you get a fair coin toss if someone hands you a coin that is weighted to come up heads more often than tails?



- (a)  $f(1) = 0$                       (b) The roots of  $g(x)$  are squares of the roots of  $f(x)$ .

Find the value of  $a^{2015} + b^{2015} + c^{2015}$

9. (12 points) Let  $ABC$  be a triangle with  $A = 90^\circ$  and  $AB = AC$ . Let  $D$  and  $E$  be points on the segment  $BC$  such that  $BD : DE : EC = 3 : 5 : 4$ . Find  $\angle DAE$ .

10. (12 points) 100 minions are lined up in a row by an assassin. The assassin puts red and blue hats on them. They can't see their own hats, but they can see the hats of the minions in front of them. The assassin starts in the back and says "What color is your hat?" the minion can only answer "red" or "blue." The minion is killed if he gives the wrong answer; then the assassin moves on to the next minion. The minions in front get to hear the answers of

the minions behind them, but not whether they live or die. They can consult and agree on a strategy before being lined up, but after being lined up and having the hats put on, they can't communicate in any way other than those already specified. What strategy should they choose to maximize the number of minions who are guaranteed to be saved?