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COMMUNICATION THEORY ASSIGNMENT-4

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(i) $\theta_i(t) = kt^2$ $\boxed{H(s)=1}$

$$\theta_e(t) = \theta_i(t) - \theta_o(t)$$

$$H_e(s) = \frac{\theta_i(s) - \theta_o(s)}{\theta_i(s)} = \left(\frac{s}{s+KH(s)}\right)$$

$$\theta_i(s) = \mathcal{L}\{kt^2\} = \boxed{\frac{2k}{s^3}}$$

$\therefore$ Let us find the value of $\theta_e(t)$ as $t\to\infty$. if it converges, we can track the signal atleast upto a constant phase or exactly (if phase $=0$)

$$\Rightarrow \lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} \theta_e(s)\cdot s$$

(Final value Theorem property of LT)

$$= \lim_{s\to 0} s \cdot H_e(s) \cdot \theta_i(s)$$

$$= \lim_{s\to 0} \cancel{s} \cdot \left(\frac{s}{s+K\cancel{H(s)}}\right) \times \frac{2k}{\cancel{s^3}\, s}$$

$(H(s)=1)$

$$= \lim_{s\to 0} \left(\frac{2k}{(s+K)s}\right) = \boxed{\infty}$$

$\Rightarrow$ $\theta_e(t)$ as $t \to \infty$ diverges.

$\Rightarrow$ The incoming signal ~~cant~~ can not be tracked.

(ii) If $H(s) = \left(\frac{s+a}{s}\right)$,

|||ly, $\lim_{t \to \infty} \theta_e(t) = ?$

$$\lim_{t \to \infty} \theta_e(t) = \lim_{s \to 0} s\, \Theta_e(s) = \lim_{s \to 0} s \cdot H_e(s) \cdot \Theta_i(s)$$

$$= \lim_{s \to 0} s \cdot \left(\frac{s}{s + K\left(H(s)\right)}\right) \cdot \frac{2k}{s^3}$$

$$= \lim_{s \to 0} s \left(\frac{s \times s}{\left(s + \frac{K(s+a)}{s}\right) \times s}\right)\left(\frac{2k}{s^3}\right)$$

$$= \lim_{s \to 0} \frac{\cancel{s^3}}{s^2 + K(s+a)}\left(\frac{2k}{\cancel{s^3}}\right)$$

$$= \frac{2k}{0^2 + K(0+a)} = \boxed{\frac{2k}{Ka}}$$

Thus, $\theta_e(t) \to \frac{2k}{Ka}$ where $t \to \infty$

$\Rightarrow$ constant-phase difference tracking is possible.

(iii) If $H(s) = \frac{s^2+as+b}{s^2}$,

$$\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} s \cdot H_e(s) \cdot \theta_i(s)$$

$$= \lim_{s\to 0} s \cdot \left(\frac{s}{s+K(H(s))}\right) \cdot \left(\frac{2k}{s^3}\right)$$

$$= \lim_{s\to 0} s \cdot \left(\frac{s}{s+K\left(\frac{s^2+as+b}{s^2}\right)}\right) \cdot \frac{2k}{s^3}$$

$$= \lim_{s\to 0} \frac{\cancel{s}^4\ \cancel{s}}{s^3+Ks^2+Kas+Kb} \cdot \frac{2k}{\cancel{s^3}}$$

$$= \boxed{0}$$

$\Rightarrow$ No phase error = 0 phase error tracking of the incoming signal.

(2)

(a)

Central freq = 1800.5 MHz

$\therefore$ for $f_{IF} = 250$ MHz, we have 2 possible f_{LO}.

(i) $f_{LO} = f_{RF} - f_{IF} = 1549.5$ MHz

or

(ii) $f_{LO} = f_{RF} + f_{IF} = 2050.5$ MHz

The f_{LO} obtained in (i) is within the tunable range of freq synth.

but (ii) does not as it is above the range and also cannot be obtained by ~~dividing by integer the~~ frequency division by integer from tunable range.

$\therefore$ we let $f_{LO} = 1549.5$ MHz

Scanned with CamScanner

$f_{RF} = f_{LO} - f_{IF} = 1800.5$ MHz and

f_{IM} (Image freq) $= f_{LO} + f_{IF} = \boxed{2300.5 \text{ MHz}}$.

RF filter must be centred ~~around~~ at 1800.5 MHz thereby passing signals in the range 1800–1801 MHz and rejecting frequency of Image at 2300.5 MHz.

A bandwidth of 50 MHz also comfortably allows for this. The IF at 250 MHz should pass message at 249.5 MHz and quickly cut off after that range.

(b) Central freq = 900.5 MHz $f_{IF} = 250$ MHz

<u>IIIly 2 choices for f_{LO}:</u>

i) $f_{LO} = f_{RF} + f_{IF} = 1150.5$ MHz

ii) $f_{LO} = f_{RF} - f_{IF} = 649.5$ MHz

Both are outside tunable range, but f_{LO} in ii) can be obtained by dividing 1948.5 MHz by 3 and (1948.5) MHz falls inside tunable range.

$\Rightarrow$ We let $f_{LO} = 649.5$ MHz.

$\Rightarrow f_{RF} = f_{LO} + f_{IF} = \underline{\underline{900.5\ MHz}}$

f_{IM} (Image freq) $= f_{LO} - f_{IF} = \underline{\underline{399.5\ MHz}}$

RF centred at 900.5 MHz
passes frequencies at 900 − 901 MHz
rejects 399.5 MHz.
20 MHz bandwidth works.

IF filter at 250 MHz must pass
message at 249.5 MHz and
sharply cut off thereafter