2/2/21

COMMUNICATION THEORY

ASSIGNMENT-2

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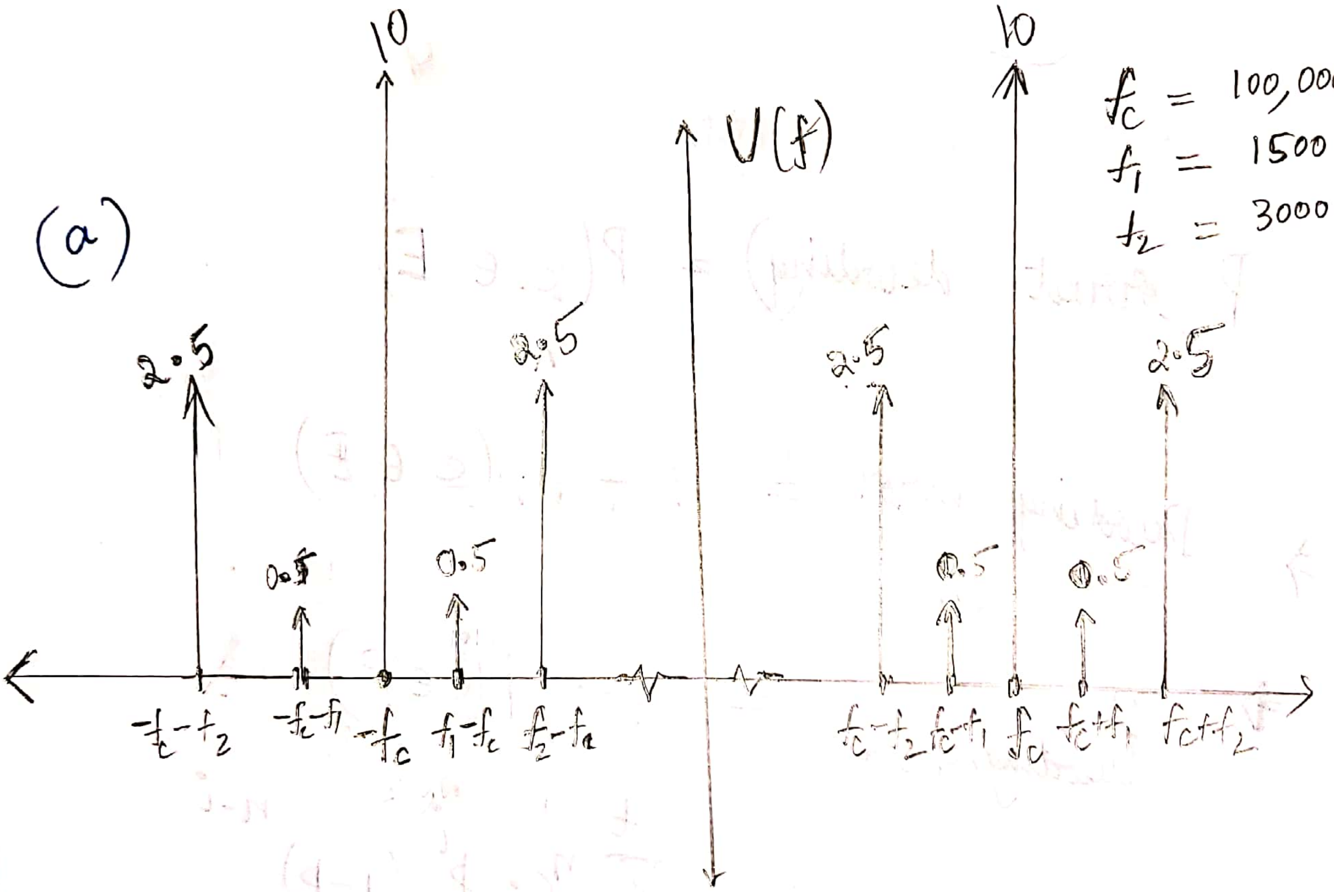
(1)

$$u(t) = \left(20 + 2\cos(3000\pi t) + 10\cos(6000\pi t)\right) \times \cos(2\pi f_c t)$$

(a)

$f_c = 100{,}000$ Hz
$f_1 = 1500$ Hz
$f_2 = 3000$ Hz



(b) $\text{Power}\left(A\cos(2\pi f_0 t)\right) = \boxed{\frac{A^2}{2}}$

$\Rightarrow$ Power in each freq. component

$(\pm f_c) = \frac{(20)^2}{2} = \boxed{200}\ W$

$\pm(f_c - f_1) = \frac{1^2}{2} = \boxed{0.5}\ W$

$\pm(f_c - f_2) = \frac{5^2}{2} = \boxed{12.5}\ W.$

(c)

$$a_{mod} = \frac{A\left|\min_t (m(t))\right|}{A_c}$$

$$\min_t (m(t)) = \left(\frac{-201}{40}\right).$$

$$\Rightarrow a_{mod} = \frac{\left|\frac{-201}{40}\right|}{10} = \frac{201}{400}$$

(d) Power in sidebands

$\pm(f_c \pm f_1) = 0.5 + 0.5 = 1$

$\pm(f_c \pm f_2) = 12.5 + 12.5 = 25$

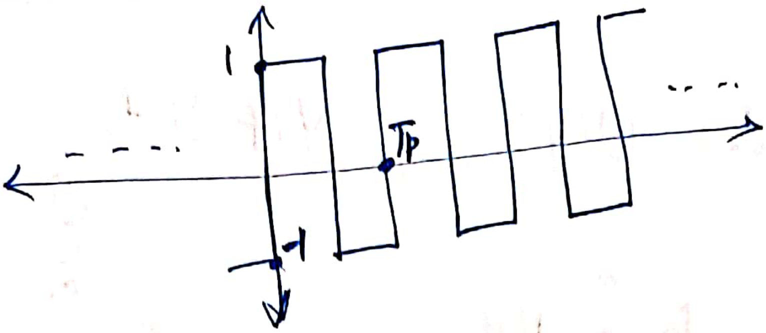
$\therefore$ Total Power $= 200 + 26$.

$\therefore$ Ratio of Side bands to total $= \frac{26}{226}$

$= \boxed{\frac{13}{113}}$

(2) $m(t)$ message signal

$s(t) =$



from the graph of $s(t)$, we can see that

$$\cancel{s(t) = \sum_{n=0}^{\infty} a_n \sin\left(2\pi n \frac{t}{T_p}\right)}$$

$$s(t) = \sum_{n=0}^{\infty} a_n \sin(2\pi n f_c t)$$

[using fourier series and the fact that $s(t)$ is odd]

$$\therefore \; m(t)\, s(t) \longleftrightarrow M(f) * S(f)$$

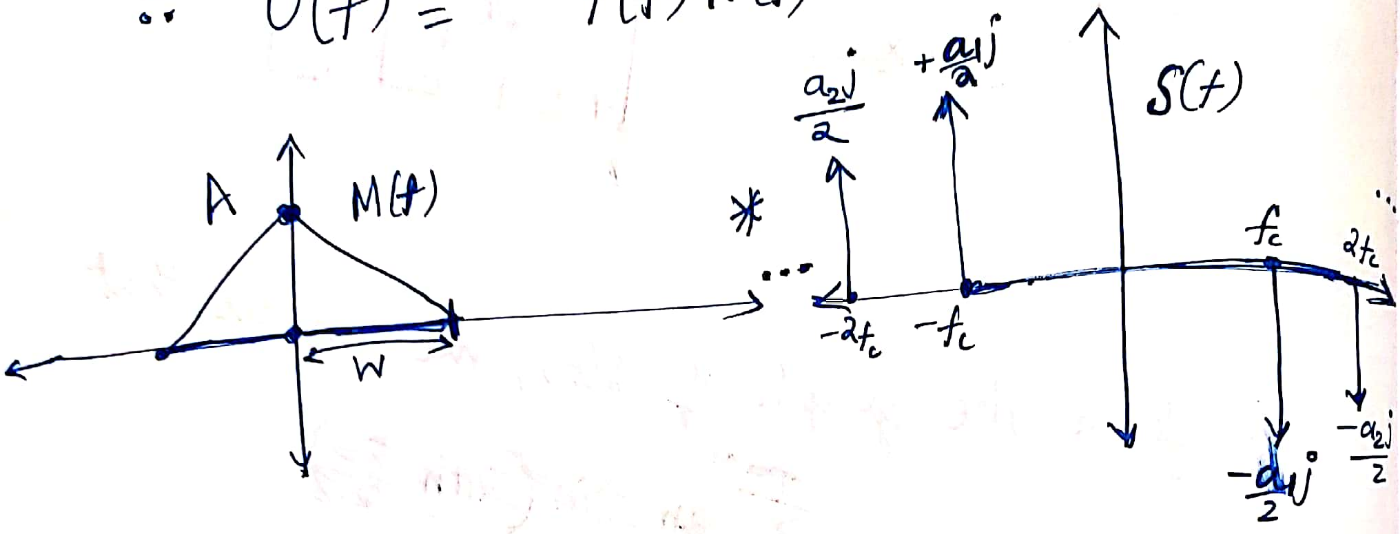
$$S(f) = \sum_{n=0}^{\infty} \frac{a_n}{2j} \left(\delta(f - n f_c) - \delta(f + f_c n) \right)$$

$$\Rightarrow M(f) * S(f) = M(f) * \left(\sum_{n=0}^{\infty} \frac{a_n}{2j} \left(\delta(f - n f_c) - \delta(f + f_c n) \right) \right)$$

$$\Rightarrow \boxed{M(f) * S(f) = \sum_{n=0}^{\infty} \frac{a_n}{2j} \left(M(f - n f_c) - M(f + n f_c) \right)}$$

let $Y(f) = M(f) * S(f)$, $H(f)$ be the impulse response of BPF

$\therefore \; U(f) = Y(f) H(f)$



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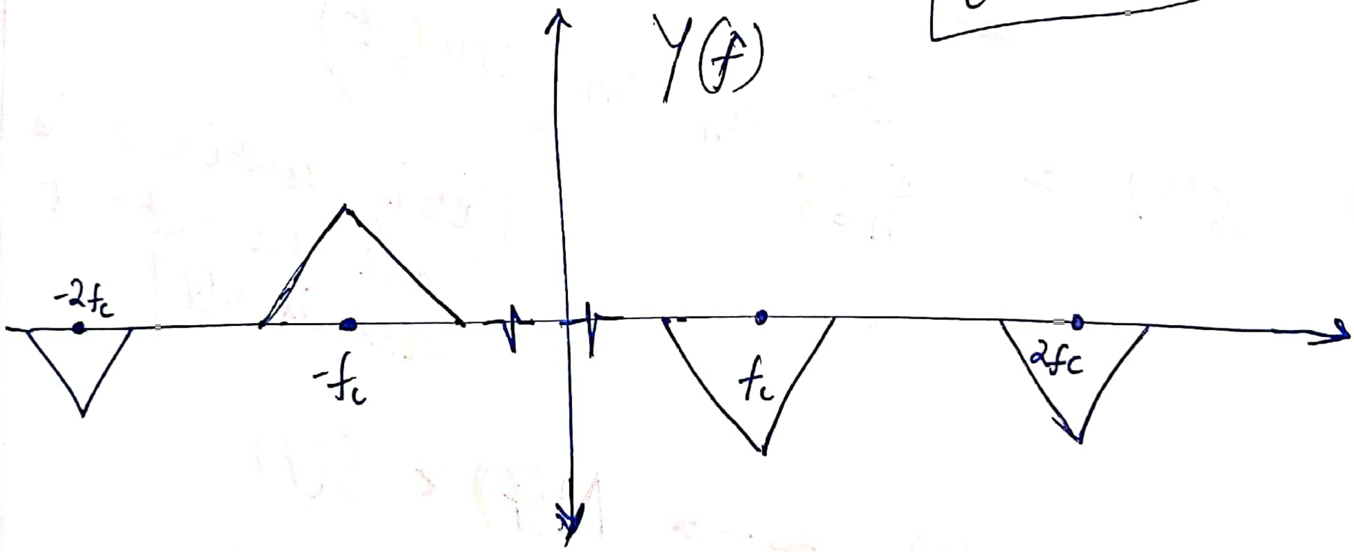
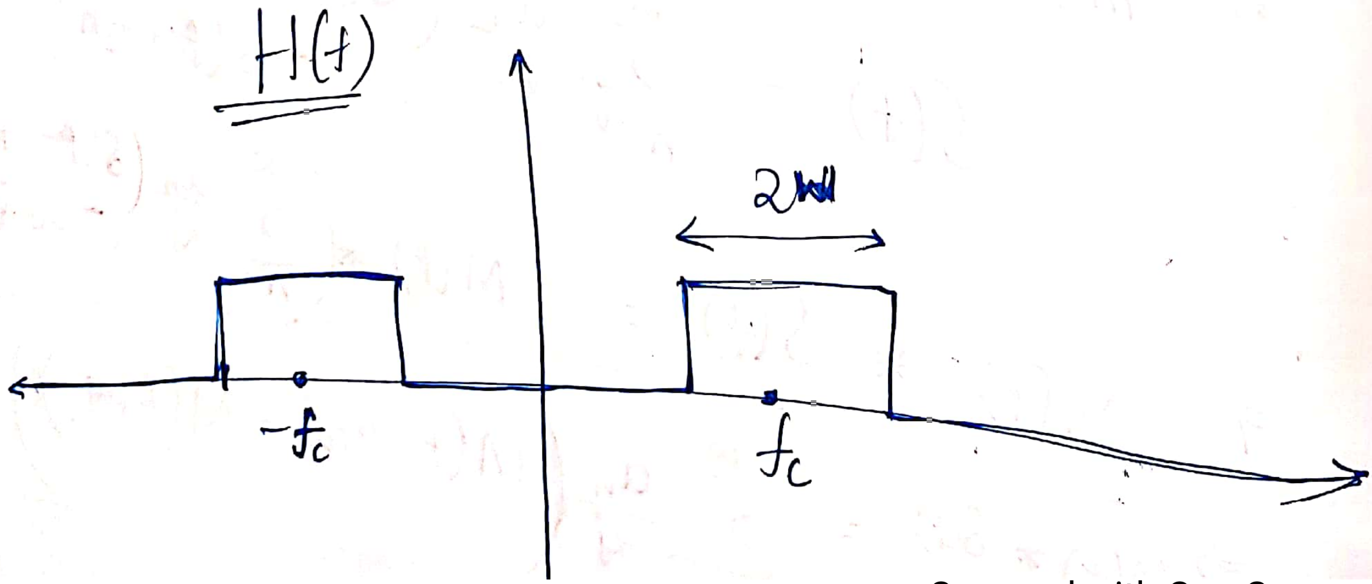
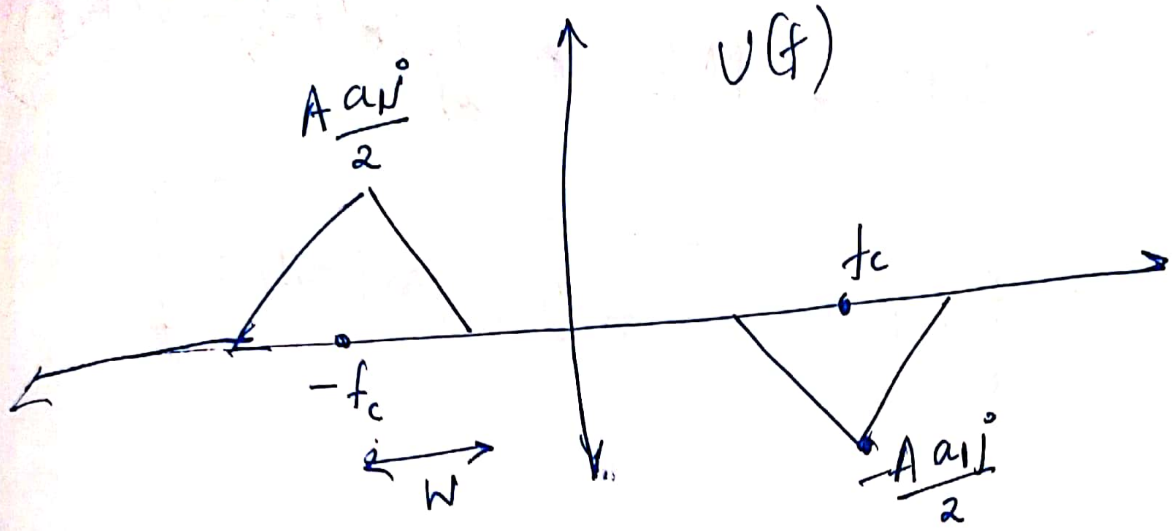
$\boxed{f_c \gg W}$







$$\therefore \quad V(f) = M(f) * \frac{a_1}{2j}\left(\delta(f-f_c) - \delta(f+f_c)\right)$$

$$a_1 = \frac{\pi}{2T_p}\int_0^{T_p} \sin(2\pi f_c t)\, s(t)\, dt$$

$$\Rightarrow \quad a_1 = \frac{1}{4}(2\pi f_c)\left(\int_0^{T_p/2} \sin(2\pi f_c t)\, dt - \int_{T_p/2}^{T_p} \sin(2\pi f_c t)\, dt\right)$$

$$\Rightarrow \quad a_1 = \frac{1}{4}\left(\left(\cos(2\pi f_c t)\right)_{\frac{T_p}{2}}^{0} + \left(\cos(2\pi f_c t)\right)_{\frac{T_p}{2}}^{T_p}\right)$$

$$\Rightarrow \quad a_1 = \frac{1}{4}\left(\cancel{\frac{(1-1)}{2}} + \cancel{\frac{(1-1)}{2}}\right)$$

$$\boxed{a_1 = 1}$$

$$\therefore \quad U(f) = M(f) * \frac{1}{2j}\left(\delta(f-f_c) - \delta(f+f_c)\right)$$

$$\Rightarrow \quad \boxed{u(t) = m(t)\sin(2\pi f_c t)}$$

(3)

$$s(t) = \sum_{n=-\infty}^{\infty} x(t - nT_p)$$

for $s(t)$ to be any periodic signal, there must exist a fourier series of the signal.

$$\Rightarrow \quad s(t) = \sum_{n=-\infty}^{\infty} C_n e^{+j2\pi f_c n t}$$

$$\Rightarrow \quad \boxed{S(f) = \sum_{n=-\infty}^{\infty} C_n \, \delta(f - f_c n)}$$

$$\Rightarrow \quad \boxed{M(f) * S(f) = \sum_{n=-\infty}^{\infty} C_n \, M(f - nf_c)}$$

If the bandpass filter has magnitude of $\frac{1}{C_1}$ and a bandwidth of $2W$, then,

$$V(f) = H(f) \left(C_1 \, M(f-f_c) \right)$$

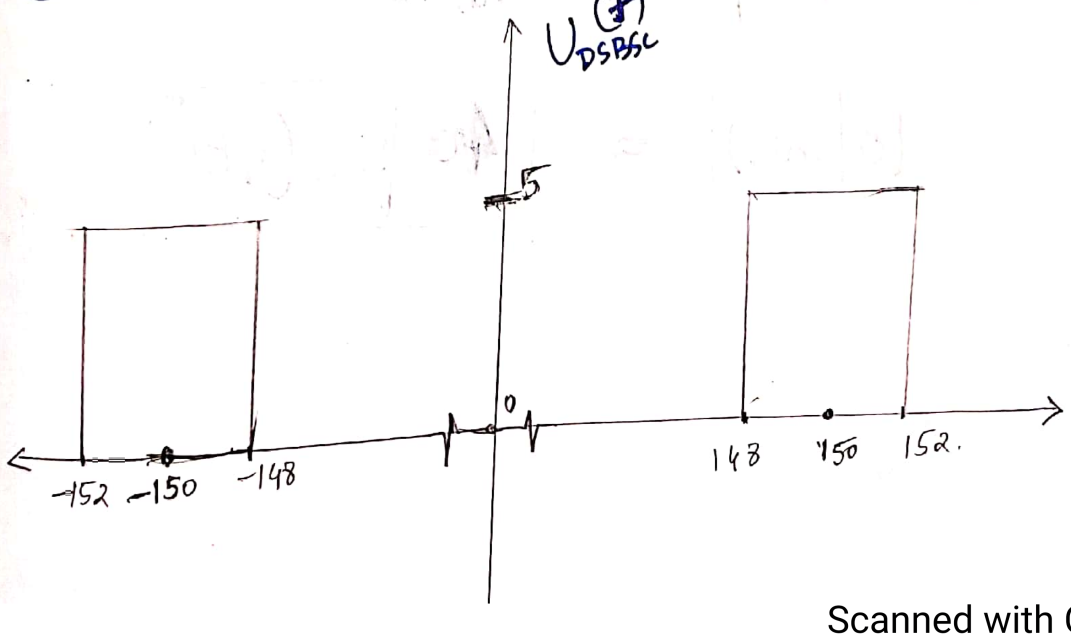
$$= M(f-f_c)$$

$$= M(f) * \delta(f-f_c)$$

$\Rightarrow$ It is possible to retrieve $\underline{m(t)\left(e^{j2\pi f_c t}\right)}$

(4) $M(f) = I_{[-2,2]}(f)$

(a) $u_{DSBSC}(t) = 10\, m(t) \cos(2\pi(150)t)$



Bandwidth $= 4Hz$ (one-sided).

Power of signal = ?

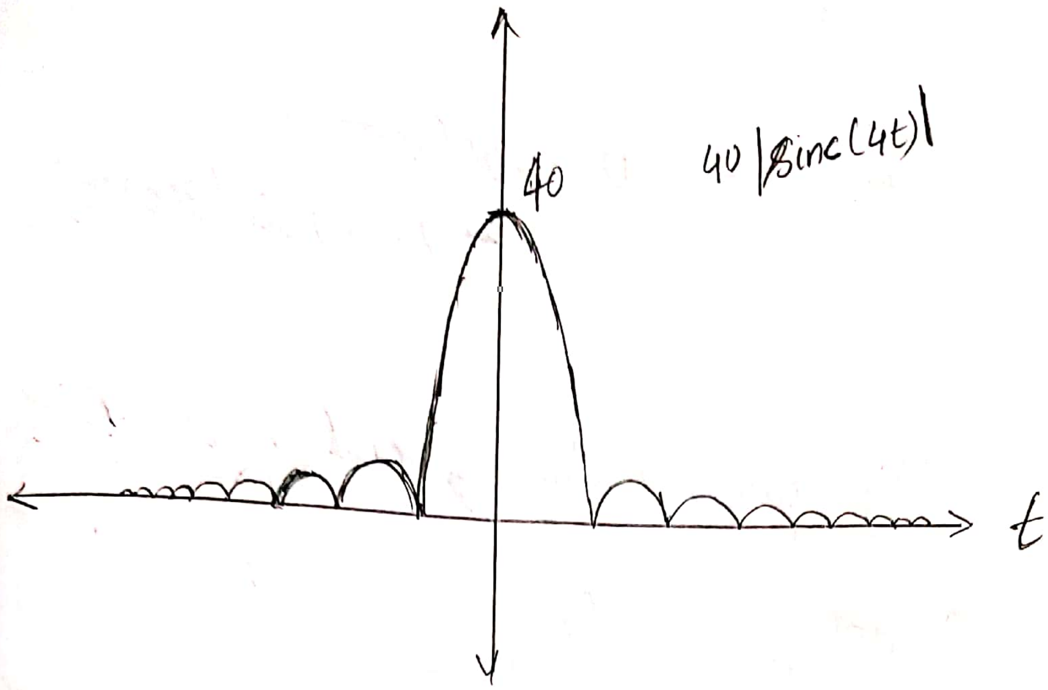
from the $V(f)$ plot it is clear that the $\int_{-\infty}^{\infty} |V(f)|^2 df$ is finite

$\Rightarrow$ $\boxed{\text{Power is zero}}$

(b) $u_{DSB-SC}(t) = 10\, m(t) \cos(300\pi t)$

$\therefore$ The envelope of $u_{DSB-SC}(t)$ will be

$$10|m(t)| = \boxed{40\, |\text{sinc}(4t)|}$$



(c) ~~For~~ $u_{AM} = |A + m(t)| \cos(300\pi t)$

$$A \geq |\min_t m(t)|$$

$$A \geq |\min_t 40\,\text{sinc}(4t)|$$

minimum at $4t = \frac{3\pi}{2}$.

$$A \geq \left| 40 \frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}} \right|$$

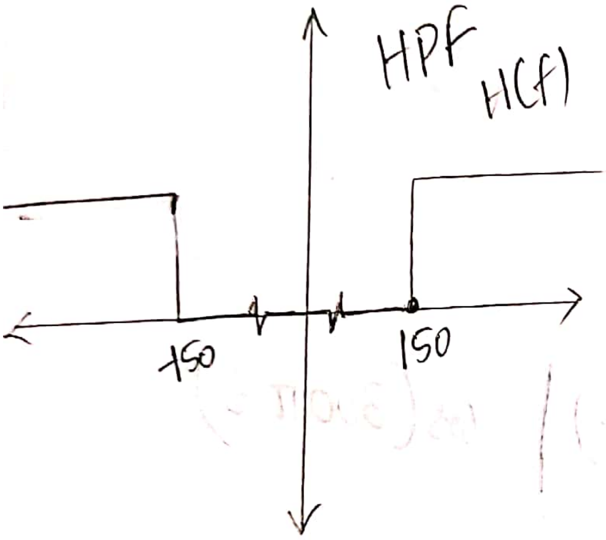
$$\Rightarrow A \geq \frac{8}{3\pi}$$

$$\Rightarrow \boxed{A \geq 0.85}$$

(d)

$$u_p(t) = u_c(t)\cos(300\pi t) - u_s(t)\sin(300\pi t)$$

High pass filtering, cutoff $\boxed{150\text{ Hz}}$



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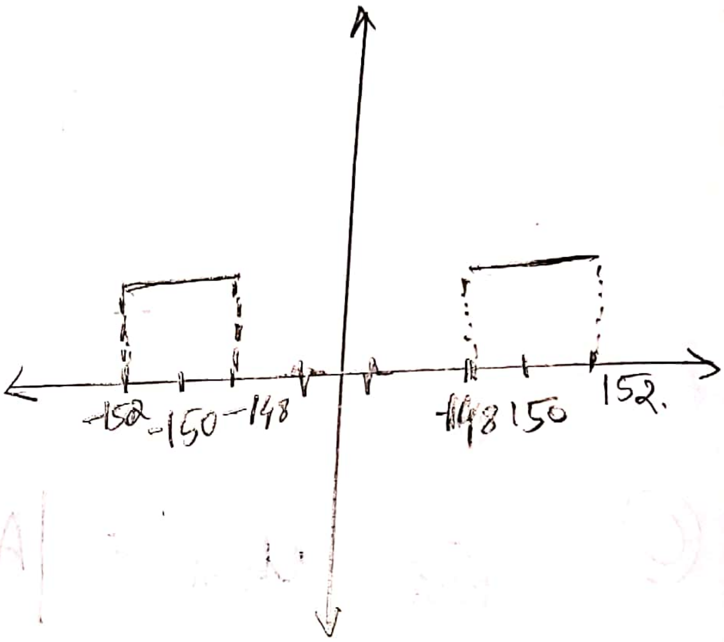

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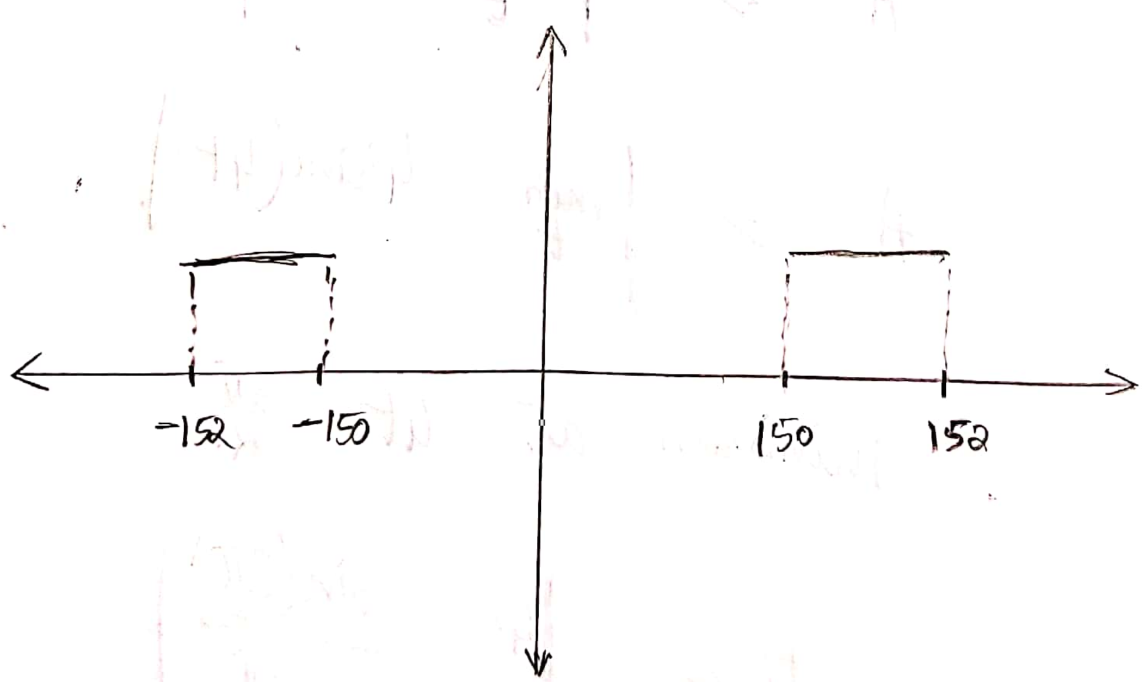

$\therefore$ output $= 5\left(I_{[-1,1]}(f-151) + I_{[-1,1]}(f+151)\right).$

$\Rightarrow \mathcal{F}^{-1}\left\{ 5\left(I_{[-1,1]}(f-151) + I_{[-1,1]}(f+151)\right)\right\}$

$\Rightarrow 20\,\mathrm{sinc}(2t)\cos(2\pi t)\cos(300\pi t)$
$- 20\,\mathrm{sinc}(2t)\sin(2\pi t)\sin(300\pi t).$

$u_c(t) = 20\,\mathrm{sinc}(2t)\cos(2\pi t)$
$u_s(t) = 20\,\mathrm{sinc}(2t)\sin(2\pi t)$

$u_p(t) = \left(20\,\mathrm{sinc}(2t)\cos(2\pi t)\right)\cos(300\pi t)$
$- \left(20\,\mathrm{sinc}(2t)\sin(2\pi t)\right)\sin(300\pi t)$

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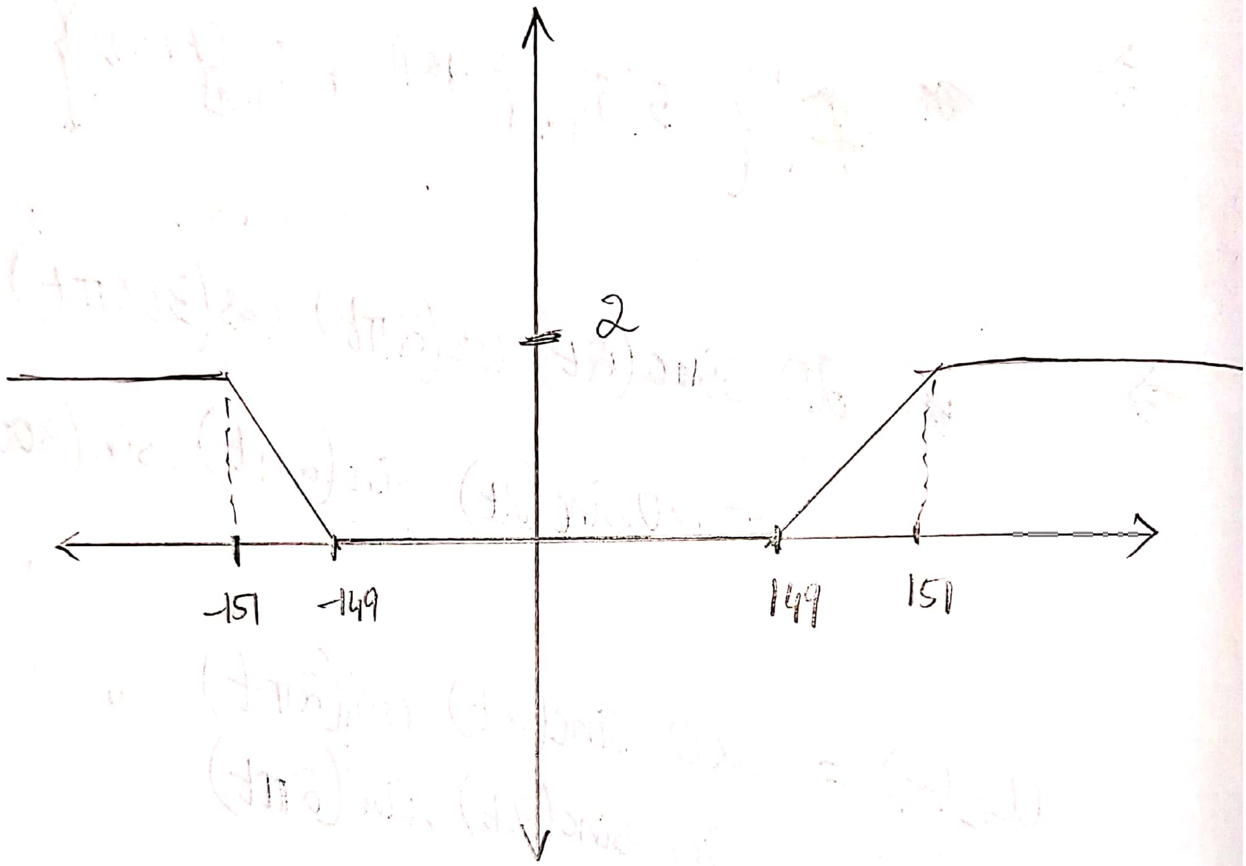
$$10\, m(t)\cos(300\pi t) \longleftrightarrow \boxed{5M(f-150) + 5M(f+150)}$$



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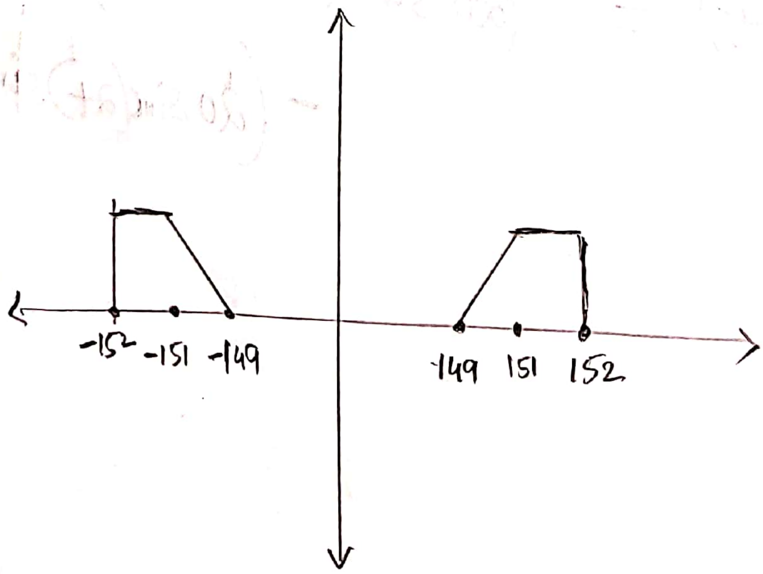

$$u_p(t) = u_c \cos(300\pi t) - u_s(t) \sin(300\pi t)$$

$$u_s(t) = \frac{5}{\pi t}\left[\operatorname{sinc}(4t) - \cos(4\pi t)\right].$$

$$u_c(t) = 20 \operatorname{sinc}(4t)$$