10/2/21

COMMUNICATION THEORY

ASSIGINMENT-3

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ECD UG2

(1) $\theta(t) = 570 \sin(10\pi t)^{\circ}$ (t in ms)

$T_0 = 0.2$ ms

$\Rightarrow f_0 = \dfrac{1}{\frac{1}{5} m}$ Hz $= \boxed{5 kHz}$

$\Rightarrow \theta(t) = 570 \sin(10\pi t)^{\circ}$

$= \dfrac{360 \times \cancel{570}^{19}}{\cancel{360}\,12} \sin(10\pi t)^{\circ}$

$= 2\pi\left(\dfrac{19}{12}\right) \sin(10\pi t)$

$= \boxed{\dfrac{19\pi}{6} \sin(10\pi t)}$

$$\Rightarrow \theta(t) = \underset{\uparrow\ \theta(0)}{0} + 2\pi\left(\frac{19}{12}\right)\sin(10\pi t)$$

$$\theta(0) + (2\pi)\, k_f' \int_0^t \underbrace{-\cos(10\pi\tau)}_{m(\tau)}(10\pi)\, d\tau$$

$$\Rightarrow \theta(t) = \theta(0) + 2\pi\left(\frac{19}{\cancel{12}\,6}\,\cancel{10}^{5}\pi\right) \cdot \int_0^t \underbrace{\left(-\cos(10\pi\tau)\right)}_{m(\tau)} d\tau$$

$$= \theta(\cancel{0})^{\,0} + 2\pi\left(\frac{95\pi}{6}\right)\int_0^t \left(-\cos(10\pi\tau)\right) d\tau$$

(a) $$\beta = \frac{\Delta f_{max}}{B}$$

$B = 5$ kHz $\qquad \Delta f_{max} = k_f \max_t |m(t)|$

$$m(t) = -\cos(10\pi t)$$

$$|m(t)| = |\cos(10\pi t)|$$

$$\max_t |m(t)| = \boxed{1}$$

$$k_f = \boxed{\frac{95\pi}{6}}$$

$$k_f \max_t |m(t)| = \frac{95\pi}{6}(1) = \boxed{\frac{95\pi}{6}}$$

$$\Rightarrow \quad \beta = \frac{\frac{\cancel{98}^{19}\pi}{6}\ \cancel{kHz}}{\cancel{5}\ \cancel{kHz}} = \boxed{\frac{19\pi}{6}}$$

(b) Message bandwidth $= B = \boxed{5\,kHz}$

(c)

$$B_{FM} \approx 2B + 2\Delta f_{max}$$ (from Carson's rule)

$$= 10 + \frac{38\pi}{6} \ kHz$$

$$\approx \boxed{29.89}\ kHz$$

Scanned with CamScanner

② $m(t)$ to FM.

$$M(f) = \begin{cases} 2\pi j f & |f| < 1 \\ 0 & \text{else.} \end{cases}$$

$$m(t) = \int_{-1}^{1} (2\pi j f)\, e^{2\pi j f t}\, df$$

$$\Rightarrow m(\tau) = \int_{-1}^{1} (2\pi j f)\, e^{2\pi j f \tau}\, df$$

$$\Rightarrow \int_0^t m(\tau)\, d\tau = \int_0^t \left(\int_{-1}^{1} 2\pi j f\, e^{2\pi j f \tau}\, df \right) d\tau$$

$$\Rightarrow \int_0^t m(\tau)\, d\tau = \int_{-1}^{1} (2\pi j f)\, df \left(\int_0^t e^{2\pi j f \tau}\, d\tau \right)$$

$$\Rightarrow \int_0^t m(\tau)\, d\tau = \int_{-1}^{1} df \left(\int_0^t d\left(e^{2\pi j f \tau}\right) \right)$$

$$\Rightarrow \int_0^t m(\tau)\, d\tau = \int_{-1}^{1} df \left(e^{2\pi j f t} - 1 \right)$$

$$\Rightarrow 2\pi \cdot 1 \cdot \int_0^t m(\tau)\, d\tau = \int_{-1}^{1} \left(e^{2\pi j f t} - 1 \right) df =$$

$$\Rightarrow \quad 2\pi \int_0^t m(\tau)\, d\tau = \int_{-1}^{1} e^{2\pi j f t}\, df - \int_{-1}^{1} df$$

$$\Rightarrow \quad 2\pi \int_0^t m(\tau)\, d\tau = \frac{e^{2\pi j t} - e^{-2\pi j t}}{2\pi j t} - 2$$

$$\Rightarrow \quad 2\pi \int_0^t m(\tau)\, d\tau = \frac{\cancel{2j} \sin(2\pi t)}{\cancel{2j}\, \pi t} - 2$$

$$\Rightarrow \quad 2\pi \int_0^t m(\tau)\, d\tau = \cancel{\operatorname{sinc}(2t)} \; \boxed{2 \operatorname{sinc}(2t) - 2}$$

$$\Rightarrow \quad \phi(t) = \phi(0) + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

$$\Rightarrow \quad \boxed{\phi(t) = 4\pi \operatorname{sinc}(2t) - 4\pi}$$

(b) $\Delta f\,(t = 1/4)$

$$\frac{1}{2\pi}\frac{d\phi(t)}{dt}\Big|_{t=1/4}$$

$$\Rightarrow \frac{1}{2\pi}\left(\frac{2\pi t\cos(2\pi t) - \sin(2\pi t)}{\pi t^2}\right)\Big|_{t=1/4}$$

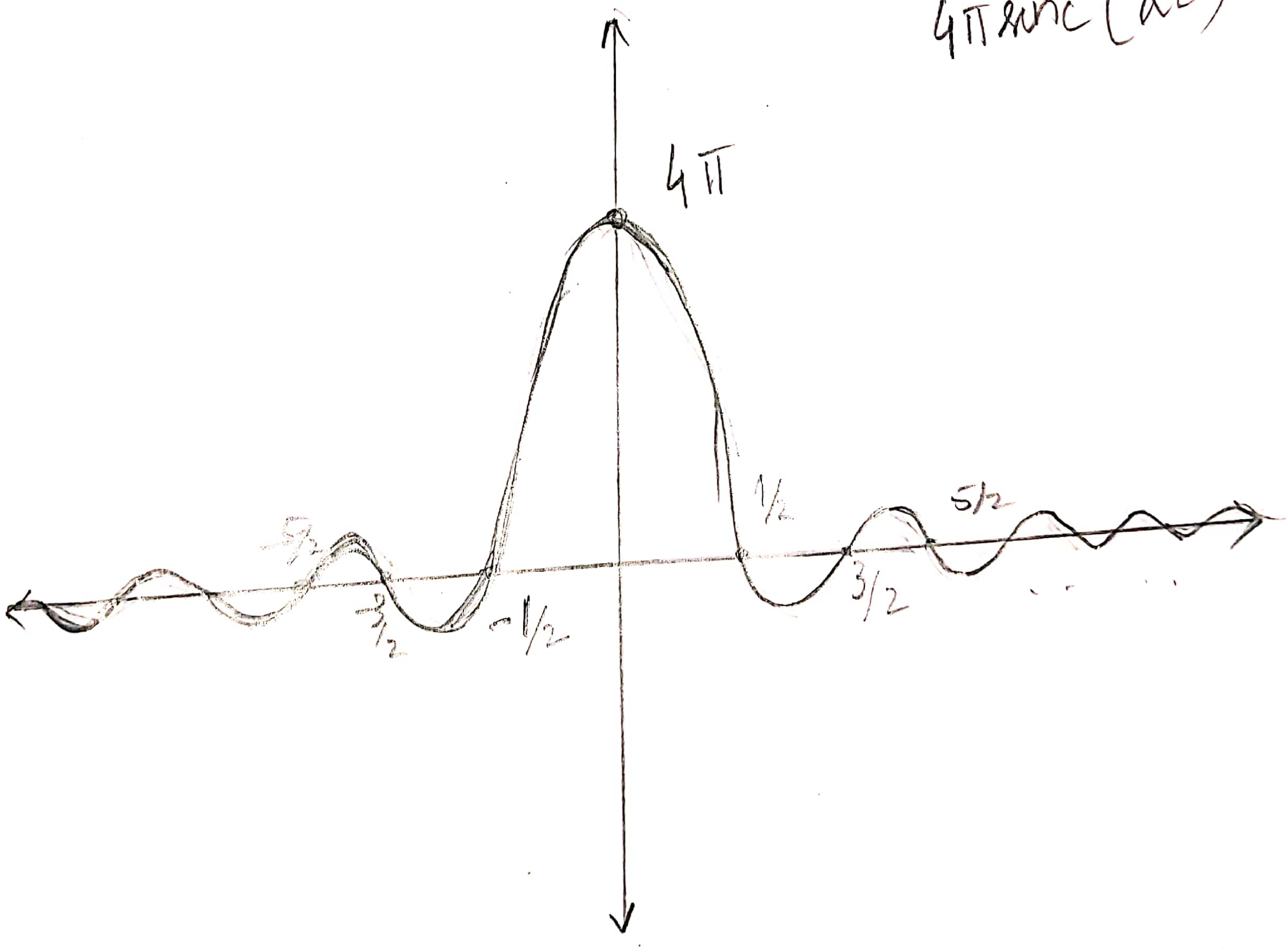
$$\Rightarrow \frac{1}{\cancel{2}\pi}\left(\frac{\frac{\pi}{2}\cancel{\cos\left(\frac{\pi}{2}\right)} - \sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{\cancel{16}_{2}}}\right)$$

$$\Rightarrow \boxed{\frac{-2}{\pi^2}}$$

$$\boxed{\left|\Delta f\,(t = 1/4)\right| = \frac{2}{\pi^2}}$$

2 a (Contd).

$4\pi \,\text{sinc}(2t) = \phi(t)$



(c) $B_{FM} = 2B + 2\Delta f_{max}$

$$= \boxed{2f_c + \frac{4}{\pi^2}}$$

(3)

$$p(t) = I_{[-1/2, 1/2]}(t)$$

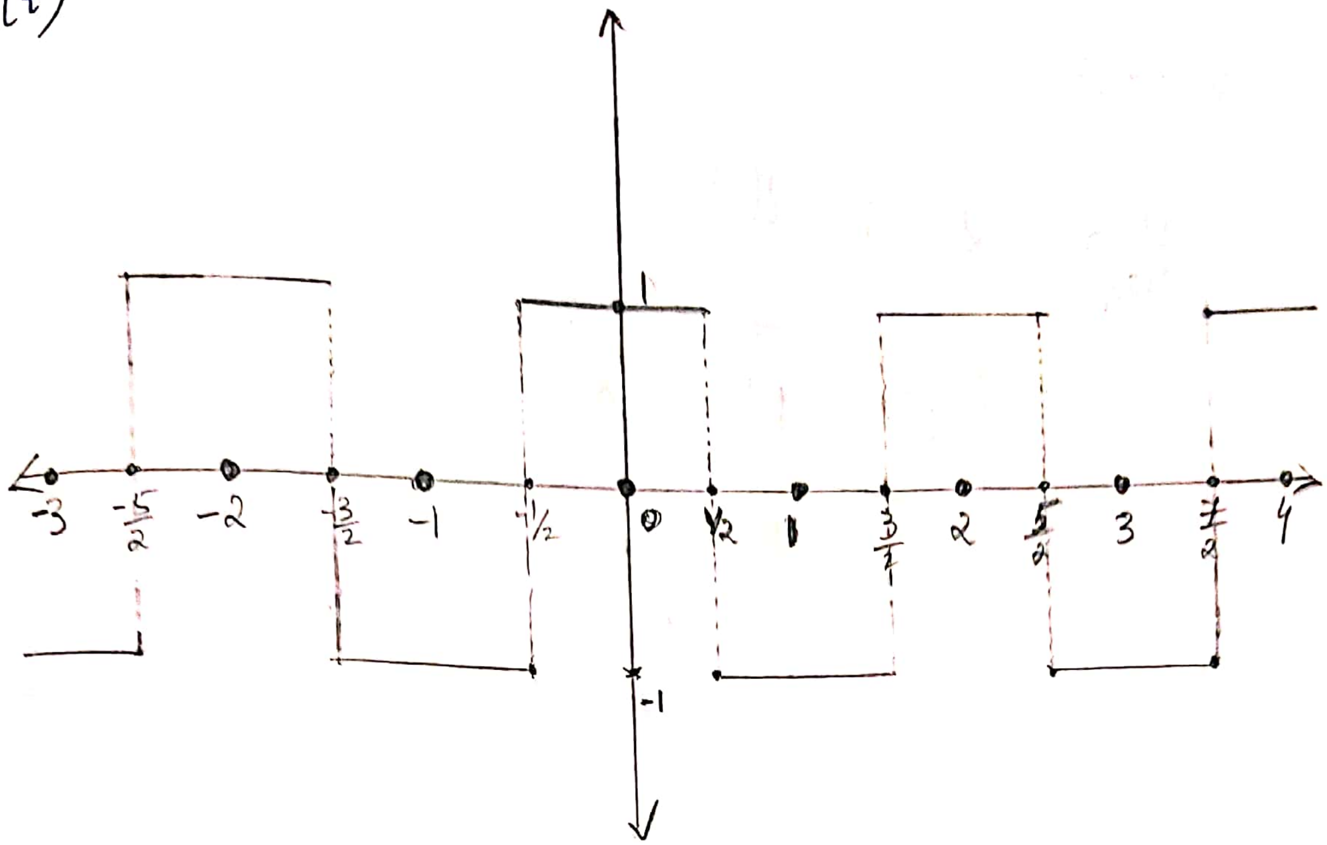
$$m(t) = \sum_{n=-\infty}^{\infty} (-1)^n \, p(t-n)$$

$$u(t) = 20 \cos\left(2\pi f_c t + \phi(t)\right)$$

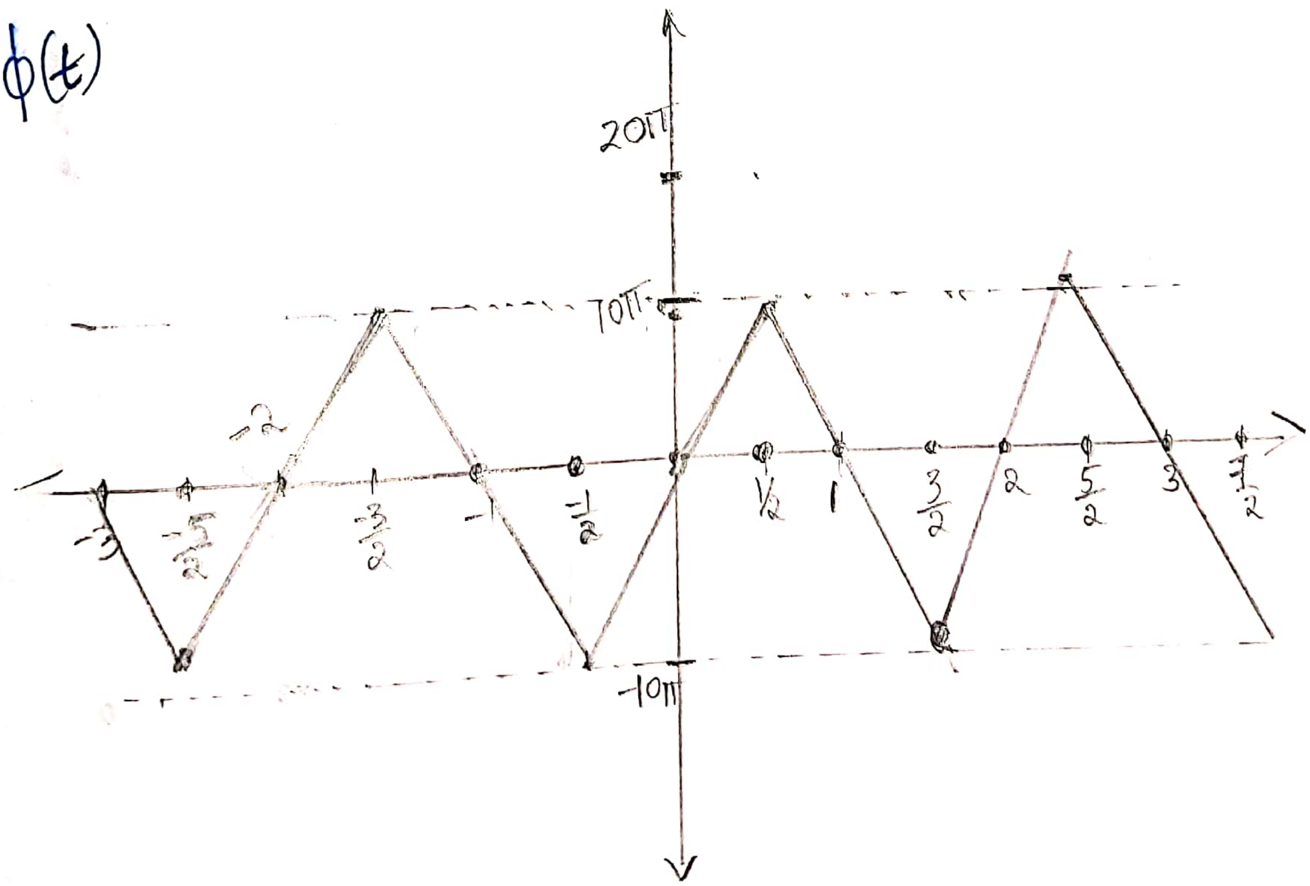
$$\phi(t) = 20\pi \int_{-\infty}^{t} m(\tau)\, d\tau + a$$

$\phi(0) = 0$, a is chosen as such.

(a) $m(t)$


1
-1
-3
-5/2
-2
-3/2
-1
-1/2
0
1/2
1
3/2
2
5/2
3
7/2
4

$\phi(t)$


20π
10π
-10π
-2
-3
-5/2
-3/2
-1
-1/2
1/2
1
3/2
2
5/2
3
7/2

(b) $W \simeq 2$

$$B_{FM} = 2f_c + 2(2)$$

$$= \boxed{2f_c + 4}$$

(c) we know that $f_m = \boxed{1/2}$ (Period $T_m = 2$)

$\therefore$ The spectrum of passband signals has components $f_c + nf_m$

$\therefore$ We get non zero powers at $f_c + nf_m$ at $\alpha = 0.8, 0.1$ but for $\alpha = 0.75$ we get zero power.