Optimal test-kit-based intervention strategy of epidemic spreading in heterogeneous complex networks

Subrata Ghosh, Abhishek Senapati, Joydev Chattopadhyay, Chittaranjan Hens, and Dibakar Ghosh

Team 9:
Hrishi, Viswanadh, Ashutosh

Recap

SUTRA

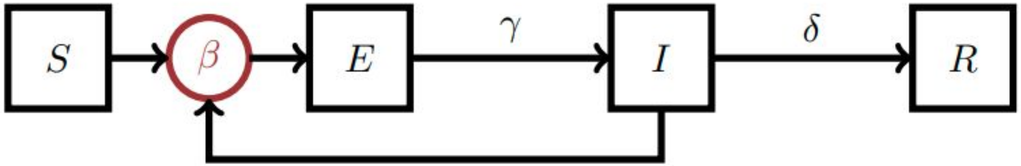
Susceptible, Undetected, Tested (positive), and Removed Approach

- This paper gives an approach to model pandemics like Covid-19.
- Authors show that this method is more appropriate than the existing pandemic models like SIR, SEIR and SAIR and apply this model for plotting active COVID-19 cases for different countries.
- The parameters for the model are calculated on the basis of current data and the model is used to predict future cases approximately by making certain assumptions.

SEIR Model

The population was divided into four compartments, as below:

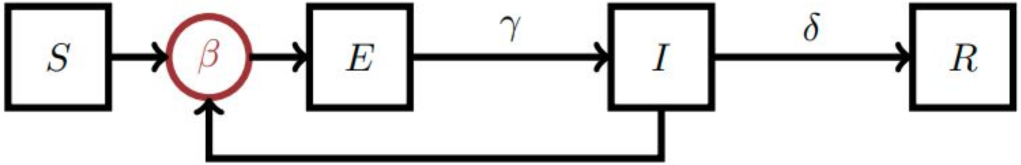
- S -> Susceptible
- E -> Exposed
- I -> Infected
- R -> Removed / Recovered



SEIR Model

$$\begin{aligned}
\frac{dS}{dt} &= \mu - \beta(t)SI - \mu S \\
\frac{dE}{dt} &= \beta(t)SI - (\mu + \alpha)E \\
\frac{dI}{dt} &= \alpha E - (\mu + \gamma)I.
\end{aligned}$$

$$S + E + I + R = 1.$$



Modified SEIR Model

$$\frac{dS}{dt} = -\beta \frac{SI}{N}, \qquad \frac{dH}{dt} = \alpha(K)I - \gamma H,$$

$$\frac{dE}{dt} = \beta \frac{SI}{N} - \sigma E, \qquad \frac{dR}{dt} = \gamma H,$$

$$\frac{dI}{dt} = \sigma E - \alpha(K)I, \qquad \frac{dK}{dt} = \xi I - \chi K.$$

Peak of Infection I_{max}

The peak of the infection will vary based on whether or not test kits are introduced:

Case 1: At first, we consider the case where test kits are not introduced; i.e., $\alpha_1 = 0$. We can analytically calculate the $I_{\max}$ as a function of $\mathcal{R}_0$

$$\boxed{I_{\max} = \frac{\sigma N}{\sigma + \alpha_0}\left(1 - \frac{1 + \ln \mathcal{R}_0}{\mathcal{R}_0}\right).}$$

Peak of Infection I_{max}

The peak of the infection will vary based on whether or not test kits are introduced:

Case 2: $\alpha_1 \neq 0$, i.e., in the presence of a test kit. The $I_{\max}$ can be captured with a transcendental equation

$$\boxed{I_{\max} = \frac{\sigma N}{\sigma + \alpha_0 + \alpha_1 a I_{\max}} \left(1 - \frac{1 + \ln \frac{\mathcal{R}_0}{1+\frac{\alpha_1}{\alpha_0} a I_{\max}}}{\frac{\mathcal{R}_0}{1+\frac{\alpha_1}{\alpha_0} a I_{\max}}} \right).}$$

Final Outbreak Size Z_{FOS}

The proportion of the population infected at the end of outbreak is also depends on whether test kits are introduced:

Case 1: $\alpha_1 = 0$, i.e., in the absence of any test kit. Z_{FOS} is given by

$$\boxed{Z_{\mathrm{FOS}} = 1 - e^{-\mathcal{R}_0\left[Z_{\mathrm{FOS}} + \frac{(E_0 + I_0)}{S_0}\right]}.}$$

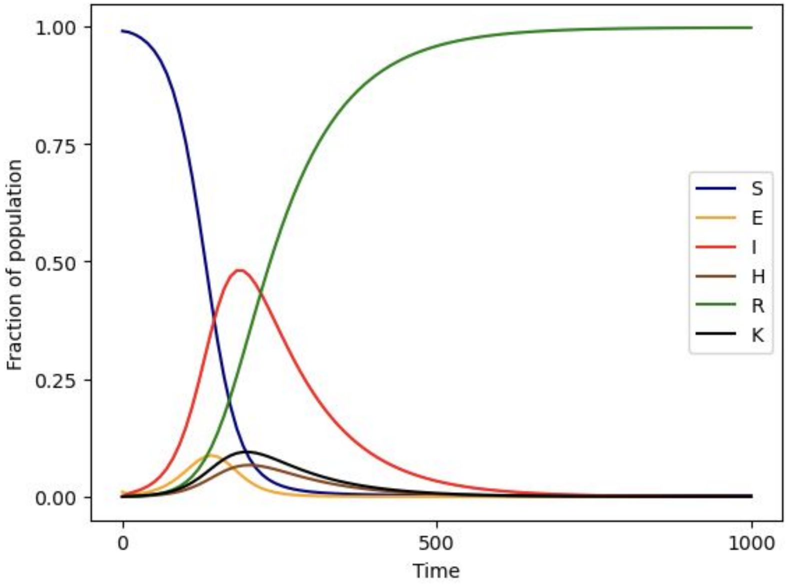
Final Outbreak Size Z_{FOS}

The proportion of the population infected at the end of outbreak is also depends on whether test kits are introduced:

Case 2 $\alpha_1 \neq 0$, i.e., in the presence of a test kit. obtain the following transcendental equation of Z_{FOS}

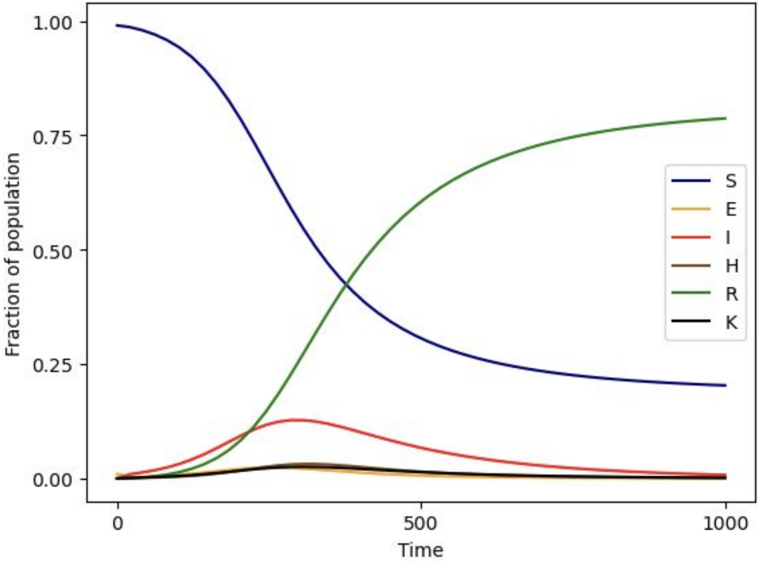
$$\boxed{Z_{FOS} \geqslant 1 - e^{-\mathcal{R}_0 \frac{\alpha_0}{(\alpha_0 + \alpha_1 \frac{N\xi}{\chi})}\left[Z_{FOS} + \frac{(E_0 + I_0)}{N} + \frac{\alpha_1}{\chi}(K_\infty - K_0)\right]}.}$$

Without Test-Kit Intervention



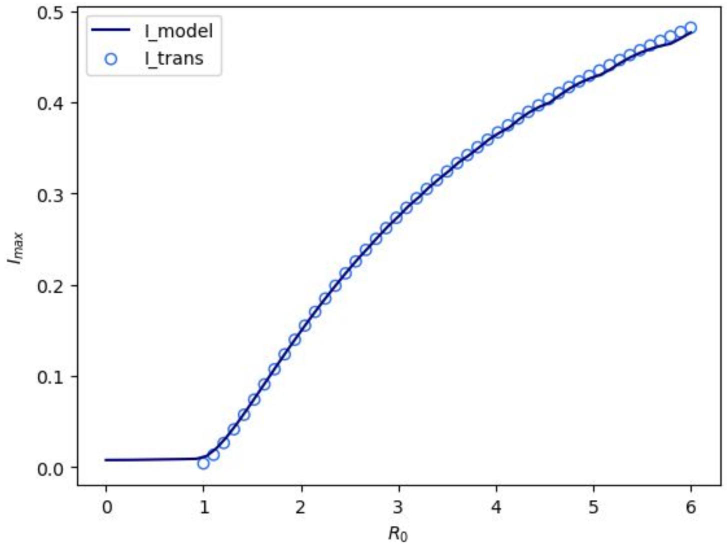
`R0 = 6, test kits = 0`

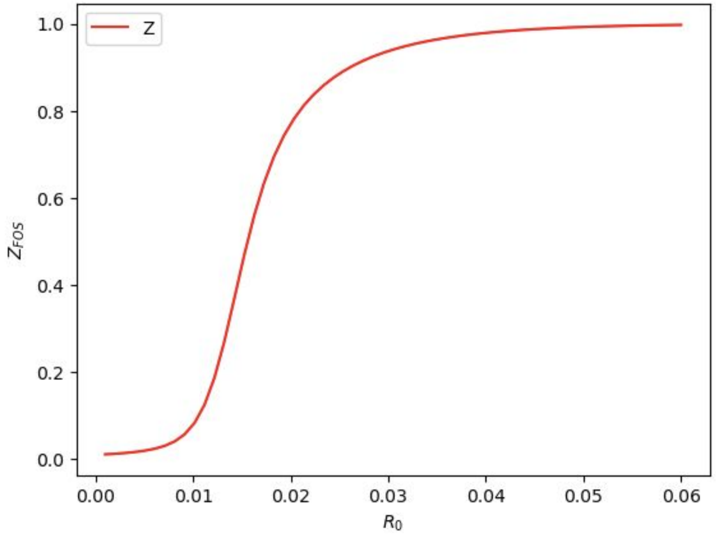
Intervention of Test-Kits



R_0 = 3 with intervention of test kits (a1 = 0.3)

Plot of I_{max} and Z_{FOS}





Intervention Strategy In Heterogeneous Networks

- Consider a heterogeneous network of M patches, i.e., M number of communities/nodes/cluster
- Each cluster is considered as an individual network for modelling. However, interactions with other clusters are also to be considered in the dynamics.
- We are also assuming that a small fraction of certain communities is infected.
- Considering the dispersion through diffusion of susceptible (S_n), exposed (E_n), infected (I_n), and recovered (R_n) individuals, the coupled network equations are considered as follows:

Dynamic Equation for Graphs

$$\frac{dS_n}{dt} = -\beta_n \left(\frac{S_n I_n}{N_n} \right) + \frac{\epsilon}{d_n} \sum_{m=1}^{M} A_{nm}(S_m - S_n),$$

$$\frac{dE_n}{dt} = \beta_n \left(\frac{S_n I_n}{N_n} \right) - \sigma E_n + \frac{\epsilon}{d_n} \sum_{m=1}^{M} A_{nm}(E_m - E_n),$$

$$\frac{dI_n}{dt} = \sigma E_n - (\alpha_0 + g_n(K)) I_n + \frac{\epsilon}{d_n} \sum_{m=1}^{M} A_{nm}(I_m - I_n),$$

$$\frac{dH_n}{dt} = (\alpha_0 + g_n(K)) I_n - \gamma H_n,$$

$$\frac{dR_n}{dt} = \gamma H_n + \frac{\epsilon}{d_n} \sum_{m=1}^{M} A_{nm}(R_m - R_n),$$

$$\frac{dK}{dt} = \xi \sum_{n=1}^{M} I_n - \chi K.$$

Strategies Test-Kits Distribution

Distribution of Test-kits is done with the following:

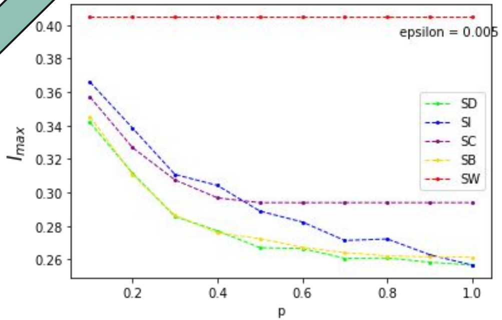
- degree-based strategy (SD)
- randomly selected and identically distributed test-kit strategy (SI)
- Betweenness centrality (SB) {Nodes that lie on shortest paths between other nodes}
- local clustering coefficient (SC) {likelihood that the neighbours of *n* are also connected}

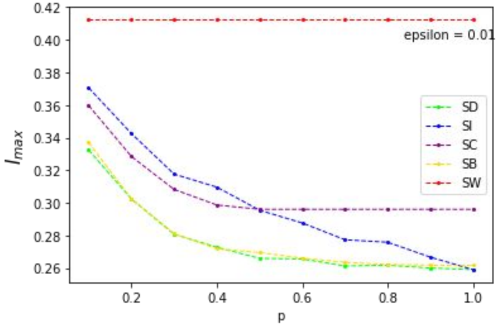
Efficiency Parameter

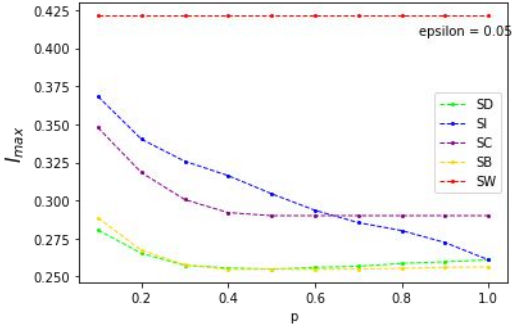
Relative Reduction (RR) of the peak of infection is defined as:

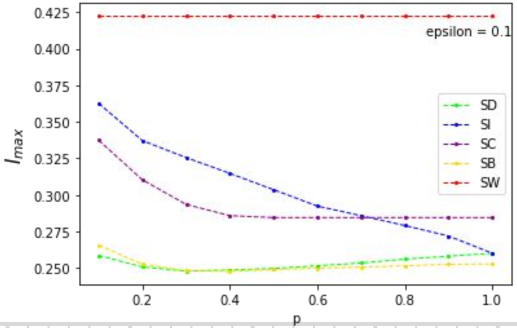
$$RR_{SI/SD} = \frac{\mathcal{I}_{\mathrm{SW}} - \mathcal{I}_{\mathrm{SI/SD}}}{\mathcal{I}_{\mathrm{SW}}} \times 100\%.$$

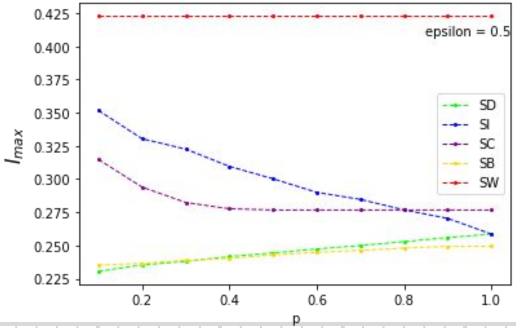
Plot of I_{max} For Erdős–Rényi Networks



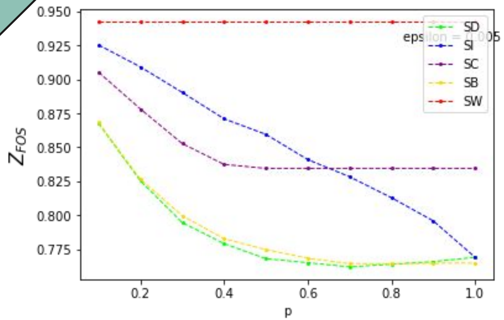


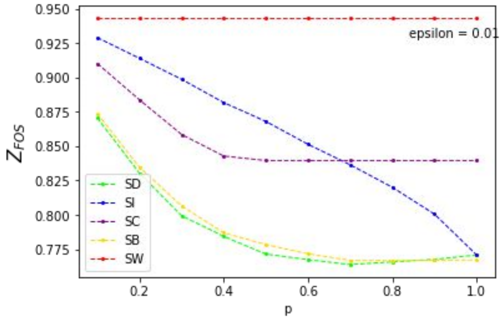


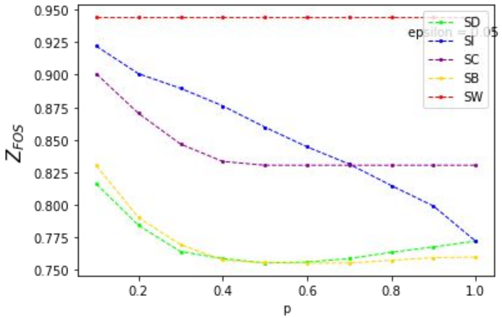


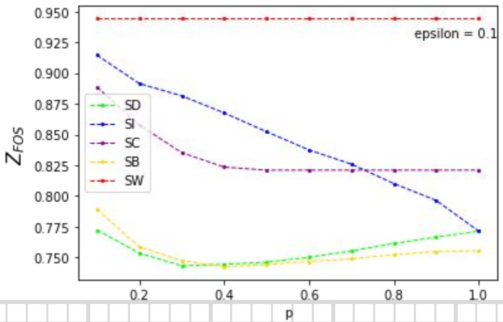


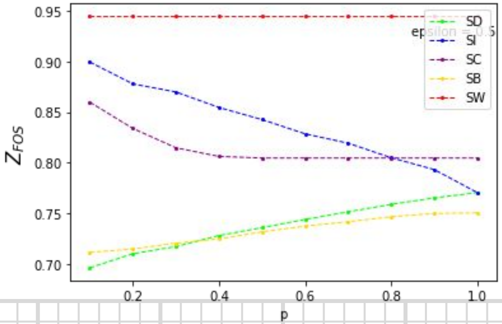
Plot of Z_{FOS} For Erdős–Rényi Networks



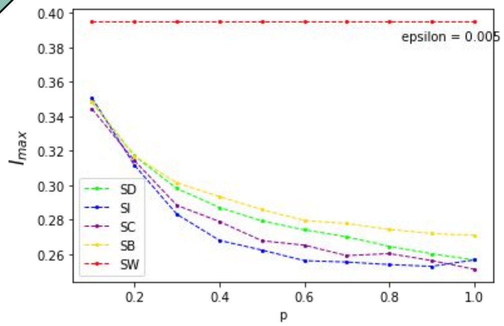


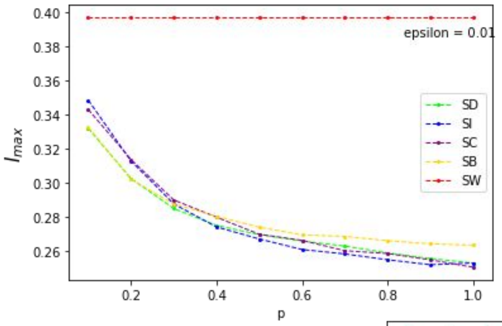


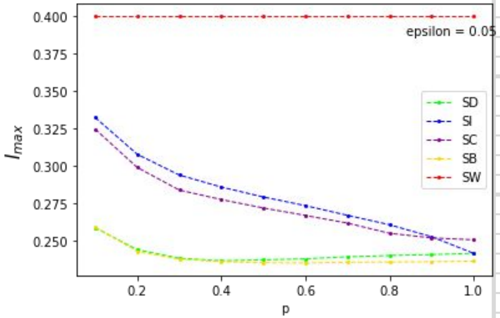


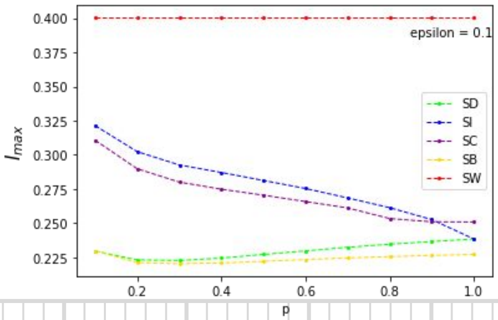


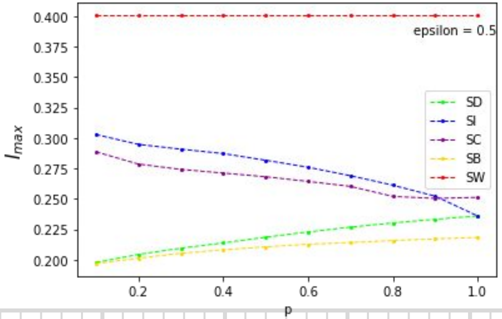
Plot of I_{max} For Scale-Free Networks



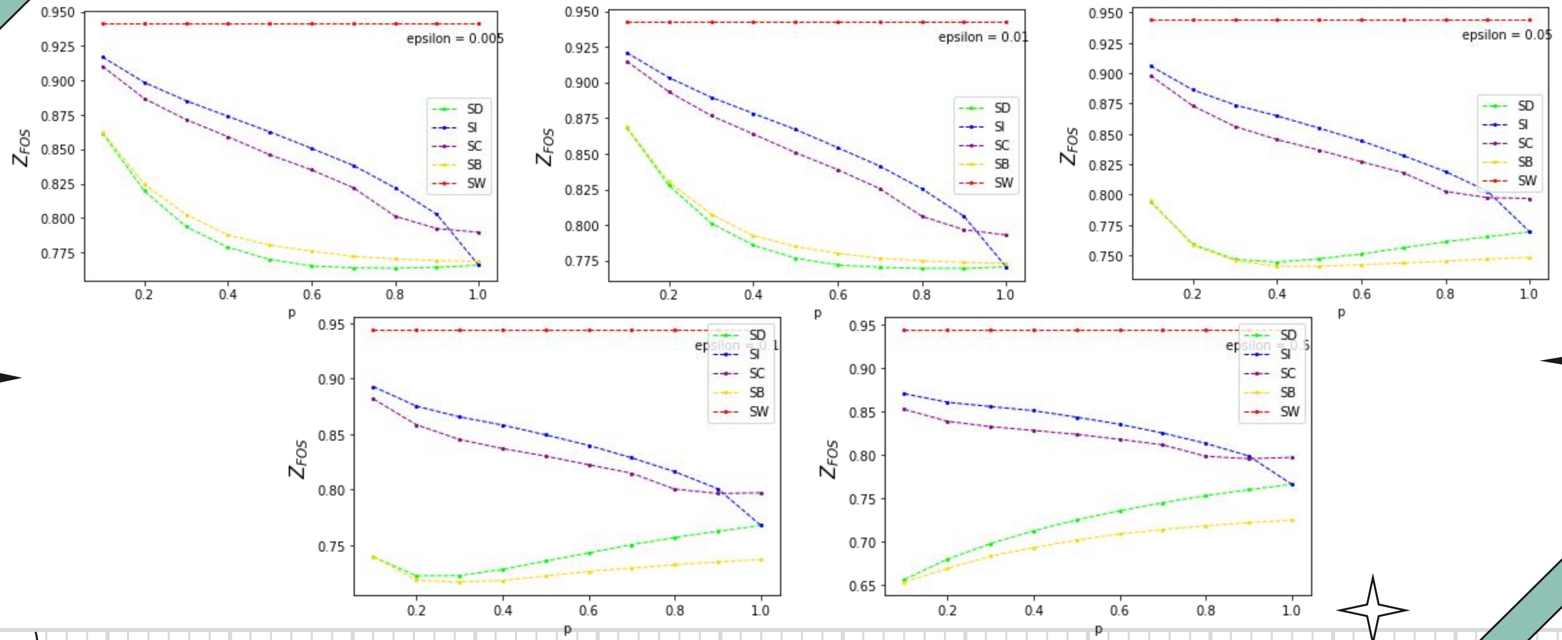








Plot of Z_{FOS} For Scale-Free Networks



Thank You

Team 9

Hrishi, Viswanadh, Ashutosh