Eigenportfolio:
Portfolio Optimization Using Principal Component Analysis

G-04

Ashutosh Anand (202318035)
Kunal Anand (202318035)
Anjali Singh (202318050)

Problem Statement

In finance, **traditional portfolio** construction methods often result in **concentrated and non-diversified portfolios** due to their sensitivity to estimation errors in expected returns and risks that may fail under market stress.

To mitigate these issues, the use of **eigenportfolios** is a valuable alternative. Eigenportfolios, derived from the principal eigenvectors of the covariance matrix of asset returns, are **capable of capturing significant market movements** and can **enhance portfolio diversification**.

Furthermore, by leveraging Principal Component Analysis (PCA), eigenportfolios **can help in improving the Sharpe Ratio**, which measures risk-adjusted returns.

Important Terminology

- **Asset:** A resource owned or controlled by an individual or business, expected to provide future economic benefits
- **Return:** The profit or loss derived from an investment, typically expressed as a percentage of the investment's initial cost.
- **Risk:** The uncertainty or variability of returns associated with an investment
- **Sharpe Ratio:** Measure of risk-adjusted return evaluating an investment's performance relative to its risk level
- **Portfolio:** Collection of investments like stocks, bonds, and other assets held by an individual or institution for optimal investment
- **Eigenportfolio:** An eigenportfolio is a specialized investment portfolio built using Principal Component Analysis (PCA)

Mathematical Formulation

- Return (of an asset)

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

- Normalized Return (of an asset)

$$\text{Normalized Return} = \frac{R_t - \mu}{\sigma}$$

- Risk (of an asset)

$$\sigma_p = \text{Standard Deviation of returns} \times \sqrt{\text{periods per year}}$$

Mathematical Formulation

- Covariance Matrix (of normalized asset return)

$$\mathrm{Cov}(X, Y) = \frac{\sum((X_i - \overline{X})(Y_i - \overline{Y}))}{N - 1}$$

$$\Sigma_{ij} = \mathrm{Cov}(R_i, R_j)$$

- Eigen Decomposition (of covariance matrix)

$$\Sigma v = \lambda v$$

Mathematical Formulation

- The weight for eigenportfolio

$$w_i = \frac{v_{ki}}{\sum_j |v_{kj}|}$$

- Sharpe Ratio (of an portfolio/investment)

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

Principal Component Analysis

- statistical technique used for dimensionality reduction while preserving as much information as possible

- Identifies pattern in the data by transforming variables into a new set of uncorrelated & orthogonal principal components

- Aim to retain maximum variance through ordering components by the amount of variance they explain

- Steps: Standardization - Covariance matrix - Eigen Decomposition - Sorting eigenvalues and eigenvectors - transformation of original matrix

- Why PCA: It has the ability to capture underlying structure and reduce noise effectively while preserving data information

Solution

- **Daily Asset Returns**: Compute the daily returns for each asset

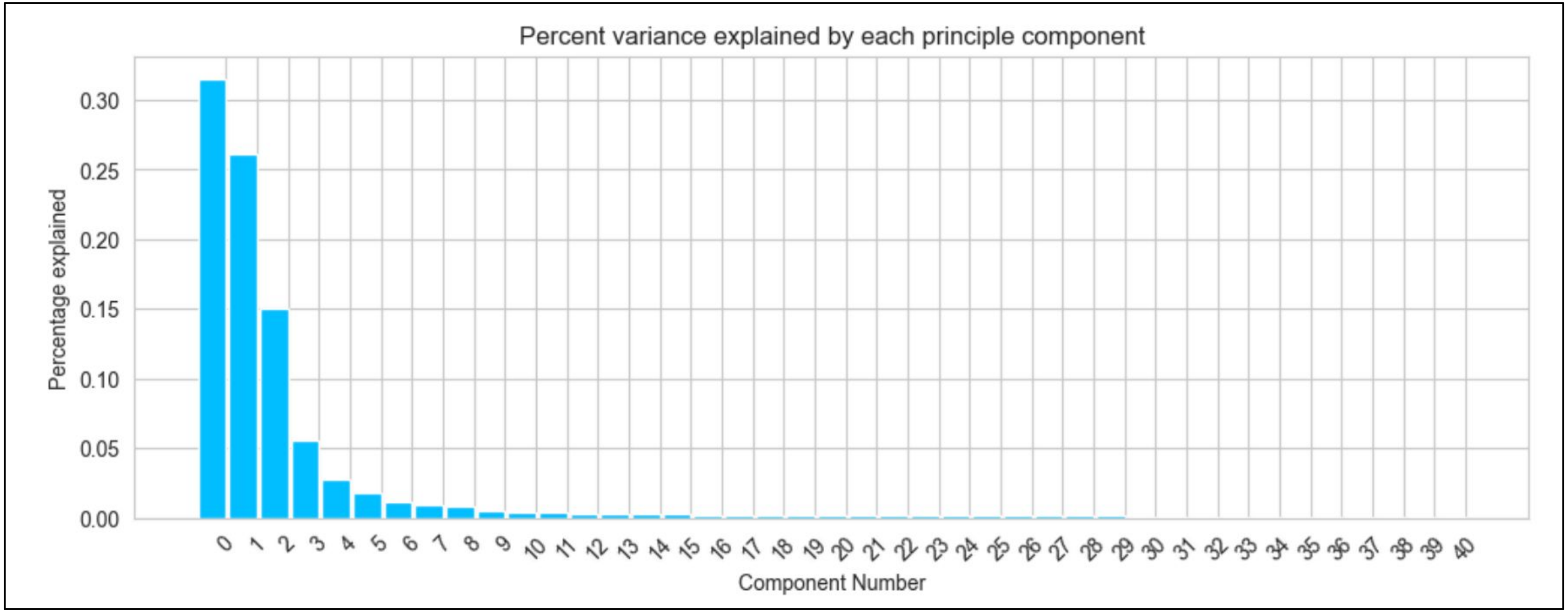
- **Normalize Returns:** Normalize the returns by subtracting the mean and dividing by the standard deviation for each asset to ensure equal contribution to the PCA.

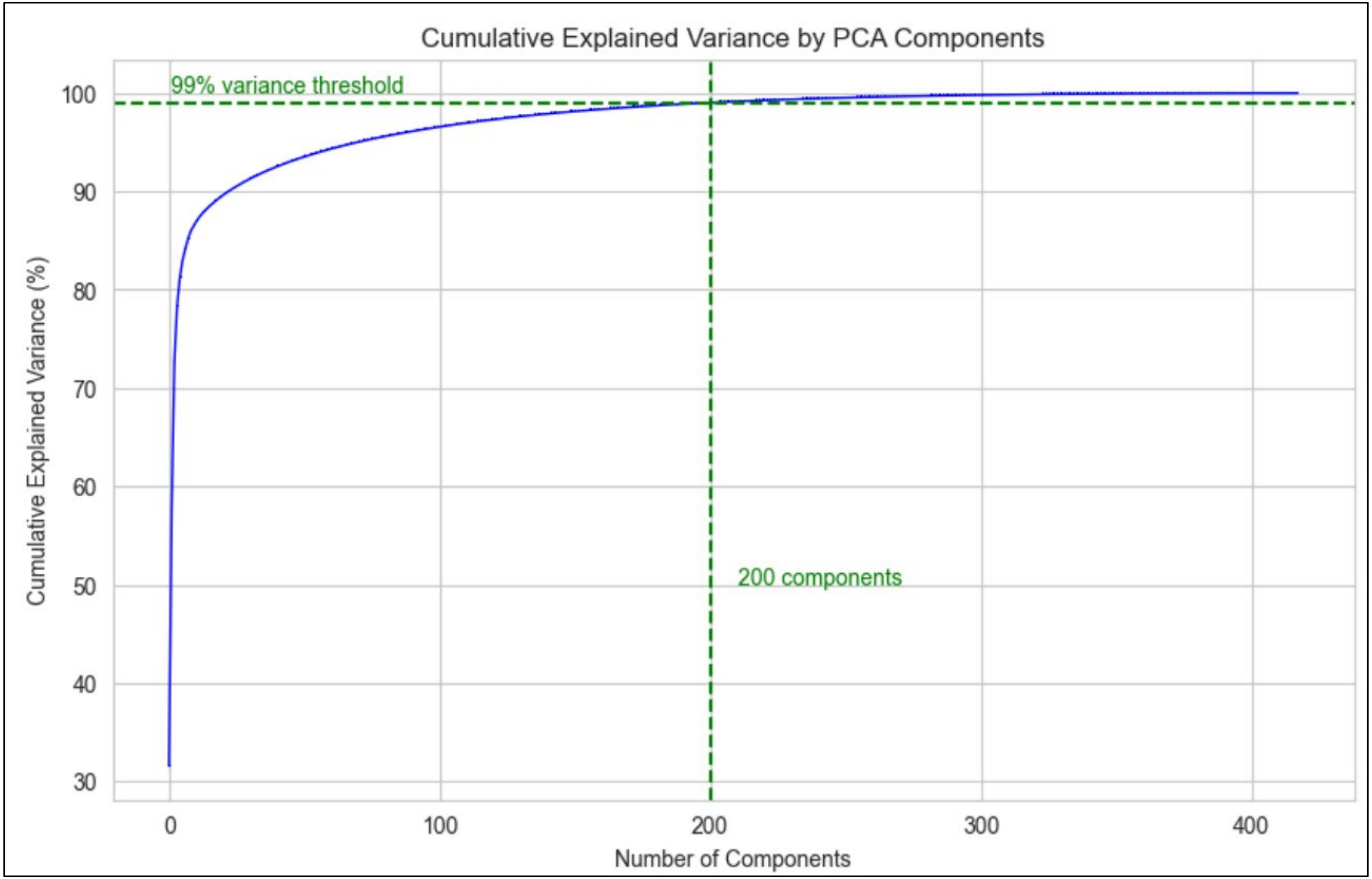
	A	AA	AAPL	ABC	ABT	ADBE	ADI	ADM	ADP	ADSK
Date										
2000-01-28	-0.190054	-0.513710	-2.714709	-0.049779	2.182933	-2.684131	-0.212461	-0.766996	-1.540731	-1.803947
2000-01-31	-0.898232	0.096888	0.688156	-1.757230	1.355644	-1.438899	-0.720771	-0.279098	-1.891884	-0.391433
2000-02-01	2.319164	1.264327	-1.227995	1.539597	0.588289	0.451774	1.606541	0.975244	0.948126	1.272129
2000-02-02	2.471738	1.221529	-0.548494	0.464060	0.339454	3.133764	-0.616815	-1.012898	-3.348109	0.177340
2000-02-03	0.510174	-1.122380	1.551619	-0.388563	0.336944	4.078966	3.310577	-0.029293	4.090396	1.217955
...	...	...	...	...	...	...	...	...	...	...
2013-12-16	0.032351	0.227351	0.149061	-0.049779	0.202478	-1.303091	0.214563	0.345160	0.374515	-0.444385
2013-12-17	0.642241	0.490683	-0.202992	-0.124572	0.113968	-0.488793	0.123138	0.922689	-0.227194	0.204113
2013-12-18	0.851202	0.483930	-0.311751	0.893983	2.616277	0.744003	0.563205	1.859793	1.437586	0.541969
2013-12-19	-0.296909	0.514485	-0.446821	-0.035070	-0.177613	-0.528601	-0.096350	0.681946	0.240632	-0.197612
2013-12-20	-0.249133	-0.109549	0.248550	0.089918	-0.346444	0.603754	0.222352	-0.492820	0.401134	0.812025

3492 rows × 419 columns

- **Covariance Matrix:** Calculate the covariance matrix of the normalized returns

- **Eigen Decomposition for PCA:** Perform eigen decomposition on the covariance matrix to find the eigenvalues and eigenvectors. The eigenvectors represent the directions of maximum variance in the data, and the eigenvalues indicate magnitude of variance in those directions.



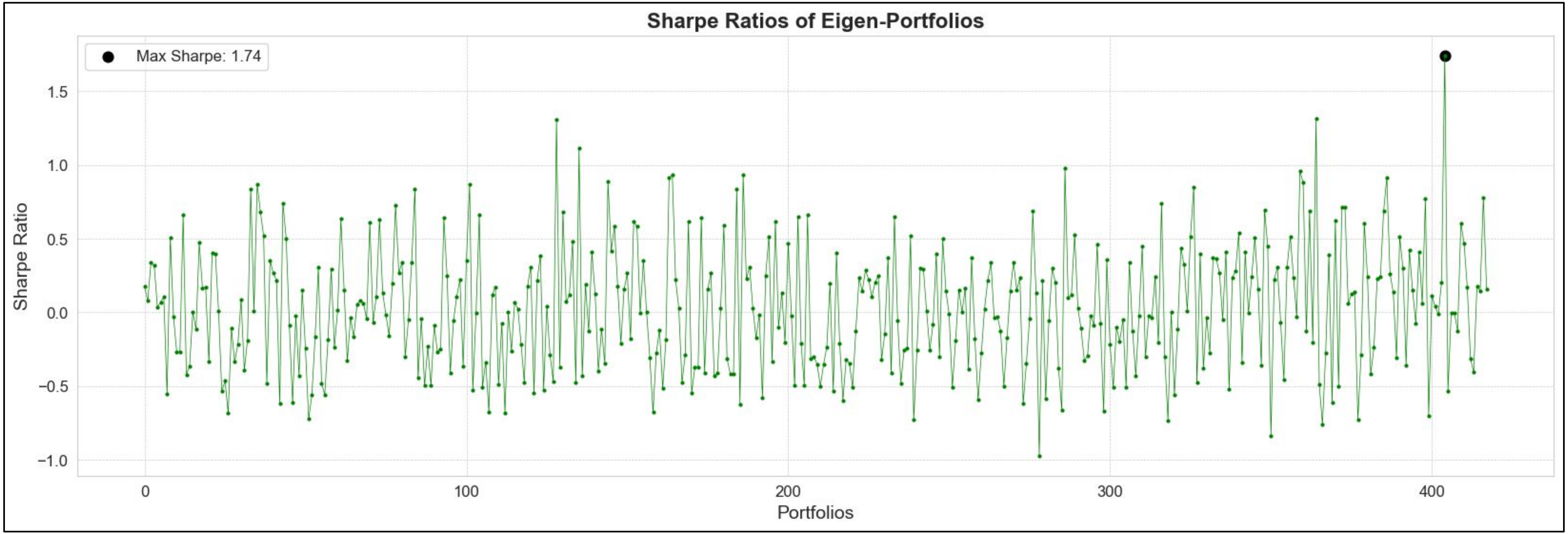

Cumulative Explained Variance by PCA Components
99% variance threshold
200 components
Cumulative Explained Variance (%)
Number of Components
30
40
50
60
70
80
90
100
0
100
200
300
400

- **Construct Eigenportfolio:** Assign weights to each asset based on their contribution to the selected principal components.

First Eigenportfolio

- CAGR = 15.47%
- Volatility = 31.45%
- Sharpe = 0.17

- **Evaluation:** Calculate the Sharpe Ratio of all eigenportfolios using the formula. The goal is to maximize this ratio to achieve higher returns per unit of risk.



Results

Best Risk-Adjusted Eigenportfolio

- CAGR = 102.15%
- Volatility = 52.91%
- Sharpe = 1.74

	Return	Vol	Sharpe
404	102.151447	52.911189	1.741625
364	160.348833	114.740593	1.310337
128	44.422096	26.358581	1.305916
135	60.058636	44.934378	1.114039
286	34.734906	25.262911	0.979100
359	64.144384	56.536644	0.957687
186	29.924647	21.394133	0.931314
164	36.343563	28.324231	0.930072
386	38.243491	30.871697	0.914867
163	35.560353	27.958513	0.914224


Returns of the market-cap weighted index vs. 405th eigen-portfolio (2009-01-02 - 2013-12-20)
PC405
SPX
Cumulative Returns (x initial investment)
0
5
10
15
20
25
30
35
2009
2010
2011
2012
2013
2014
Date

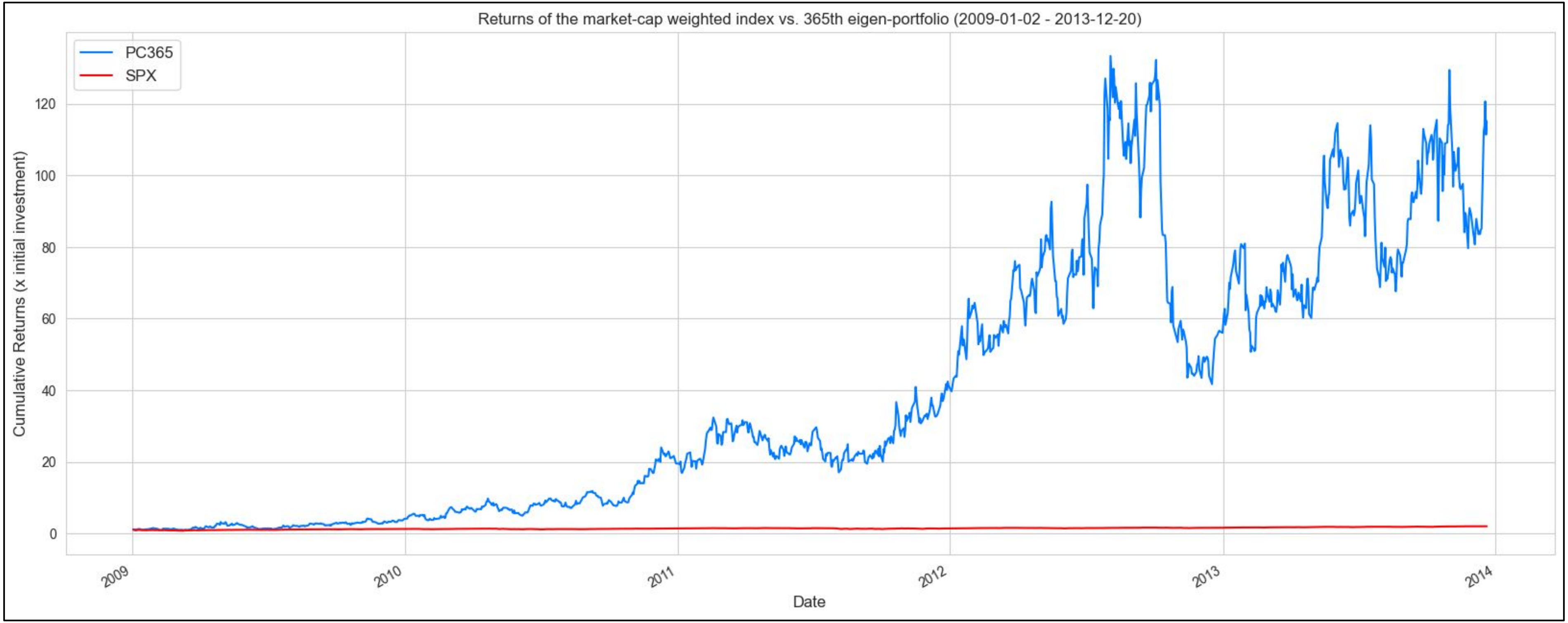
Custom Metric

$$\text{custom metric} = \begin{cases} \text{Return} \times \text{Sharpe}, & \text{if Return} > 0 \text{ and Sharpe} > 0 \\ 0, & \text{otherwise} \end{cases}$$

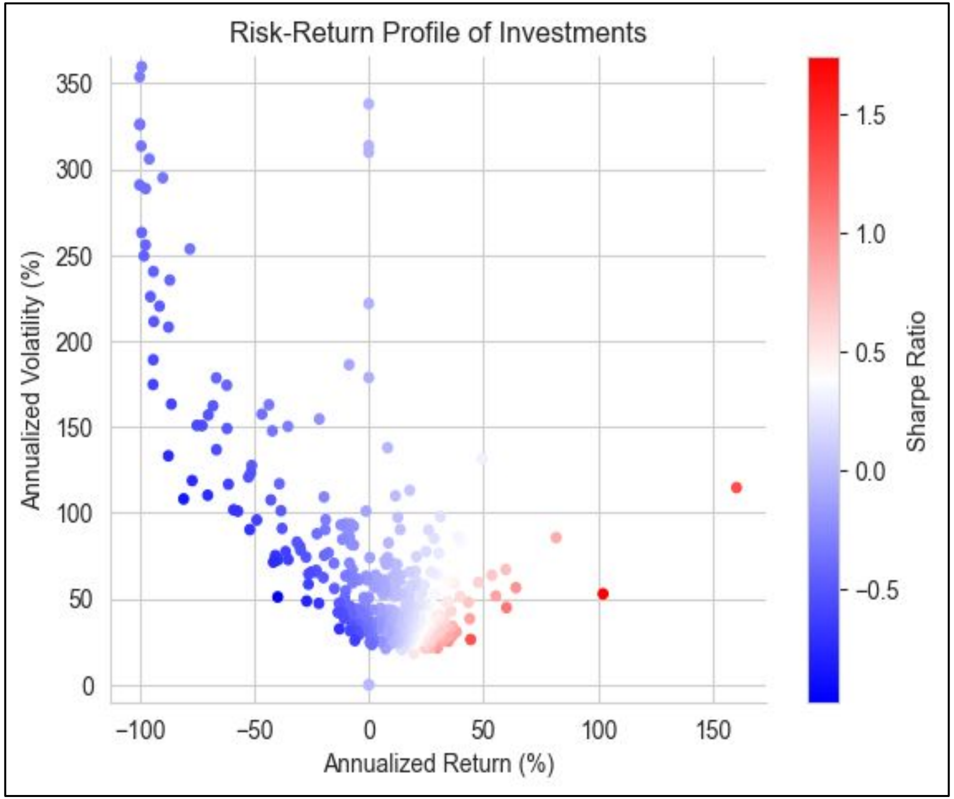
	Return	Vol	Sharpe	metric
364	160.348833	114.740593	1.310337	210.110992
404	102.151447	52.911189	1.741625	177.909508
184	81.731764	85.690210	0.837106	68.418126
135	60.058636	44.934378	1.114039	66.907646
359	64.144384	56.536644	0.957687	61.430215
...	...	...	...	...
148	-0.518050	50.461539	-0.208437	0.000000
204	-8.347542	86.056048	-0.213205	0.000000
216	-9.124076	89.291140	-0.214177	0.000000
300	-10.159739	93.738824	-0.215063	0.000000
278	-39.681154	51.124730	-0.971764	0.000000

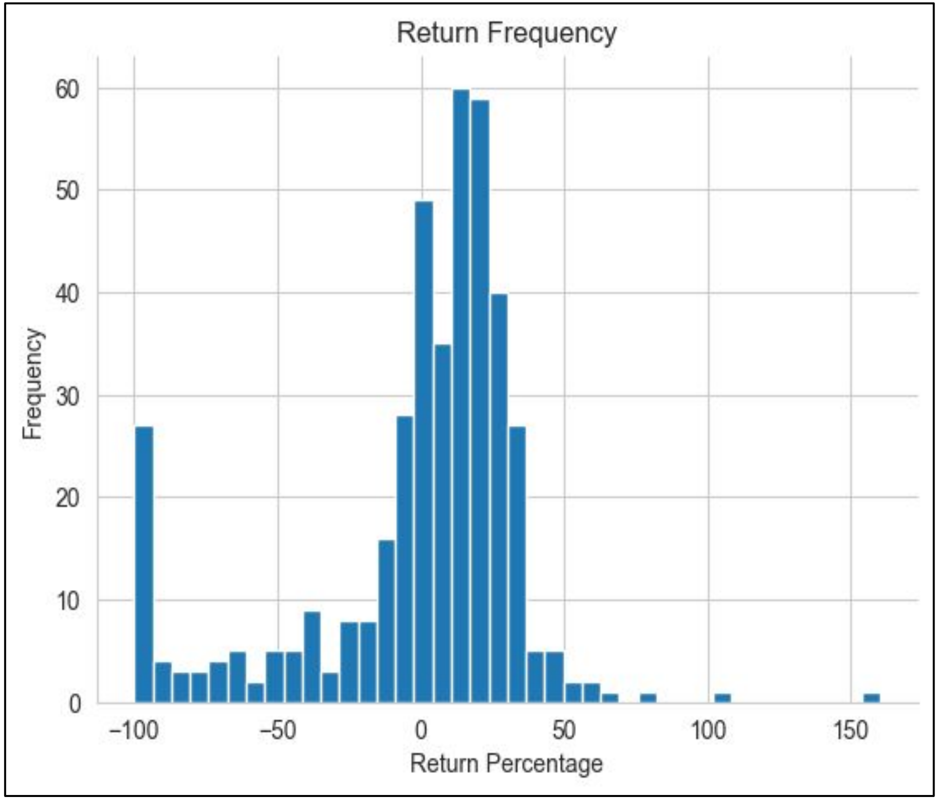
Best Custom Metric Adjusted Eigenportfolio

- CAGR = 160.35%
- Volatility = 114.74%
- Sharpe = 1.31

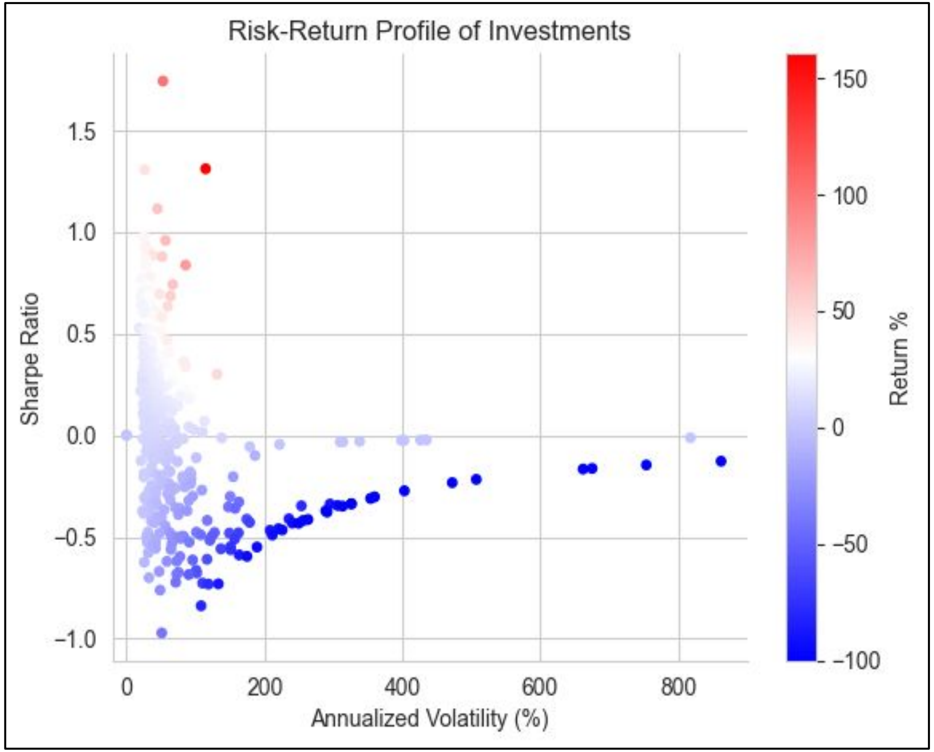

Returns of the market-cap weighted index vs. 365th eigen-portfolio (2009-01-02 - 2013-12-20)
PC365
SPX
Cumulative Returns (x initial investment)
0
20
40
60
80
100
120
2009
2010
2011
2012
2013
2014
Date

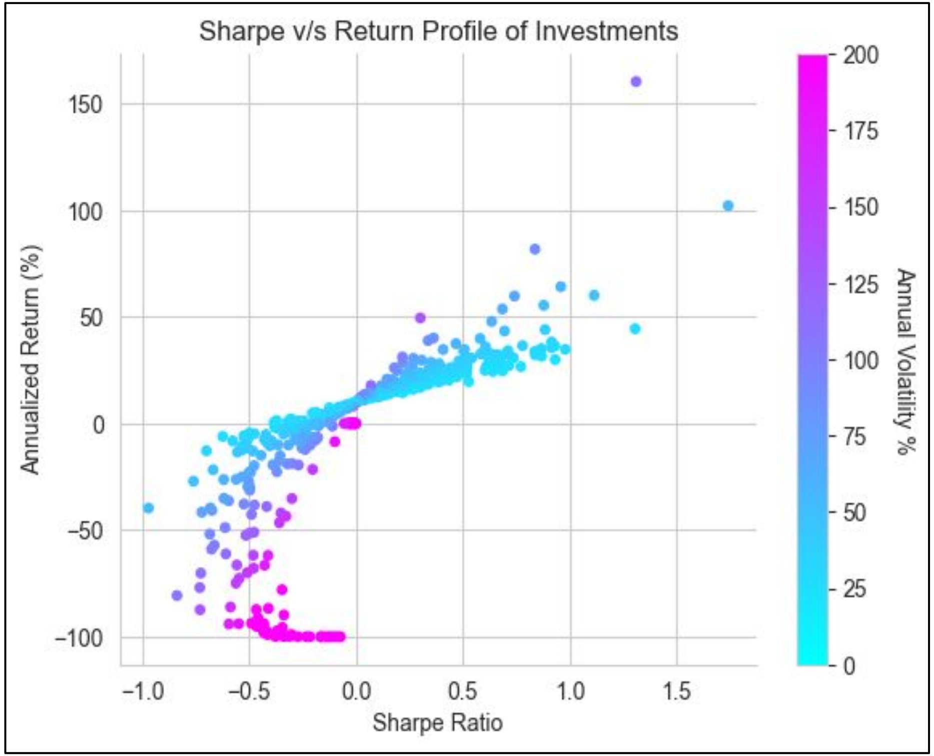
Insights





- Investment with **moderately high** volatility can offer **higher returns**
- Majority of portfolios have annual CAGR around -50 to 50%





- Investments with higher volatility do not always result in higher Sharpe Ratios, which suggests that higher risk does not uniformly translate to better risk-adjusted returns.
- Investments with higher Sharpe Ratios tend to offer higher returns.

Conclusion

- PCA reduces dataset complexity, with 200 principal components explaining 99% variance
- PCA effectively constructs eigenportfolios for optimizing risk-adjusted returns
- Diverse distribution through eigenportfolio weights, including negative weights for short positions, indicate complex strategies
- Several eigenportfolios surpass market-cap weighted index (SPX) returns from 2009 to 2013

- 405th and 365th eigenportfolios demonstrate sophisticated asset allocation for risk-return balance

- Positive correlation suggests higher Sharpe ratios lead to higher Returns

- Novel Custom metric approach combines Return and Sharpe Ratio favoring risk-adjusted performance over high Sharpe ratios

- The Custom metric approach is particularly beneficial for risk-tolerant investors aiming to maximize returns