Modelling

⇒ Considering a small satellite, ~~spherical~~ irregular in shape ~~or~~ moving in a circular orbit which surface charge density σ.

⇒ Trying to derive dynamics and kinematics equation and to find numerical solution on MatLAB and Observe angular velocity and Quaternion ~~observation~~ variation through time.

We have Euler Equation of Attitude [Standard]

$$I\dot{\omega} = S(\omega)\, I\omega + \underbrace{3\omega_0^2\, S(I c_z^b) c_x^b}_{\text{Torque due to Gravity}} + \underbrace{T_{coul}}_{\text{Coulomb Torque}}$$

where $\underbrace{S(\omega)}_{\text{used for cross product}} = (-\bar{\omega}\times)$

$$= \begin{bmatrix} 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_x & 0 \end{bmatrix}$$

⇒ Main step is to calculate Torque.

Torque due to Lorentz force → Calculation

$$\vec{F} = q(\vec{v} \times \vec{B})$$ ⇒ σ ⇒ Surface Charge Density

$$\boxed{T_L = \int_S \sigma \rho \times (V_{rel} \times B)\, dS}$$ ⇒ Integration over surface
⇒ ρ ⇒ distance of ~~each~~ dS from ~~CO~~ Centre of Mass

(from Griffiths)

from Tikhonov (2011) ⇒ $\boxed{T_L = q\, r_0 \times A_{rot} (V_{rel} \times B_0)}$

(r_0 ⇒ Centre of charge distance from COM)

⇒ As satellite is very small w.r.t, we can assume whole charge at a point to calculate torque

$$r_0 = \frac{\int_S (\sigma\, ds)\, r}{q_{total}}$$ (its like calculating Com $\int \frac{r\,dm}{m}$ also here, $\int \frac{dq\, r}{q}$)

V_{rel} = Velocity of point of Centre of Charge w.r.t. geomagnetic field

$$V_{rel} = V - \omega_e \times R$$

Assumption: Assumed velocity of Centre of Charge same as COM as satellite is very small w.r.t. Earth.

From Gangestad $\cancel{V_{rel} = R(\Omega - \omega_E \cos i) \times \hat{i} + R\omega_E} \sin i$

$$\boxed{V_{rel} = R(\omega_0 - \omega_E \cos i)\, \hat{e}_{tangential} + R\omega_E \sin i \cos u\, \hat{e}_{normal}}$$

where u = argument of latitude

As we are considering circular orbit,

$$u = \nu \text{ (true anomaly)} = \omega_0 t$$

$$V_{rel} = R\left(\overset{\omega_0}{\cancel{\Omega}} - \omega_E \cos i\right)\hat{e}_t + R\omega_E \sin i \cos \omega_0 t \ \hat{e}_n$$

This term is due to effect of rotation of Earth component in direction of ω_0

This term is due Earth rotation component perpendicular to orbital plane

$$\boxed{V_{rel} = R\left(\omega_0 - \omega_E \cos i\right)(\hat{y}_0) + R\omega_E \sin i \cos \omega_0 t \,(\hat{x}_0)}$$

~~Converted into~~. written $\hat{e}_t$ and $\hat{e}_n$ ~~to~~ in orbital coordinates with direction as in your paper, $\hat{x}_0 \Rightarrow$ Normal i.e, directed towards Earth

$\hat{y}_0 \Rightarrow$ Tangential

Now, considered B_0 from the paper we got

$$\vec{B} = \frac{B_0}{r^5}\left(3(\vec{N}\vec{r})\vec{r} - r^2\vec{N}\right) \qquad N = (0, 0, -1)^T$$

Calculated

$$\boxed{T_L = q\, r_0 \times A_{rot}\left(V_{rel} \times B_0\right)}$$

$A_{rot} \Rightarrow$ from your paper

$q = \int \sigma\, ds \qquad r_0 = \dfrac{\int r\, \sigma\, ds}{q_{total}}$

Our Euler equation

$$\boxed{I\dot{\omega} = S(\omega) I\omega + 3\omega_0^2 S(I e_x^b) e_x^b + q\, r_0 \times A_{rot} (v_{rel} \times B_0)} \quad -①$$

for Kinematics, I used your paper

$$\boxed{\dot{q} = R(q)\,\omega_r} \quad ⊝②$$

⇒ ~~Initially~~ for Numerical Analysis, Initially I considered $q = [0,0,0,1]$ body axis aligning with Orbital one

⇒ at time t

$$\vec{R} = \begin{bmatrix} R\cos\omega_0 t \\ R\sin\omega_0 t \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ & & \\ & & \end{bmatrix}$$

$$\vec{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix} \begin{bmatrix} R\cos\omega_0 t \\ R\sin\omega_0 t \\ 0 \end{bmatrix}$$ ⇒ Circular orbit with inclination i

$\boxed{i = 40^\circ}$

⇒ Considered Centre of Charge at (2, 2, 2) and radius of orbit = 7000 km

⇒ Considered initial $\text{omega} = [0,0,0]$

⇒ Used satellite Norms as ϕ in our given paper

⇒ Implemented these two equations on MatLAB and attached results were produced.