DT Hash

B Tech Project

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What is Generic Programming?

the adjective "generic" is heavily overloaded!

- Java / C# generics
- C++ templates
- Ada generic packages

What is Generic Programming?

> The goal is often the same.
>
> `A higher level of abstraction than "normally" available.`

> The technique is also often the same.
>
> `Some form of parametrization and instantiations.`

Examples of Generic Programming

Java / C#

```
public class Stack<T>
{
    public void push (T item) {..}
    public T pop () {..}
}
```

Examples of Generic Programming

C++

```
template <typename T, typename Compare>
T & min (T&a, T&b, Compare comp) {
    if (comp (b, a))
        return b;
    return a;
}
```

Generic Programming

It is apparant that:

- Java-stype generics ~ parametric polymorphism
- C++ templates ~ ad-hoc polymorphism

In Haskell:

- Both forms of polymorphism already exist.
- We don't call them generics because they are sort of native to the language.

Can there be a yet higher level of abstraction?

YES!

Generic Programming: Haskell.

Datatype-generic Programming:

- Abstract over the structure of the datatype.
- Also known as "polytypism" and "shape / structure polymorphism".

Algebraic DataTypes and Generics

```haskell
data D p = Alt1 | Alt2 Int p
```

A datatype can have:

- Parameters: type variables (≥ 0)
- Alternatives: unique constructors (≥ 0)
- Fields: types for each constructor (≥ 0)

Non-syntactic features

- Recursion
- Nesting

Structure of Datatypes: Sums

Structure of Datatypes: Sums

```haskell
data AltEx = A1 Int | A2 Char
```

Note the similarity with Either

```haskell
data Either a b = Left a | Right b
```

In fact, AltEx can be modelled as:

```haskell
type AltEx' = Either Int Char
a1 :: Int -> AltEx'                a2 :: Char -> AltEx'
a1 = Left                          a2 = Right
```

Structure of Datatypes: Sums

Alternatives are often called as **sums**. We use another *identical* sum type to represent it, instead of Either.

```
data a :+: b = L a | R b
```

What about a type with > 2 alternatives?

```
data AltEx2 = B1 Int | B2 Char | B3 Float
```

We nest them.

```
type AltEx2 = Int :+: (Char :+: Float)
-- Note that:
b3 :: Float -> AltEx2
b3 = R . R
```

Structure of Datatypes: Products

Structure of Datatypes: Products

```haskell
data FldEx = FldEx Int Char
```

Note the similarity with pair `(,)`.

```haskell
data (,) a b = (,) a b
```

So, we can model FldEx similarly.

```haskell
type FldEx' = (,) Int Char
fldEx' :: Int -> Char-> FldEx'
fldEx' = (,)
```

Structure of Datatypes: Products

The pair type is the basic binary product type. We use the following identical type instead.

```
data a :*: b = a :*: b
```

And more than two fields can be handled using nesting.

```
data FldEx2 = FldEx2 Int Char Float
```

```
type FldEx2' = Int :*: (Char :*: Float)
-- smart constructors
fldEx2' :: Int -> Char -> Float -> FldEx2'
fldEx2' x y z = x :*: (y :*: x)
```

Structure of Datatypes: Sums of Products

Structure of Datatypes: Sums of Products

To "sum" it all up, recall the first example.

```
data D p = Alt1 | Alt2 Int p
```

We can define an identical type using the sum and product types.

```
type RepD p = U :+: Int :*: p
```

Notes:

- We use *unit* type data U = U, *(identical to standard type ())* to represent an alternatice without fields.
- `:+:` is `infix 5` and `:*:` is `infix 6`, so no parantheses.

Proof?

Structure of Datatypes: Isomorphism

How do we know that RepD accurately models D?

We define an Isomorphism as follows.

```
fromD :: D p -> RepD p
fromD Alt1 = LU
fromD (Alt2 i p) = R (i :*: p)
toD :: RepD p D p
toD (L U) = Alt1
toD (R (i :*: p)) = Alt2 i p
```

This allows us to convert between the familiar datatype and the *structural representation* used for generic operations.

Something missing?

Structure of Datatypes: Constructors

The representation lacked any information about the constructors (e.g. the names).

That's easily repaired with another datatype:

```
data C a = C String a
```

```
type RepD p = C U :+: C (Int :*: p)
fromD Alt1 = L (C "Alt1" U)
fromD (Alt2 i p) = R (C "Alt2" (i :*: p))
```

Generic Functions

- Defined on each possible case of the structure representation.
- Work for every datatype that has an isomorphism with a structure representation.

Example: `show :: a -> String`

Generic Functions: show

Let's define the show function for each possible structure case.

- Unit:

```haskell
showU :: U -> String
showU U = ""
```

- Constructor name:

```haskell
showC :: (a -> String) -> C a -> String
showC sA (C name a) = "(" ++ name ++ " " ++ sA a ++ ")"
```

Generic Functions: show

- Binary Product:

```
showP :: (a -> String)->(b -> String) -> a:*:b -> String
showP sA sB (a:*:b) = sA a ++ " " ++ sB b
```

- Binary Sum:

```
showS :: (a -> String)->(b -> String) -> a:+:b -> String
showS sA _ (L a) = sA a
showS _ sB (R b) = sB b
```

Generic Functions: show

Now `show` for RepD can be defined as:

```
-- assumming showInt is known.
showRepD :: (p -> String) -> RepD p -> String
showRepD sP = showS (showC showU) (showC (showP sInt sP))
```

Some observations:

- Predictable pattern.
- Recursive functions, but argument types differ.

Recursive, but differing arguments..

Recursive, but differing arguments..

Recursive, but differing arguments..

Recursive, but differing arguments..

Recursive, but differing arguments..

Recursive, but differing arguments..

Generic Functions, Generically

Let's explore "true" genericity, where the structure is a parameter instead of a pattern.

- **Polymorphic Recursion** - functions with comon scheme that reference each other and allows types to change in the calls.

```
showU :: U -> String
showC :: (.) => C a -> String
ShowP :: ...
...
```

- A common encoding for Isomorphisms

```
data T = ..       -- User defined datatype
type RepT = --    -- Structure representation
from :: T -> RepT
to   :: RepT -> T
```

Polymorphic Recursion

We can encode polymorphic recursion in several ways. Most obvious one is the type classes.

- Standard classes already use polymorphic recursion for deriving instances.
- Class declaration specifies type signature.
- Each recursive case can be specified by an instance of the class.

Show class:

```
class Show a where
  show a :: a -> String
```

Polymorphic Recursion: Show

Unit:

```
instance Show U where
  show = showU
```

Constructor name:

```
instance Show a => Show (C a) where
  show = showC show
```

Polymorphic Recursion: Show

Binary product:

```
instance (Show a, Show b) => Show (a :*: b) where
  show = showP show show
```

Binary sum:

```
instance (Show a, Show b) => Show (a :+: b) where
  show = showS show show
```

Polymorphic Recursion

Recall `showRepD`:

```
showRepD :: (p -> String) -> RepD p -> String
showRepD sP = showS (showC showU) (showC (showP sInt sP))
```

Compare to new version that's possible.

Polymorphic Recursion

Recall `showRepD`:

```
showRepD :: (p -> String) -> RepD p -> String
showRepD sP = showS (showC showU) (showC (showP sInt sP))
```

Compare to new version that's possible.

```
show'RepD :: Show p => RepD p -> String
show'RepD = show
```

Encoding Isomorphisms

To define show function for `D`, we still need to define another function:

```
show'D :: Show p => D p -> String
show'D = show'RepD . fromD
```

How about a `show` function that knows how to convert any type `T` to its structure representation type `RepT`, given an isomorphism between `T` and `RepT`.

Encoding Isomorphisms

We define a class of function pairs.

- Type class, but addition of a *type family*.
- The function pair implements an isomorphism.

```
from :: T -> RepT                    to :: RepT -> T
```

- The functions require two types, so each instance must have two types.
- `RepT` is precisely determined by `T`, so really we only need one unique type and a secondtype derivable from the first.

Encoding Isomorphisms

The type class:

```
class Generic a where
  type Rep a
  from :: a -> Rep a
  to :: Rep a -> a
```

- `Rep` is a type family, an associated type synonym.
- `Rep` can be thought as a function on types. Given a unique type (index) `T` you get a type synonym, `Rep T`.
- Also, two datatypes may have the same representation.

Encoding Isomorphisms

The instance for `D` is:

```
instance Generic (D p) where
  type Rep (D p) = RepD p
  from = fromD
  to = toD
```

Generic Show Function

Finally,

```
gshow :: (Show (Rep a), Generic a) => a -> String
gshow = show . from
```

Hashing*

** Cryptographic*

Hashing

A hash function h with some interesting properties.

$$h : \{0,1\}^* \to \{0,1\}^n$$

- It is extremely easy to calculate $h(x)$.
- It is extremely computationally difficult to calculate $h^{-1}(y)$.
- It is extremely unlikely that two slightly different messages have the same hash.

Cryptographic Hash Functions *under the hood*

> Pour the **initial value** in a big cauldron and place it over a nice fire. Now slowly add salt if needed and stir well. Marinade your input string by **appending some strengthened padding**. Now chop the resulting bit string into nice **small pieces of the same size** and stretch each piece to at least four times its original length. Slowly add each single piece while continually stirring at the speed given by the rotation constants and spicing it up with some addition constants. When the **hash stew** is ready, extract a portion of **at least 128 bits** and present this hash value on a warm plate with some garnish.
> **'Attacks on Hash functions and Applications.'**

'Attacks on Hash functions and Applications' by Marc Stevens, Universiteit Leiden, 2012.

Constructing a Hash Function

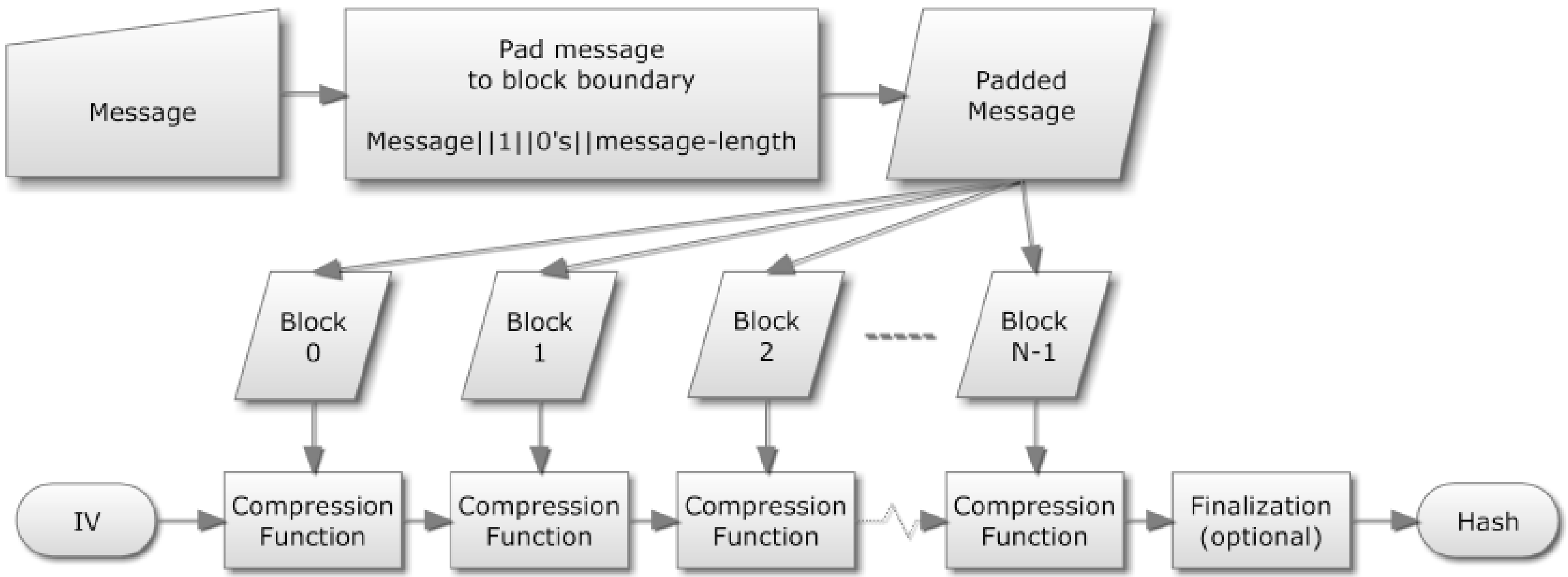
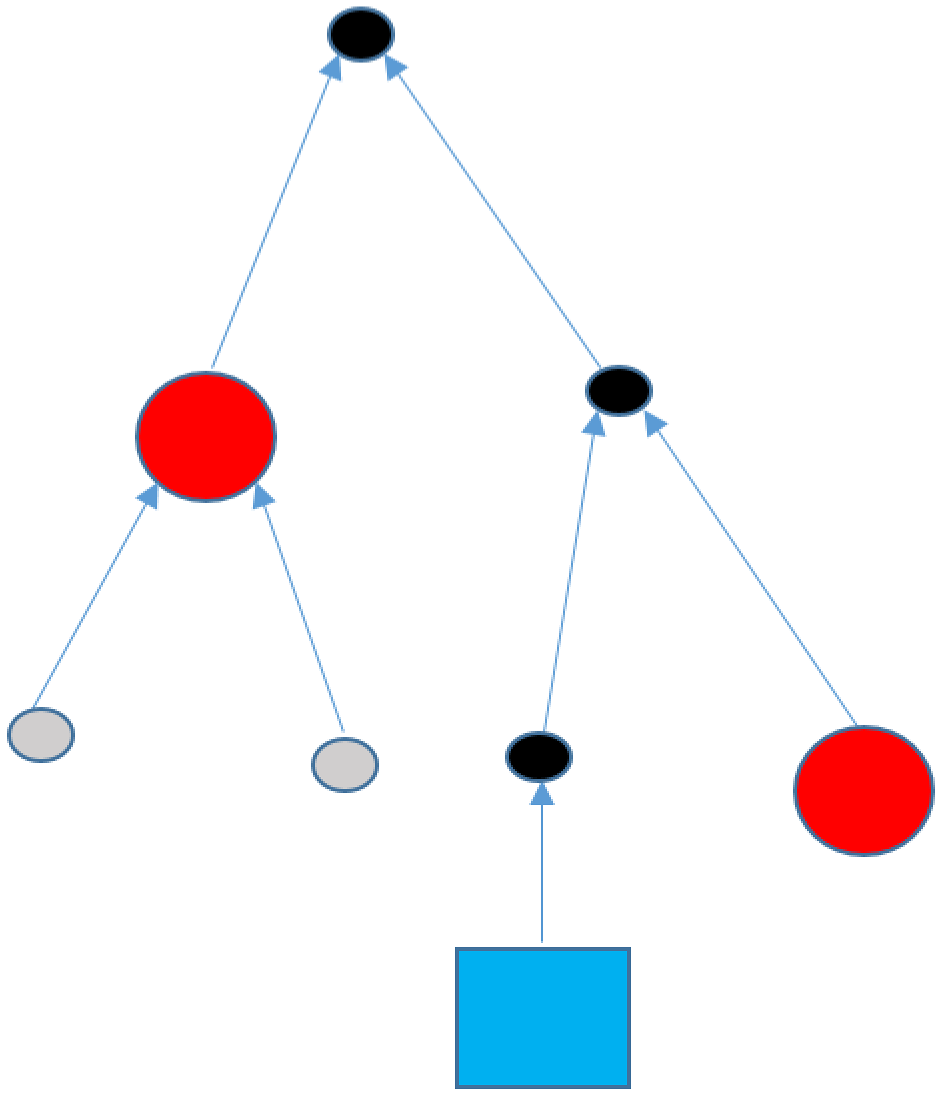


Figure 3: *Merkle-Damgård Construction*

Tree Hashing

- Merkle (1980): authenticate any leaf w.r.t. the hash at the root with a logarithmic number of hash computations.
- Enables:
  - Parallel Computation of nodes.
  - Incremental update to the root-hash after a leaf changes. *(old hash values are stored on the nodes.)*

Merkle Tree

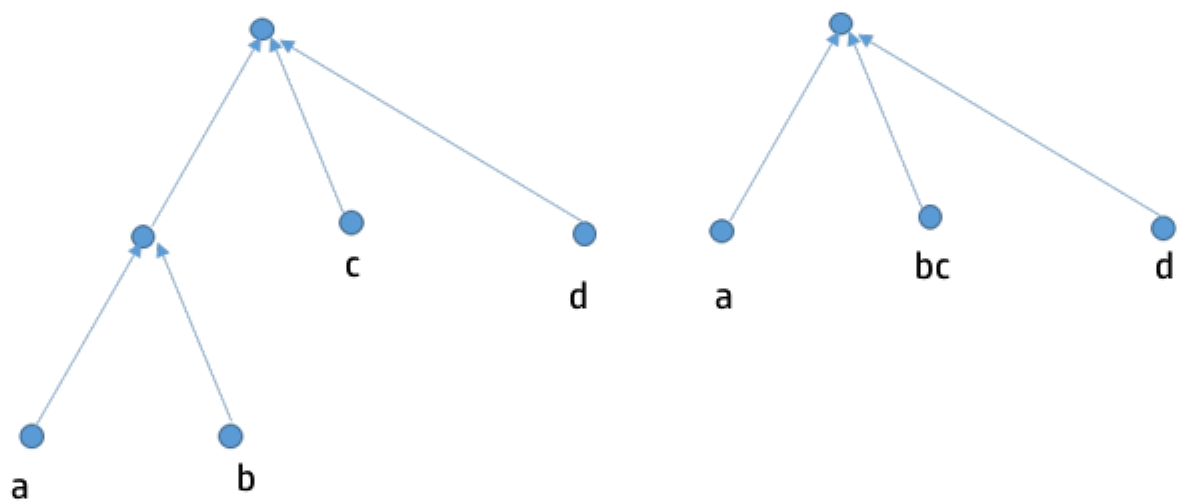


About Sakura

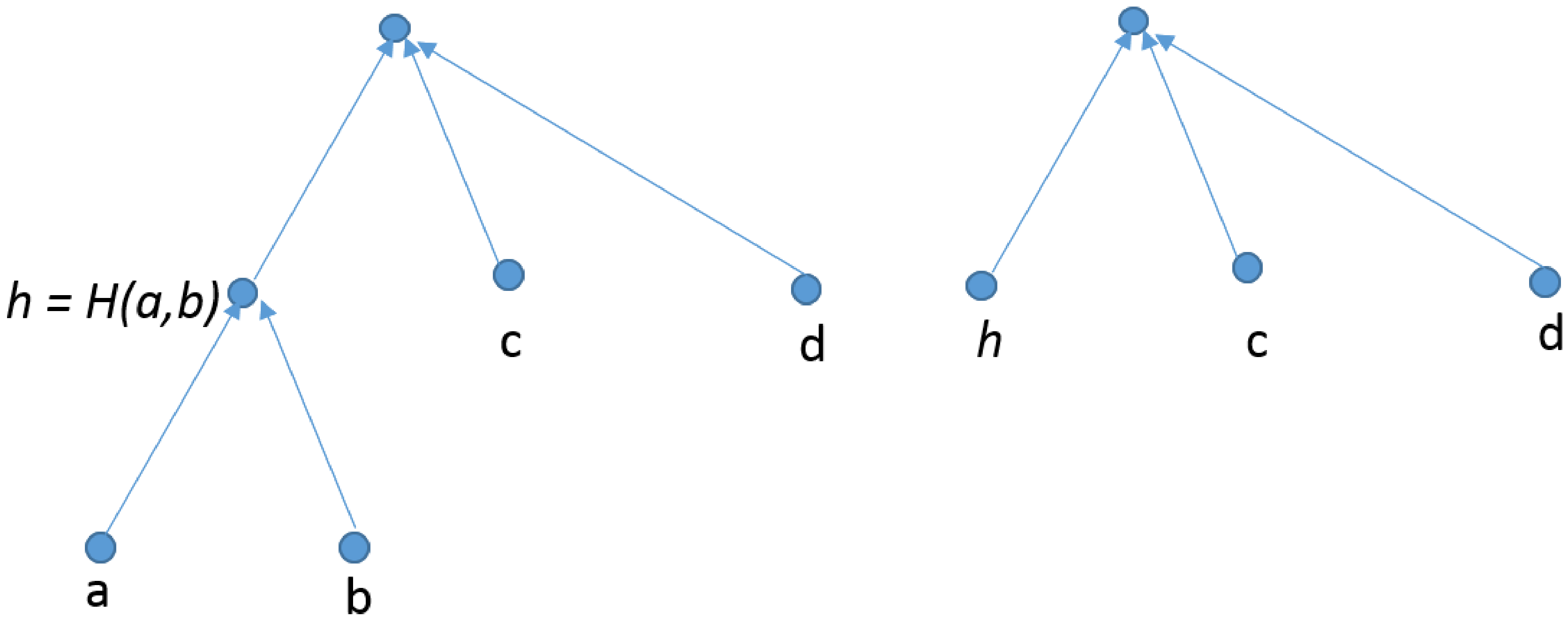
- Tree Hash Mode.
    - More flexible than other tree hash modes.
    - multiple shapes of trees possible.
- Takes an inner hash function as a paramater.

$$Sakura :: Mode \rightarrow Innerhashfunction \rightarrow Input \rightarrow hash$$

Tree Shapes



Stupid Collision



`abcd` and `hcd` should not have the same tree hash, otherwise, you have a collision.

Detailed Example *Hop Tree*

An example hop tree from *Sakura*.

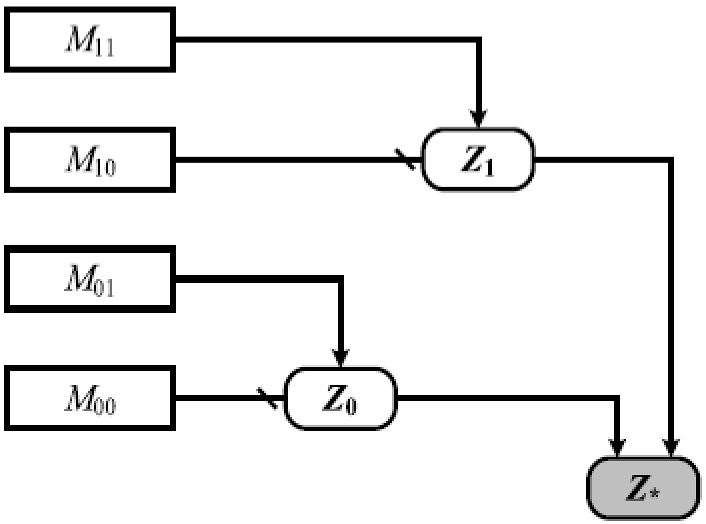
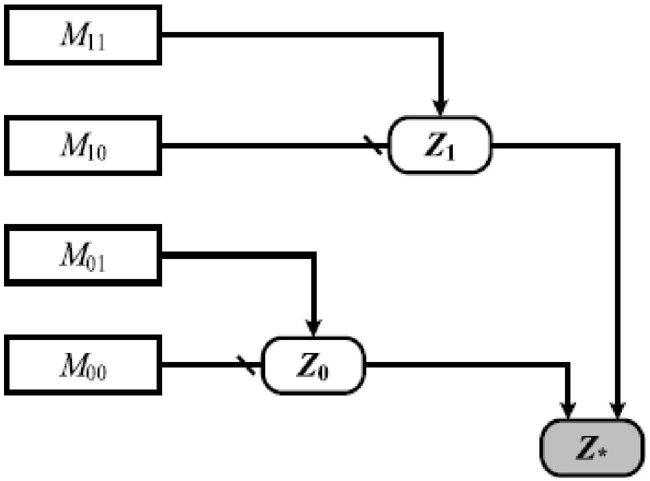
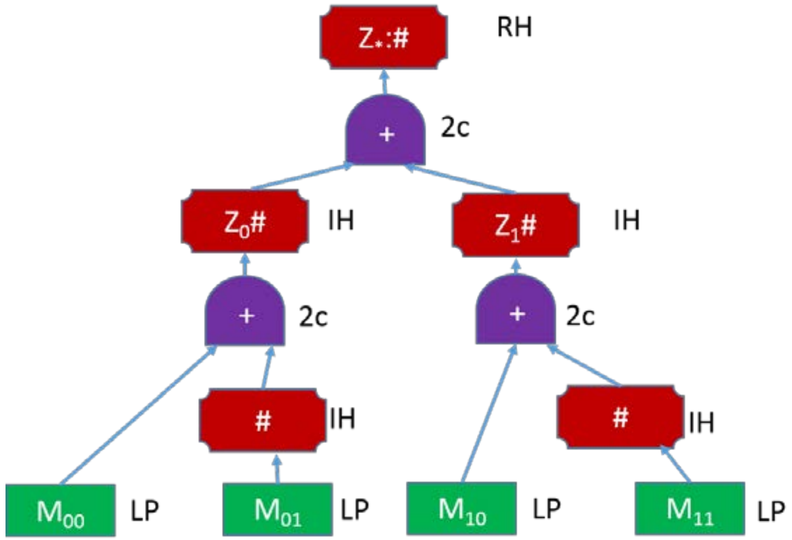


Image from Sakura Paper

Encoding the Example





Implementation of Sakura

Capturing the Shape

- InnerHash, Concat, Interleave, Slice, Pad

```
data HShape = InnerHash HShape
            | Concat [HShape]
            | Interleaving [HShape]
            | Slice Int Int
            | Pad  BStr
```

Serial Hash Computation

```
type BStr  = [Word8]
type HashF = [Word8] -> [Word8]
```

```
my_slice :: Int -> Int -> BStr -> BStr
my_slice from to = (drop from).(take to)
```

```
s :: HashF -> HShape -> BStr -> BStr
-- Serial Hash Function
s h (InnerHash aShape) bStr = h $ s h aShape bStr
s h (Concat l) bStr = concat $ map (\x -> s h x bStr) l
s _ (Slice from to) bStr = my_slice from to bStr
s _ (Pad x) _ = x
```

Parallel Hash Computation

```
p :: HashF -> HShape -> BStr -> BStr
-- Parallel Hash Function
p h (InnerHash aShape) bStr = h $ p h aShape  bStr
p h (Concat l) bStr = concat $ parMap rpar (\x -> p h x bS
p _ (Slice from to) bStr = my_slice from to bStr
p _ (Pad x) _ = x
```

Algebraic DataTypes and Hashes

Basic Idea

```
class Hashable a where
    data ID a
    type Node a
    hash :: Node a -> ID a
```

```
instance Hashable a => Hashable [a] where
    ID [a] = ListID
    Node [a] = Nil
             | (ID a) : (ID [a])
```

A Novel Idea

```
class Hashable a where
    toHShape :: a -> HShape
    default toHShape :: (Generic a, GHashable (Rep a)) =>a
    toHShape = gtoHShape . from
```

Let's explore the code.