

B Tech Project

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Functional Programming in Haskell

- Everything is an equation.
- That's pretty much all. JK.
- *Cannot* have multi argument functions.
- Think of functions and datatypes as equals.
- Lazy Evaluation (*evaluates only when necessary*).
- Algebraic DataTypes.

Algebraic DataTypes

- has one or more data constructors.
- Each data constructor can have zero or more arguments.
- Can be recursive too.
- Pattern matching:
 - Match values against patterns
 - Bind variables to successful matches.

Example

```
data Shape = Rectangle Int Int  
           | Square Int
```

```
area :: Shape -> Int  
area (Rectangle len breadth) = len * breadth  
area (Square side) = side * side
```

```
rec = Rectangle 3 4  
main = print $ area rec
```

Some Data Structures in Functional Programming

Binary Tree

```
BTree a = NullBTree  
        | BNode a (BTree a) (BTree a)
```

Rose Tree

```
RTree a = NullRTree  
        | RNode a [RTree a]
```

List

```
List a = Nil  
      | Cons a (List a)
```

Looks unfamiliar?

Just with some special notation: `[a]` for `List a`, `[]` for `Nil` and `(:)` for `Cons`.

```
[a]      = []  
      | a : [a]
```

Hashing

A hash function h with some interesting properties.

$$h : \{0, 1\}^* \rightarrow \{0, 1\}^n$$

- It is extremely easy to calculate $h(x)$.
- It is extremely computationally difficult to calculate $h^{-1}(y)$.
- It is extremely unlikely that two slightly different messages have the same hash.

Cryptographic Hash Functions *under the hood*

Pour the **initial value** in a big cauldron and place it over a nice fire. Now slowly add salt if needed and stir well. Marinate your input string by **appending some strengthened padding**. Now chop the resulting bit string into nice **small pieces of the same size** and stretch each piece to at least four times its original length. Slowly add each single piece while continually stirring at the speed given by the rotation constants and spicing it up with some addition constants. When the **hash stew** is ready, extract a portion of **at least 128 bits** and present this hash value on a warm plate with some garnish.

— **Attacks on Hash functions and Applications.**

Constructing a Hash Function

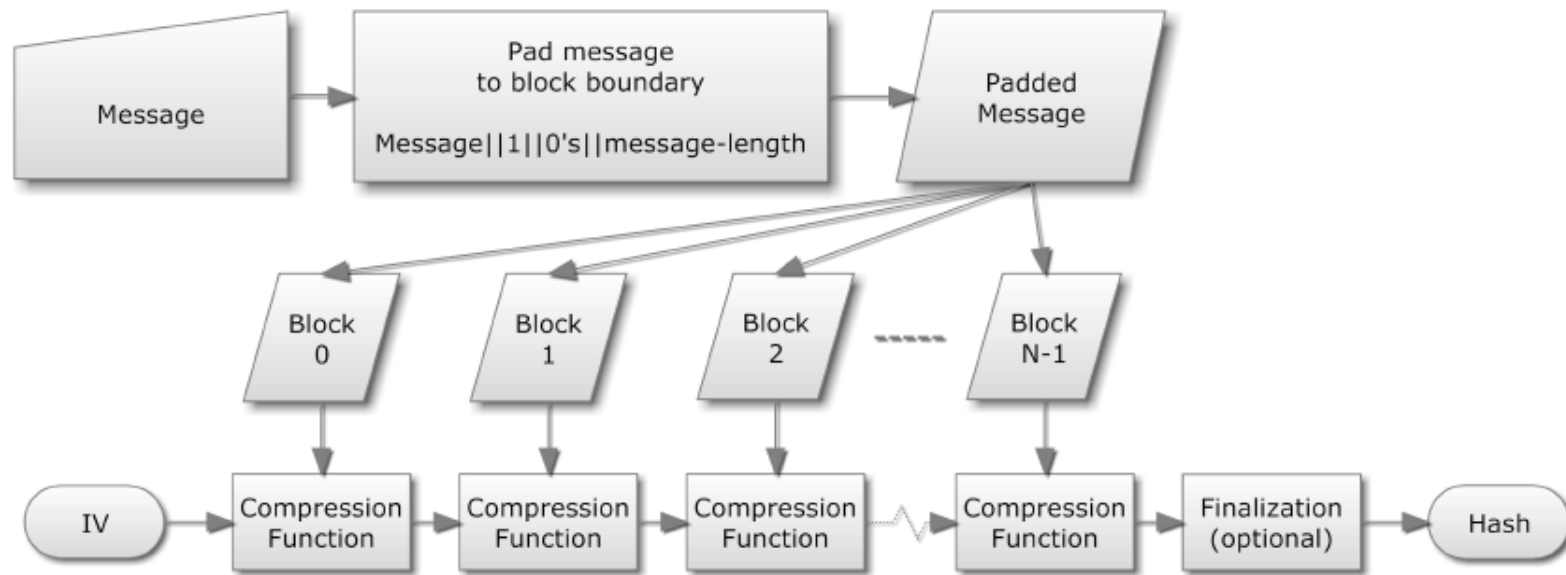
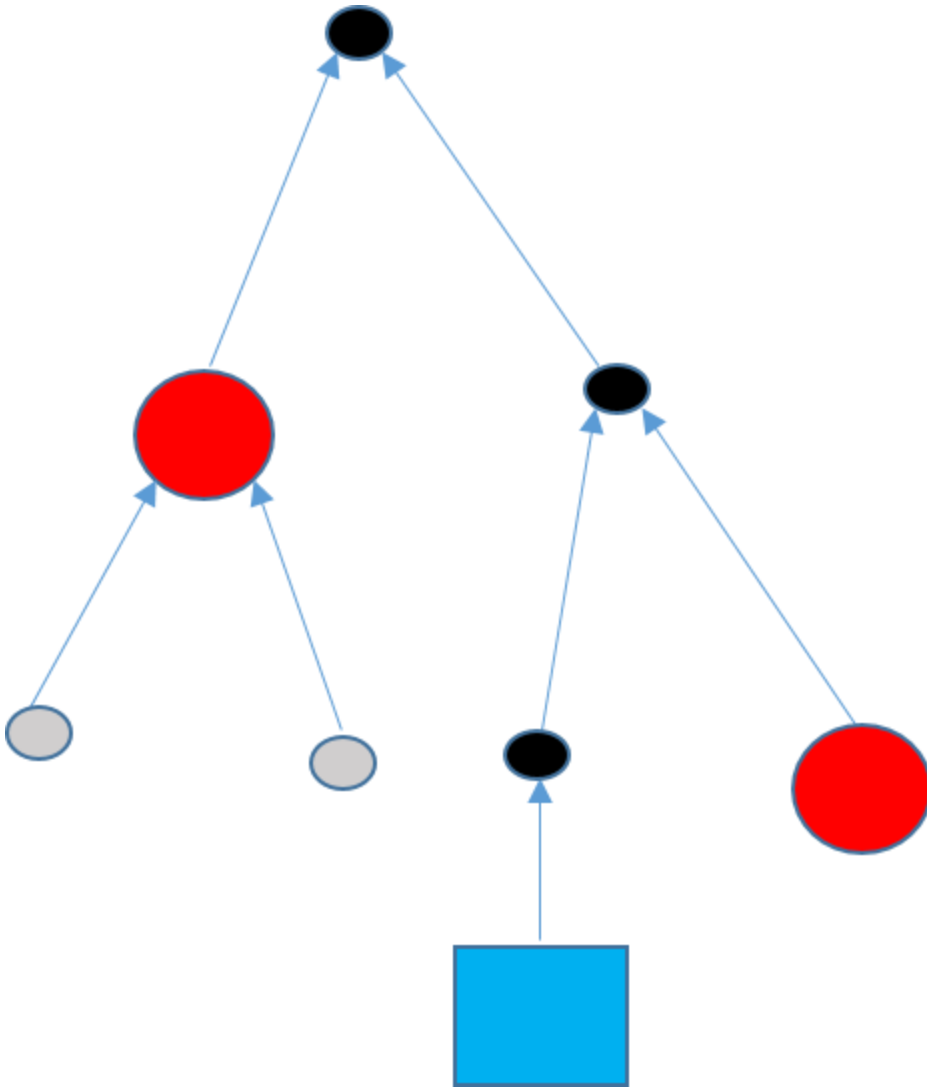


Figure 3: *Merkle-Damgård Construction*

Tree Hashing

- Merkle et al(1980): authenticate any leaf w.r.t. the hash at the root with a logarithmic number of hash computations.
- Enables:
 - Parallel Computation of nodes.
 - Incremental update to the root-hash after a leaf changes.
(old hash values are stored on the nodes.)

Merkle Tree

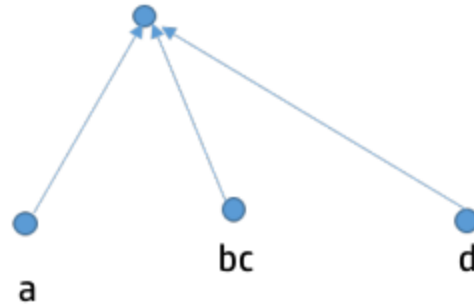
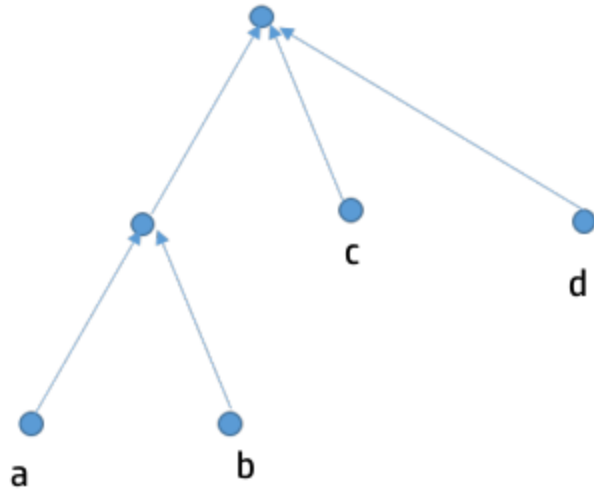


About Sakura

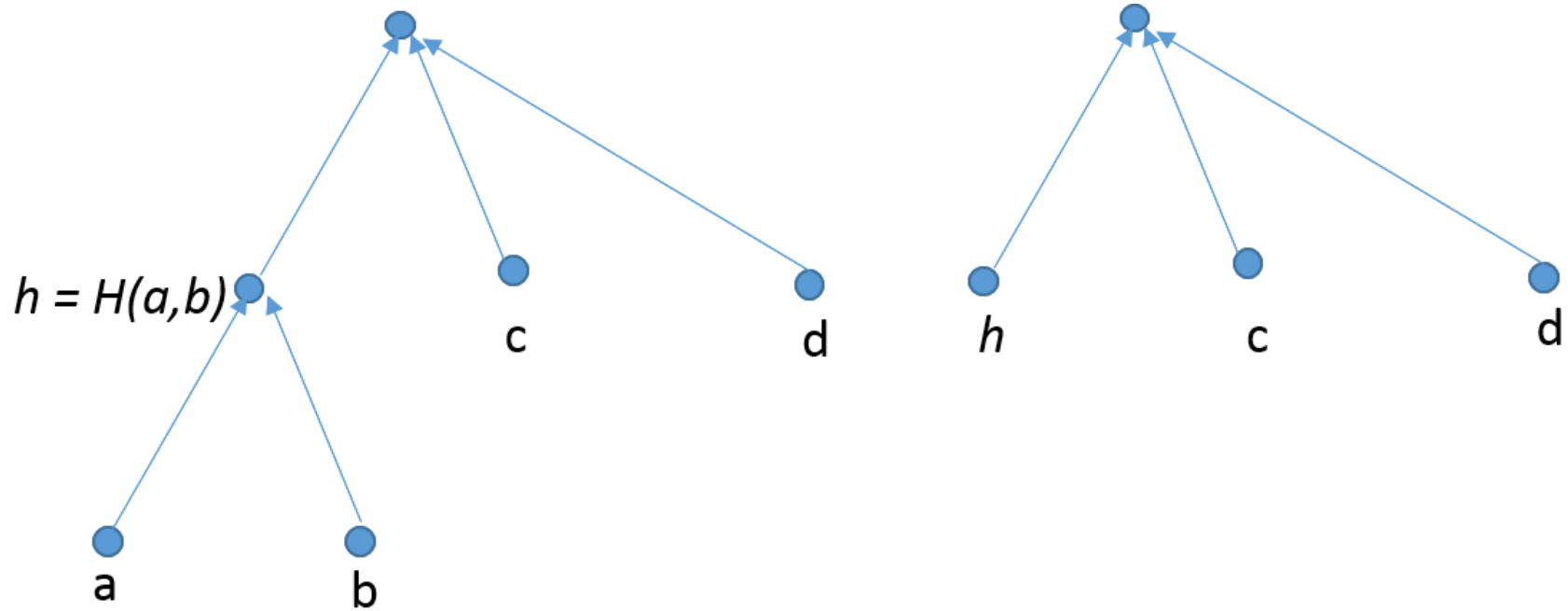
- Tree Hash Mode.
 - More flexible than other tree hash modes.
 - multiple shapes of trees possible.
- Takes an inner hash function as a parameter.

Sakura :: Mode → Innerhashfunction → Input → hash

Tree Shapes



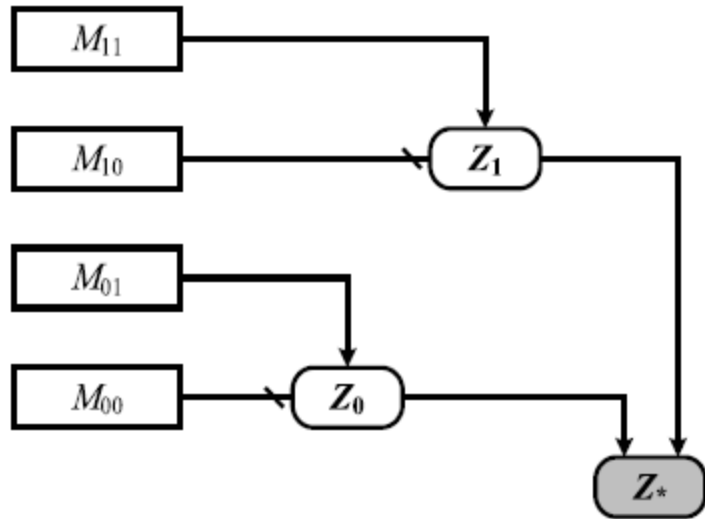
Stupid Collision



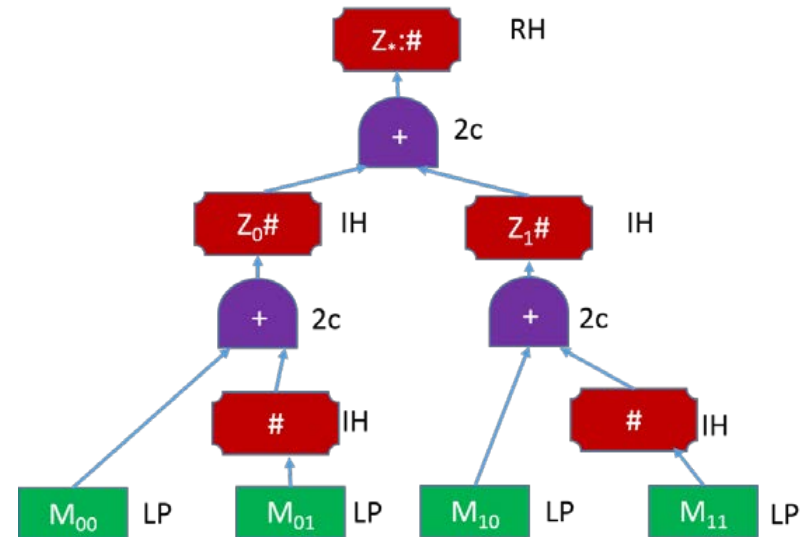
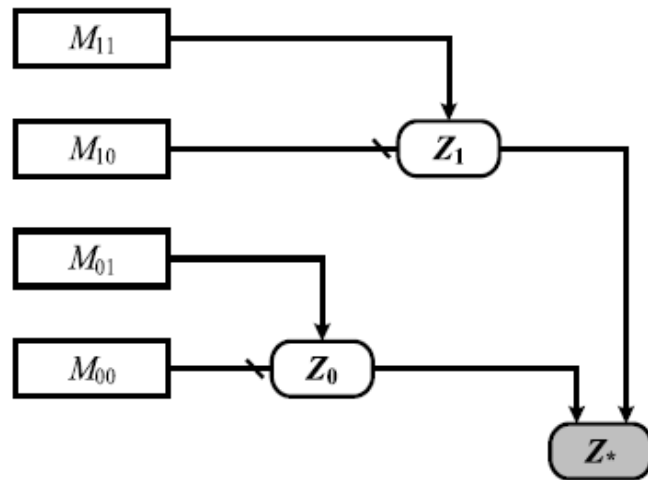
`abcd` and `hcd` should not have the same tree hash, otherwise, you have a collision.

Detailed Example *Hop Tree*

An example hop tree from *Sakura*.



Encoding the Example



Implementation of Sakura

Capturing the Shape

- InnerHash, Concat, Interleave, Slice, Pad

```
data HShape = InnerHash HShape
            | Concat [HShape]
            | Interleaving [HShape]
            | Slice Int Int
            | Pad BStr
```

Serial Hash Computation

```
type BStr  = [Word8]
type HashF = [Word8] -> [Word8]
```

```
my_slice :: Int -> Int -> BStr -> BStr
my_slice from to = (drop from).(take to)
```

```
s :: HashF -> HShape -> BStr -> BStr
-- Serial Hash Function
s h (InnerHash aShape) bStr = h $ s h aShape bStr
s h (Concat l) bStr = concat $ map (\x -> s h x bStr) l
s _ (Slice from to) bStr = my_slice from to bStr
s _ (Pad x) _ = x
```

Parallel Hash Computation

```
p :: HashF -> HShape -> BStr -> BStr
-- Parallel Hash Function
p h (InnerHash aShape) bStr = h $ p h aShape bStr
p h (Concat l) bStr =
    concat $ parMap rpar (\x -> p h x bStr) l
p _ (Slice from to) bStr = my_slice from to bStr
p _ (Pad x) _ = x
```

Algebraic DataTypes and Hashes

Basic Idea

```
class Hashable a where  
  data ID a  
  type Node a  
  hash :: Node a -> ID a
```

```
instance Hashable a => Hashable [a] where  
  ID [a] = ListID  
  Node [a] = Nil  
             | (ID a) : (ID [a])
```