

## Numerical Methods & Analysis Assignments

### Integration

#### Goal:

- Implement different integration algorithms
- Apply these algorithms to calculate the integral of a function  $f(x)$
- Understand and calculate a ground truth and then calculate and compare the accuracy of the different methods
- Investigate and understand the role of digitization/sampling and computational errors

#### The Problem:

In their work, a team needs to have some quantifiable metrics about the effect of data sampling (or digitization). In these studies:

- They measure a physical property  $F$  that changes over time and it is described by a known function  $F(t)$  over time

To investigate this physical property they need to calculate integrals and derivatives of this function, i.e.

- $F(t)$ , over a defined time interval, say from  $T_a$  to  $T_b$ .
- The physical property  $F$  can be measured experimentally with a sensor with selectable rate of digitization; one can select the frequency of data collection or the duration in between the samples, or the number of samples over a time period  $T_a$  to  $T_b$ )

$$F(t) = A \exp(kt) \cos(\omega t)$$

$$A = 1.000 \quad k = 0.055 \quad \omega = 2.0 \quad T_a = 0.001 \quad \text{and} \quad T_b = 2.0$$

1. Implement the algorithms to calculate integrals based on the three methods we discussed in class: midpoint, trapezoid, Simpson
2. Implement the algorithms to calculate derivatives based on the two of the methods we discussed in class: Forward difference and Central difference
3. Generate the Ground Truth; in your answer discuss why this is the ground truth (or the "real" value) versus which you will compare your numerical results. *Hint:* the ground truth is the "truth" thus must be independent of the algorithm you use to calculate the integral!
4. Generate the sampled data; assuming that (a) the sensor is pre-set to collect  $N$  equidistant samples between  $T_a$  to  $T_b$ . Consider that  $N$  takes the values  $N = 2^2, 2^4, \dots, 2^{10}$ .
5. Integration - calculate: (a) the integral of the sampled data between  $T_a$  and  $T_b$  with the three methods and (b) the Error of the three integrals calculated from the experimental data (relative to the Ground Truth). Then, (c) plot the error vs  $N$  (decide if you use linear or log scales in your graphs) and (d) Comment on the effect of  $N$  and method of calculations
6. Differentiation - calculate: Do the same tasks as in question (5) above for the first and second derivative. Important is task (d)!