Ashutosh Kumar Singh 19CS30008

10. Here, on using the generic least squares solution for $Ax = b$ by taking $\hat{x} = (A^T A)^{-1} A^T b$ does not work because the matrix A comes out to be singular, i.e., it has linearly dependent columns. So, to solve this issue, we use Tikhonov Regularized Inversion.

$$\hat{x} = \underset{x \in R^n}{\operatorname{argmin}} \|Ax - b\|_2^2 + \lambda \|x\|_2^2$$

We construct $\tilde{A} = \begin{bmatrix} A \\ \sqrt{\lambda} I \end{bmatrix}$

Now here, $\tilde{A}$ has linearly independent columns without any condition on A.

So, $\hat{x} = (\tilde{A}^T \tilde{A})^{-1} \tilde{A}^T b$

On simplifying this expression, and considering $\lambda = 1$, we get,

$$\hat{x} = (A^T A + I)^{-1} A^T b.$$

This is our least squares solution.

We get a test accuracy of 84.9000%

The confusion matrix is also shown.

mnist_ls

October 21, 2021

[106]:
```
import numpy as np
from tensorflow.keras.datasets import mnist
import seaborn as sns
import matplotlib.pyplot as plt
```

[107]:
```
def load_mnist_data():
    (x_train, y_train), (x_test, y_test) = mnist.load_data()

    # reshape and normalize training and test data
    x_train = x_train.reshape(x_train.shape[0], -1) / 255
    x_test = x_test.reshape(x_test.shape[0], -1) / 255

    digit_images = []
    labels = []
    for i in range(10):
        images = x_train[y_train == i]
        digit_images.append(images[np.random.choice(len(images),
                            1000, replace=False)])
        labels.append(np.full((1000,), i))

    x_train = np.vstack(digit_images)
    y_train = np.hstack(labels)

    # shuffle training data
    order = np.random.permutation(x_train.shape[0])
    x_train = x_train[order]
    y_train = y_train[order]

    # take 1000 examples for the test set
    test_indices = np.random.choice(x_test.shape[0], 1000)
    x_test = x_test[test_indices]
    y_test = y_test[test_indices]
    return (x_train, y_train), (x_test, y_test)
```

[108]:
```
def least_squares(A, b, factor=1.0):
    return np.linalg.inv((A.T @ A) + factor * np.eye(A.shape[1])) @ (A.T @ b)
```

[109]:

```python
def confusion_matrix(y_true, y_pred, num_labels):
    matrix = np.zeros(shape=(num_labels, num_labels), dtype=int)
    for i in range(len(y_pred)):
        matrix[y_true[i], y_pred[i]] += 1
    return matrix
```

[110]:

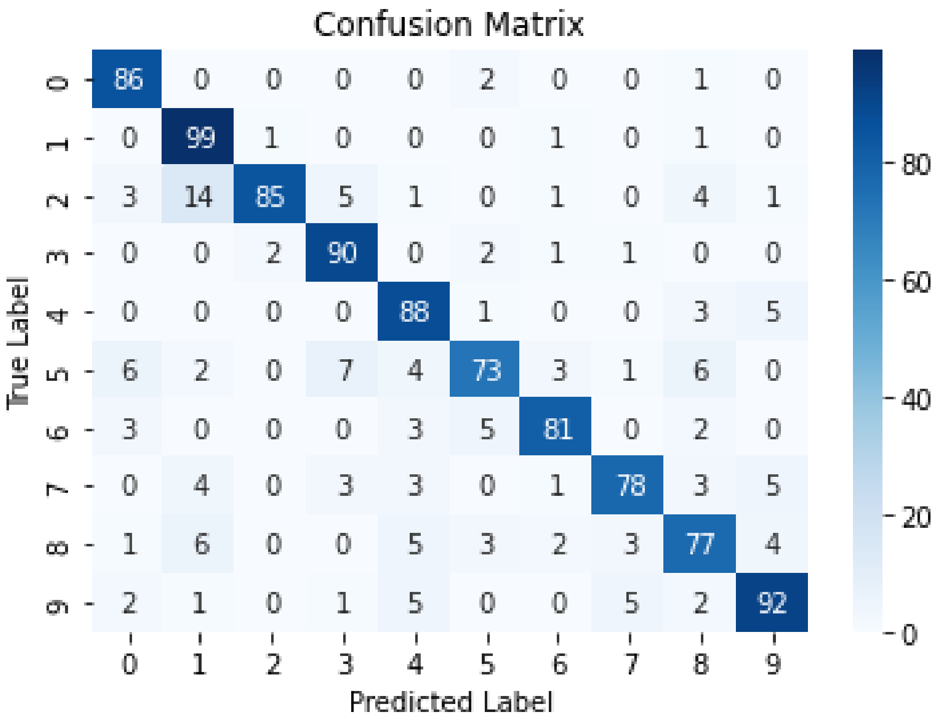
```python
def main():
    (x_train, y_train), (x_test, y_test) = load_mnist_data()
    coeff = np.zeros((x_train.shape[1], 10))
    for i in range(10):
        b = 2 * (y_train == i) - 1
        coeff[:, i] = least_squares(x_train, b)
    preds = np.argmax(x_test @ coeff, axis=1).astype(int)
    print(f'Accuracy : {(np.mean(preds == y_test) * 100):.4f}%\n')

    cnf_mat = confusion_matrix(y_test, preds, 10)
    sns.heatmap(cnf_mat, annot=True, cmap='Blues', fmt='d')
    plt.xlabel('Predicted Label')
    plt.ylabel('True Label')
    plt.title('Confusion Matrix')
    plt.show()
```

[111]:

```python
if __name__ == "__main__":
    main()
```

```
Accuracy : 84.9000%
```


Confusion Matrix
True Label
Predicted Label