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5. Given the system of linear equations $Ax = b$, where $A \in R^{n \times n}$ is an invertible matrix & $b \in R^n$. Also, A is orthogonal.

Since the matrix A is orthogonal, it implies that the columns of A are linearly independent and also, more importantly, ~~the column~~ we have n linearly independent vectors in R^n, hence they can be said to form a basis of R^n.

Thus, we can say that $b \in$ colspace (A) always.

Since A is invertible, we have a solution

$$x = A^{-1}b.$$

But as A is orthogonal,

$$AA^T = A^TA = I,$$

hence, $A^{-1} = A^T$

So, the solution will be $\boxed{x = A^Tb}$.

Thus, the advantages that we get from A being orthogonal are that there is always guarantee to be a solution x to $Ax = b$, and more importantly, we are saved from the burden of calculating the inverse, as the inverse of an orthogonal matrix is simply its transpose.