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Q2. (a) To prove :- $\operatorname{avg}(\alpha x + \beta \mathbb{1}_n) = \alpha \operatorname{avg}(x) + \beta$

We know, $\operatorname{avg}(x) = \left(\frac{\mathbb{1}}{n}\right)^T x$,

So,

$$\operatorname{avg}(\alpha x + \beta \mathbb{1}_n) = \left(\frac{\mathbb{1}_n}{n}\right)^T (\alpha x + \beta \mathbb{1}_n)$$

$$= \left(\frac{\mathbb{1}_n}{n}\right)^T (\alpha x) + \left(\frac{\mathbb{1}_n}{n}\right)^T (\beta \mathbb{1}_n)$$

[by properties of inner product, $(a^T \alpha x) = \alpha (a^T x)$]

$$= \alpha \left(\frac{\mathbb{1}_n}{n}\right)^T x + \frac{\beta}{n} \cdot (\mathbb{1}_n)^T \cdot \mathbb{1}_n$$

$$= \alpha \operatorname{avg}(x) + \frac{\beta}{n} \cdot \|\mathbb{1}_n\|^2$$

$$= \alpha \operatorname{avg}(x) + \frac{\beta}{n} \cdot n$$

$$= \alpha \operatorname{avg}(x) + \beta.$$

(b) We know, $\operatorname{std}(x) = \frac{\|x - \operatorname{avg}(x) \cdot \mathbb{1}_n\|_2}{\sqrt{n}}$

To prove :- $\operatorname{std}(\alpha x + \beta \mathbb{1}_n) = |\alpha| \operatorname{std}(x)$

$$\operatorname{std}(\alpha x + \beta \mathbb{1}_n) = \frac{\|(\alpha x + \beta \mathbb{1}_n) - \operatorname{avg}(\alpha x + \beta \mathbb{1}_n) \cdot \mathbb{1}_n\|_2}{\sqrt{n}}$$

[using part (a)]

$$= \frac{\|\alpha x + \cancel{\beta \mathbb{1}_n} - \alpha \operatorname{avg}(x) \cdot \mathbb{1}_n - \cancel{\beta \mathbb{1}_n}\|_2}{\sqrt{n}}$$

$$= \frac{\|\alpha (x - \operatorname{avg}(x) \cdot \mathbb{1}_n)\|_2}{\sqrt{n}}$$

[by properties of norm, $\|\alpha x\| = |\alpha| \|x\|$]

$$= |\alpha| \frac{\|x - \operatorname{avg}(x) \cdot \mathbb{1}_n\|_2}{\sqrt{n}}$$

$$= |\alpha| \operatorname{std}(x)$$