Ashutosh Kumar Singh 19CS30008

Q4. $Ax = b$, $A \in R^{m \times n}$, $b \in R^n$.

We need to find $x \in R^n$

Let $A = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

We can write the matrix - vector product Ax as:-

$$Ax = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= x_1 \begin{bmatrix} | \\ a_1 \\ | \end{bmatrix} + x_2 \begin{bmatrix} | \\ a_2 \\ | \end{bmatrix} + \cdots + x_n \begin{bmatrix} | \\ a_n \\ | \end{bmatrix}$$

Thus, Ax can be viewed as a linear combination of the columns of A, and we want this to be equal to b.

Thus, Ax is a vector in the column space of A, so, we can say that if $b \in$ colspace(A), then there will exist atleast one solution of the equation $Ax = b$.

So, we have said that columns of $A = \{a_1, \ldots, a_m\}$ if they span R^m, then we will have a soln. because in that case we can represent b as a linear combination of columns of A.

For this linear combination to be unique, we can see that columns of A will have to form a basis of R^m.

For that to happen, we will also need $n = m$.
Thus, for uniqueness, ~~colspace (A)~~ columns of A must form a basis of colspace (A).

So, to summarize,

i) If $b \in$ colspace (A), then there exists a soln. of $Ax = b$.

ii) If columns of A form a basis of colspace (A), then this solution is unique.