Assignment 1

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Question 1

a.) Future Value of $amount$ after n years = amount (1 + $interest$) ** $years$

b.) APR = 0.04 quarterly compounding

periodic_rate = 0.04 / 4 = 0.01

future_value = $amount$ * (1 + periodic_rate) ** (4 * $years$)

c.) APR = 0.04 monthly compounding

periodic_rate = 0.04 / 12 = 0.00333

future_value = $amount$ * (1 + periodic_rate) ** (12 * $years$)

```
# question 1a
amount = 200000 # investment amount
n = 2 # number of years
interest = 0.04 # interest offered by the bank on the Certificate of Deposit
fv = amount * (1 + interest)** n
print("Future value of ${:.2f} after {} years if compunded annually is: ${:.2f}".format(amount, n, fv))
```

```
Future value of $200000.00 after 2 years if compunded annually is: $216320.00
```

```
# question 1b
apr = 0.04
compounding_period = "quarterly"
periodic_rate_quarterly = apr / 4  # for quarterly compuding frequency we divide the interest by 4
fv = amount * (1 + periodic_rate_quarterly) ** (4 * n)
print("Future value of ${:.2f} after {} years if compunded quarterly is: ${:.2f}".format(amount, n, fv))
```

```
Future value of $200000.00 after 2 years if compunded quarterly is: $216571.34
```

```
# question 1c
apr = 0.04
compounding_period = "monthly"
periodic_rate_quarterly = apr / 12 # for monthly compuding frequency we divide the interest by 12
fv = amount * (1 + periodic_rate_quarterly) ** (12 * n)
print("Future value of ${:.2f} after {} years if compunded quarterly is: ${:.2f}".format(amount, n, fv))
```

```
Future value of $200000.00 after 2 years if compunded quarterly is: $216628.59
```

Question 2

Annual earnings is $100,000 for the next 35 years. Savings are 45% of the earnings. Thus, effective annual invetsment amount is $45,000.

rate of interest is (r) = 2%

n = 35 years

a.) Future Value = amount * (((1 + r)**n - 1) / r)

```
# Question 2a

n = 35
r = 0.02
amount = 45000
fv = amount * (((1 + r)**n - 1) / r)
print("${:.2f} saved annually at a rate of {}% sums up to ${:.2f} after {} years".format(amount, r*100, fv, n))
```

```
$45000.00 saved annually at a rate of 2.0% sums up to $2249751.49 after 35 years
```

```
# Question 2b
```

Question 3

(a) $EAR = (1 + \frac{r}{n})^n - 1$

(b) $c = \frac{500000\frac{y}{12}}{1-\frac{1}{(1+\frac{y}{12})^{360}}} = 2108$

Noting that c is thr monthly payment and y equals 0.03.

(c) $fv = \frac{c}{\frac{y}{12}}[1 - \frac{1}{(1+\frac{y}{12})^{340}}]$

Noting that fv is the present value of the remaining loan after paying the 20th monthly payment.

```
# Question 3

# (a)
y=(1+0.03/12)**12-1
print(f"(a) The effective annual rate is {round(y, 4)}")
# (b)
y=0.03
c=500000*(y/12)/(1-1/((1+y/12)**360))
print(f"(b) The monthly payment is {round(c, 2)}")
# (c)
fv=c*(1-1/((1+y/12)**340))/(y/12)
print(f"(c) After the 20th monthly payment, I still owe the bank {round(fv, 2)}")
```

```
(a) The effective annual rate is 0.0304
(b) The monthly payment is 2108.02
(c) After the 20th monthly payment, I still owe the bank 482425.86
```

Question 4

(a) $p = \frac{100}{(1+0.03)^3} = 91.51$

(b) $p = \frac{1}{(1+0.02)} + \frac{101}{(1+0.025)^2} = 97.11$

(c) $p = \frac{13}{(1+0.02)} + \frac{13}{(1+0.025)^2} + \frac{13}{(1+0.03)^3} + \frac{113}{(1+0.035)^4} = 135.49$

```
#(a)
p1=100/(1+0.03)**3
print(f"(a) The price of the bond is {round(p1, 2)}")
```

```
(a) The price of the bond is 91.51
```

```
#(b)
p2=(1/1.02)+(101/1.025**2)
print(f"(a) The price of the bond is {round(p2, 2)}")
```

```
(a) The price of the bond is 97.11
```

```
#(c)
p3=(13/1.02)+(13/(1.025)**2)+(13/(1.03)**3)+(113/(1.035)**4)
print(f"(c) The price of the bond is {round(p3, 2)}")
```

```
(c) The price of the bond is 135.49
```

Question 5

(a) Present value of X :

$$pv_x = 98.98 = \frac{101.5}{1+\frac{sARPa}{2}}$$

Noting $sAPR_a$ the APR of X with semi annual coupounding :

$$sARP_a = 2(\frac{101.5}{98.98} - 1)$$

(b) Present value of Y :

$$pv_y = 98.59 = \frac{2}{(1+\frac{r_a}{2})^2} + \frac{102}{(1+\frac{r_b}{2})^2}$$

Then

$$r_b = 2(\sqrt{\frac{102}{98.59-\frac{2}{1+\frac{r_a}{2}}}} - 1)$$

```
# Question 5 - Numerical application

# (a)
r_a = 2*(101.5/98.98 - 1)
print(f"(a) APR with semi-annual coupounding is {round(r_a, 4)}")

# (b)
r_b = 2 * ( (102/(98.59 - 2/(1+r_a/2)))**0.5 - 1)
print(f"(b) APR with semi-annual coupounding is {round(r_b, 4)}")
```

```
(a) APR with semi-annual coupounding is 0.0509
(b) APR with semi-annual coupounding is 0.0547
```

Question 6

(a) Bond price:

$$p = \frac{5}{(1+0.05)} + \frac{5}{(1+0.06)^2} + \frac{105}{(1+0.07)^3} = 94.92$$

(b) Forward rate:

$$f_{01} = 0.05$$

$$f_{12} = \frac{(1+0.06)^2}{(1+0.05)} - 1 = 0.07$$

$$f_{23} = \frac{(1+0.07)^3}{(1+0.06)^2} - 1 = 0.09$$

(c) Guaranteed 3-year return:

We should use the forward rate from stage 1 to stage 2 and the forward rate from stage 2 to stage 3 to reinvest the coupons.

$$p = 5 * (1 + f_12) * (1 + f_23) + 5 * (1 + f_23) + 105$$

$$r = \frac{p-100}{100} = 0.1628$$

Noting that p is the final payoff at maturity and r is the return of the whole investment.

```
# (a)
P = 5 / (1 + 0.05) + 5 / (1 + 0.06) ** 2 + 105 / (1 + 0.07) ** 3
print(f"(a) Bond price is {round(P, 4)}")

from scipy.optimize import fsolve
y = fsolve(lambda y:5 / (1 + y) + 5 / (1 + y) ** 2 + 105 / (1 + y) ** 3 - P, 0.01)[0]
print(f"    Yield to maturity is {round(y, 4)}")

# (b)
f01 = 0.05
f12 = (1 + 0.06) ** 2 / (1 + 0.05) - 1
f23 = (1 + 0.07) ** 3 / (1 + 0.06) ** 2 - 1
print(f"(b) Forward rate from stage 0 to stage 1 is {round(f01, 4)}")
print(f"    Forward rate from stage 1 to stage 2 is {round(f12, 4)}")
print(f"    Forward rate from stage 2 to stage 3 is {round(f23, 4)}")

# (c)
p = 5 * (1 + f12) * (1 + f23) + 5 * (1 + f23) + 105
r = (p - 100) / 100
print(f"(c) Guaranteed 3-year return is {round(r, 4)}")
```

```
(a) Bond price is 94.9232
    Yield to maturity is 0.0693
(b) Forward rate from stage 0 to stage 1 is 0.05
    Forward rate from stage 1 to stage 2 is 0.0701
    Forward rate from stage 2 to stage 3 is 0.0903
(c) Guaranteed 3-year return is 0.1628
```