

Fixed Income Homework2

In []: Name: Yuetong Li, Zechen Liu, Jessica Ghai, Ashutosh Ikade

```
In [145... import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
import statsmodels.api as sm
```

```
In [148... file_path='/Users/liyuetong/Desktop/Fixed Income Markets HW2(1:23 11pm due)/Hor
strips=pd.read_excel(file_path,sheet_name='STRIPS')
tnote=pd.read_excel(file_path,sheet_name='T-Note')
```

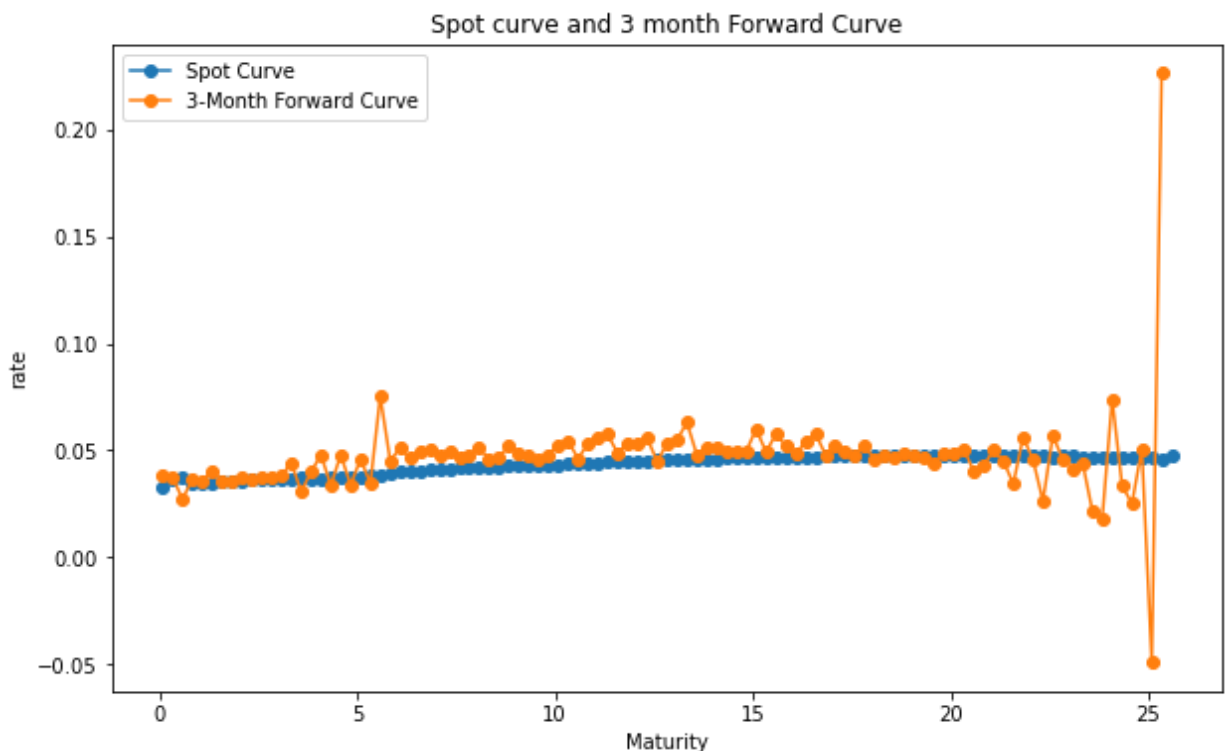
Question1:

```
In [150... strips['discount']=strips['Price']/100
```

```
In [151... strips['spot']=(1/(strips['discount'])*(1/(2*strips['Maturity']))-1)*2
```

```
In [152... strips['forward']=((strips['discount']/strips['discount'].shift(-1))-1)*4
```

```
In [153... plt.figure(figsize=(10, 6))
plt.plot(strips['Maturity'],strips['spot'],label='Spot Curve',marker='o')
plt.plot(strips['Maturity'],strips['forward'],label='3-Month Forward Curve',ma
plt.title('Spot curve and 3 month Forward Curve')
plt.xlabel('Maturity')
plt.ylabel('rate')
plt.legend()
plt.show()
```



Question2:

```
In [154... reg=pd.DataFrame()
reg['T']=strips['Maturity']
reg['T^2']=strips['Maturity']**2
reg['T^3']=strips['Maturity']**3
reg['T^4']=strips['Maturity']**4
reg['T^5']=strips['Maturity']**5
reg['Log_D']=np.log(strips['discount'])
```

```
In [155... X=reg[['T','T^2','T^3','T^4','T^5']]
model=sm.OLS(reg['Log_D'],X)
result=model.fit()
result.summary()
```

Out [155]:

OLS Regression Results

Dep. Variable:	Log_D	R-squared (uncentered):	1.000
Model:	OLS	Adj. R-squared (uncentered):	1.000
Method:	Least Squares	F-statistic:	6.635e+05
Date:	Sun, 21 Jan 2024	Prob (F-statistic):	3.01e-220
Time:	22:19:10	Log-Likelihood:	430.35
No. Observations:	103	AIC:	-850.7
Df Residuals:	98	BIC:	-837.5
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
T	-0.0326	0.001	-36.890	0.000	-0.034	-0.031
T^2	-0.0011	0.000	-3.780	0.000	-0.002	-0.001
T^3	-1.981e-05	3.16e-05	-0.628	0.532	-8.24e-05	4.28e-05
T^4	2.824e-06	1.44e-06	1.956	0.053	-4.1e-08	5.69e-06
T^5	-4.682e-08	2.33e-08	-2.011	0.047	-9.3e-08	-6.12e-10

Omnibus:	47.731	Durbin-Watson:	2.038
Prob(Omnibus):	0.000	Jarque-Bera (JB):	2709.260
Skew:	-0.141	Prob(JB):	0.00
Kurtosis:	28.124	Cond. No.	8.38e+06

Notes:

[1] R^2 is computed without centering (uncentered) since the model does not contain a constant.

[2] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[3] The condition number is large, 8.38e+06. This might indicate that there are strong multicollinearity or other numerical problems.

```
In [156... print('The estimate coefficients a,b,c,d,e are:', '\n', result.params)
```

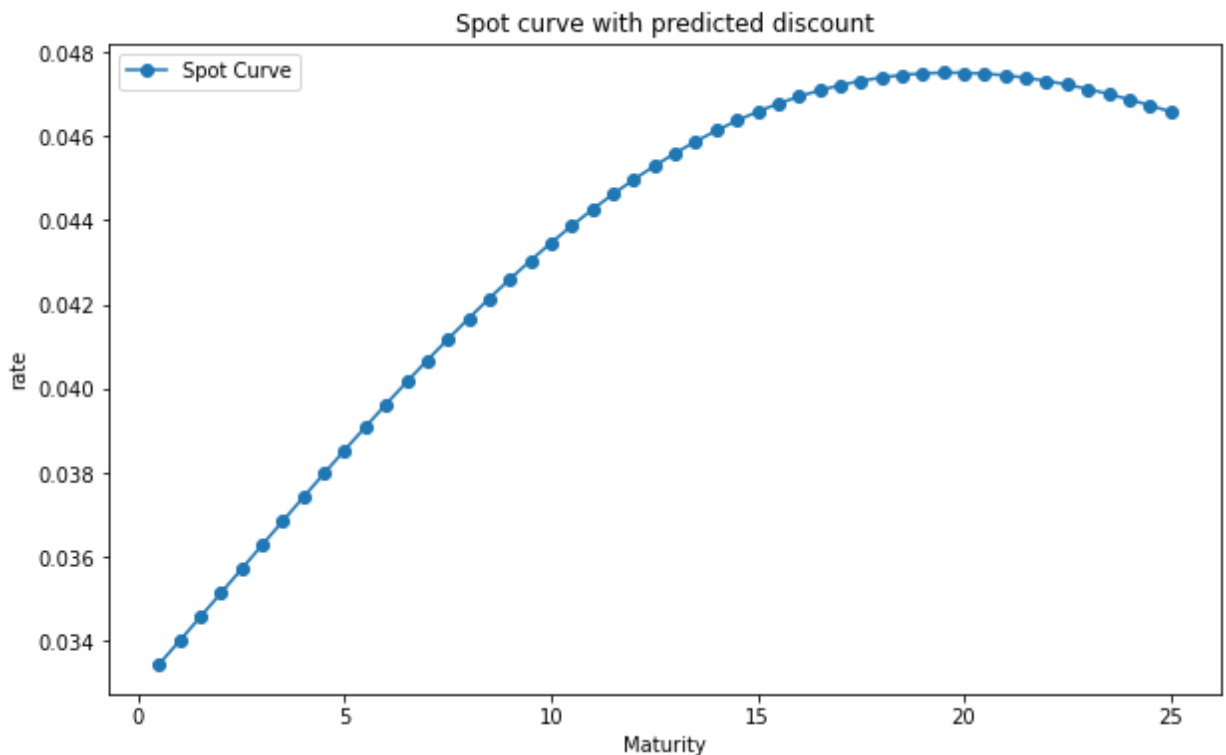
The estimate coefficients a,b,c,d,e are:

```
T      -3.262805e-02
T^2    -1.074704e-03
T^3    -1.981201e-05
T^4     2.823672e-06
T^5    -4.681743e-08
dtype: float64
```

Question3:

```
In [157... coef=result.params
T=np.arange(0,25.1,0.5)
prediction=pd.DataFrame()
prediction['T']=T
prediction['Predict_D']=np.exp(coef[0]*T+coef[1]*T**2+coef[2]*T**3+coef[3]*T**4)
prediction['spot']=(1/(prediction['Predict_D']))*(1/(2*prediction['T']))-1)*2
```

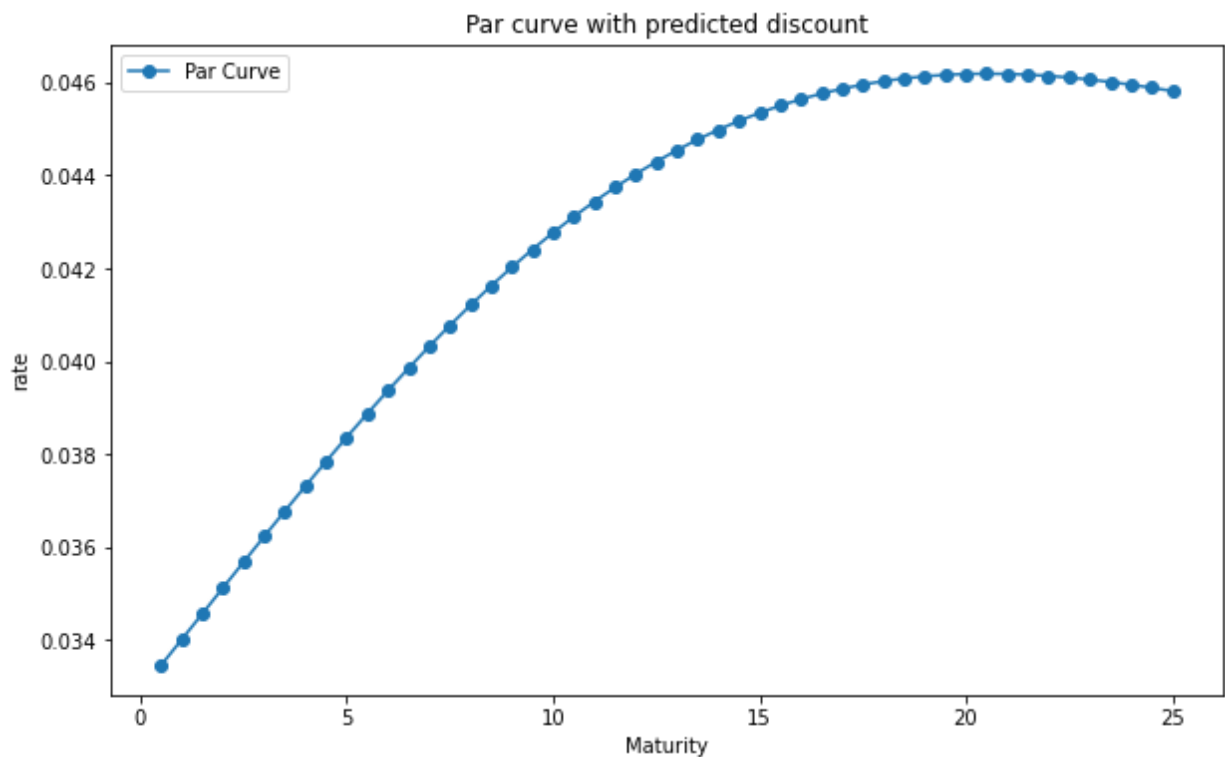
```
In [158... plt.figure(figsize=(10, 6))
plt.plot(prediction['T'].iloc[1:],prediction['spot'].iloc[1:],label='Spot Curve')
plt.title('Spot curve with predicted discount')
plt.xlabel('Maturity')
plt.ylabel('rate')
plt.legend()
plt.show()
```



Question4:

```
In [160... prediction=prediction.iloc[1:]
prediction['par']=2*(1-1*prediction['Predict_D'])/prediction['Predict_D'].cumsum()
```

```
In [161... plt.figure(figsize=(10, 6))
plt.plot(prediction['T'],prediction['par'],label='Par Curve',marker='o')
plt.title('Par curve with predicted discount')
plt.xlabel('Maturity')
plt.ylabel('rate')
plt.legend()
plt.show()
```

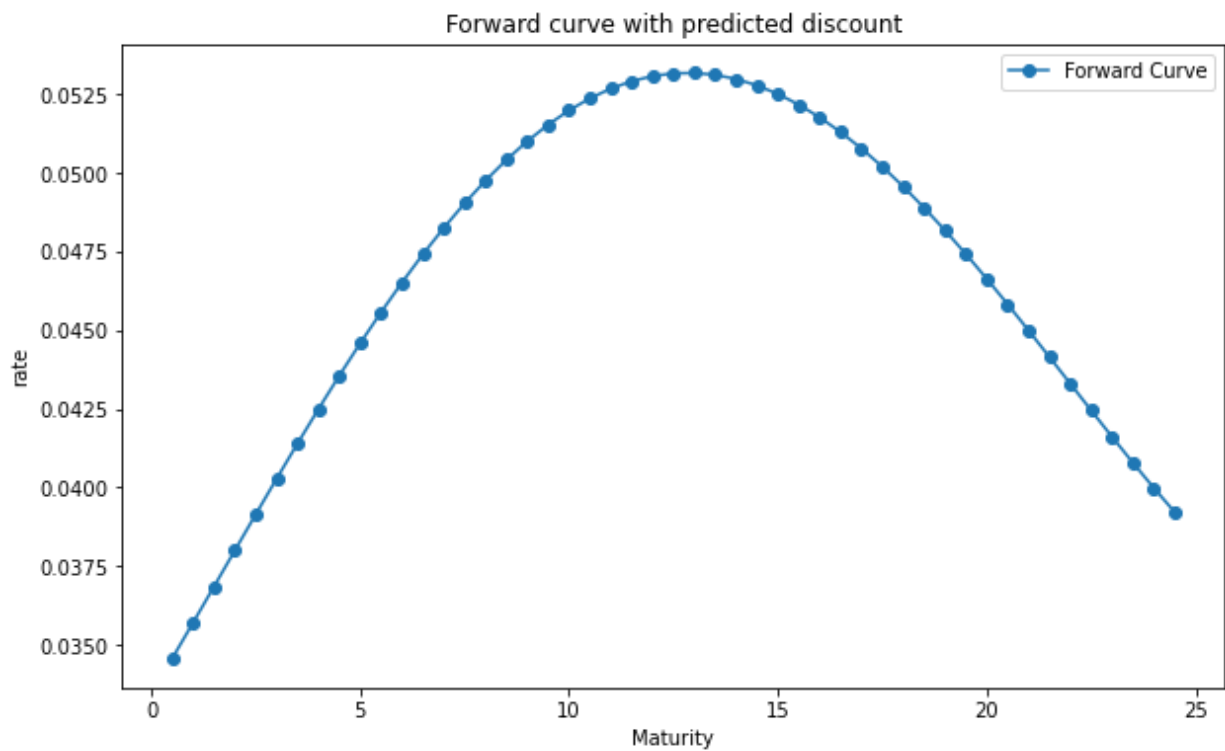


Question5:

```
In [162... prediction['forward']=((prediction['Predict_D']/prediction['Predict_D'].shift(.

```

```
In [163... plt.figure(figsize=(10, 6))
plt.plot(prediction['T'],prediction['forward'],label='Forward Curve',marker='o')
plt.title('Forward curve with predicted discount')
plt.xlabel('Maturity')
plt.ylabel('rate')
plt.legend()
plt.show()
```



Question6:

In [165...

```
reg2=pd.DataFrame()
reg2['Yield']=tnote['Yield']
reg2['T']=tnote['Maturity']
reg2['T^2']=tnote['Maturity']**2
reg2['T^3']=tnote['Maturity']**3
reg2['T^4']=tnote['Maturity']**4
reg2['T^5']=tnote['Maturity']**5
reg2['intercept']=1
```

In [166...

```
X2=reg2[['intercept','T','T^2','T^3','T^4','T^5']]
model2=sm.OLS(reg2['Yield'],X2)
result2=model2.fit()
result2.summary()
```

Out[166]:

OLS Regression Results

Dep. Variable:	Yield	R-squared:	0.989
Model:	OLS	Adj. R-squared:	0.989
Method:	Least Squares	F-statistic:	1199.
Date:	Sun, 21 Jan 2024	Prob (F-statistic):	8.18e-62
Time:	22:19:55	Log-Likelihood:	104.77
No. Observations:	70	AIC:	-197.5
Df Residuals:	64	BIC:	-184.1
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
intercept	2.5944	0.034	76.748	0.000	2.527	2.662
T	0.5127	0.029	17.951	0.000	0.456	0.570
T^2	-0.0794	0.007	-10.861	0.000	-0.094	-0.065
T^3	0.0065	0.001	8.576	0.000	0.005	0.008
T^4	-0.0003	3.45e-05	-7.298	0.000	-0.000	-0.000
T^5	3.57e-06	5.6e-07	6.370	0.000	2.45e-06	4.69e-06

Omnibus:	54.065	Durbin-Watson:	0.476
Prob(Omnibus):	0.000	Jarque-Bera (JB):	397.444
Skew:	-2.025	Prob(JB):	4.97e-87
Kurtosis:	13.948	Cond. No.	1.60e+07

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

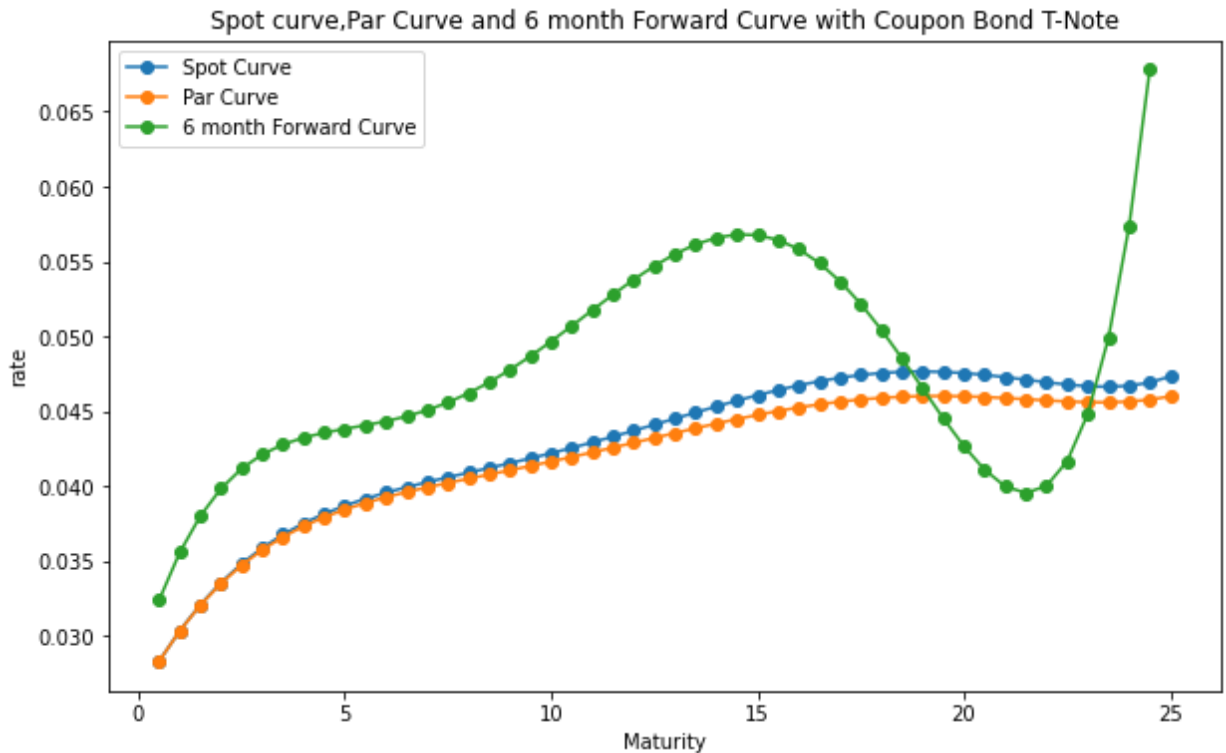
[2] The condition number is large, 1.6e+07. This might indicate that there are strong multicollinearity or other numerical problems.

```
In [167... coef=result2.params
T=np.arange(0.5,25.1,0.5)
prediction2=pd.DataFrame()
prediction2['T']=T
prediction2['intercept']=1
prediction2['Predict_Y']=coef[0]*prediction2['intercept']+coef[1]*T+coef[2]*T**2
```

```
In [168... prediction2['discount'] = 0
prediction2.loc[0,'discount']=100/(100 + prediction2.loc[0, 'Predict_Y'] / 2)
for i in range(1, len(prediction2['discount'])):
    prediction2.loc[i, 'discount'] = (100 - prediction2.loc[i, 'Predict_Y'] / 2)
```

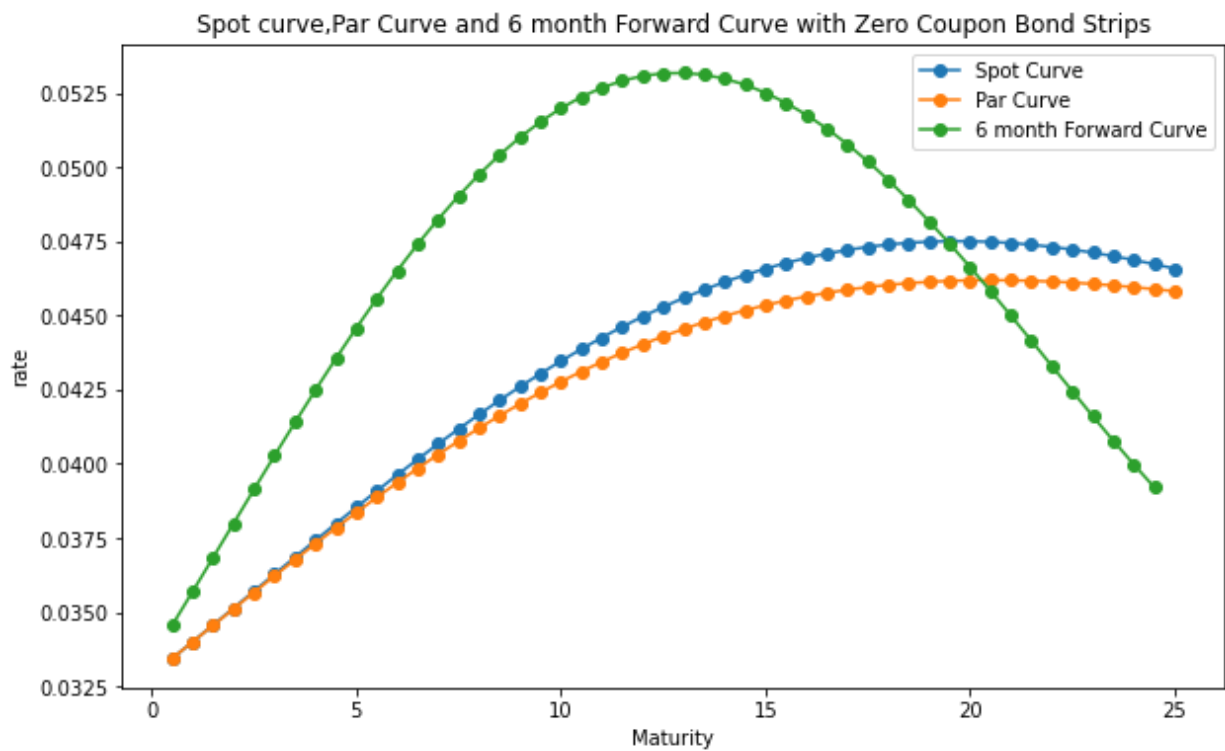
```
In [169... prediction2['spot']=(1/(prediction2['discount']))*(1/(2*prediction2['T']))-1)*
prediction2['par']=2*(1-1*prediction2['discount'])/prediction2['discount'].cum
prediction2['forward']=((prediction2['discount']/prediction2['discount']).shift
```

```
In [172... plt.figure(figsize=(10, 6))
plt.plot(prediction2['T'],prediction2['spot'],label='Spot Curve',marker='o')
plt.plot(prediction2['T'],prediction2['par'],label='Par Curve',marker='o')
plt.plot(prediction2['T'],prediction2['forward'],label='6 month Forward Curve'
plt.title('Spot curve,Par Curve and 6 month Forward Curve with Coupon Bond T-No
plt.xlabel('Maturity')
plt.ylabel('rate')
plt.legend()
plt.show()
```



Question7:

```
In [171... plt.figure(figsize=(10, 6))
plt.plot(prediction['T'],prediction['spot'],label='Spot Curve',marker='o')
plt.plot(prediction['T'],prediction['par'],label='Par Curve',marker='o')
plt.plot(prediction['T'],prediction['forward'],label='6 month Forward Curve',ma
plt.title('Spot curve,Par Curve and 6 month Forward Curve with Zero Coupon Bond
plt.xlabel('Maturity')
plt.ylabel('rate')
plt.legend()
plt.show()
```

Observation: Forward curve is different between coupon bond and zero coupon bond while the par curve and spot curve look pretty similar

Reason: The forward curve correlated with current yield curve at different points in time. The difference in the forward curves between coupon bonds and zero-coupon bonds caused by that coupon bonds have periodic coupon payments, while zero-coupon bonds only have a single cash flow at maturity.