Katholieke Universiteit Leuven

Design and Hedging of a Structured Product - Bonus Certificate (Costco)

Financial Engineering

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1 Introduction

This report presents the design, pricing, and hedging of a Bonus Certificate linked to Costco (COST) stock - a structured product, offering capped upside and conditional downside protection through a predefined bonus level and barrier. The product is targeted at investors with a moderately bullish or range-bound short-term outlook, while also enabling the issuing bank to generate a transparent, risk-managed margin. Pricing is based on the Bates model, which extends the Heston framework by incorporating jump processes to better reflect volatility smiles and tail risk. A two-stage calibration is performed: first in multiple maturities to estimate structural market dynamics, and then to refine them for short-term accuracy at the 3-month product horizon. Monte Carlo simulation is employed for pricing the path-dependent payoffs. The report outlines the full methodology, from data filtering and model calibration to exotic valuation and hedging. The full technical implementation is provided.[1]

2 Data

2.1 Reference Date and Data Sources

The reference date selected for the analysis is April 1, 2025. Option chain data for Costco (COST) stock was retrieved using `yfinance`, a Python interface to Yahoo Finance. On the reference date, the observed spot price was $S_0 = \$954.00$.

2.2 Dividend Yield Estimation

The continuous dividend yield q is estimated based on the most recent four quarterly dividend payments, resulting in a total annualized dividend amount:

$$D = 4.78 \text{ USD}$$

[1] All modeling and simulation code is available at: https://github.com/ashutoshjha3103/exotic_option_pricing_fe2025

Accordingly, the dividend yield is computed as:

$$q = \frac{D}{S_0} = \frac{4.78}{954.00} \approx 0.005 \quad (0.5\%)$$

2.3 Interest Rate Assumption

Although interest rates can be implied from option market data using put-call parity, such estimates are often unstable due to market imperfections such as bid-ask spreads, stale quotes, or illiquid contracts. The implied interest rate r_{impl} for a given strike is computed as:

$$r_{\text{impl}} = -\frac{1}{T} \ln\left(\frac{K}{S_0} \cdot \frac{C-P}{e^{-qT}}\right)$$

where C and P represent the market prices of call and put options at strike K, and q is the dividend yield. Using strikes within $\pm 10\%$ of the spot price, average implied rates were calculated as shown in Table 1.

Table 1: Implied vs. US Treasury Bill Interest Rates

Maturity (T)	Implied Rate from Parity	Treasury Yield (Used)
0.25 years	24.8%	4.22%
0.50 years	16.1%	4.28%
1.00 years	10.7%	4.05%

Given the unrealistic nature of these implied rates, U.S. Treasury yields were used instead as more reliable and representative estimates of the risk-free rate.

2.4 No-Arbitrage Filtering and Dataset Summary

To ensure consistency and filter out mispriced options, a structured no-arbitrage filtering routine was applied across maturities $T = 0.25,\ 0.50,\ 1.00$. Liquidity screening was first applied to retain only options with nonzero volume or open interest. Monotonicity was then enforced: call prices had to decrease and put prices increase with strike, within a $0.50 tolerance. Convexity was checked using the butterfly condition:

$$\pi_C(K') \leq \frac{1}{2}[\pi_C(K) + \pi_C(K'')], \quad \text{for } K < K' < K''$$

To detect pricing inconsistencies between matched call-put pairs, a relaxed version of put-call parity was used. For each strike K, the following tolerance-based condition had to hold:

$$\frac{\left|\pi_C(K,T) - \pi_P(K,T) - \left(S_0 e^{-qT} - K e^{-rT}\right)\right|}{|S_0 e^{-qT} - K e^{-rT}|} < 15\%$$

This threshold accounts for bid-ask spreads and market noise. Only strikes passing all checks were retained, and only out-of-the-money (OTM) options were used for model calibration, as summarized in Table 6 in the Appendix.

3 Valuation

3.1 Bates Model with FFT and Carr-Madan Formula

The Bonus Certificate is priced under the Bates model, which extends the Heston stochastic volatility framework by incorporating a jump-diffusion component in the style of Merton. This model captures volatility smiles and fat tails commonly observed in equity options markets. The formulation is attributed to Bates [1]. The asset dynamics are governed by:

$$dS_t = (r - \lambda \mathbb{E}[J-1]) S_t\, dt + \sqrt{v_t} S_t\, dW_t^S + (J-1) S_t\, dN_t$$

$$dv_t = \kappa(\theta - v_t)\, dt + \sigma\sqrt{v_t}\, dW_t^v, \quad \text{Corr}(dW_t^S, dW_t^v) = \rho$$

where N_t is a Poisson process with intensity λ, and jump sizes $J \sim \log\mathcal{N}(\mu_J, \sigma_J^2)$. European option prices under this model are computed using the Carr-Madan FFT method, which enables efficient pricing over a dense strike grid. A comparison with Black-Scholes prices at the money is provided in Appendix Table 7, serving as a sanity check for the numerical implementation.

3.2 Calibration

To ensure accurate pricing of the Bonus Certificate, Bates model parameters are calibrated in two stages. A joint calibration is first performed using vanilla options across maturities

$T = 0.25, 0.50, 1.00$ to estimate structural features of the market - such as mean-reversion (κ), long-run variance (θ), vol-of-vol (σ), correlation (ρ). These parameters are considered long-term and are later used as tightly bounded anchors in the short-term calibration. The final product is designed with a 3-month maturity to cater to investor who prefer short-term exposure and to suit Costco's relatively stable, liquid equity profile, which supports daily hedging and efficient pricing. Investors benefit from a capped upside and conditional downside protection over a short horizon, while the issuer (the bank) can manage risk effectively and retain a well-defined margin. The second calibration stage focuses exclusively on $T = 0.25$ data to improve local fit. Short-term variance v_0 and other near-term dynamics are allowed more flexibility to better reflect the skew and smile that directly impact exotic pricing.

Joint Calibration Across All Maturities

The jointly calibrated parameters, which provide a long-run structural profile, are presented in Table 2.

Table 2: Joint Calibration Parameters (T = 0.25, 0.50, 1.00)

Parameter	Value	Interpretation
v_0	0.0552	Initial variance
κ	0.4000	Mean-reversion speed
θ	0.3020	Long-run variance
σ	1.5000	Vol-of-vol
ρ	-0.6391	Correlation (price-vol)
λ	0.5000	Jump intensity
μ_J	0.0400	Jump mean (log)
σ_J	0.1500	Jump standard deviation

A final loss value of 1647.05 is obtained for the joint calibration. As shown in Figure 1, the Bates model captures the general shape of market call prices across maturities, with particularly close alignment in far out-of-the-money (OTM) call regions. However, this fit is somewhat deceptive - deep OTM options have low absolute values, and small price differences appear minor. And for at-the-money (ATM) and in-the-money (ITM) ranges, visible pricing gaps remain, showing the limits of a single global parameter set.

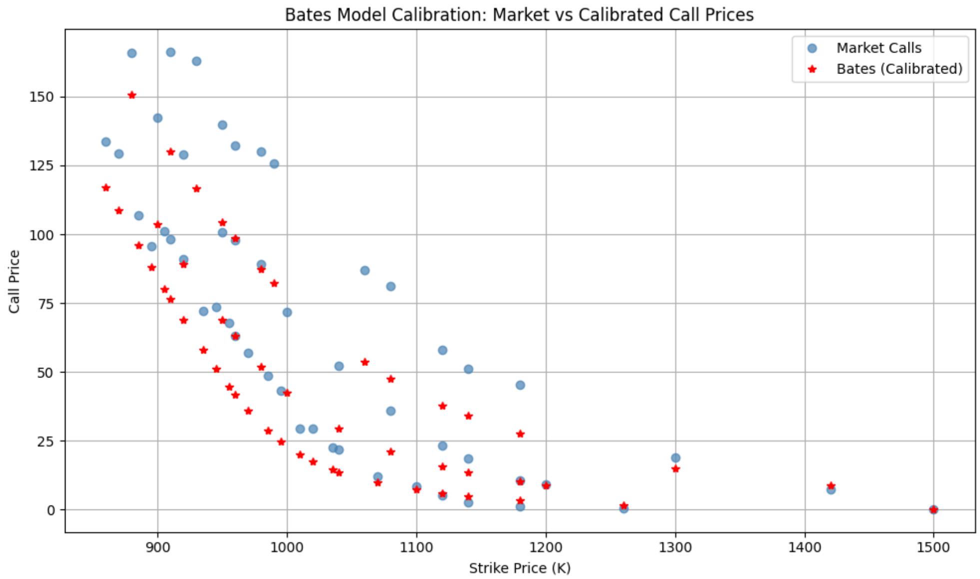

Figure 1: Joint Calibration: Bates vs Market Call Prices

Residuals in Figure 2 show systematic underpricing of OTM puts, especially at lower strikes - critical for exotic payoff structures like Bonus Certificates that are sensitive to downside risk. Residuals on call options exhibit clustering near specific strike bands. This behavior is problematic for pricing accuracy.

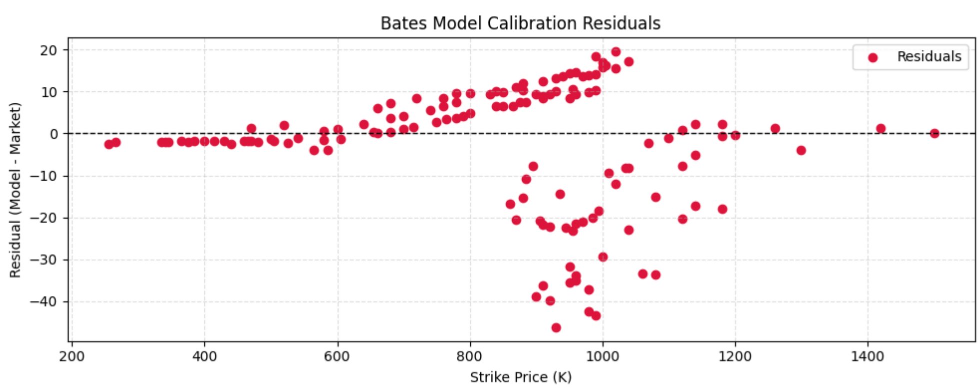

Figure 2: Joint Calibration Residuals Across Maturities

Additional calibration diagnostics in the Appendix reinforce these points. Figure 7 shows persistent discrepancies in OTM put pricing. The volatility smiles in Figures 8 - 10 reveal that while the model captures skew directionally, it fails to recover the full convexity of market smiles. These findings motivate further fine-tuning with short-term calibration tailored to the 3-month maturity of the product.

Fine Tuning / Short-Term Calibration for $T = 0.25$

To address the shortcomings identified in the joint calibration - especially the underpricing of downside risk - a dedicated short-term calibration is performed using only the $T = 0.25$ data. This step is motivated by our choice of offering a product with 3-month maturity, where precise local fit is critical. Long term parameters from the joint calibration are used as tightly bounded anchors (initial guesses) to preserve structural consistency, while short-horizon variables such as v_0, σ_J, λ, and μ_J are allowed to vary more freely.

The resulting parameter estimates are summarized in Table 3.

Table 3: Short-Term Calibration Parameters (T = 0.25)

Parameter	**Value**	**Interpretation**
v_0	0.0156	Lower short-term variance
κ	0.6000	Faster mean-reversion
θ	0.4000	Higher long-run variance
σ	1.0000	Reduced vol-of-vol
ρ	-0.5500	Price-vol correlation
λ	0.7000	Increased jump frequency
μ_J	0.0460	Slightly positive jump mean
σ_J	0.1127	Reduced jump uncertainty

The short-term calibration achieves a lower loss of 159.60, reflecting a significant improvement in fit over the joint calibration. The comparison between market and model prices for both calls and puts in Figure 3 shows tight alignment, particularly in the OTM Put region where the joint calibration underperformed. Additional call price comparisons are provided in Appendix Figure 11.

Residuals shown in Figure 4 further confirm that the model now captures left-tail risk more effectively. The downward bias in OTM puts observed previously has been corrected, and

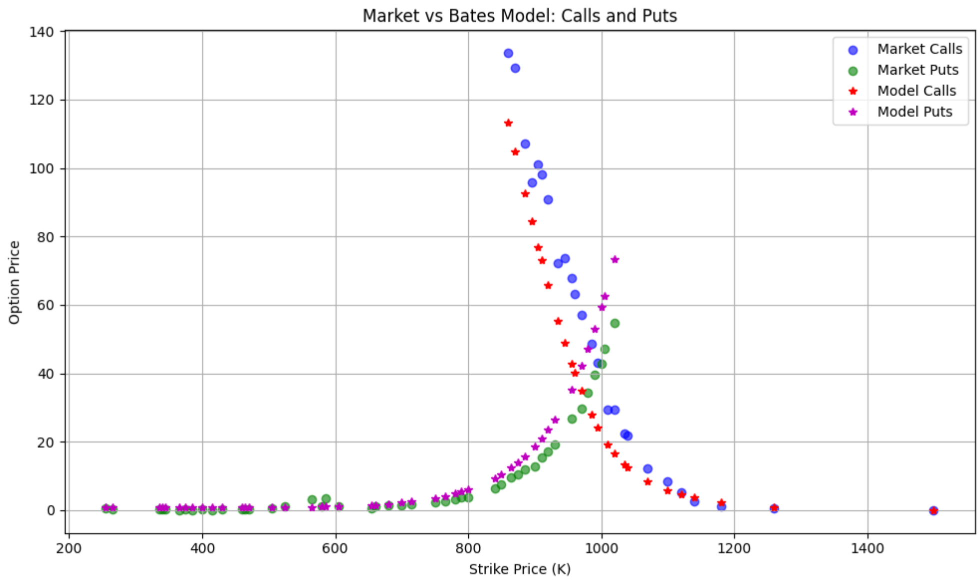

Figure 3: Short-Term Calibration: Market vs Bates Model (Calls and Puts)

the residuals are more symmetrically distributed.

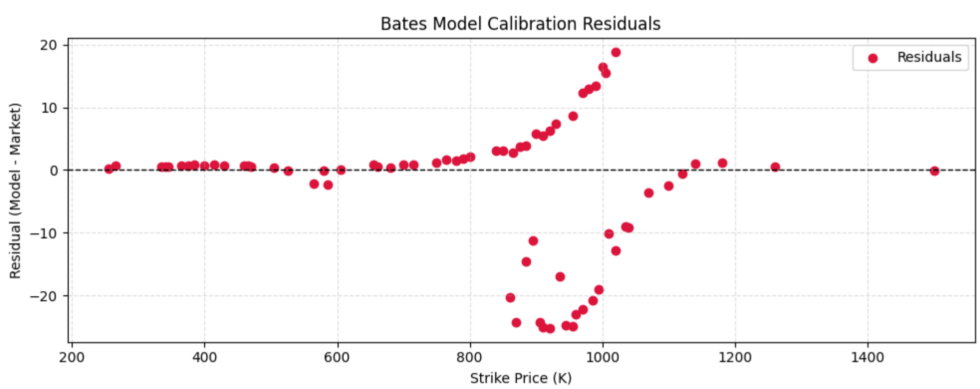

Figure 4: Short-Term Calibration Residuals (T = 0.25)

Lastly, the refined volatility smile in Figure 5 illustrates that the Bates model now successfully captures both skew and convexity for the 3-month maturity. This improvement is essential for accurately pricing products such as Bonus Certificates that are sensitive to downside movements and the local volatility surface.

3.3 Exotic Product: Bonus Certificate - Costco

The Bonus Certificate presented in this report is linked to Costco (COST) stock and has a maturity of 3 months ($T = 0.25$ years). It is structured to provide capped upside while

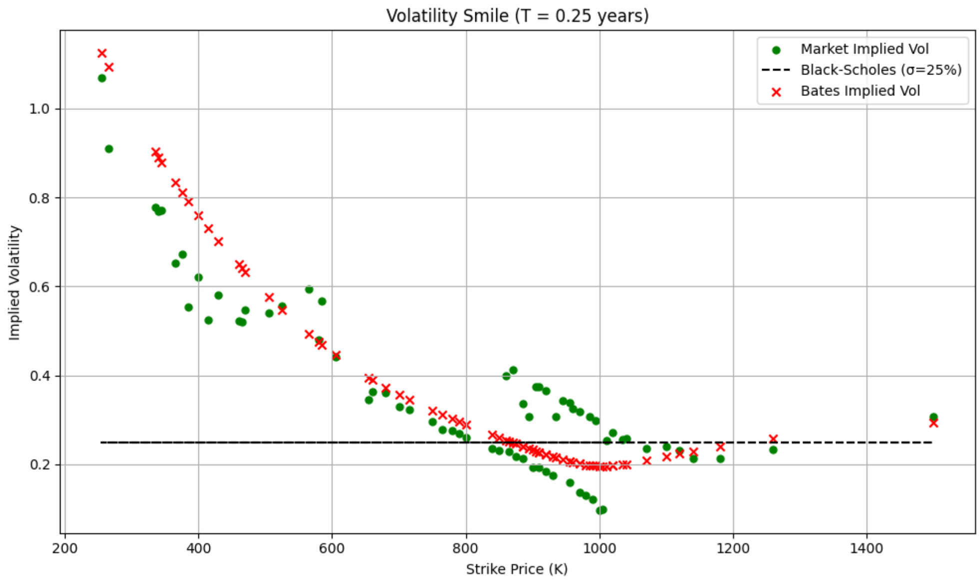

Figure 5: Implied Volatility Smile at T = 0.25 (Refined Calibration)

offering conditional protection against downside risk, depending on the stock's path. If the stock price ends above the bonus level or breaches the barrier at any point, the final payoff equals S_T. Otherwise, if the price stays above the barrier and ends below the bonus level, a fixed bonus payout is delivered. Formally, the payoff Payoff_{BC} is defined as:

$$\text{Payoff}_{BC} = \begin{cases} S_T, & \text{if } S_T \geq B \text{ or } \min_{t\in[0,T]} S_t < H \\ B, & \text{if } H \leq \min_{t\in[0,T]} S_t \leq S_T < B \end{cases}$$

The product specifications are summarized in Table 4.

Table 4: Bonus Certificate Product Parameters

Parameter	**Value**
Spot Price S_0	\$954.00
Date Offered	April 1, 2025
Bonus Level B	\$975.00
Barrier Level H	\$600.00
Maturity T	0.25 years (3 months)
Model Price	\$991.44
Sale Price	\$1015.00
Bank Margin	\$23.56 (2.38%)

Pricing is performed using Monte Carlo simulation with 100,000 asset paths and 252 time steps, under the refined short-term calibrated Bates model. Simulated asset trajectories illustrating the stochastic nature of returns can be found in Appendix Figure 12.

The payoff distribution is shown in Figure 6. Most scenarios cluster around the bonus level, reflecting stable market performance. The right tail reflects full equity participation, while the left tail represents rare losses due to barrier breaches.

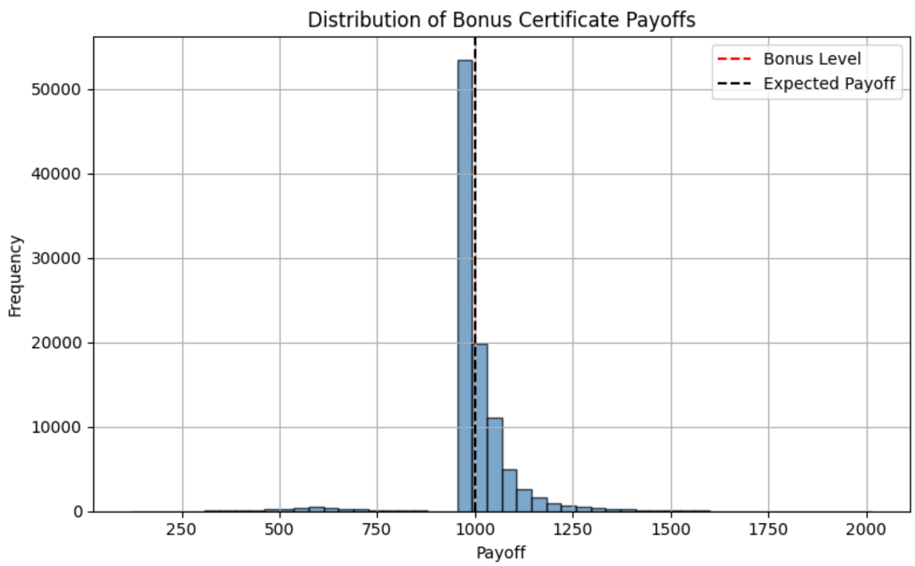

Figure 6: Histogram of Simulated Payoffs - Bonus Certificate

Why buy this exotic product? For investors, the Bonus Certificate allows limited equity participation in upward moves, while offering conditional protection if the barrier remains untouched. This makes the product suitable for clients with a stable or moderately bullish outlook on Costco's short-term performance, a reasonable outlook for Costco.

Why issue this exotic product? From the issuer's perspective, the structure provides a modest and transparent margin of $23.56 (2.38%) over the model price. This markup reflects standard practice and helps cover model and operational risk.

4 Hedging

As the issuing bank is short the Bonus Certificate at inception, risk exposure arises due to potential payouts linked to the underlying asset's performance. To mitigate directional exposure, a delta-hedging strategy is implemented at time $t = 0$. The delta of the Bonus Certificate is computed numerically using a central finite difference approximation on the Monte Carlo pricing engine. Let Δ denote the sensitivity of the product price with respect to the spot price S_0. The approximation is given by:

$$\Delta \approx \frac{V(S_0 + \varepsilon) - V(S_0 - \varepsilon)}{2\varepsilon}$$

where ε is a small perturbation, and $V(\cdot)$ denotes the simulated price of the Bonus Certificate. Using this method with $\varepsilon = 1$, a delta of approximately $\Delta = 0.6448$ is obtained.

The hedging parameters are summarized in Table 5.

Table 5: Delta Hedging Summary (at $t = 0$)

Parameter	**Value**
Notional Value	$1,000,000
Spot Price S_0	$954.00
Computed Delta Δ	0.6448
Finite Difference Step ε	1.00
Hedge Position (Shares)	676

Based on the computed delta, the bank takes a long position in 676 shares of Costco to hedge the short exposure embedded in the Bonus Certificate. This hedge partially offsets the sensitivity of the product's payoff to fluctuations in the underlying stock.

The delta hedge provides protection against small, continuous price movements but does not fully capture path-dependence or jump risk. It is therefore partial and may require dynamic rebalancing. Still, it allows the bank to manage first-order risk.

5 Conclusion

The Bates model provides a flexible and realistic framework for pricing structured products by accounting for both stochastic volatility and jump risks, features commonly observed in equity markets. The Bonus Certificate built on this foundation offers investors an appealing risk-return profile, combining limited downside exposure with the potential for enhanced returns in stable or moderately bullish markets. From the bank's perspective, risk exposure is actively managed through delta hedging and conservative product design, ensuring that profitability is achieved without assuming excessive market risk.

References

[1] Bates, D. S. (1996). Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche Mark options. *The Review of Financial Studies*, 9(1), 69-107.

Appendix

This appendix contains supplementary figures and tables that support the analysis and results discussed in the main body of the report. These materials provide additional insights into model diagnostics, calibration performance, and simulation behavior.

Table 6: Number of clean option datapoints retained per maturity after Arbitrage Checks

Maturity (T)	OTM Calls Retained	OTM Puts Retained
0.25 years	70 / 176	49 / 167
0.50 years	25 / 69	26 / 50
1.00 years	25 / 66	22 / 52

The results in Table 7 confirm that Bates prices fall within expected ranges w.r.t. Black Scholes prices, reflecting the model's incorporation of volatility skew and jump risk.

Table 7: Sanity Check: Bates vs Black-Scholes Pricing (ATM, K = 954.40)

Maturity (T)	Bates Call	BS Call
0.25 years	\$37.51	\$51.83
0.50 years	\$51.46	\$75.71
1.00 years	\$69.31	\$110.30

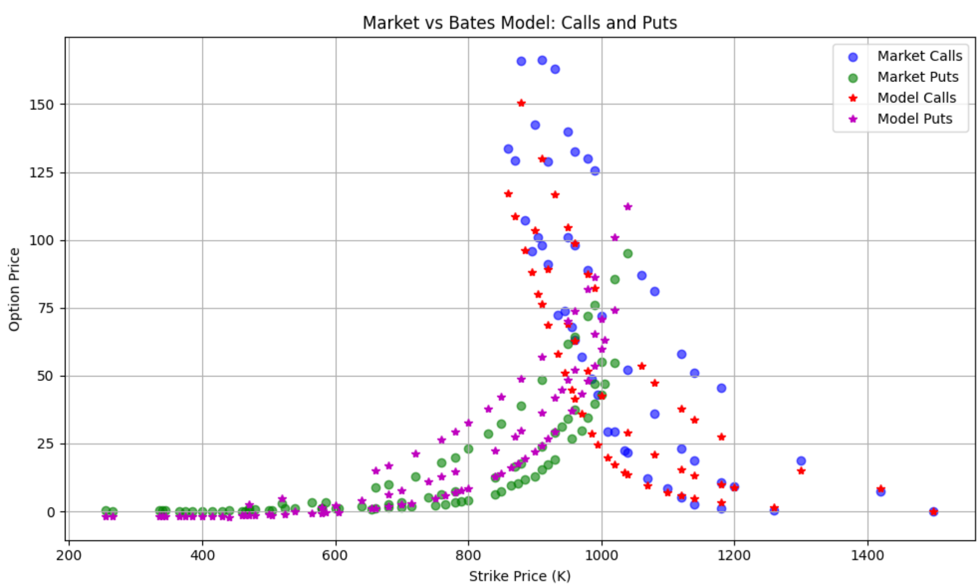

Figure 7: Market vs Bates Model (Calibrated on ALL Maturities): Call and Put Prices

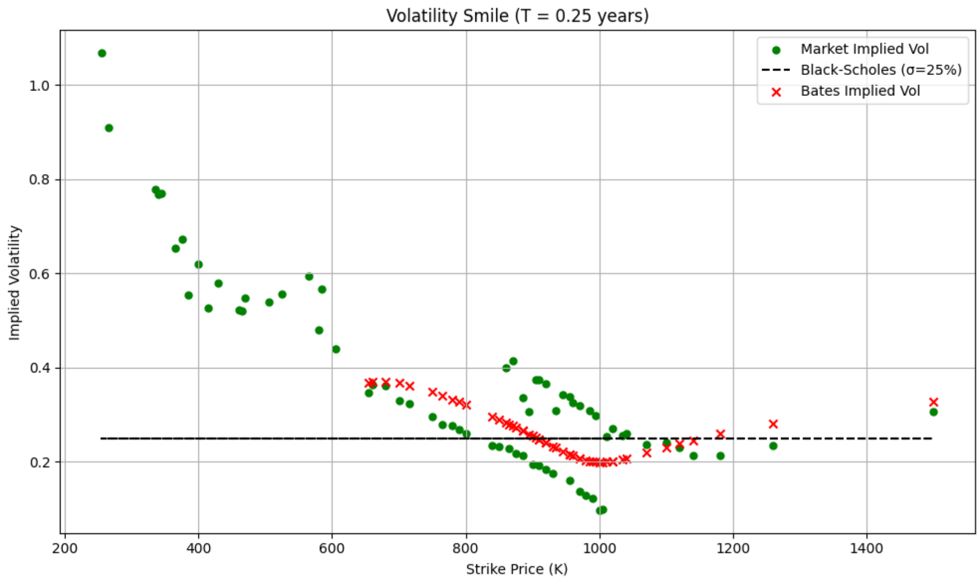

Figure 8: Volatility Smile for 3-month Maturity with Bates Model Parameters Jointly Calibrated on ALL Maturities

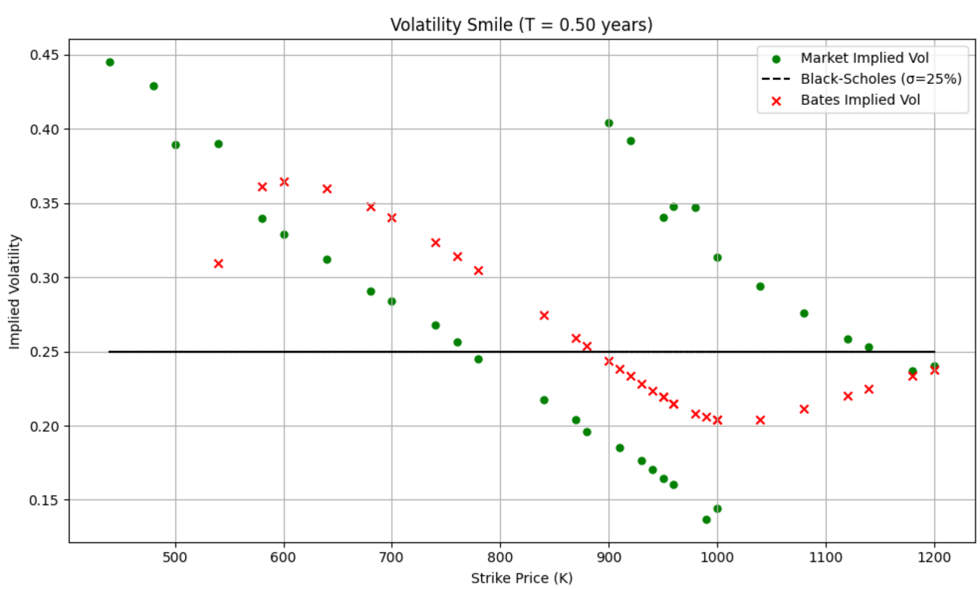

Figure 9: Volatility Smile for 6-month Maturity with Bates Model Parameters Jointly Calibrated on ALL Maturities

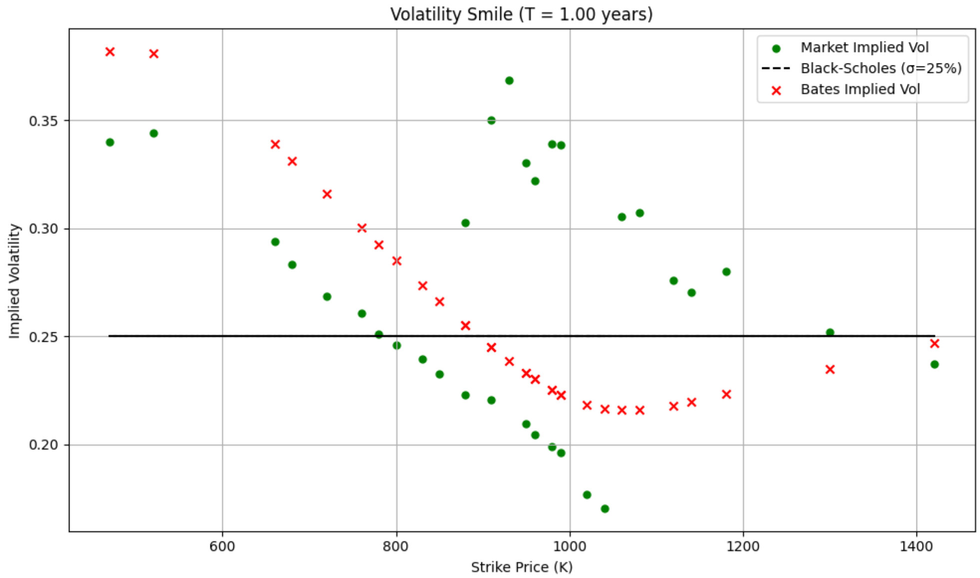

Figure 10: Volatility Smile for 12-month Maturity with Bates Model Parameters Jointly Calibrated on ALL Maturities

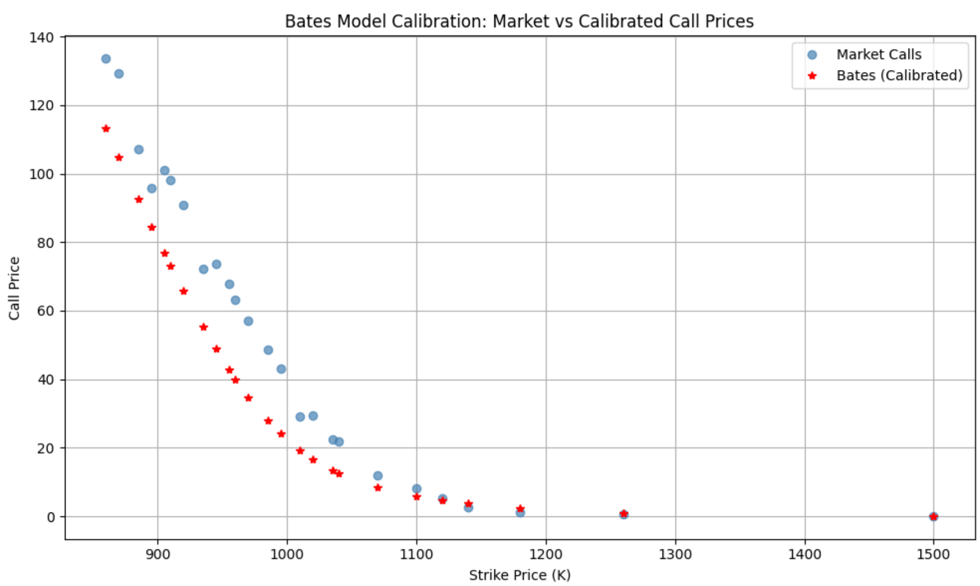

Figure 11: Short-Term Calibration: Bates vs Market Call Prices (T = 0.25)

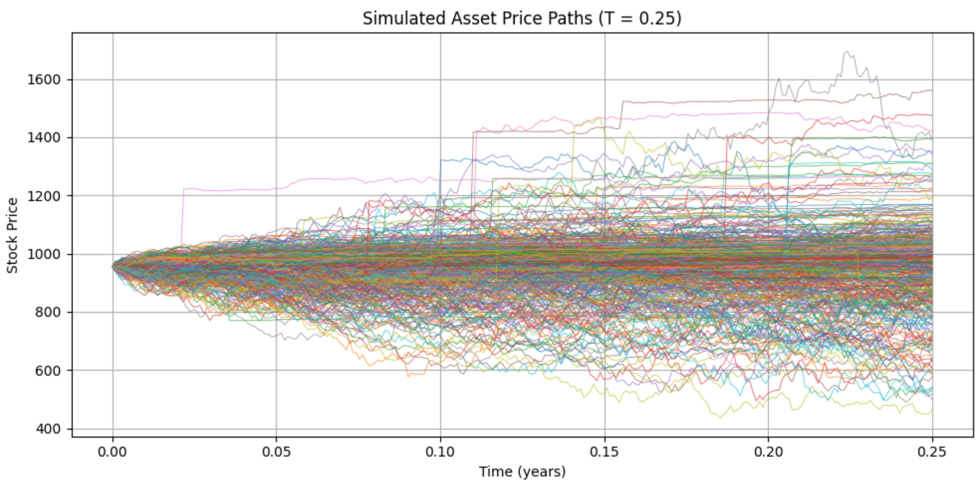

Figure 12: Simulated Bates Model Asset Paths for Costco (T = 0.25)