

# Studies of granularity of a hadronic calorimeter for tens-of-TeV jets at a 100 TeV $pp$ collider

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## Abstract

Jet substructure variables for hadronic jets with transverse momenta in the range from 2.5 TeV to 20 TeV were studied using several designs for spacial size of calorimeter cells. The studies used the full Geant4 simulation of calorimeter response combined with realistic reconstruction of calorimeter clusters. In most cases considered in this study the results indicate that the performance of jet-substructure reconstruction improves with reducing cell sizes of a hadronic calorimeter from  $\Delta\eta \times \Delta\phi = 0.087 \times 0.087$  to  $0.022 \times 0.022$ .

*Keywords:* multi-TeV physics,  $pp$  collider, future hadron colliders, FCC, SppC

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## 1. Introduction

Particle collisions at energies beyond those attained at the LHC will lead to many challenges for detector technologies. Future experiments, such as high-energy LHC (HE-LHC), future circular  $pp$  colliders of the European initiative, FCC-hh [1] and the Chinese initiative, SppC [2] will be required to measure high-momentum bosons ( $W$ ,  $Z$ ,  $H$ ) and top quarks with strongly collimated decay products that form jets. Jet substructure techniques are used to identify such highly boosted particles, and thus can maximize the physics potential of the future colliders.

The reconstruction of jet substructure variables for collimated jets with transverse momentum above 10 TeV requires an appropriate detector design. The most important for reconstruction of such jets are tracking and calorimetry. Recently, a number of studies [3, 4, 5] have been discussed using various fast simulation tools, such as Delphes [6], in which momenta of particles are smeared to mimic detector response.

A major step towards the usage of full Geant4 simulation to verify the granularity requirements for calorimeters was made in [7]. These studies have illustrated

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16 a significant impact of granularity of electromagnetic (ECAL) and hadronic (HCAL)  
 17 calorimeters on the shape of hadronic showers calculated using calorimeter hits for two  
 18 particles separated by some angle. It was concluded that high granularity is essential  
 19 in resolving two close-by particles for energies above 100 GeV.

20 This paper makes another step in understanding of this problem in terms of high-  
 21 level physics quantities typically used in physics analyses. Similar to the studies pre-  
 22 sented in [7], this paper is based on a full Geant4 simulation with realistic jet recon-  
 23 struction.

## 24 2. Simulation of detector response

25 The description of the detector and software used for this study is discussed in [7].  
 26 We use the SiFCC detector geometry with a software package that represents a versatile  
 27 environment for simulations of detector performance, testing new technology options,  
 28 and event reconstruction techniques for future 100 TeV colliders.

29 The baseline detector discussed in [7] uses a steel-scintillator hadronic calorimeter  
 30 with a transverse cell size of  $5 \times 5 \text{ cm}^2$ , which corresponds to  $\Delta\eta \times \Delta\phi = 0.022 \times 0.022$ ,  
 31 where  $\eta$  is the pseudorapidity,  $\eta \equiv -\ln \tan(\theta/2)$ , and  $\phi$  is the azimuthal angle. The  
 32 depth of the HCAL in the barrel region is about 11.25 interaction lengths ( $\lambda_I$ ). The  
 33 HCAL has 64 longitudinal layers in the barrel and the endcap regions.

34 In addition, to the baseline HCAL geometry, several geometry variations were con-  
 35 sidered. We used the HCAL with the cells that have the transverse size of  $20 \times 20 \text{ cm}^2$ ,  
 36  $2 \times 2 \text{ cm}^2$  and  $1 \times 1 \text{ cm}^2$ . In the terms of  $\Delta\eta \times \Delta\phi$ , such cell sizes correspond to  
 37  $0.087 \times 0.087$ ,  $0.0087 \times 0.0087$  and  $0.0043 \times 0.0043$ , respectively.

38 The GEANT4 (version 10.3) [8] simulation of calorimeter response was complemented  
 39 with the full reconstruction of calorimeter clusters formed by the Pandora algorithm  
 40 [9, 10]. Calorimeter clusters were built from calorimeter hits in the ECAL and HCAL  
 41 after applying the corresponding sampling fractions. No other corrections are applied.  
 42 Hadronic jets were reconstructed with the FASTJET package [11] using the anti- $k_T$   
 43 algorithm [12] with a distance parameter of 0.5.

44 In the following discussion, we use the simulations of a heavy  $Z'$  boson, a hypo-  
 45 theoretical gauge boson that arises from extensions of the electroweak symmetry of the  
 46 Standard Model. The  $Z'$  bosons were simulated with the masses  $M = 5, 10, 20$  and  
 47 40 TeV. The lowest value represents a typical mass that is within the reach of the LHC  
 48 experiments. The value 40 TeV represents the physics reach for a 100 TeV collider. The  
 49  $Z'$  particles are forced to decay to two light-flavor jets ( $q\bar{q}$ ),  $W^+W^-$  or  $t\bar{t}$ , where  $W$  and  
 50  $t$  decay hadronically. In all such scenarios, two highly boosted jets are produced, which  
 51 are typically back-to-back in the laboratory frame. The typical transverse momenta  
 52 of such jets are  $\simeq M/2$ . The main difference between considered decay types lays in  
 53 different jet substructure. In the case of the  $q\bar{q}$  decays, jets do not have any internal  
 54 structure. In the case of  $W^+W^-$ , each jet originates from  $W$ , thus it has two subjets  
 55 because of the decay  $W \rightarrow q\bar{q}$ . In the case of hadronic top decays, jets have three  
 56 subjets due to the decay  $t \rightarrow W^+ b \rightarrow q\bar{q}b$ . The signal events were generated using the  
 57 PYTHIA8 generator with the default settings, ignoring interference with SM processes.  
 58 The event samples used in this paper are available from the HepSim database [13].

### 59 3. Studies of jet properties

60 We consider several variables that represent jet substructure using different types of  
 61 calorimeter granularity. The question we want to answer is how close the reconstructed  
 62 jet substructure variables reflect the input “truth” values that are reconstructed using  
 63 input particles directly from the PYTHIA8 generator.

64 In this study we use the jet effective radius and jet splitting scales as benchmark  
 65 variables to study jet substructure properties for different calorimeter granularity sce-  
 66 narios. The effective radius is the average of the energy weighted radial distance  $\delta R_i$  in  
 67  $\eta - \phi$  space of jet constituents. It is defined as  $(1/E) \sum_i e_i \delta R_i$ , where  $E$  is the energy  
 68 of the jet and  $e_i$  is the energy of a calorimeter cluster  $i$  at the distance  $\delta R_i$  from the jet  
 69 center. The sum runs over all constituents of the jet. Recently, it has been studied for  
 70 multi-TeV jets in Ref.[14]. A jet  $k_T$  splitting scale [15] is defined as a distance measure  
 71 used to form jets by the  $k_T$  recombination algorithm [16, 17]. This variable has been  
 72 studied by ATLAS [18], and more recently in the context of 100 TeV physics [14]. The  
 73 splitting scale is defined as  $\sqrt{d_{12}} = \min(p_T^1, p_T^2) \times \delta R_{12}$  [18] at the final stage of the  $k_T$   
 74 clustering, where two subjets are merged into the final one.

75 Figures 1 and 2 show the distributions of the jet effective radius and jet splitting  
 76 scale for different jet transverse momenta and HCAL granularities. The reconstructed-  
 77 level distributions significantly disagree with the distributions reconstructed using truth-  
 78 level particles. The distribution reconstructed with the cell sizes  $1 \times 1 \text{ cm}^2$  are clos-  
 79 est to the truth-level variables. The distributions reconstructed using the cell size of  
 80  $20 \times 20 \text{ cm}^2$ , show the largest discrepancy with the truth-level variables. Note that, in  
 81 terms of closeness of reconstructed distributions to the truth level, there is no significant  
 82 difference between  $5 \times 5 \text{ cm}^2$ ,  $2 \times 2 \text{ cm}^2$  and  $1 \times 1 \text{ cm}^2$  choices.

83 Thus this study confirms the baseline SiFCC detector geometry [7] that uses  $5 \times$   
 84  $5 \text{ cm}^2$  cells, corresponding to  $\Delta\eta \times \Delta\phi = 0.022 \times 0.022$ . Note that the ATLAS and CMS  
 85 detectors use the HCAL cell sizes in the barrel region which are close to  $\Delta\eta \times \Delta\phi =$   
 86  $0.087 \times 0.087$ . According to this study, such HCAL cell sizes are not optimal in terms  
 87 of performance for tens-of-TeV jets.

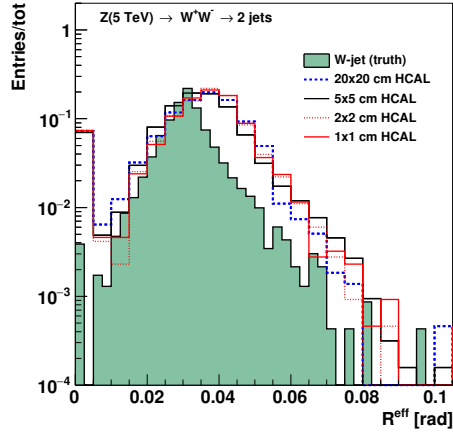
88 In the next few sections we will consider several other physics-motivated variables  
 89 that can shed light upon the performance of the HCAL for tens-of-TeV jets.

### 90 4. Detector performance with soft drop mass

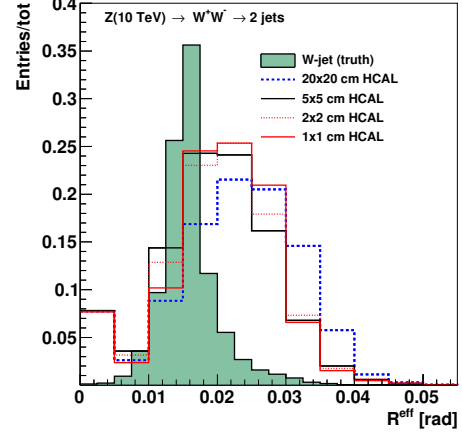
91 In this section, we use the jet mass computed with a specific algorithm, soft drop  
 92 declustering, to study the performance of detector with various detector cell sizes and  
 93 center-of-mass (c.m.) energies.

#### 94 4.1. The technique of soft drop declustering

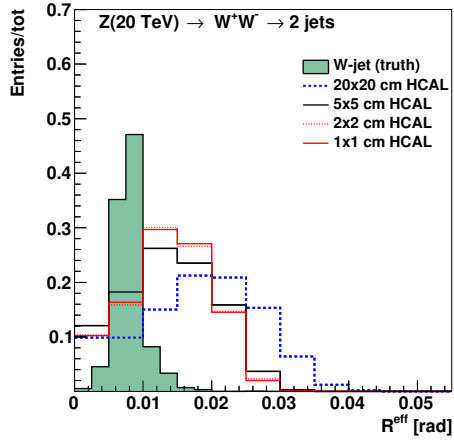
95 The soft drop declustering [19] is a grooming method that removes soft wide-  
 96 angle radiation from a jet. The constituents of a jet  $j_0$  are first reclustered using  
 97 the Cambridge-Aachen (C/A) algorithm [20, 21]. Then, the jet  $j_0$  is broken into two  
 98 subjets  $j_1$  and  $j_2$  by undoing the last stage of C/A clustering. If the subjets pass  
 99 the following soft drop condition, jet  $j_0$  is the final soft-drop jet. Otherwise, the algo-  
 100 rithm redefines  $j_0$  to be the subjet with larger  $p_T$  (among  $j_1$  and  $j_2$ ) and iterates the



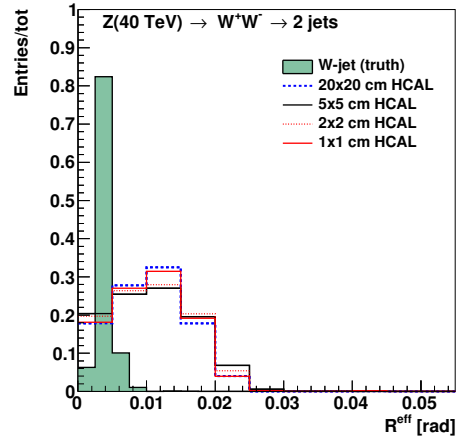
(a) 5 TeV



(b) 10 TeV

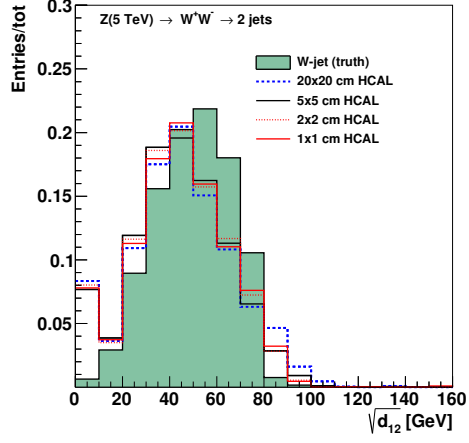


(c) 20 TeV

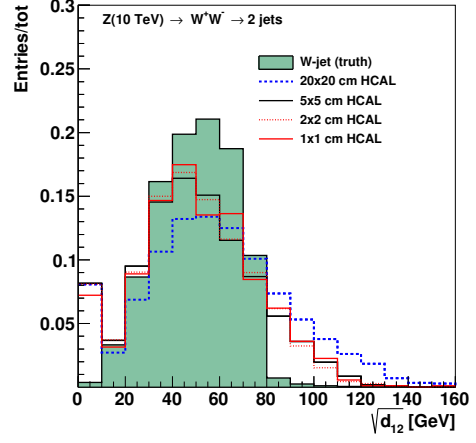


(d) 40 TeV

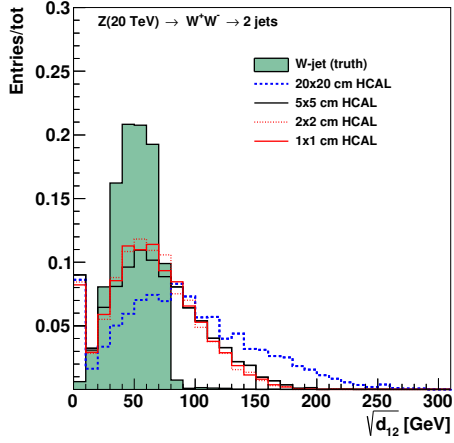
Figure 1: Jet effective radius for different jet transverse momenta and HCAL granularities.



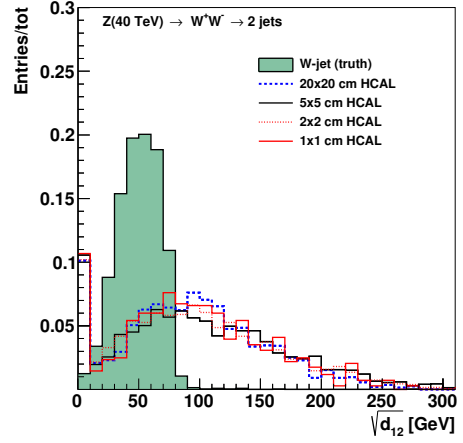
(a) 5 TeV



(b) 10 TeV



(c) 20 TeV



(d) 40 TeV

Figure 2: Jet splitting scale for different jet transverse momenta and HCAL granularity.

101 procedure.

$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\text{cut}} \left( \frac{\Delta R_{12}}{R_0} \right)^\beta, \quad (1)$$

102 where  $p_{T1}$  and  $p_{T2}$  are the transverse momenta of the two subjets,  $z_{\text{cut}}$  is soft drop  
103 threshold,  $\Delta R_{12}$  is the distance between the two subjets in the rapidity-azimuth angle  
104 plane ( $y$ - $\phi$ ),  $R_0$  is the characteristic radius of the original jet, and  $\beta$  is the angular  
105 exponent.

106 In our study, we compare the HCAL performance by comparing the soft drop mass  
107 with  $\beta = 0$  and  $\beta = 2$ . For  $\beta = 0$ , the soft drop condition depends only on the  $z_{\text{cut}}$ .  
108 For  $\beta = 2$ , the condition depends on the angular distance between the two subjets and  
109  $z_{\text{cut}}$  and the algorithm becomes infrared and collinear safe.

#### 110 4.2. Analysis method

111 We employ the following method to quantify the detector performance and de-  
112 termine the cell size that gives the best separation power to distinguish signal from  
113 background. For each configuration of detector and c.m. energy, we draw the receiver  
114 operating characteristic (ROC) curves in which the x-axis is the signal efficiency ( $\epsilon_{\text{sig}}$ )  
115 and y-axis is the inverse of background efficiency ( $1/\epsilon_{\text{bkg}}$ ). In order to scan the efficien-  
116 cies of soft drop mass cuts, we vary the mass window as follows. We first look for the  
117 median bin  $i_{\text{med}}$ <sup>1</sup> of the soft drop mass histogram from simulated signal events. Taking  
118 the right boundary of bin  $i_{\text{med}}$  as the center of mass window  $x_{\text{center}}$ , we start increasing  
119 the width of mass window symmetrically on the left and on the right of  $x_{\text{center}}$ , in steps  
120 of 5 GeV, i.e. the narrowest mass window is  $[x_{\text{center}} - 5, x_{\text{center}} + 5]$ . If one side reaches  
121 the boundary of the mass histogram, we only increase the width on the other side, also  
122 in steps of 5 GeV. For each mass window, there will be corresponding  $\epsilon_{\text{sig}}$  and  $\epsilon_{\text{bkg}}$ ,  
123 which gives a point in the ROC curves.

#### 124 4.3. Results and conclusion

125 Figures 3, 5, 7 and 9 show the distributions for the soft drop mass for  $\beta = 0$  and  
126  $\beta = 2$  with different c.m. energies and detector cell sizes; the signals considered are  
127  $Z' \rightarrow WW$  and  $Z' \rightarrow t\bar{t}$ . Figures 4, 6, 8, and 10 show the ROC curves for different  
128 detector cell sizes and c.m. energies.

129 These studies show that the reconstruction of soft drop mass improves with decrease  
130 of the HCAL cell sizes. Figures 4 and 6 show that for  $\beta = 0$  the smallest detector cell  
131 size,  $1 \times 1 \text{ cm}^2$ , has the best separation power at  $\sqrt{s} = 5, 10$ , and 20 TeV when the  
132 signal is  $Z' \rightarrow WW$ , and at  $\sqrt{s} = 10$  and 20 TeV when the signal is  $Z' \rightarrow t\bar{t}$ . On the  
133 contrary, Figs. 8 and 10 show that for  $\beta = 2$  the smallest detector cell size does not  
134 have improvements in the separation power with respect to those with larger cell sizes.  
135 In fact, the performances of the three cell sizes are similar. In addition, sometimes  
136 bigger detector cell sizes,  $5 \times 5 \text{ cm}^2$  or even  $20 \times 20 \text{ cm}^2$  have the best separation power.

137 Note that the separation between ROC curves depends on the physics variable and  
138 on the boost of top-quarks (or  $W$ ). For example, the absence of difference between

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<sup>1</sup>The integral from bin 0 to bin  $i_{\text{med}}$  ( $i_{\text{med}} - 1$ ) should be greater (less) than half of the total number of events. Note, the bin width is 5 GeV.

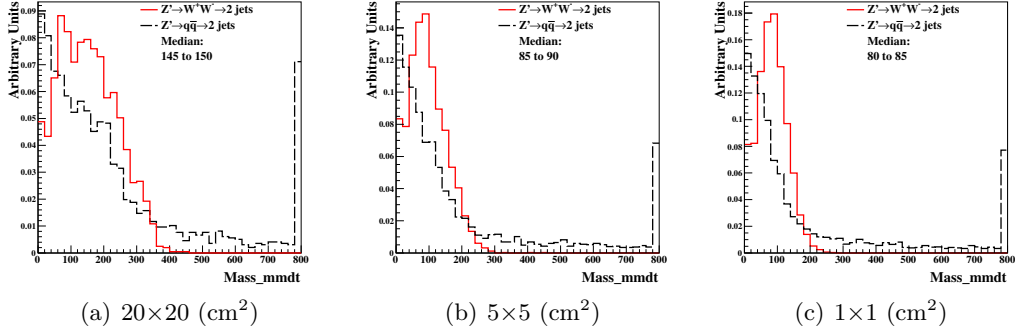


Figure 3: Distributions of soft drop mass for  $\beta=0$ , with 20 TeV c.m. energies and three different detector cell sizes:  $20\times 20$ ,  $5\times 5$  and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow WW$  ( $Z' \rightarrow q\bar{q}$ ).

the ROC curves shown in Fig. 6(a) is due to the insufficient boost of the top quarks. From the other hand, Fig. 6(d) does not show a separation between the ROC lines since the boost is too high, therefore, all the considered detector geometries show the same performance.

We also find that the soft drop mass with  $\beta = 0$  has better performance for distinguishing signal from background than for  $\beta = 2$ . Therefore, we will apply requirements on this variable when studying the other jet substructure variables.

## 5. Detector performance with jet substructure variables

In this section, we use several jet substructure variables to study the performance of detector with various detector cell sizes and c.m. energies.

### 5.1. $N$ -subjettiness

The variable  $N$ -subjettiness [22], denoted by  $\tau_N$ , is designed to “count” the number of subjet(s) in a large radius jet so to separate signal jets from decays of heavy bosons and background jets from QCD processes. The  $\tau_N$  is the  $p_T$ -weighted angular distance between each jet constituent and the closest subjet axis:

$$\tau_N = \frac{1}{d_0} \sum_k p_{T,k} \min\{\Delta R_{1,k}, \Delta R_{2,k}, \dots, \Delta R_{N,k}\}, \quad (2)$$

with a normalization factor  $d_0$ :

$$d_0 = \sum_k p_{T,k} R_0.$$

The  $k$  runs over all constituent particles in a given large radius jet,  $p_{T,k}$  is the transverse momentum of each individual constituent particle,  $\Delta R_{j,k} = \sqrt{(\Delta y)^2 + (\Delta \phi)^2}$  is the distance between the constituent particle  $k$  and the candidate subjet axis  $j$  in the  $y - \phi$  plane. The  $R_0$  is the characteristic jet radius used in the anti- $k_t$  jet algorithm.

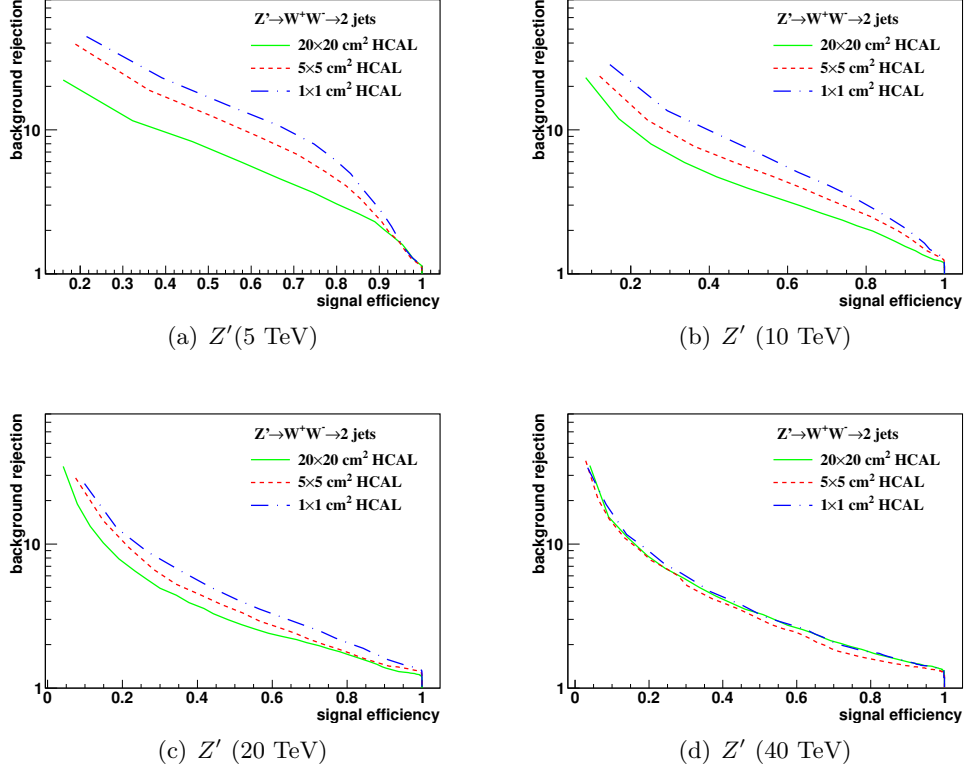


Figure 4: The ROC curves of soft drop mass selection for  $\beta=0$  with 5, 10, 20, 40 TeV c.m. energies. Three different detector cell sizes are compared:  $20\times 20$ ,  $5\times 5$ , and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow WW$  ( $Z' \rightarrow q\bar{q}$ ).

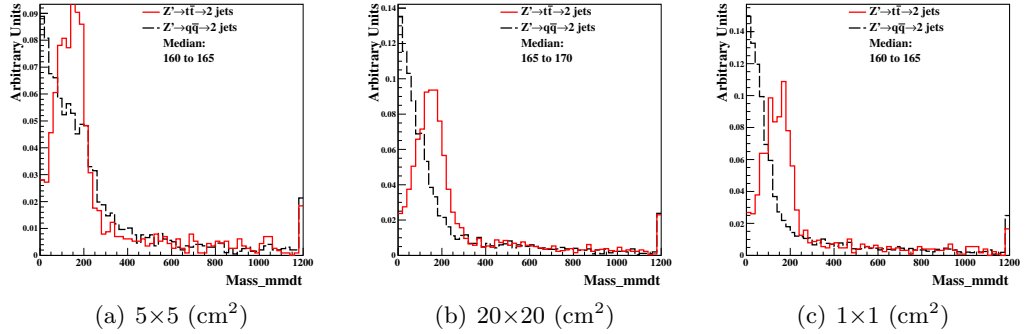


Figure 5: Distributions of soft drop mass for  $\beta=0$ , with 20 TeV c.m. energies and three different detector cell sizes:  $20\times 20$ ,  $5\times 5$ , and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow t\bar{t}$  ( $Z' \rightarrow q\bar{q}$ ).



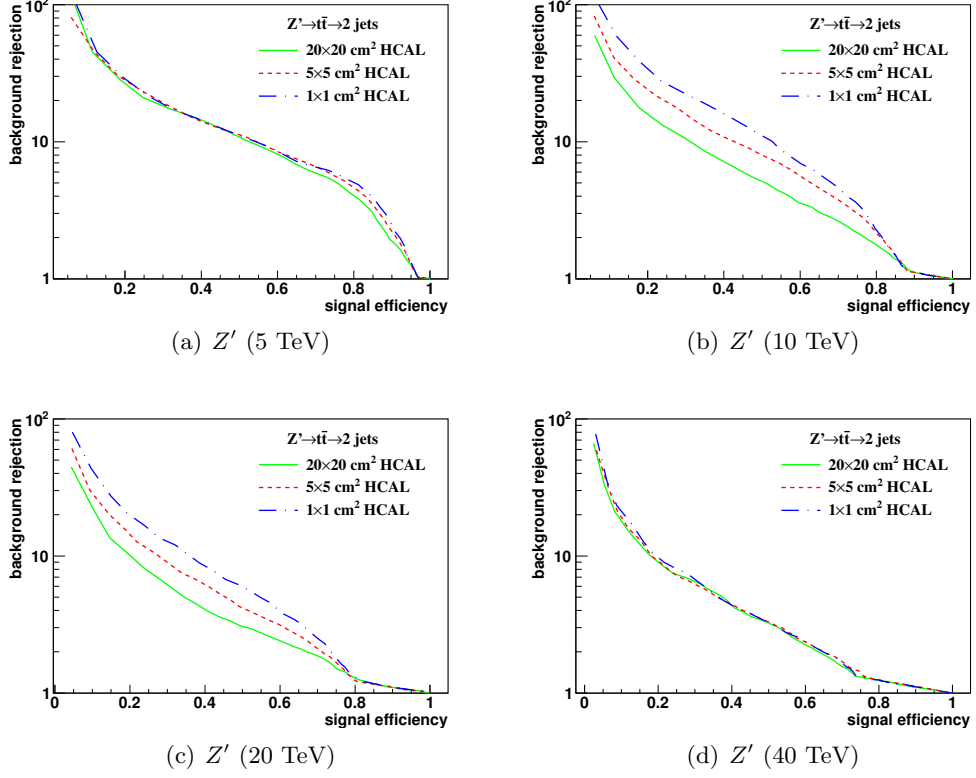


Figure 6: The ROC curves of soft drop mass selection for  $\beta=0$  with 5,10, 20, 40 TeV c.m. energies. Three different detector cell sizes are compared:  $20\times 20$ ,  $5\times 5$ , and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow t\bar{t}$  ( $Z' \rightarrow q\bar{q}$ ).

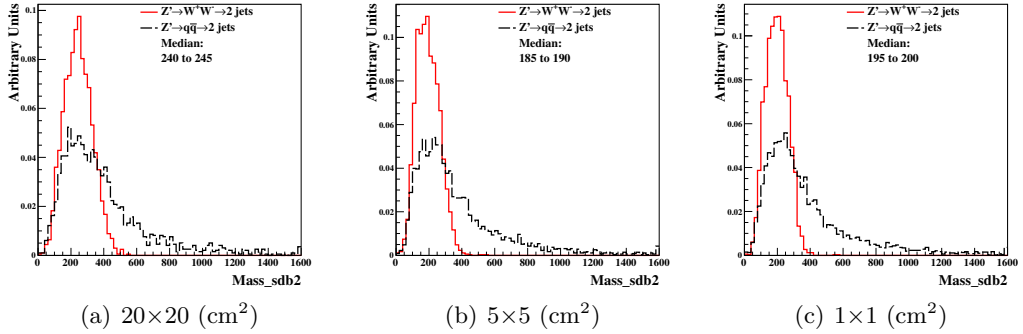


Figure 7: Distributions of soft drop mass for  $\beta=2$ , with 20 TeV c.m. energies and three different detector cell sizes:  $20\times 20$ ,  $5\times 5$  and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow WW$  ( $Z' \rightarrow q\bar{q}$ ).

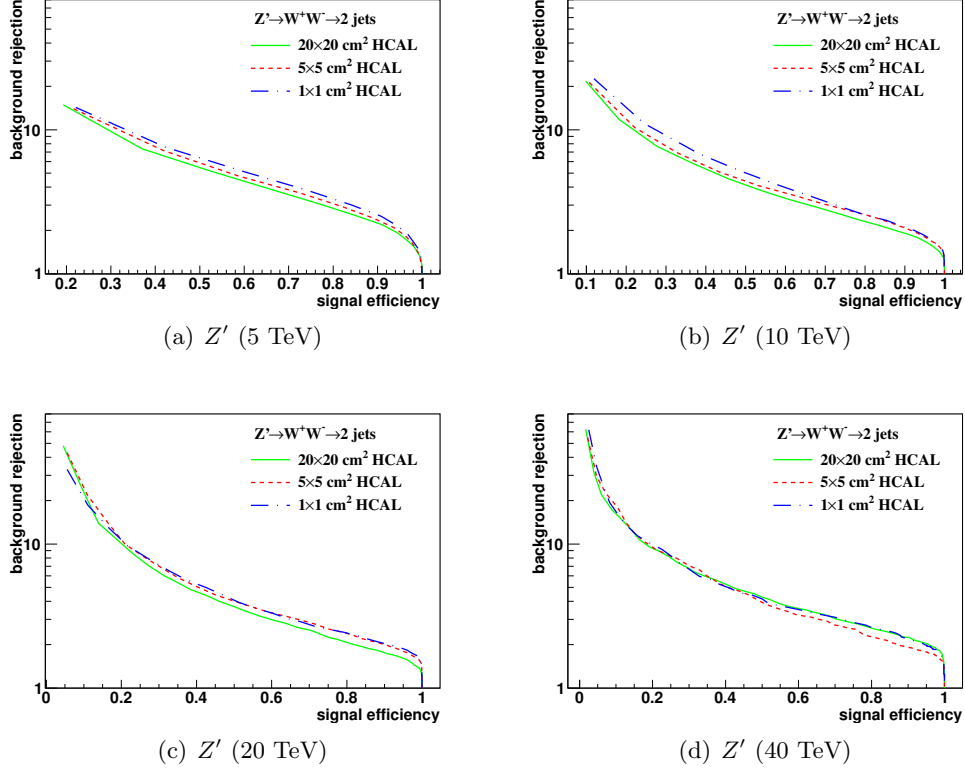


Figure 8: The ROC curves of soft drop mass selection for  $\beta=2$  with 5, 10, 20, 40 TeV c.m. energies. Three different detector cell sizes are compared:  $20\times 20$ ,  $5\times 5$ , and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow WW$  ( $Z' \rightarrow q\bar{q}$ ).

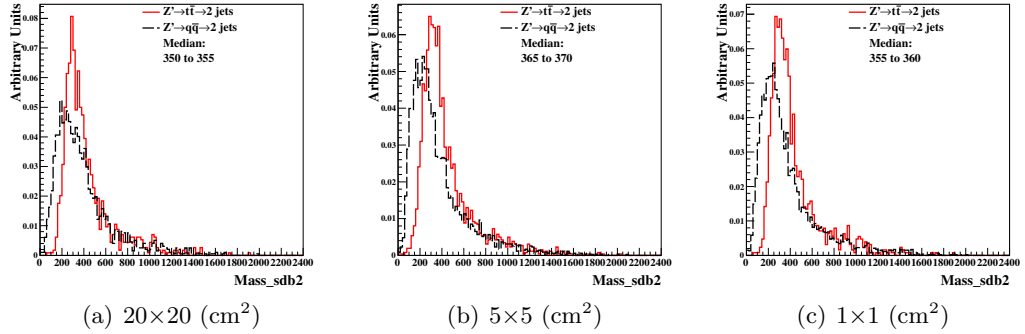


Figure 9: Distributions of soft drop mass for  $\beta=2$ , with 20 TeV c.m. energies and three different detector cell sizes:  $20\times 20$ ,  $5\times 5$ , and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow t\bar{t}$  ( $Z' \rightarrow q\bar{q}$ ).

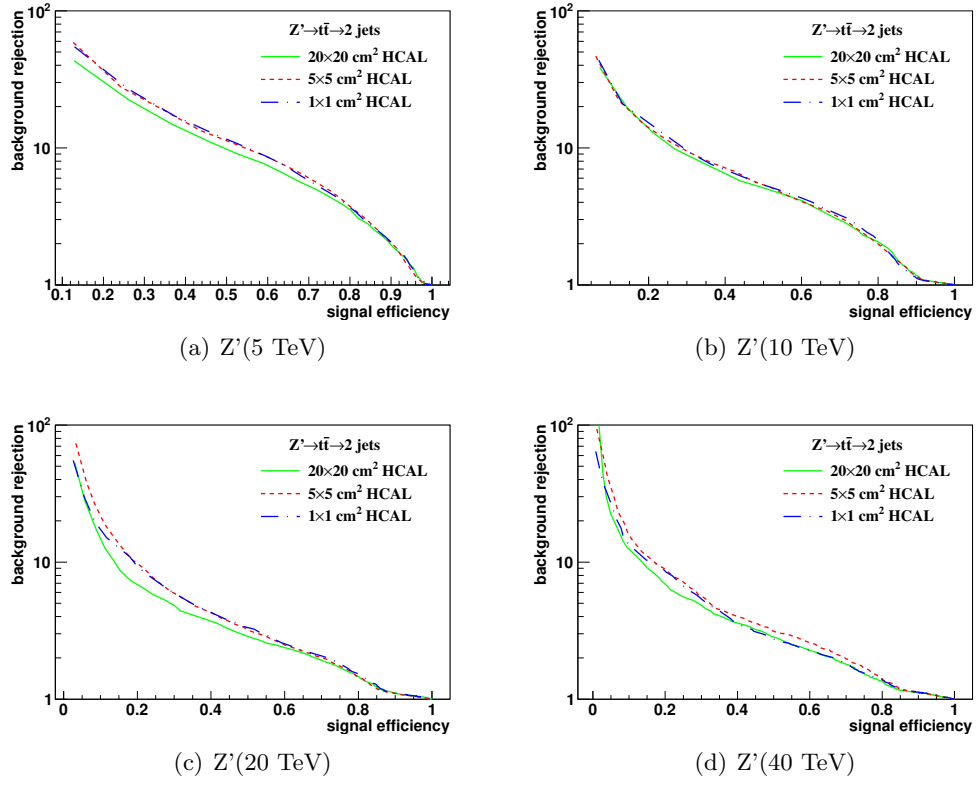


Figure 10: The ROC curves of soft drop mass selection for  $\beta=2$  with 5, 10, 20 and 40 TeV c.m. energies. Three different detector cell sizes are compared:  $20 \times 20$ ,  $5 \times 5$  and  $1 \times 1 \text{ cm}^2$ . The signal (background) process is  $Z' \rightarrow t\bar{t}$  ( $Z' \rightarrow q\bar{q}$ ).

This analysis uses the jet reconstruction described in Sect. 2. The subjet axes are obtained by running the exclusive  $k_t$  algorithm [23] and reversing the last  $N$  clustering steps. Namely, when  $\tau_N$  is computed, the  $k_t$  algorithm is forced to return exactly  $N$  jets. If a large radius jet has  $N$  subjet(s), its  $\tau_N$  is smaller than  $\tau_{N-1}$ . Therefore, in our analysis, the ratio of the  $\tau_N$  variables,  $\tau_{21}$  ( $\tau_2/\tau_1$ ) and  $\tau_{32}$  ( $\tau_3/\tau_2$ ), are used to distinguish the one-prong background jets and the two-prong jets from  $W$  or the three-prong jets from top.

We use the ROC curves as described in Section 4.2 to analyze the detector performance and determine the cell size that gives the best separation power to distinguish signal from background. Following the suggestion by Ref. [24], the requirement on the soft drop mass with  $\beta = 0$  is applied before the study of  $N$ -subjettiness. For each detector configuration and c.m. energy, the soft drop mass selection is determined as follows. First, we look for the median bin of the soft drop mass histogram from simulated signal events as described in Section 4.2. Then, we compare the numbers of events in the bins adjacent to the medium bin (bin  $i_{\text{med}} - 1$  and bin  $i_{\text{med}} + 1$ ). The bin with larger number of events is added, in addition to the medium bin, to extend the mass window. The procedure is repeated until the window contains at least 75% of the total number of signal events.

In order to obtain the signal and background efficiencies, various ranges of  $\tau_{21}$  and  $\tau_{32}$  are scanned. Since some of the background distributions have long tails and leak into the signal-dominated region, we use the following method as suggested by the Pearson Lemma Method to determine the ranges of  $\tau$  variables. First, we take the ratio of the signal to background  $\tau_{21}$  ( $\tau_{32}$ ) histograms. The boundaries of the bin (seed bin) with maximum signal to background ratio (S/N) give us the first range of  $\tau$  selection:  $x_{\text{low}}^{\text{seedbin}} < \tau_{21} < x_{\text{high}}^{\text{seedbin}}$ . Then, we compare the S/N in the bins adjacent to the seed bin. The bin with larger S/N is added, in addition to the seed bin, to extend the  $\tau_{21}$  selection window. Every window has its corresponding  $\epsilon_{\text{sig}}$  and  $1/\epsilon_{\text{bkg}}$  and an ROC curve is mapped out.

In addition to the ROC curves, we use the so-called "Mann-Whitney" test to quantify the detector performance. The value of Mann-Whitney is related to the integrated area under the ROC curve: if the value is bigger, it indicates the signal and background distributions have similar shapes and can not be well separated from each other. Vice versa, if the value is smaller, we can achieve a better signal and background separation.

Figures 11 and 13 show the distributions of  $\tau_{21}$  and  $\tau_{32}$  for  $\sqrt{s} = 20$  TeV after applying the requirement on the soft drop mass. The signals considered are  $Z' \rightarrow WW$  ( $\tau_{21}$ ) and  $Z' \rightarrow t\bar{t}$  ( $\tau_{32}$ ). Figures 12 and 14 present the ROC curves from different detector cell sizes and c.m. energies, respectively. The smallest detector cell size ( $1 \times 1 \text{ cm}^2$ ) does not have the best separation power. In fact, in some cases, the best separation power comes from a detector with bigger cell sizes ( $5 \times 5 \text{ cm}^2$  and  $20 \times 20 \text{ cm}^2$ ).

Figures 17 (a) and (b) present the summary plots of  $\tau_{21}$  and  $\tau_{32}$  with various detector cell sizes and c.m. energies using Mann Whitney U test. For  $\tau_{21}$  at smaller c.m. energies, when the cell size is smaller, the detector performance improves. However, when c.m. energy increases, no improvement is observed using the smallest detector cell size ( $1 \times 1 \text{ cm}^2$ ). For  $\tau_{32}$ , the case is similar to  $\tau_{21}$ . It is interesting to note that at very large c.m. energies, the large detector cell sizes ( $5 \times 5 \text{ cm}^2$  and  $20 \times 20 \text{ cm}^2$ ) have

205 a better separation power than the smallest cell size considered in this analysis.

## 206 5.2. Energy correlation function

207 The energy correlation function (ECF) [25] is defined as follows:

$$ECF(N, \beta) = \sum_{i_1 < i_2 < \dots < i_N \in J} \left( \prod_{a=1}^N p_{Tia} \right) \left( \prod_{b=1}^{N-1} \prod_{c=b+1}^N R_{i_b i_c} \right)^\beta, \quad (3)$$

208 where the sum is looped all particles in the jet  $J$ ,  $p_T$  is the transverse momentum of  
 209 each individual particle, and  $R$  is the distance between two particles in the  $y$ - $\phi$  plane.  
 210 In order to use a dimensionless variable, a parameter  $r_N$  is defined:

$$r_N^{(\beta)} \equiv \frac{ECF(N+1, \beta)}{ECF(N, \beta)}. \quad (4)$$

211 The idea of  $r_N$  comes from  $N$ -subjettiness  $\tau_N$ . Both  $r_N$  and  $\tau_N$  are linear in the  
 212 energy of the soft radiation for a system of  $N$  partons with soft radiation. In gen-  
 213 eral, if the system has  $N$  subjets,  $ECF(N+1, \beta)$  should be significantly smaller than  
 214  $ECF(N, \beta)$ . Therefore, we can use this feature to distinguish jets with different num-  
 215 bers of subjets. As in Section 5.1, the ratio  $r_N/r_{N-1}$ , denoted by  $C_N$ , (double ratios  
 216 of ECFs) is used to study the detector performance:

$$C_N^{(\beta)} \equiv \frac{r_N^{(\beta)}}{r_{N-1}^{(\beta)}} = \frac{ECF(N-1, \beta) ECF(N+1, \beta)}{ECF(N, \beta)^2}. \quad (5)$$

217 In our analysis, we set  $N = 2$  and  $\beta = 1$  ( $C_2^1$ ).

218 Figure 15 presents the histograms of  $C_2^1$  with  $\sqrt{s} = 20$  TeV after making the re-  
 219 quirement on the soft drop mass. The signal considered is  $Z' \rightarrow WW$ . Figure 16 shows  
 220 the ROC curves from different detector cell sizes for each c.m. energy, respectively.  
 221 One can see that the smallest detector cell size ( $1 \times 1$  cm<sup>2</sup>) does not have the best  
 222 signal/background separation power. Figure 17(c) summarizes the result of the Mann  
 223 Whitney U test for  $C_2^1$ . When c.m. energy increases, no improvement is observed from  
 224 detector with the smallest cell size.

## 225 6. Conclusions

226 The studies presented in this paper show that the reconstruction of jet substructure  
 227 variables for future particle colliders will benefit from small cell sizes of the hadronic  
 228 calorimeters. This conclusion was obtained using the realistic GEANT4simulation of  
 229 calorimeter responses combined with reconstruction of calorimeter clusters used as in-  
 230 puts for jet reconstruction. Hadronic calorimeters that use the cell sizes of  $20 \times 20$  cm<sup>2</sup>  
 231 ( $\Delta\eta \times \Delta\phi = 0.087 \times 0.087$ ) are least performat almost for every substructure variables  
 232 considered in this analysis for jet transverse momenta between 2.5 to 10 TeV. Such  
 233 cell sizes are close to those used for the ATLAS and CMS detectors at the LHC. In  
 234 terms of the reconstruction of the physics-motivated quantities used for jet substruc-  
 235 ture studies, the performance of a hadronic calorimeter with  $\Delta\eta \times \Delta\phi = 0.022 \times 0.022$

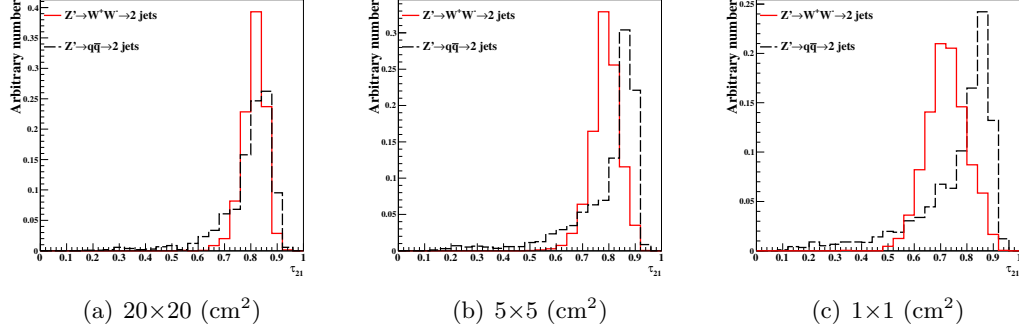


Figure 11: Distributions of  $\tau_{21}$  in 20 TeV energy collision for different detector sizes. Cell sizes in  $20 \times 20$ ,  $5 \times 5$ , and  $1 \times 1 \text{ cm}^2$  are shown here.

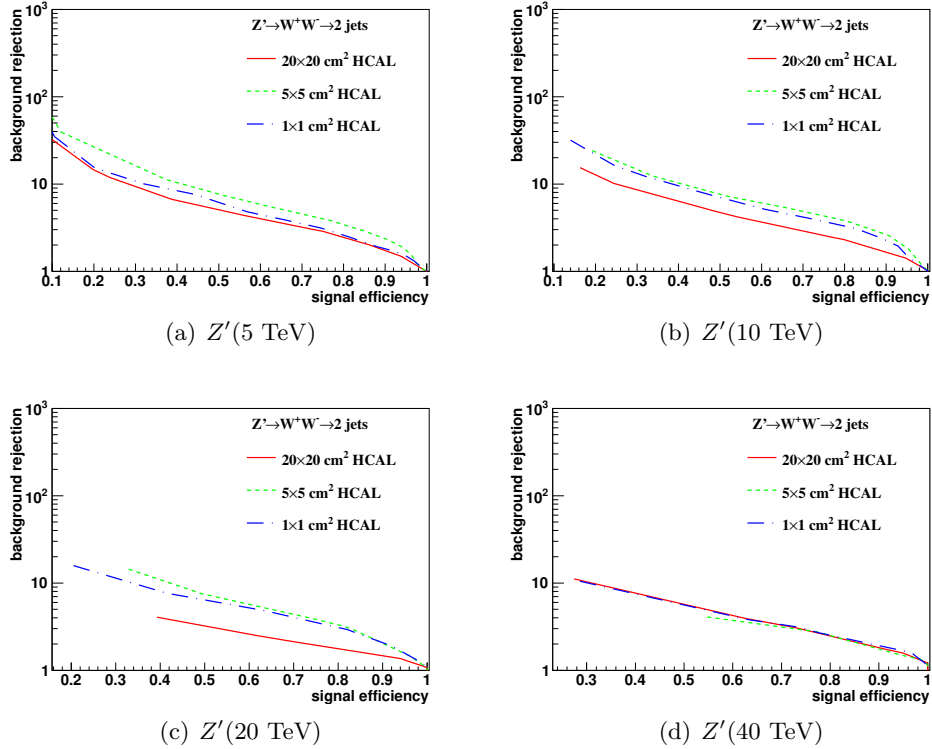


Figure 12: Signal efficiency versus background rejection rate using  $\tau_{21}$ . The energies of collision at (a) 5, (b) 10, (c) 20 and (d) 40 TeV are shown here. In each figure, the three ROC curves correspond to different detector sizes.

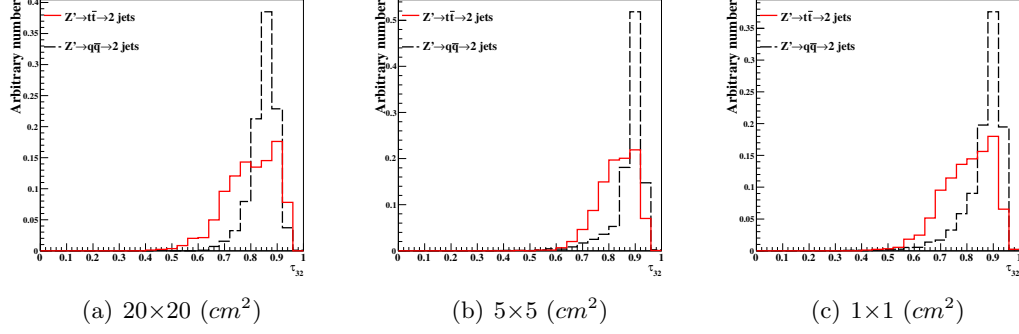


Figure 13: Distributions of  $\tau_{32}$  in 20 TeV energy collision for different detector sizes. Cell sizes in  $20 \times 20$ ,  $5 \times 5$ , and  $1 \times 1 \text{ cm}^2$  are shown here.

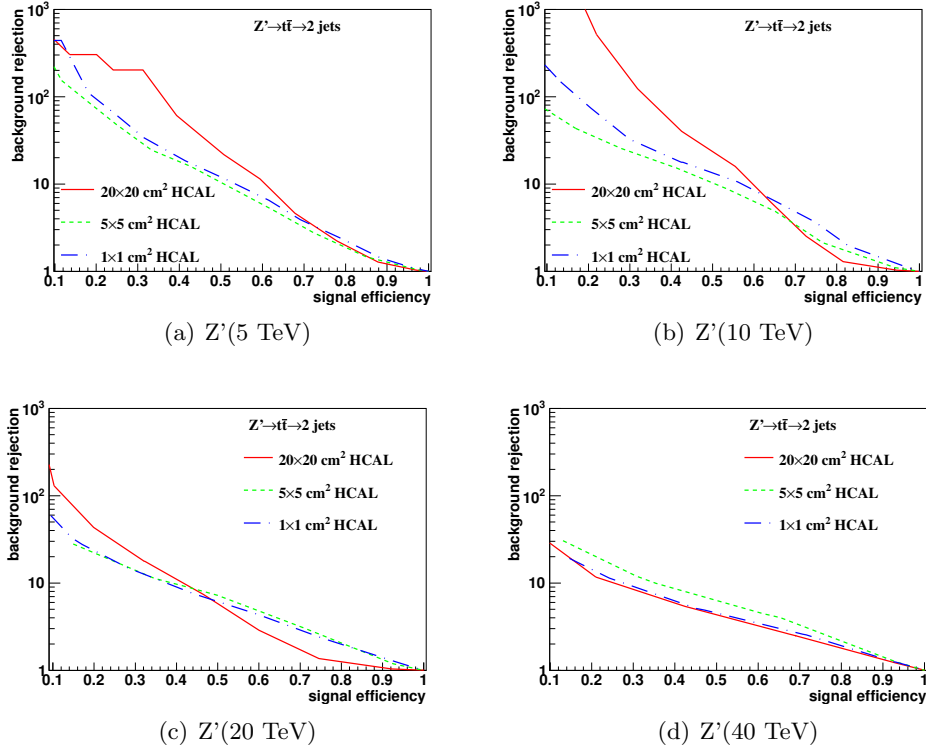


Figure 14: Signal efficiency versus background rejection rate using  $\tau_{32}$ . The energies of collision at (a) 5, (b) 10, (c) 20 and (d) 40 TeV are shown here. In each figure, the three ROC curves correspond to different detector sizes.

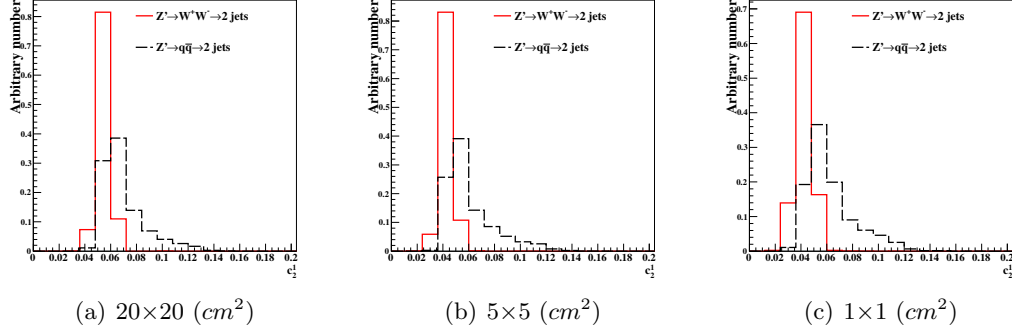


Figure 15: Distributions of  $C_2^1$  in 20 TeV energy collision for different detector sizes. Cell sizes in  $20 \times 20$ ,  $5 \times 5$ , and  $1 \times 1 \text{ cm}^2$  are shown here.

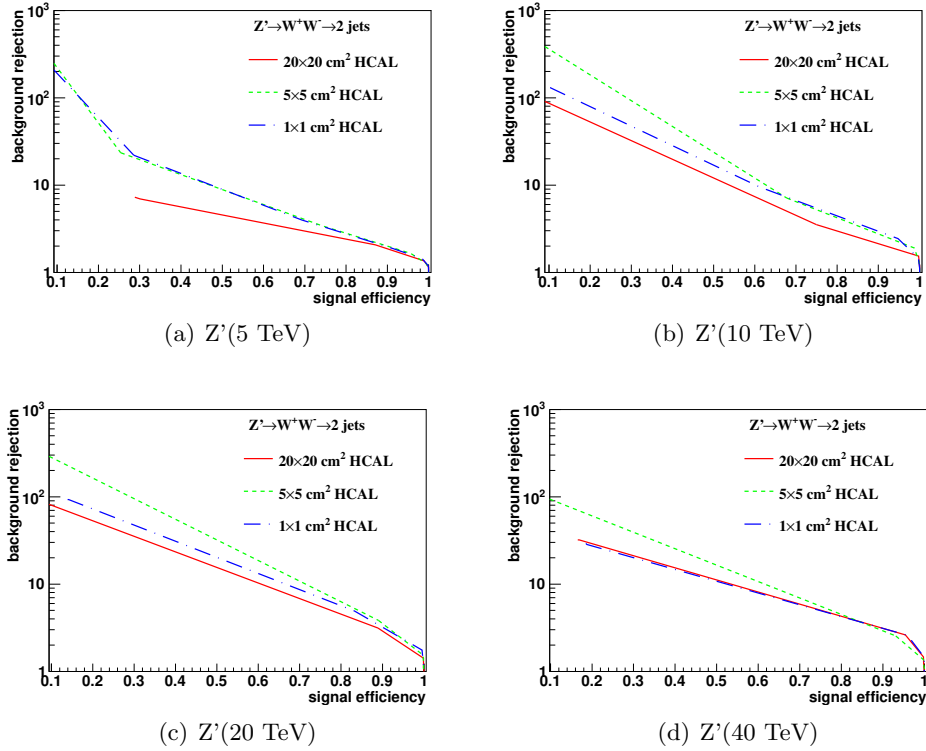


Figure 16: Signal efficiency versus background rejection rate using  $C_2^1$ . The energies of collision at (a) 5, (b) 10, (c) 20, and (d) 40 TeV are shown here. In each figure, the three ROC curves correspond to different detector sizes.



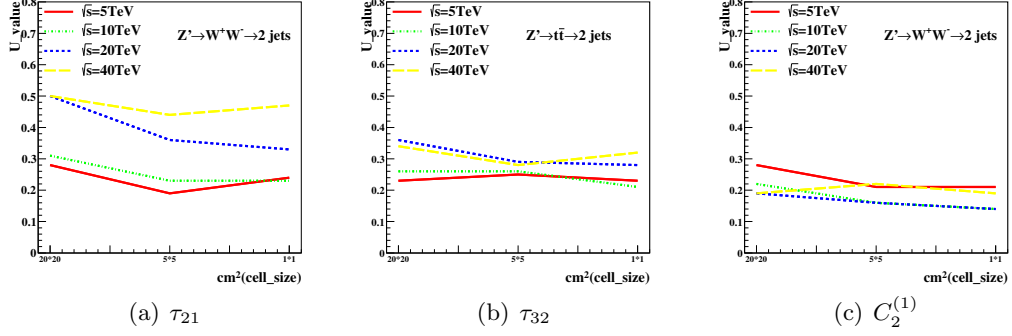


Figure 17: The Mann-Whitney U values for  $\tau_{21}$ ,  $\tau_{32}$ , and  $C_2^{(1)}$  reconstructed with different collision energies and detector cell sizes.

is, in most cases, better than for a detector with  $0.087 \times 0.087$  cells. Thus this study confirms the baseline SiFCC detector geometry [7] that uses  $\Delta\eta \times \Delta\phi = 0.022 \times 0.022$  HCAL cells. The performance of the HCAL with cells  $\Delta\eta \times \Delta\phi = 0.0087 \times 0.0087$  and  $\Delta\eta \times \Delta\phi = 0.0043 \times 0.0043$  were found to be similar.

It interesting to note that, for very boosted jets with transverse momenta close to 20 TeV, no significant improvement with the decrease of cell sizes was observed. This result needs to be understood in terms of various type of simulations and different options for construction of the calorimeter clusters.

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