

# Studies of granularity of a hadronic calorimeter for tens-of-TeV jets at a 100 TeV $pp$ collider

C.-H. Yeh<sup>a</sup>, S.V. Chekanov<sup>b</sup>, A.V. Kotwal<sup>c,d</sup>, J. Proudfoot<sup>b</sup>, S. Sen<sup>c</sup>, N.V. Tran<sup>d</sup>, S.-S. Yu<sup>a</sup>

<sup>a</sup> *Department of Physics, National Central University, Chung-Li, Taoyuan City 32001, Taiwan*

<sup>b</sup> *HEP Division, Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, IL 60439, USA.*

<sup>c</sup> *Department of Physics, Duke University, USA*

<sup>d</sup> *Fermi National Accelerator Laboratory*

<sup>e</sup> *Department of Physics, Michigan State University, 220 Trowbridge Road, East Lansing, MI 48824*

---

## Abstract

Jet substructure variables for hadronic jets with transverse momenta in the range from 2.5 TeV to 20 TeV were studied using several designs for spacial size of calorimeter cells. The studies used the full Geant4 simulation of calorimeter response combined with realistic reconstruction of calorimeter clusters. In most cases considered in this study the results indicate that the performance of jet-substructure reconstruction improves with reducing cell sizes of a hadronic calorimeter from  $\Delta\eta \times \Delta\phi = 0.087 \times 0.087$  to  $0.022 \times 0.022$ .

*Keywords:* multi-TeV physics,  $pp$  collider, future hadron colliders, FCC, SppC

---

## 1. Introduction

Particle collisions at energies beyond those attained at the LHC will lead to many challenges for detector technologies. Future experiments, such as high-energy LHC (HE-LHC), future circular  $pp$  colliders of the European initiative, FCC-hh [1] and the Chinese initiative, SppC [2] will be required to measure high-momentum bosons ( $W$ ,  $Z$ ,  $H$ ) and top quarks with strongly collimated decay products that form jets. Jet substructure techniques are used to identify such highly boosted particles, and thus can maximize the physics potential of the future colliders.

The reconstruction of jet substructure variables for collimated jets with transverse momentum above 10 TeV requires an appropriate detector design. The most important for reconstruction of such jets are tracking and calorimetry. Recently, a number of studies [3, 4, 5] have been discussed using various fast simulation tools, such as Delphes [6], in which momenta of particles are smeared to mimic detector response.

A major step towards the usage of full Geant4 simulation to verify the granularity requirements for calorimeters was made in [7]. These studies have illustrated

---

*Email addresses:* jwuzelski18@gmail.com (C.-H. Yeh), chekanov@anl.gov (S.V. Chekanov), ashutosh.kotwal@duke.edu (A.V. Kotwal), proudfoot@anl.gov (J. Proudfoot), sourav.sen@duke.edu (S. Sen), ntran@fnal.gov (N.V. Tran), syu@cern.ch (S.-S. Yu)

16 a significant impact of granularity of electromagnetic (ECAL) and hadronic (HCAL)  
 17 calorimeters on the shape of hadronic showers calculated using calorimeter hits for two  
 18 particles separated by some angle. It was concluded that high granularity is essential  
 19 in resolving two close-by particles for energies above 100 GeV.

20 This paper makes another step in understanding of this problem in terms of high-  
 21 level physics quantities typically used in physics analyses. Similar to the studies pre-  
 22 sented in [7], this paper is based on a full Geant4 simulation with realistic jet recon-  
 23 struction.

## 24 2. Simulation of detector response

25 The description of the detector and software used for this study is discussed in [7].  
 26 We use the SiFCC detector geometry with a software package that represents a versatile  
 27 environment for simulations of detector performance, testing new technology options,  
 28 and event reconstruction techniques for future 100 TeV colliders.

29 The baseline detector discussed in [7] uses a steel-scintillator hadronic calorimeter  
 30 with a transverse cell size of  $5 \times 5 \text{ cm}^2$ , which corresponds to  $\Delta\eta \times \Delta\phi = 0.022 \times 0.022$ ,  
 31 where  $\eta$  is the pseudorapidity,  $\eta \equiv -\ln \tan(\theta/2)$ , and  $\phi$  is the azimuthal angle. The  
 32 depth of the HCAL in the barrel region is about 11.25 interaction lengths ( $\lambda_I$ ). The  
 33 HCAL has 64 longitudinal layers in the barrel and the endcap regions.

34 In addition, to the baseline HCAL geometry, several geometry variations were con-  
 35 sidered. We used the HCAL with the cells that have the transverse size of  $20 \times 20 \text{ cm}^2$ ,  
 36  $2 \times 2 \text{ cm}^2$  and  $1 \times 1 \text{ cm}^2$ . In the terms of  $\Delta\eta \times \Delta\phi$ , such cell sizes correspond to  
 37  $0.087 \times 0.087$ ,  $0.0087 \times 0.0087$  and  $0.0043 \times 0.0043$ , respectively.

38 The GEANT4 (version 10.3) [8] simulation of calorimeter response was complemented  
 39 with the full reconstruction of calorimeter clusters formed by the Pandora algorithm  
 40 [9, 10]. Calorimeter clusters were built from calorimeter hits in the ECAL and HCAL  
 41 after applying the corresponding sampling fractions. No other corrections are applied.  
 42 Hadronic jets were reconstructed with the FASTJET package [11] using the anti- $k_T$   
 43 algorithm [12] with a distance parameter of 0.5.

44 In the following discussion, we use the simulations of a heavy  $Z'$  boson, a hypo-  
 45 thetical gauge boson that arises from extensions of the electroweak symmetry of the  
 46 Standard Model. The  $Z'$  bosons were simulated with the masses  $M = 5, 10, 20$  and  
 47 40 TeV. The lowest value represents a typical mass that is within the reach of the LHC  
 48 experiments. The value 40 TeV represents the physics reach for a 100 TeV collider. The  
 49  $Z'$  particles are forced to decay to two light-flavor jets ( $q\bar{q}$ ),  $W^+W^-$  or  $t\bar{t}$ , where  $W$  and  
 50  $t$  decay hadronically. In all such scenarios, two highly boosted jets are produced, which  
 51 are typically back-to-back in the laboratory frame. The typical transverse momenta  
 52 of such jets are  $\simeq M/2$ . The main difference between considered decay types lays in  
 53 different jet substructure. In the case of the  $q\bar{q}$  decays, jets do not have any internal  
 54 structure. In the case of  $W^+W^-$ , each jet originates from  $W$ , thus it has two subjets  
 55 because of the decay  $W \rightarrow q\bar{q}$ . In the case of hadronic top decays, jets have three  
 56 subjets due to the decay  $t \rightarrow W^+ b \rightarrow q\bar{q}b$ . The signal events were generated using the  
 57 PYTHIA8 generator with the default settings, ignoring interference with SM processes.  
 58 The event samples used in this paper are available from the HepSim database [13].

### 59 3. Studies of jet properties

60 We consider several variables that represent jet substructure using different types of  
 61 calorimeter granularity. The question we want to answer is how close the reconstructed  
 62 jet substructure variables reflect the input “truth” values that are reconstructed using  
 63 input particles directly from the PYTHIA8 generator.

64 In this study we use the jet effective radius and jet splitting scales as benchmark  
 65 variables to study jet substructure properties for different calorimeter granularity sce-  
 66 narios. The effective radius is the average of the energy weighted radial distance  $\delta R_i$  in  
 67  $\eta - \phi$  space of jet constituents. It is defined as  $(1/E) \sum_i e_i \delta R_i$ , where  $E$  is the energy  
 68 of the jet and  $e_i$  is the energy of a calorimeter cluster  $i$  at the distance  $\delta R_i$  from the jet  
 69 center. The sum runs over all constituents of the jet. Recently, it has been studied for  
 70 multi-TeV jets in Ref.[14]. A jet  $k_T$  splitting scale [15] is defined as a distance measure  
 71 used to form jets by the  $k_T$  recombination algorithm [16, 17]. This variable has been  
 72 studied by ATLAS [18], and more recently in the context of 100 TeV physics [14]. The  
 73 splitting scale is defined as  $\sqrt{d_{12}} = \min(p_T^1, p_T^2) \times \delta R_{12}$  [18] at the final stage of the  $k_T$   
 74 clustering, where two subjects are merged into the final one.

75 Figures 1 and 2 show the distributions of the jet effective radius and jet splitting  
 76 scale for different jet transverse momenta and HCAL granularities. The reconstructed-  
 77 level distributions significantly disagree with the distributions reconstructed using truth-  
 78 level particles. The distribution reconstructed with the cell sizes  $1 \times 1 \text{ cm}^2$  are clos-  
 79 est to the truth-level variables. The distributions reconstructed using the cell size of  
 80  $20 \times 20 \text{ cm}^2$ , show the largest discrepancy with the truth-level variables. Note that, in  
 81 terms of closeness of reconstructed distributions to the truth level, there is no significant  
 82 difference between  $5 \times 5 \text{ cm}^2$ ,  $2 \times 2 \text{ cm}^2$  and  $1 \times 1 \text{ cm}^2$  choices.

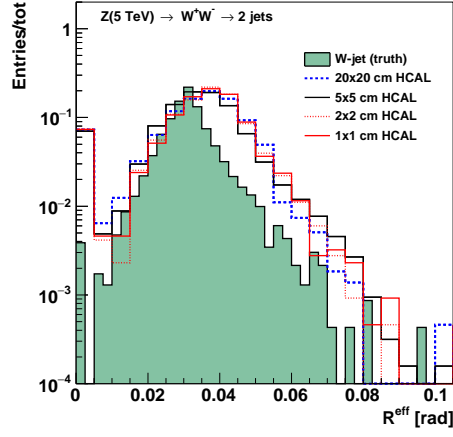
83 Thus this study confirms the baseline SiFCC detector geometry [7] that uses  $5 \times$   
 84  $5 \text{ cm}^2$  HCAL cells, corresponding to  $\Delta\eta \times \Delta\phi = 0.022 \times 0.022$ . Similar HCAL cell  
 85 sizes,  $0.025 \times 0.025$ , were recently adopted for the baseline FCC-hh detector [19, 20]  
 86 planned at CERN. Before the publication [7], such a choice for the HCAL cells was  
 87 motivated by the studies of jet substructure using a fast detector simulation of boosted  
 88 jets. In addition to the improvements in physics performance, the smaller HCAL cells  
 89 reduce the required dynamic range for signal reconstruction [4], and thus can simplify  
 90 the calorimeter readout.

91 It should be noted that the ATLAS and CMS detectors use the HCAL cell sizes in  
 92 the barrel region which are close to  $\Delta\eta \times \Delta\phi = 0.087 \times 0.087$ . According to this study,  
 93 such HCAL cell sizes are not optimal in terms of performance for tens-of-TeV jets.

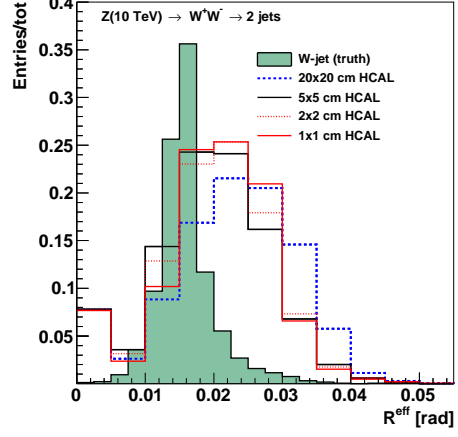
94 In the next few sections we will consider several other physics-motivated variables  
 95 that can shed light upon the performance of the HCAL for tens-of-TeV jets.

### 96 4. Detector performance with soft drop mass

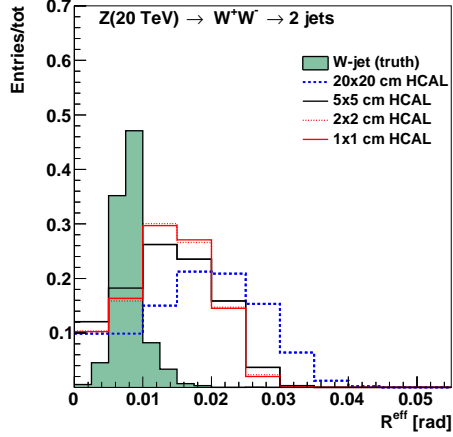
97 In this section, we use the jet mass computed with a specific algorithm, soft drop  
 98 declustering, to study the performance of detector with various detector cell sizes and  
 99 center-of-mass (c.m.) energies.



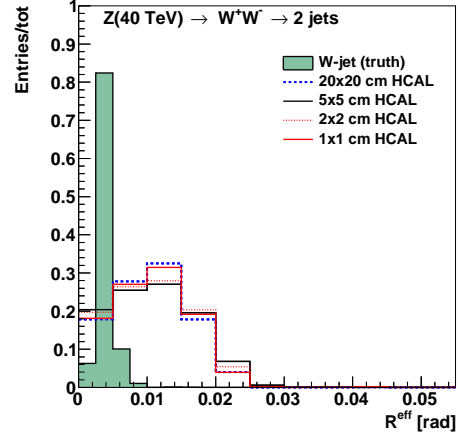
(a) 5 TeV



(b) 10 TeV

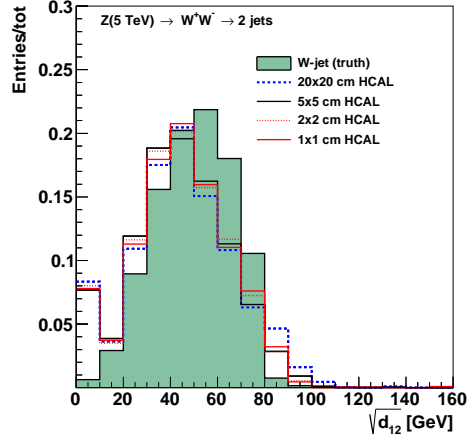


(c) 20 TeV

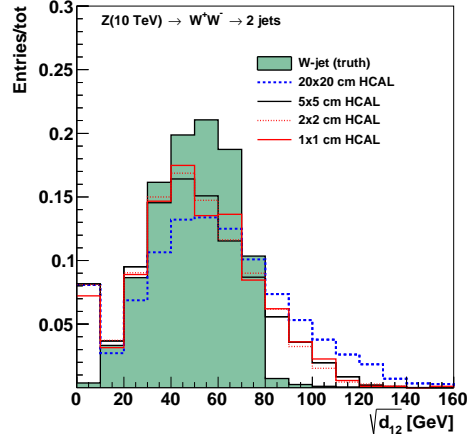


(d) 40 TeV

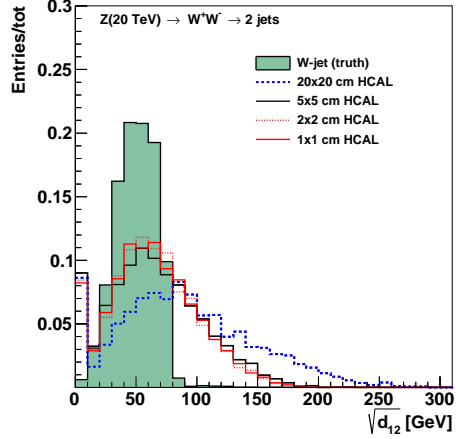
Figure 1: Jet effective radius for different jet transverse momenta and HCAL granularities.



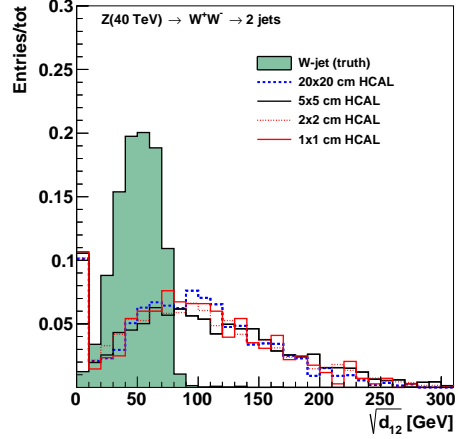
(a) 5 TeV



(b) 10 TeV



(c) 20 TeV



(d) 40 TeV

Figure 2: Jet splitting scale for different jet transverse momenta and HCAL granularity.

#### 4.1. The technique of soft drop declustering

The soft drop declustering [21] is a grooming method that removes soft wide-angle radiation from a jet. The constituents of a jet  $j_0$  are first reclustered using the Cambridge-Aachen (C/A) algorithm [22, 23]. Then, the jet  $j_0$  is broken into two subjets  $j_1$  and  $j_2$  by undoing the last stage of C/A clustering. If the subjets pass the following soft drop condition, jet  $j_0$  is the final soft-drop jet. Otherwise, the algorithm redefines  $j_0$  to be the subjet with larger  $p_T$  (among  $j_1$  and  $j_2$ ) and iterates the procedure.

$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\text{cut}} \left( \frac{\Delta R_{12}}{R_0} \right)^\beta, \quad (1)$$

where  $p_{T1}$  and  $p_{T2}$  are the transverse momenta of the two subjets,  $z_{\text{cut}}$  is soft drop threshold,  $\Delta R_{12}$  is the distance between the two subjets in the rapidity-azimuth angle plane ( $y$ - $\phi$ ),  $R_0$  is the characteristic radius of the original jet, and  $\beta$  is the angular exponent.

In our study, we compare the HCAL performance by comparing the soft drop mass with  $\beta = 0$  and  $\beta = 2$ . For  $\beta = 0$ , the soft drop condition depends only on the  $z_{\text{cut}}$ . For  $\beta = 2$ , the condition depends on the angular distance between the two subjets and  $z_{\text{cut}}$  and the algorithm becomes infrared and collinear safe.

#### 4.2. Analysis method

We employ the following method to quantify the detector performance and determine the cell size that gives the best separation power to distinguish signal from background. For each configuration of detector and c.m. energy, we draw the receiver operating characteristic (ROC) curves in which the x-axis is the signal efficiency ( $\epsilon_{\text{sig}}$ ) and y-axis is the inverse of background efficiency ( $1/\epsilon_{\text{bkg}}$ ). In order to scan the efficiencies of soft drop mass cuts, we vary the mass window as follows. We first look for the median bin  $i_{\text{med}}$ <sup>1</sup> of the soft drop mass histogram from simulated signal events. Taking the right boundary of bin  $i_{\text{med}}$  as the center of mass window  $x_{\text{center}}$ , we start increasing the width of mass window symmetrically on the left and on the right of  $x_{\text{center}}$ , in steps of 5 GeV, i.e. the narrowest mass window is  $[x_{\text{center}} - 5, x_{\text{center}} + 5]$ . If one side reaches the boundary of the mass histogram, we only increase the width on the other side, also in steps of 5 GeV. For each mass window, there will be corresponding  $\epsilon_{\text{sig}}$  and  $\epsilon_{\text{bkg}}$ , which gives a point in the ROC curves.

#### 4.3. Results and conclusion

Figures 3, 5, 7 and 9 show the distributions for the soft drop mass for  $\beta = 0$  and  $\beta = 2$  with different c.m. energies and detector cell sizes; the signals considered are  $Z' \rightarrow WW$  and  $Z' \rightarrow t\bar{t}$ . Figures 4, 6, 8, and 10 show the ROC curves for different detector cell sizes and c.m. energies.

These studies show that the reconstruction of soft drop mass improves with decrease of the HCAL cell sizes. Figures 4 and 6 show that for  $\beta = 0$  the smallest detector cell size,  $1 \times 1 \text{ cm}^2$ , has the best separation power at  $\sqrt{s} = 5, 10, \text{ and } 20 \text{ TeV}$  when the

<sup>1</sup>The integral from bin 0 to bin  $i_{\text{med}}$  ( $i_{\text{med}} - 1$ ) should be greater (less) than half of the total number of events. Note, the bin width is 5 GeV.

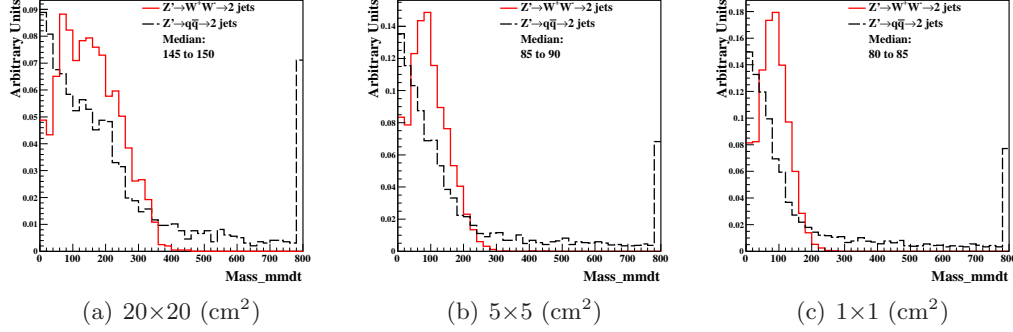


Figure 3: Distributions of soft drop mass for  $\beta=0$ , with 20 TeV c.m. energies and three different detector cell sizes:  $20 \times 20$ ,  $5 \times 5$  and  $1 \times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow WW$  ( $Z' \rightarrow q\bar{q}$ ).

signal is  $Z' \rightarrow WW$ , and at  $\sqrt{s}=10$  and 20 TeV when the signal is  $Z' \rightarrow t\bar{t}$ . On the contrary, Figs. 8 and 10 show that for  $\beta = 2$  the smallest detector cell size does not have improvements in the separation power with respect to those with larger cell sizes. In fact, the performances of the three cell sizes are similar. In addition, sometimes bigger detector cell sizes,  $5 \times 5$   $\text{cm}^2$  or even  $20 \times 20$   $\text{cm}^2$  have the best separation power.

Note that the separation between ROC curves depends on the physics variable and on the boost of top-quarks (or  $W$ ). For example, the absence of difference between the ROC curves shown in Fig. 6(a) is due to the insufficient boost of the top quarks. From the other hand, Fig. 6(d) does not show a separation between the ROC lines since the boost is too high, therefore, all the considered detector geometries show the same performance.

We also find that the soft drop mass with  $\beta = 0$  has better performance for distinguishing signal from background than for  $\beta = 2$ . Therefore, we will apply requirements on this variable when studying the other jet substructure variables.

## 5. Detector performance with jet substructure variables

In this section, we use several jet substructure variables to study the performance of detector with various detector cell sizes and c.m. energies.

### 5.1. $N$ -subjettiness

The variable  $N$ -subjettiness [24], denoted by  $\tau_N$ , is designed to “count” the number of subjet(s) in a large radius jet so to separate signal jets from decays of heavy bosons and background jets from QCD processes. The  $\tau_N$  is the  $p_T$ -weighted angular distance between each jet constituent and the closest subjet axis:

$$\tau_N = \frac{1}{d_0} \sum_k p_{T,k} \min\{\Delta R_{1,k}, \Delta R_{2,k}, \dots, \Delta R_{N,k}\}, \quad (2)$$

with a normalization factor  $d_0$ :

$$d_0 = \sum_k p_{T,k} R_0.$$

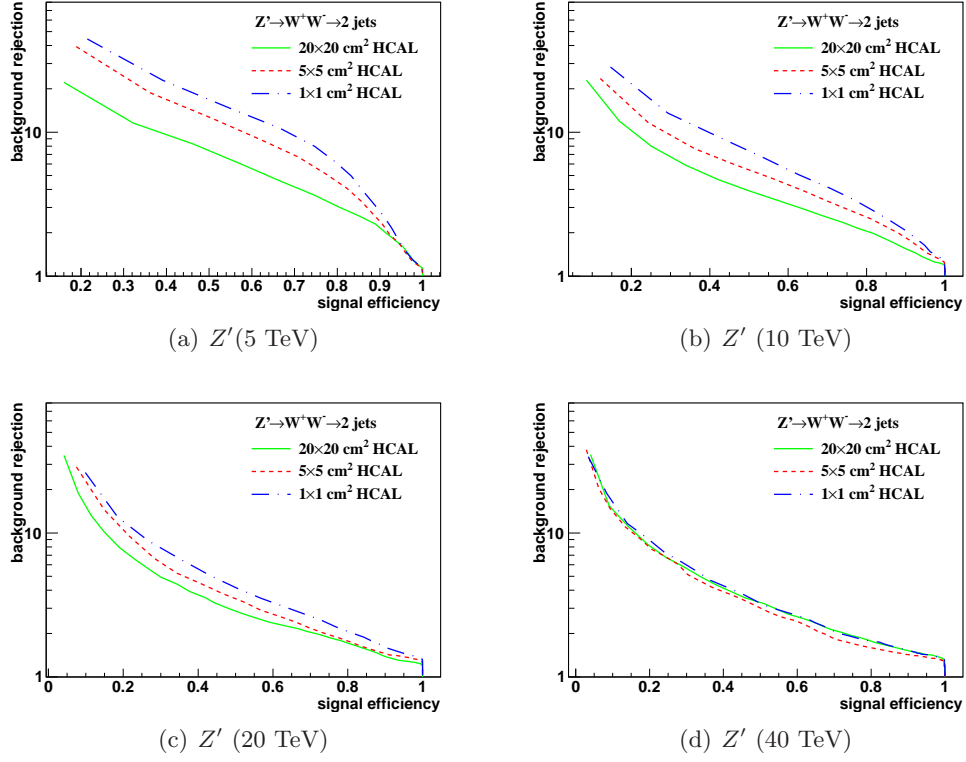


Figure 4: The ROC curves of soft drop mass selection for  $\beta=0$  with 5, 10, 20, 40 TeV c.m. energies. Three different detector cell sizes are compared:  $20\times 20$ ,  $5\times 5$ , and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow WW$  ( $Z' \rightarrow q\bar{q}$ ).

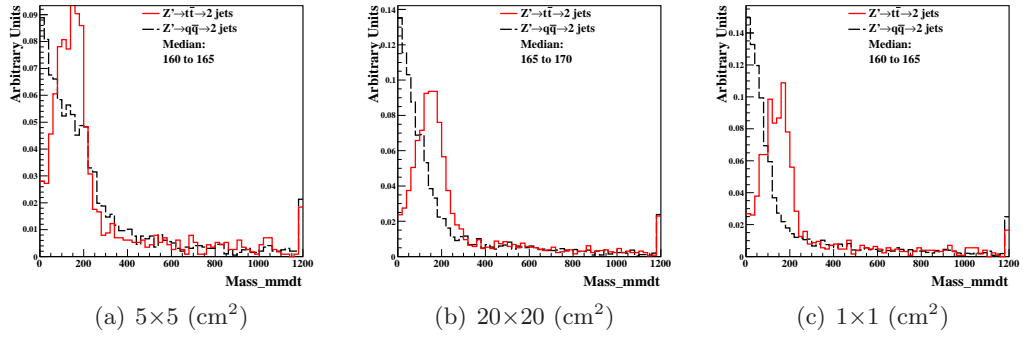


Figure 5: Distributions of soft drop mass for  $\beta=0$ , with 20 TeV c.m. energies and three different detector cell sizes:  $20\times 20$ ,  $5\times 5$ , and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow t\bar{t}$  ( $Z' \rightarrow q\bar{q}$ ).



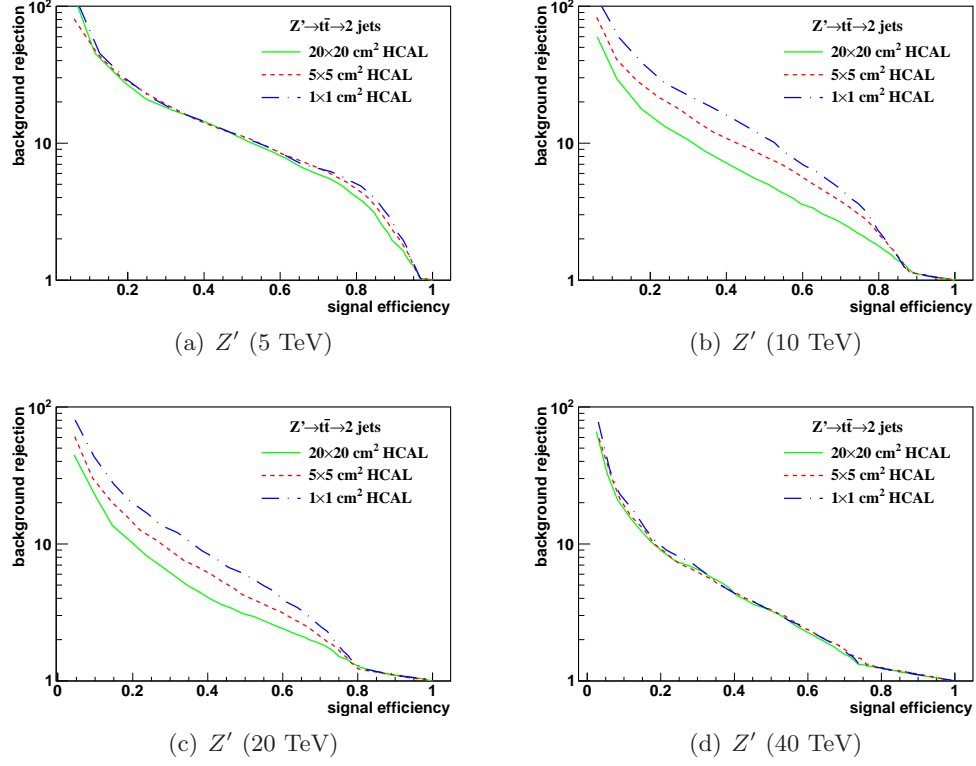


Figure 6: The ROC curves of soft drop mass selection for  $\beta=0$  with 5,10, 20, 40 TeV c.m. energies. Three different detector cell sizes are compared:  $20\times 20$ ,  $5\times 5$ , and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow t\bar{t}$  ( $Z' \rightarrow q\bar{q}$ ).

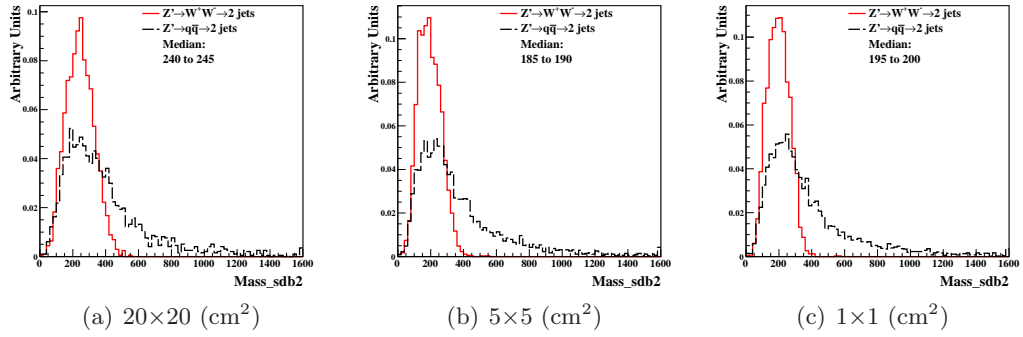


Figure 7: Distributions of soft drop mass for  $\beta=2$ , with 20 TeV c.m. energies and three different detector cell sizes:  $20\times 20$ ,  $5\times 5$  and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow W W$  ( $Z' \rightarrow q\bar{q}$ ).

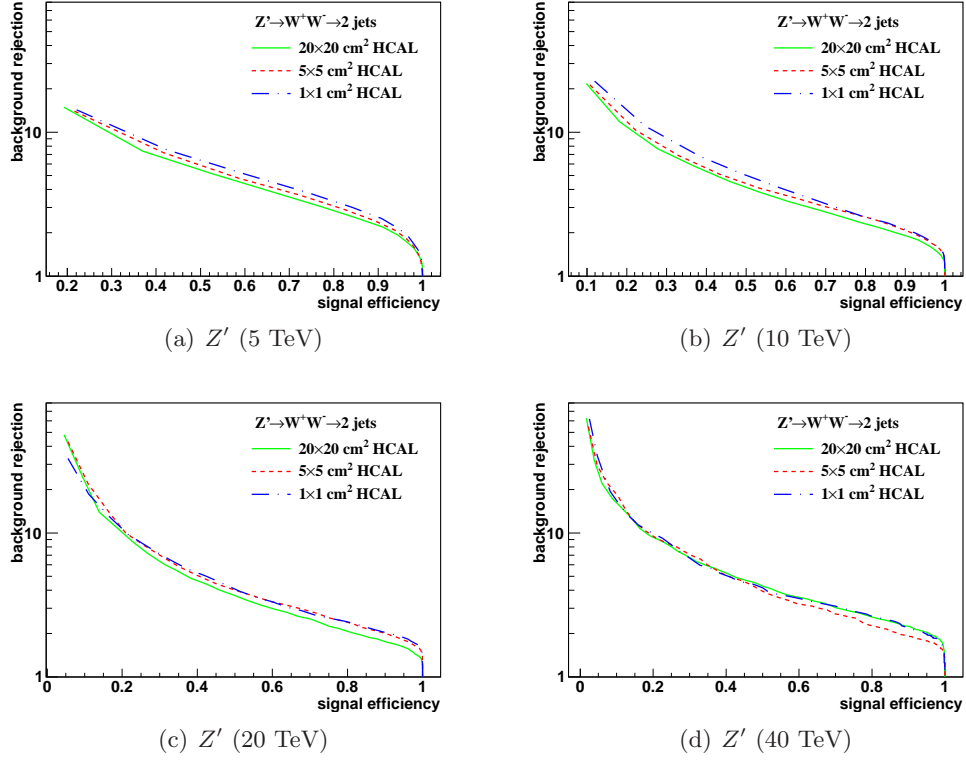


Figure 8: The ROC curves of soft drop mass selection for  $\beta=2$  with 5, 10, 20, 40 TeV c.m. energies. Three different detector cell sizes are compared:  $20\times 20$ ,  $5\times 5$ , and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow WW$  ( $Z' \rightarrow q\bar{q}$ ).

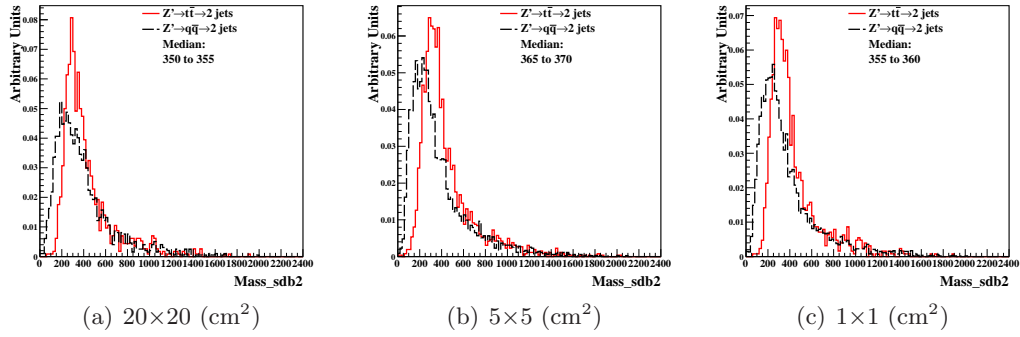


Figure 9: Distributions of soft drop mass for  $\beta=2$ , with 20 TeV c.m. energies and three different detector cell sizes:  $20\times 20$ ,  $5\times 5$ , and  $1\times 1$   $\text{cm}^2$ . The signal (background) process is  $Z' \rightarrow t\bar{t}$  ( $Z' \rightarrow q\bar{q}$ ).

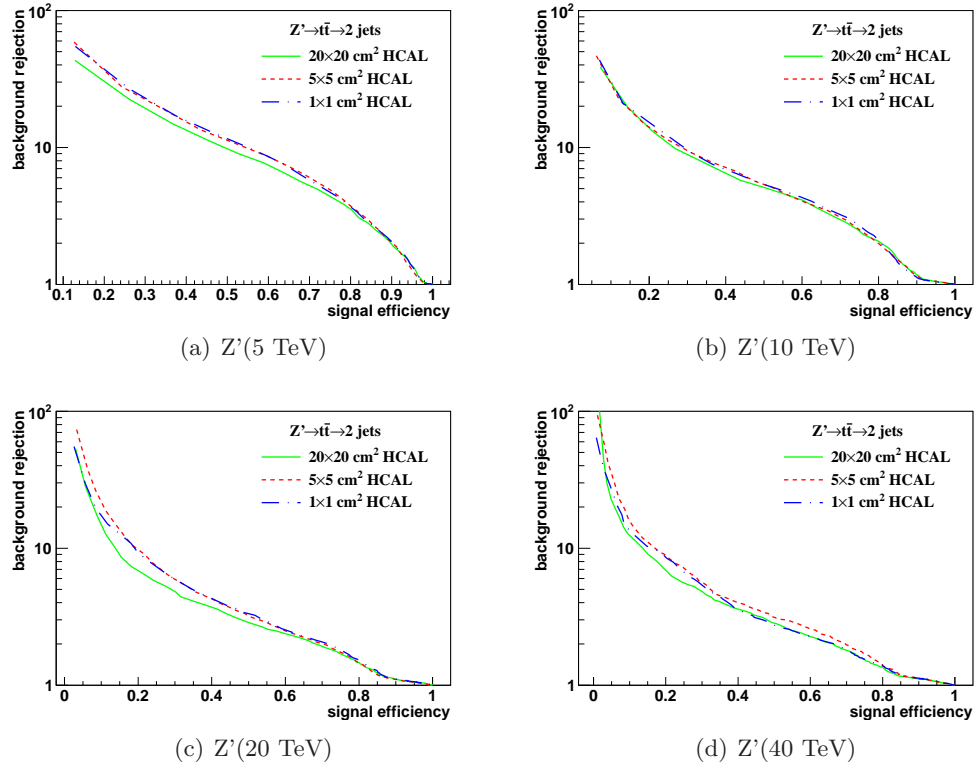


Figure 10: The ROC curves of soft drop mass selection for  $\beta=2$  with 5, 10, 20 and 40 TeV c.m. energies. Three different detector cell sizes are compared:  $20 \times 20$ ,  $5 \times 5$  and  $1 \times 1 \text{ cm}^2$ . The signal (background) process is  $Z' \rightarrow t\bar{t}$  ( $Z' \rightarrow q\bar{q}$ ).

161 The  $k$  runs over all constituent particles in a given large radius jet,  $p_{T,k}$  is the transverse  
 162 momentum of each individual constituent particle,  $\Delta R_{j,k} = \sqrt{(\Delta y)^2 + (\Delta \phi)^2}$  is the  
 163 distance between the constituent particle  $k$  and the candidate subjet axis  $j$  in the  $y - \phi$   
 164 plane. The  $R_0$  is the characteristic jet radius used in the anti- $k_t$  jet algorithm.

165 This analysis uses the jet reconstruction described in Sect. 2. The subjet axes are  
 166 obtained by running the exclusive  $k_t$  algorithm [25] and reversing the last  $N$  clustering  
 167 steps. Namely, when  $\tau_N$  is computed, the  $k_t$  algorithm is forced to return exactly  $N$   
 168 jets. If a large radius jet has  $N$  subjet(s), its  $\tau_N$  is smaller than  $\tau_{N-1}$ . Therefore, in our  
 169 analysis, the ratio of the  $\tau_N$  variables,  $\tau_{21}$  ( $\tau_2/\tau_1$ ) and  $\tau_{32}$  ( $\tau_3/\tau_2$ ), are used to distinguish  
 170 the one-prong background jets and the two-prong jets from  $W$  or the three-prong jets  
 171 from top.

172 We use the ROC curves as described in Section 4.2 to analyze the detector perfor-  
 173 mance and determine the cell size that gives the best separation power to distinguish  
 174 signal from background. Following the suggestion by Ref. [26], the requirement on the  
 175 soft drop mass with  $\beta = 0$  is applied before the study of  $N$ -subjettiness. For each  
 176 detector configuration and c.m. energy, the soft drop mass selection is determined as  
 177 follows. First, we look for the median bin of the soft drop mass histogram from sim-  
 178 ulated signal events as described in Section 4.2. Then, we compare the numbers of  
 179 events in the bins adjacent to the medium bin (bin  $i_{\text{med}} - 1$  and bin  $i_{\text{med}} + 1$ ). The bin  
 180 with larger number of events is added, in addition to the medium bin, to extend the  
 181 mass window. The procedure is repeated until the window contains at least 75% of the  
 182 total number of signal events.

183 In order to obtain the signal and background efficiencies, various ranges of  $\tau_{21}$  and  
 184  $\tau_{32}$  are scanned. Since some of the background distributions have long tails and leak into  
 185 the signal-dominated region, we use the following method as suggested by the Pearson  
 186 Lemma Method to determine the ranges of  $\tau$  variables. First, we take the ratio of  
 187 the signal to background  $\tau_{21}$  ( $\tau_{32}$ ) histograms. The boundaries of the bin (seed bin)  
 188 with maximum signal to background ratio (S/N) give us the first range of  $\tau$  selection:  
 189  $x_{\text{low}}^{\text{seedbin}} < \tau_{21} < x_{\text{high}}^{\text{seedbin}}$ . Then, we compare the S/N in the bins adjacent to the seed  
 190 bin. The bin with larger S/N is added, in addition to the seed bin, to extend the  $\tau_{21}$   
 191 selection window. Every window has its corresponding  $\epsilon_{\text{sig}}$  and  $1/\epsilon_{\text{bkg}}$  and an ROC  
 192 curve is mapped out.

193 In addition to the ROC curves, we use the so-called "Mann-Whitney" test [27]  
 194 to quantify the detector performance. The value of Mann-Whitney is related to the  
 195 integrated area under the ROC curve: if the value is bigger, it indicates the signal and  
 196 background distributions have similar shapes and can not be well separated from each  
 197 other. Vice versa, if the value is smaller, we can achieve a better signal and background  
 198 separation.

199 Figures 11 and 13 show the distributions of  $\tau_{21}$  and  $\tau_{32}$  for  $\sqrt{s} = 20$  TeV after  
 200 applying the requirement on the soft drop mass. The signals considered are  $Z' \rightarrow WW$   
 201 ( $\tau_{21}$ ) and  $Z' \rightarrow t\bar{t}$  ( $\tau_{32}$ ). Figures 12 and 14 present the ROC curves from different  
 202 detector cell sizes and c.m. energies, respectively. The smallest detector cell size  
 203 ( $1 \times 1$  cm<sup>2</sup>) does not have the best separation power. In fact, in some cases, the  
 204 best separation power comes from a detector with bigger cell sizes ( $5 \times 5$  cm<sup>2</sup> and  
 205  $20 \times 20$  cm<sup>2</sup>).

206 Figures 17 (a) and (b) present the summary plots of  $\tau_{21}$  and  $\tau_{32}$  with various detector

cell sizes and c.m. energies using Mann Whitney U test. For  $\tau_{21}$  at smaller c.m. energies, when the cell size is smaller, the detector performance improves. However, when c.m. energy increases, no improvement is observed using the smallest detector cell size ( $1 \times 1 \text{ cm}^2$ ). For  $\tau_{32}$ , the case is similar to  $\tau_{21}$ . It is interesting to note that at very large c.m. energies, the large detector cell sizes ( $5 \times 5 \text{ cm}^2$  and  $20 \times 20 \text{ cm}^2$ ) have a better separation power than the smallest cell size considered in this analysis.

## 5.2. Energy correlation function

The energy correlation function (ECF) [28] is defined as follows:

$$ECF(N, \beta) = \sum_{i_1 < i_2 < \dots < i_N \in J} \left( \prod_{a=1}^N p_{Tia} \right) \left( \prod_{b=1}^{N-1} \prod_{c=b+1}^N R_{i_b i_c} \right)^\beta, \quad (3)$$

where the sum is looped all particles in the jet  $J$ ,  $p_T$  is the transverse momentum of each individual particle, and  $R$  is the distance between two particles in the  $y$ - $\phi$  plane. In order to use a dimensionless variable, a parameter  $r_N$  is defined:

$$r_N^{(\beta)} \equiv \frac{ECF(N+1, \beta)}{ECF(N, \beta)}. \quad (4)$$

The idea of  $r_N$  comes from N-subjettiness  $\tau_N$ . Both  $r_N$  and  $\tau_N$  are linear in the energy of the soft radiation for a system of  $N$  partons with soft radiation. In general, if the system has  $N$  subjets,  $ECF(N+1, \beta)$  should be significantly smaller than  $ECF(N, \beta)$ . Therefore, we can use this feature to distinguish jets with different numbers of subjets. As in Section 5.1, the ratio  $r_N/r_{N-1}$ , denoted by  $C_N$ , (double ratios of ECFs) is used to study the detector performance:

$$C_N^{(\beta)} \equiv \frac{r_N^{(\beta)}}{r_{N-1}^{(\beta)}} = \frac{ECF(N-1, \beta) ECF(N+1, \beta)}{ECF(N, \beta)^2}. \quad (5)$$

In our analysis, we set  $N = 2$  and  $\beta = 1$  ( $C_2^1$ ).

Figure 15 presents the histograms of  $C_2^1$  with  $\sqrt{s} = 20 \text{ TeV}$  after making the requirement on the soft drop mass. The signal considered is  $Z' \rightarrow WW$ . Figure 16 shows the ROC curves from different detector cell sizes for each c.m. energy, respectively. One can see that the smallest detector cell size ( $1 \times 1 \text{ cm}^2$ ) does not have the best signal/background separation power. Figure 17(c) summarizes the result of the Mann Whitney U test for  $C_2^1$ . When c.m. energy increases, no improvement is observed from detector with the smallest cell size.

## 6. Conclusions

The studies presented in this paper show that the reconstruction of jet substructure variables for future particle colliders will benefit from small cell sizes of the hadronic calorimeters. This conclusion was obtained using the realistic GEANT4 simulation of calorimeter responses combined with reconstruction of calorimeter clusters used as inputs for jet reconstruction. Hadronic calorimeters that use the cell sizes of  $20 \times 20 \text{ cm}^2$

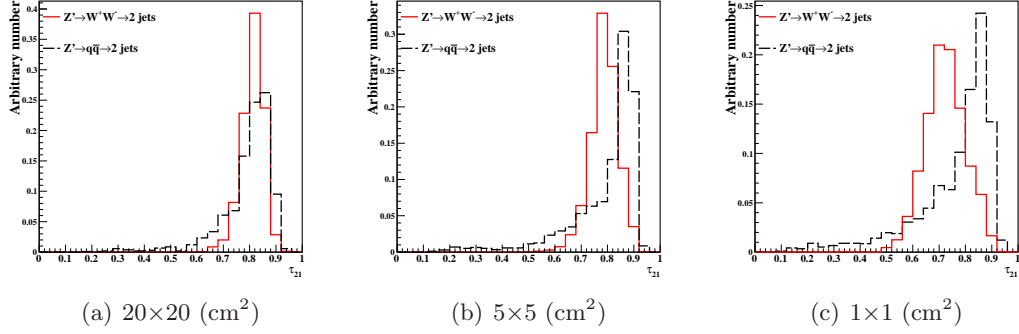


Figure 11: Distributions of  $\tau_{21}$  in 20 TeV energy collision for different detector sizes. Cell sizes in  $20 \times 20$ ,  $5 \times 5$ , and  $1 \times 1 \text{ cm}^2$  are shown here.

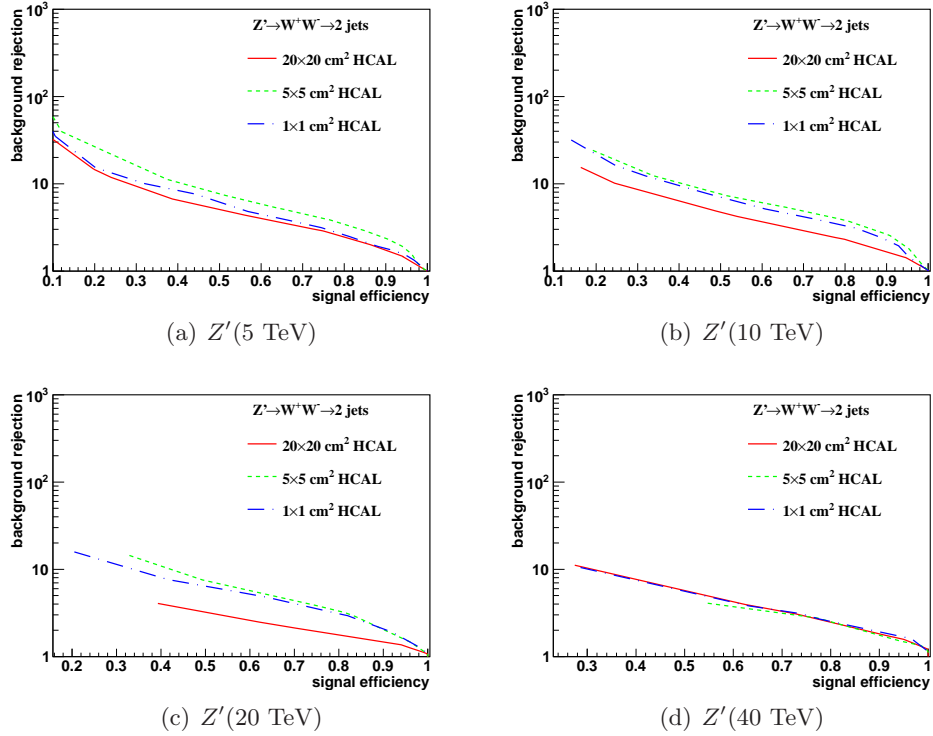


Figure 12: Signal efficiency versus background rejection rate using  $\tau_{21}$ . The energies of collision at (a) 5, (b) 10, (c) 20 and (d) 40 TeV are shown here. In each figure, the three ROC curves correspond to different detector sizes.

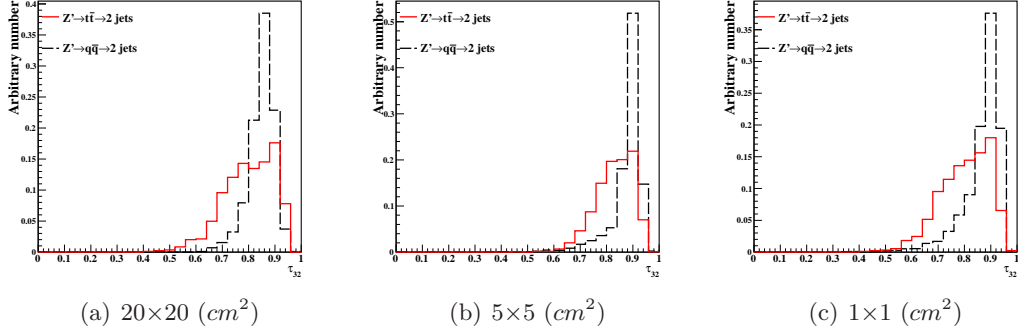


Figure 13: Distributions of  $\tau_{32}$  in 20 TeV energy collision for different detector sizes. Cell sizes in  $20 \times 20$ ,  $5 \times 5$ , and  $1 \times 1$   $\text{cm}^2$  are shown here.

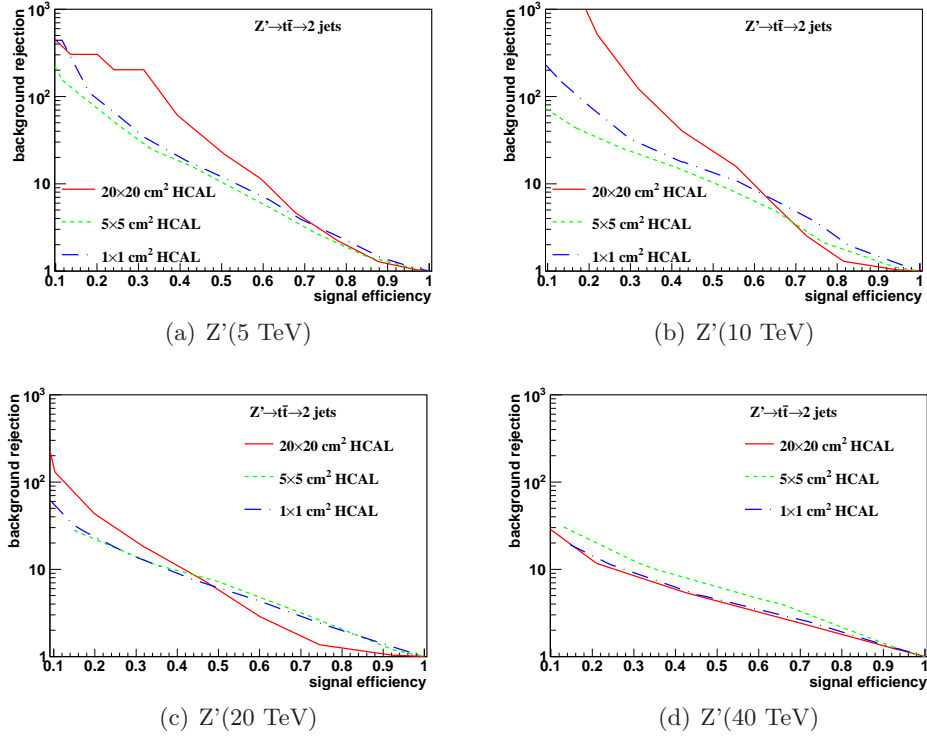


Figure 14: Signal efficiency versus background rejection rate using  $\tau_{32}$ . The energies of collision at (a) 5, (b) 10, (c) 20 and (d) 40 TeV are shown here. In each figure, the three ROC curves correspond to different detector sizes.

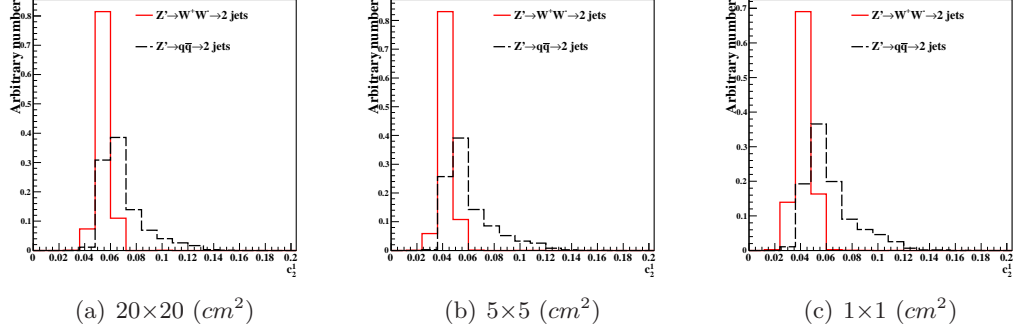


Figure 15: Distributions of  $C_2^1$  in 20 TeV energy collision for different detector sizes. Cell sizes in  $20 \times 20$ ,  $5 \times 5$ , and  $1 \times 1$   $\text{cm}^2$  are shown here.

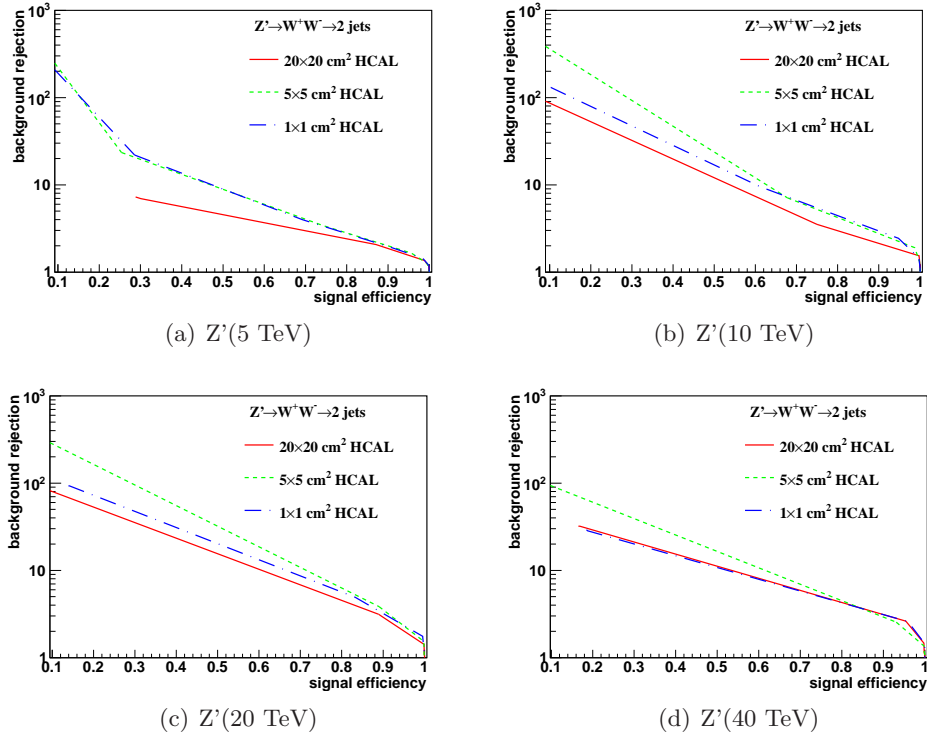


Figure 16: Signal efficiency versus background rejection rate using  $C_2^1$ . The energies of collision at (a) 5, (b) 10, (c) 20, and (d) 40 TeV are shown here. In each figure, the three ROC curves correspond to different detector sizes.



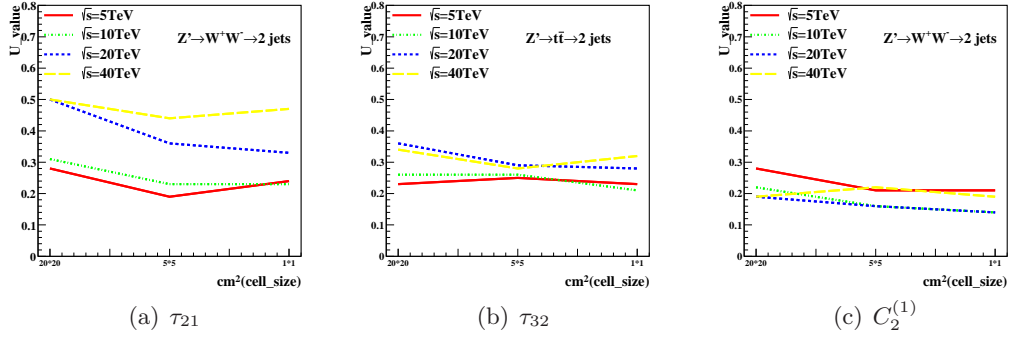


Figure 17: The Mann-Whitney U values for  $\tau_{21}$ ,  $\tau_{32}$ , and  $C_2^{(1)}$  reconstructed with different collision energies and detector cell sizes.

( $\Delta\eta \times \Delta\phi = 0.087 \times 0.087$ ) are least performant almost for every substructure variables considered in this analysis for jet transverse momenta between 2.5 to 10 TeV. Such cell sizes are close to those used for the ATLAS and CMS detectors at the LHC. In terms of the reconstruction of the physics-motivated quantities used for jet substructure studies, the performance of a hadronic calorimeter with  $\Delta\eta \times \Delta\phi = 0.022 \times 0.022$  is, in most cases, better than for a detector with  $0.087 \times 0.087$  cells. The performance of the HCAL with cells  $\Delta\eta \times \Delta\phi = 0.0087 \times 0.0087$  and  $\Delta\eta \times \Delta\phi = 0.0043 \times 0.0043$  were found to be similar.

Thus this study confirms the HCAL geometry of the SiFCC detector [7], with the  $\Delta\eta \times \Delta\phi = 0.022 \times 0.022$  HCAL cells. It also confirms the HCAL design of the baseline FCC-hh [19, 20] detector with  $\Delta\eta \times \Delta\phi = 0.025 \times 0.025$  HCAL cells.

It is interesting to note that, for very boosted jets with transverse momenta close to 20 TeV, no significant improvement with the decrease of cell sizes was observed. This result needs to be understood in terms of various type of simulations and different options for construction of the calorimeter clusters.

## Acknowledgements

This research was performed using resources provided by the Open Science Grid, which is supported by the National Science Foundation and the U.S. Department of Energy's Office of Science. We gratefully acknowledge the computing resources provided on Blues, a high-performance computing cluster operated by the Laboratory Computing Resource Center at Argonne National Laboratory. Argonne National Laboratory's work was supported by the U.S. Department of Energy, Office of Science under contract DE-AC02-06CH11357. The Fermi National Accelerator Laboratory (Fermilab) is operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the United States Department of Energy.

## References

- [1] M. Benedikt, [The Global Future Circular Colliders Effort](#) CERN-ACC-SLIDES-2016-0016. Presented at P5 Workshop on the Future of High Energy Physics, BNL, USA, Dec. 15-18, 2013. URL <http://cds.cern.ch/record/2206376>
- [2] J. Tang, et al., Concept for a Future Super Proton-Proton Collider (2015). [arXiv:1507.03224](#).
- [3] R. Calkins, et al., [Reconstructing top quarks at the upgraded LHC and at future accelerators](#), in: Proceedings, Community Summer Study 2013: Snowmass on the Mississippi (CSS2013): Minneapolis, MN, USA, July 29-August 6, 2013. [arXiv:1307.6908](#). URL <https://inspirehep.net/record/1244676/files/arXiv:1307.6908.pdf>
- [4] S. V. Chekanov, J. Dull, Energy range of hadronic calorimeter towers and cells for high-pT jets at a 100 TeV collider. <http://arxiv.org/abs/1511.01468> [arXiv:1511.01468](#).
- [5] E. Coleman, M. Freytsis, A. Hinzmann, M. Narain, J. Thaler, N. Tran, C. Vernieri, The importance of calorimetry for highly-boosted jet substructure [arXiv:1709.08705](#).
- [6] DELPHES 3 Collaboration, J. de Favereau, C. Delaere, P. Demin, A. Giammanco, V. Lematre, A. Mertens, M. Selvaggi, DELPHES 3, A modular framework for fast simulation of a generic collider experiment, JHEP 02 (2014) 057. [arXiv:1307.6346](#), [doi:10.1007/JHEP02\(2014\)057](#).
- [7] S. V. Chekanov, M. Beydler, A. V. Kotwal, L. Gray, S. Sen, N. V. Tran, S. S. Yu, J. Zuzelski, Initial performance studies of a general-purpose detector for multi-TeV physics at a 100 TeV pp collider, JINST 12 (06) (2017) P06009. [arXiv:1612.07291](#), [doi:10.1088/1748-0221/12/06/P06009](#).
- [8] J. Allison, et al., Recent developments in Geant4, Nuclear Instruments and Methods in Physics Research A 835 (2016) 186.
- [9] M. J. Charles, PFA Performance for SiD, in: Linear colliders. Proceedings, International Linear Collider Workshop, LCWS08, and International Linear Collider Meeting, ILC08, Chicago, USA, November 16-20, 2008, 2009. [arXiv:0901.4670](#).
- [10] J. S. Marshall, M. A. Thomson, Pandora Particle Flow Algorithm, in: Proceedings, International Conference on Calorimetry for the High Energy Frontier (CHEF 2013), 2013, pp. 305–315. [arXiv:1308.4537](#).
- [11] G. P. S. M. Cacciari, G. Soyez, FastJet user manual CERN-PH-TH/2011-297. [arXiv:1111.6097](#).
- [12] M. Cacciari, G. P. Salam, G. Soyez, The anti-kt jet clustering algorithm, JHEP 0804 (2008) 063. [arXiv:0802.1189](#).
- [13] S. Chekanov, HepSim: a repository with predictions for high-energy physics experiments, Advances in High Energy Physics 2015 (2015) 136093, available as <http://atlaswww.hep.anl.gov/hepsim/>.
- [14] B. Auerbach, S. Chekanov, J. Love, J. Proudfoot, A. Kotwal, Sensitivity to new high-mass states decaying to  $t\bar{t}b\bar{a}$  at a 100 TeV collider <http://arxiv.org/abs/1412.5951> [arXiv:1412.5951](#).
- [15] J. Butterworth, B. Cox, J. R. Forshaw,  $WW$  scattering at the CERN LHC, Phys.Rev. D65 (2002) 096014. [arXiv:hep-ph/0201098](#), <http://dx.doi.org/10.1103/PhysRevD.65.096014> [doi:10.1103/PhysRevD.65.096014](#).
- [16] S. Catani, Y. L. Dokshitzer, M. H. Seymour, B. R. Webber, [Longitudinally-invariant k-clustering algorithms for hadron-hadron collisions](#), Nuclear Physics B 406 (12) (1993) 187 – 224. URL <http://www.sciencedirect.com/science/article/pii/055032139390166M>
- [17] S. D. Ellis, D. E. Soper, Successive combination jet algorithm for hadron collisions, Phys. Rev. D48 (1993) 3160–3166. [arXiv:hep-ph/9305266](#), <http://dx.doi.org/10.1103/PhysRevD.48.3160> [doi:10.1103/PhysRevD.48.3160](#).
- [18] ATLAS Collaboration Collaboration, G. Aad, et al., Jet mass and substructure of inclusive jets in  $\sqrt{s} = 7$  TeV  $pp$  collisions with the ATLAS experiment, JHEP 1205 (2012) 128. [arXiv:1203.4606](#), [doi:10.1007/JHEP05\(2012\)128](#).
- [19] C. Neubüser, Performance studies and requirements on the calorimeters for a fcc-hh experiment, in: Z.-A. Liu (Ed.), Proceedings of International Conference on Technology and Instrumentation in Particle Physics 2017, Springer Singapore, Singapore, 2018, pp. 37–43.
- [20] J. Faltova, [Design and performance studies of a hadronic calorimeter for a fcc-hh experiment](#), Journal of Instrumentation 13 (03) (2018) C03016. URL <http://stacks.iop.org/1748-0221/13/i=03/a=C03016>
- [21] A. J. Larkoski, S. Marzani, G. Soyez, J. Thaler, Soft Drop, JHEP 05 (2014) 146. [arXiv:1402.2657](#), [doi:10.1007/JHEP05\(2014\)146](#).
- [22] Y. L. Dokshitzer, G. D. Leder, S. Moretti, B. R. Webber, Better jet clustering algorithms, JHEP

- 08 (1997) 001. [arXiv:hep-ph/9707323](#), <http://dx.doi.org/10.1088/1126-6708/1997/08/001> doi:  
10.1088/1126-6708/1997/08/001.
- [23] M. Wobisch, T. Wengler, Hadronization corrections to jet cross-sections in deep inelastic scattering, in: Monte Carlo generators for HERA physics. Proceedings, Workshop, Hamburg, Germany, 1998-1999, 1998, pp. 270–279. [arXiv:hep-ph/9907280](#).
- [24] J. Thaler, K. Van Tilburg, Identifying Boosted Objects with N-subjettiness, JHEP 03 (2011) 015. [arXiv:1011.2268](#), doi:[10.1007/JHEP03\(2011\)015](#).
- [25] S. Catani, Y. L. Dokshitzer, M. H. Seymour, B. R. Webber, Longitudinally-invariant  $k_{\perp}$ -clustering algorithms for hadron-hadron collisions, Nucl. Phys. B 406 (CERN-TH-6775-93. LU-TP-93-2) (1993) 187–224.
- [26] F. A. Dreyer, L. Necib, G. Soyez, J. Thaler, Recursive Soft Drop, JHEP 06 (2018) 093. [arXiv:1804.03657](#), doi:[10.1007/JHEP06\(2018\)093](#).
- [27] H. B. Mann, D. R. Whitney, On a test of whether one of two random variables is stochastically larger than the other, Ann. Math. Statist. 18 (1) (1947) 50–60. doi:[10.1214/aoms/1177730491](#).  
URL <https://doi.org/10.1214/aoms/1177730491>
- [28] A. J. Larkoski, G. P. Salam, J. Thaler, Energy Correlation Functions for Jet Substructure, JHEP 06 (2013) 108. [arXiv:1305.0007](#), doi:[10.1007/JHEP06\(2013\)108](#).