# Studies of granularity of a hadronic calorimeter for tens-of-TeV jets at a 100 TeV $pp$ collider


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## Abstract

Jet substructure variables for hadronic jets with transverse momenta in the range from 2.5 TeV to 20 TeV were studied using several designs for spacial size of calorimeter cells. The studies used the full Geant4 simulation of calorimeter response combined with realistic reconstruction of calorimeter clusters used in jet reconstruction. The results unambiguously indicate that performance of jet-substructure reconstruction improves with reducing cell sizes.



*Keywords:* multi-TeV physics, $pp$ collider, future hadron colliders, FCC, SppC


## 1. Introduction

Particle collisions at energies beyond those attained at the LHC will lead to many challenges for detector technologies. Future experiments, such as high-energy LHC (HE-LHC), future circular $pp$ colliders of the European initiative, FCC-hh [1] and the Chinese initiative, SppC [2] will be required to measure high-momentum bosons ($W$, $Z$, $H$) and top quarks with strongly collimated decay products that form jets. Studies of jet substructure can help identify such particles.

The reconstruction of jet substructure variables for collimated jets with transverse momentum above 10 TeV require an appropriate detector design. The most important for reconstruction of such jets are tracking and calorimeter. Recently, a number of studies [3, 4, 5] have been discussed using various fast simulation tools, such as Delphes [6], in which momenta of particles are smeared to mimic detector response.

A major step towards the usage of full Geant4 simulation to verify the granularity requirements for calorimeters was made in [7]. The studies included in this paper have illustrated a significant impact of granularity of electromagnetic (ECAL) and hadronic (HCAL) calorimeters on the shape of hadronic showers calculated using calorimeter


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hits for two particles separated by some angle. It was concluded that high granularity is essential in resolving two close-by particles for energies above 100 GeV.

This paper makes another step in understanding of this problem in terms of high-level physics quantities typically used in physics analyses. Similar to the studies presented in [7], this paper is based on a full Geant4 simulation with realistic jet reconstruction.

## 2. Simulation of detector response and event reconstruction

The description of the detector and software used for this study is discussed in [7]. We use the SiFCC detector geometry with a software package that represents a versatile environment for simulations of detector performance, testing new technology options, event reconstruction techniques for future 100 TeV colliders.

The GEANT4 (version 10.3) [8] simulation of calorimeter response was complemented with the full reconstruction of calorimeter clusters formed by the Pandora algorithm [9, 10]. Calorimeter clusters were built from calorimeter hits in the ECAL and HCAL after applying the corresponding sampling fractions. No other corrections are applied. Hadronic jets were reconstructed with the FASTJET package [11] using the anti-$k_T$ algorithm [12] with a distance parameter of 0.5.

In the following discussion, we use the simulations of a heavy $Z'$ boson, a hypothetical gauge boson that arises from extensions of the electroweak symmetry of the Standard Model. The $Z'$ bosons were simulated with the masses, $M = 5$, 10, 20 and 40 TeV. The lowest value represents a typical mass that is within the reach of the LHC experiments. The value 40 TeV represents the physics reach for a 100 TeV collider. The $Z'$ particles are forced to decay to two light-flavor jets ($q\bar{q}$), $W^+W^-$ or $t\bar{t}$, where $W$ and $t$ decay hadronically. In all such scenarios, two highly boosted jets are produced, which are typically back-to-back in the laboratory frame. Typical transverse momenta of such jets are $\simeq M/2$. The main difference between considered decay types lays in different jet substructure. In the case of the $q\bar{q}$ decays, jets do not have any internal structure. In the case of $W^+W^-$, each jet originates from $W$, thus it has two subjects because of the decay $W \rightarrow q\bar{q}$. In the case of hadronic top decays, jets have three subjects due to the decay $t \rightarrow W^+ b \rightarrow q\bar{q}b$. The signal events were generated using the PYTHIA8 generator with the default settings, ignoring interference with SM processes. The event samples used in this paper are available from the HepSim database [13].

## 3. Studies of jet properties

First let us consider several variables that represent jet substructure using different types of calorimeter granularity. The question we want to answer is how close the reconstructed jet substructure variables reflect the input "truth" values that are reconstructed using input particles directly from the PYTHIA8 generator.

In this study we use the jet effective radius and jet splitting scales as benchmark variables to study jet substructure properties for different calorimeter granularity scenarios. The effective radius is the average of the energy weighted radial distance in $\eta-\phi$ space of jet constituents. Recently, it has been studied for multi-TeV jets in Ref.[14]. A jet $k_T$ splitting scale [15] is defined as a distance measure used to form jets by the

$k_T$ recombination algorithm [16, 17]. This variable has been studied by ATLAS [18], and more recently in the context of 100 TeV physics [14]. The splitting scale is defined as $\sqrt{d_{12}} = \min(p_T^1, p_T^2) \times \delta R_{12}$ [18] at the final stage of the $k_T$ clustering, where two subjets are merged into the final one.

Figures 1 and 2 show the distributions of the jet effective radius and jet splitting scale for different jet transverse momenta and HCAL granularities. The reconstructed-level distributions significantly disagree with the distributions reconstructed using truth-level particles. The distribution reconstructed with the cell sizes 1 cm×1 cm are closest to the truth-level variables. The distributions reconstructed using the cell size of 20 cm×20 cm, which is similar to the nuclear interaction length of $Fe$ ($\lambda_I \simeq 17$ cm) the SiFCC calorimeter [7], show the largest discrepancy with the truth-level variables. Note that, in terms of closeness of reconstructed distributions to the truth level, there is no significant difference between 5 cm×5 cm, 2 cm×2 cm and 1 cm×1 cm choices. Both the ATLAS and CMS detectors use cell sizes which are close to the nuclear interaction length of the HCAL cells.

## 4. Study of detector performance with soft drop mass

In this section, we use the jet mass computed with a specific algorithm, soft drop declustering, to study the performance of detector with various detector cell sizes and center-of-mass (c.m.) energies.

### *4.1. The technique of soft drop declustering*

The soft drop declustering [19] is a grooming method that removes soft wide-angle radiation from a jet. The constituents of a jet $j_0$ are first reclustered using the Cambridge-Aachen (C/A) algorithm [20, 21]. Then, the jet $j_0$ is broken into two subjets $j_1$ and $j_2$ by undoing the last stage of C/A clustering. If the subjets pass the following soft drop condition, jet $j_0$ is the final soft-drop jet. Otherwise, the algorithm redefines $j_0$ to be the subjet with larger $p_T$ (among $j_1$ and $j_2$) and iterates the procedure.

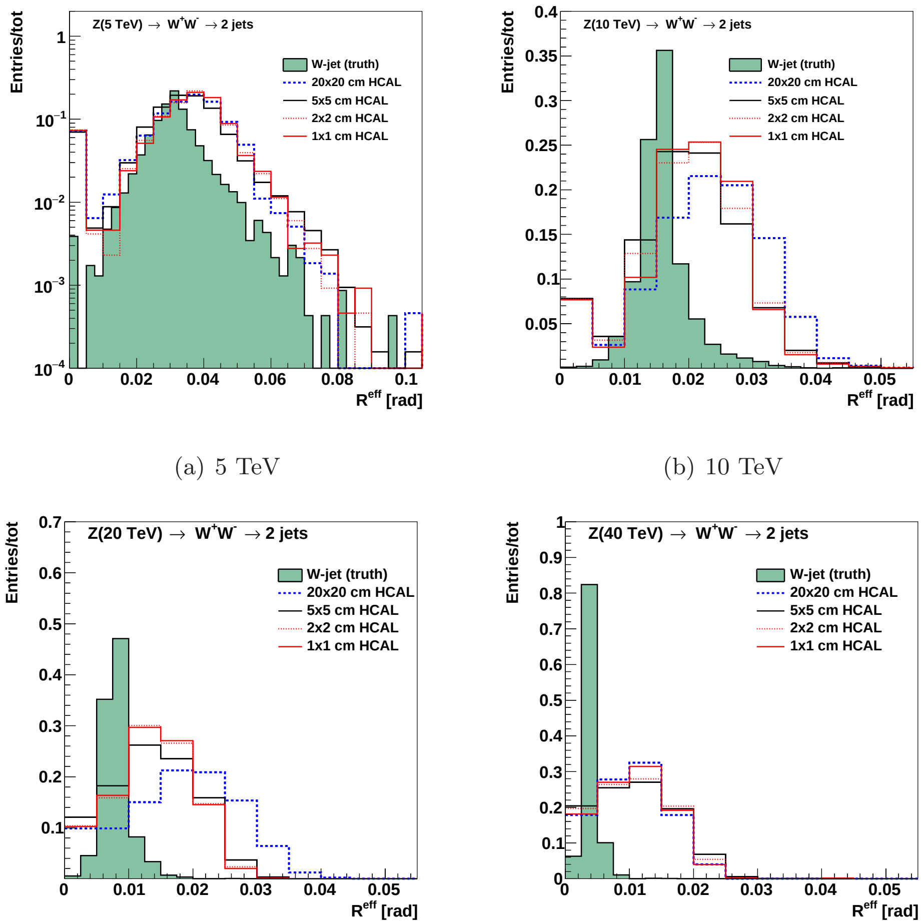
$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\text{cut}}\left(\frac{\Delta R_{12}}{R_0}\right)^{\beta}, \tag{1}$$

where $p_{T1}$ and $p_{T2}$ are the transverse momenta of the two subjets, $z_{\text{cut}}$ is soft drop threshold, $\Delta R_{12}$ is the distance between the two subjets in the rapidity-azimuth angle plane ($y$-$\phi$), $R_0$ is the characteristic radius of the original jet, and $\beta$ is the angular exponent.

In our study, we compare the performance of future detector when setting $\beta = 0$ versus when setting $\beta = 2$. For $\beta = 0$, the soft drop condition depends only on the $z_{\text{cut}}$. For $\beta = 2$, the condition depends on the angular distance between the two subjets and $z_{\text{cut}}$ and the algorithm becomes infrared and collinear safe.

### *4.2. Analysis method*

We employ the following method to quantify the detector performance and find out the cell size that gives the best separation power to distinguish signal from background. For each configuration of detector and c.m. energy, we draw the receiver operating characteristic (ROC) curves in which the x-axis is the signal efficiency ($\epsilon_{\text{sig}}$) and y-axis



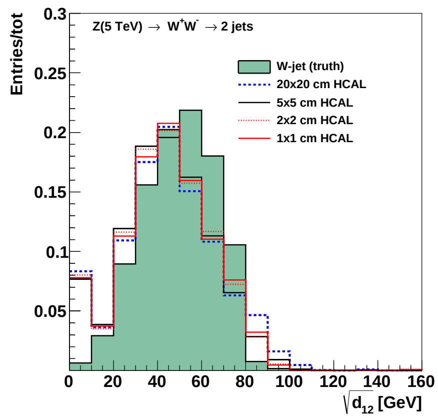

(a) 5 TeV

(b) 10 TeV

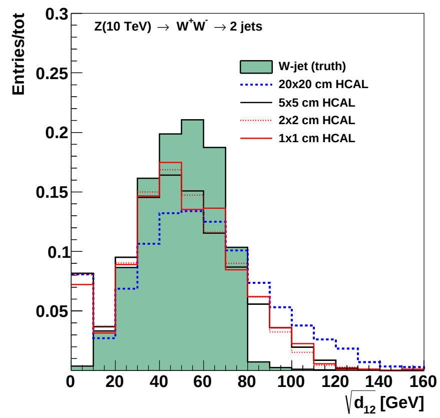
(c) 20 TeV

(d) 40 TeV

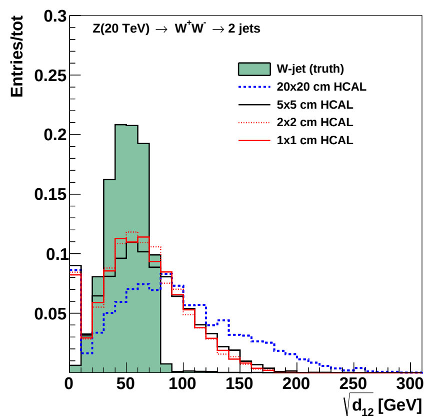
Figure 1: Jet effective radius for different jet transverse momenta and HCAL granularities.



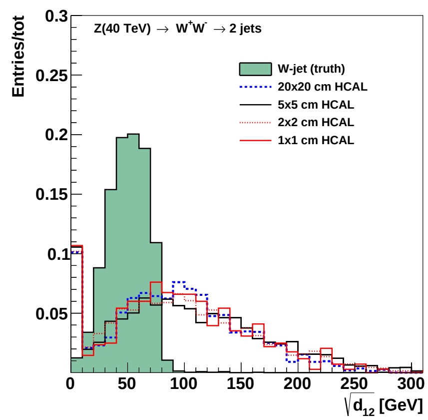

(a) 5 TeV




(b) 10 TeV

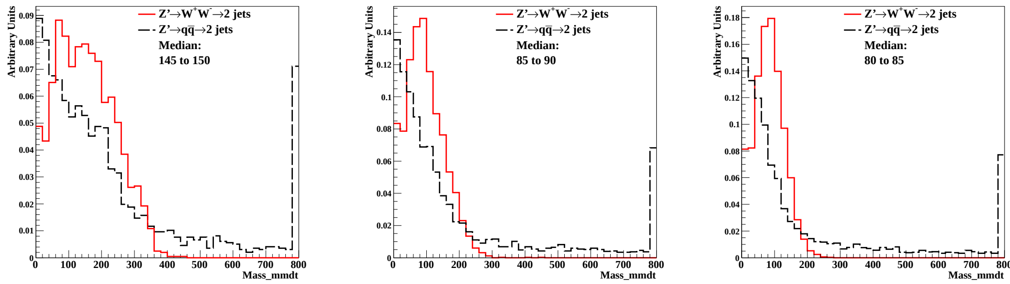



(c) 20 TeV




(d) 40 TeV

Figure 2: Jet splitting scale for different jet transverse momenta and HCAL granularity.




(a) $20\times20(\mathrm{cm}^2)$ (b) $5\times5(\mathrm{cm}^2)$ (c) $1\times1(\mathrm{cm}^2)$

Figure 3: Distributions of soft drop mass for $\beta$=0, with 20 TeV c.m. energies and three different detector cell sizes: 20×20, 5×5, and 1×1 (cm$^2$). The signal (background) process is $Z' \to WW$ ($Z' \to q\bar{q}$).

is the inverse of background efficiency ($1/\epsilon_{\mathrm{bkg}}$). In order to scan the efficiencies of soft drop mass cuts, we vary the mass window as follows. We first look for the median bin $i_{\mathrm{med}}$[1] of the soft drop mass histogram from simulated signal events. Taking the right boundary of bin $i_{\mathrm{med}}$ as the center of mass window $x_{\mathrm{center}}$, we start increasing the width of mass window symmetrically on the left and on the right of $x_{\mathrm{center}}$, in steps of 5 GeV, i.e. the narrowest mass window is $[x_{\mathrm{center}} - 5, x_{\mathrm{center}} + 5]$. If one side reaches the boundary of the mass histogram, we only increase the width on the other side, also in steps of 5 GeV. For each mass window, there will be corresponding $\epsilon_{\mathrm{sig}}$ and $\epsilon_{\mathrm{bkg}}$, which gives a point in the ROC curves.

### *4.3. Results and conclusion*

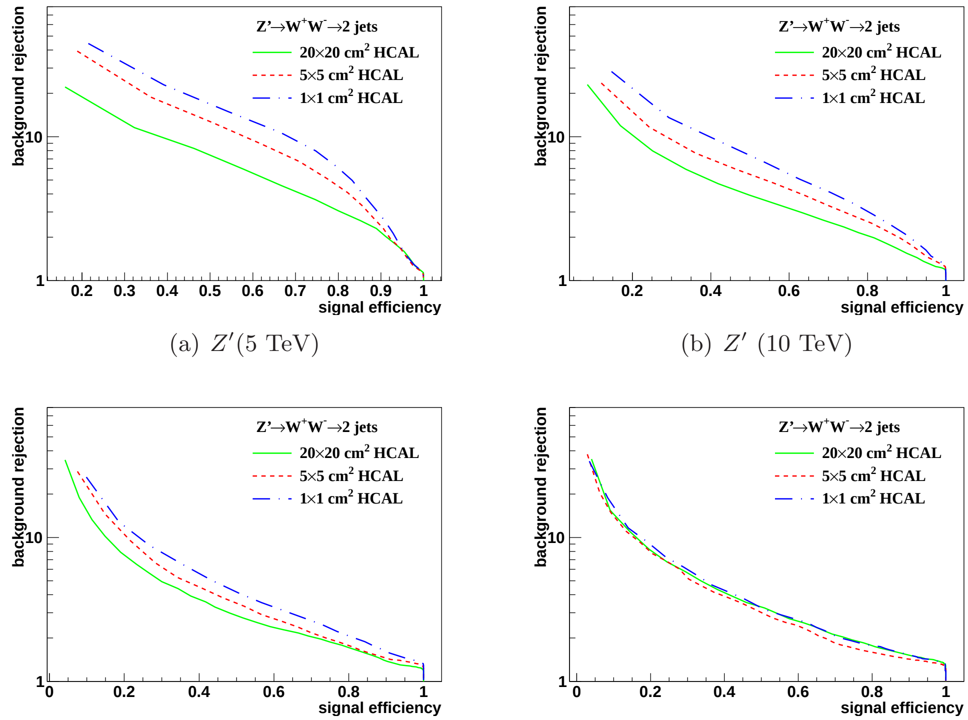
Figures 3, 5, 7, and 9 show a few representative distributions for the soft drop mass for $\beta = 0$ and $\beta = 2$ with different c.m. energies and detector cell sizes; the signals considered are $Z' \to WW$ and $Z' \to t\bar{t}$.

Figures 4, 6, 8, and 10 show the ROC curves for different detector cell sizes and c.m. energies.

These studies show that the reconstruction of soft drop mass improves with decrease of the HCAL cell sizes. Figures 4 and 6 show that for $\beta = 0$ the smallest detector cell size, 1 cm × 1 cm, has the best separation power at $\sqrt{s}$ =5, 10, and 20 TeV when the signal is $Z' \to WW$ and at $\sqrt{s}$ =10 and 20 TeV when the signal is $Z' \to t\bar{t}$. On the contrary, Figs. 8 and 10 show that for $\beta = 2$ the smallest detector cell size does not have improvements in the separation power with respect to those with larger cell sizes. In fact, the performances of the three cell sizes are similar. In addition, sometimes bigger detector cell sizes, 5 cm × 5 cm or 20 cm × 20 cm have the best separation power.

We also find that the soft drop mass with $\beta = 0$ has better performance for distinguishing signal from background than for $\beta = 2$. Therefore, we will apply requirements on this variable when studying the other jet substructure variables.

[1]The integral from bin 0 to bin $i_{\mathrm{med}}$ ($i_{\mathrm{med}} - 1$) should be greater (less) than half of the total number of events. Note, the bin width is 5 GeV.



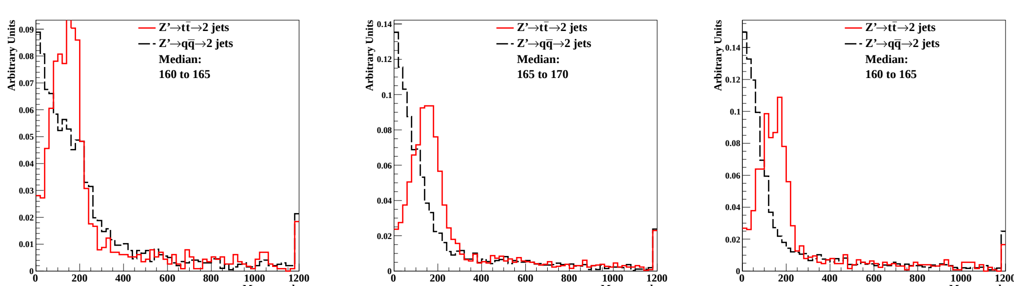

(a) $Z'$(5 TeV)

(b) $Z'$ (10 TeV)

(c) $Z'$ (20 TeV)

(d) $Z'$ (40 TeV)

Figure 4: The ROC curves of soft drop mass selection for $\beta$=0 with 5, 10, 20, 40 TeV c.m. energies. Three different detector cell sizes are compared: 20×20, 5×5, and 1×1 (cm$^2$). The signal (background) process is $Z' \to WW$ ($Z' \to q\bar{q}$).

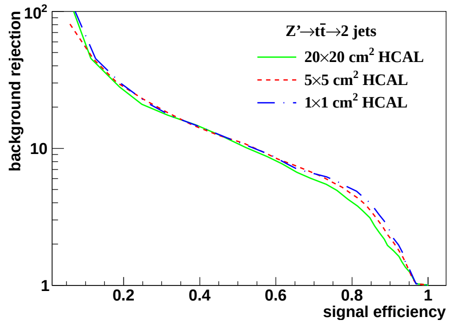



(a) 5×5 (cm$^2$)

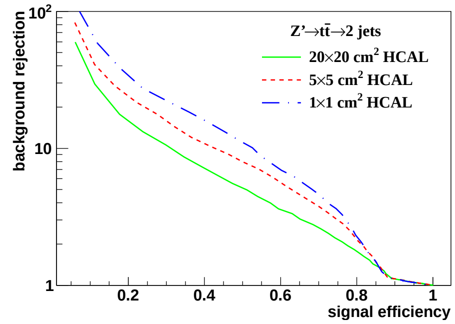
(b) 20×20 (cm$^2$)

(c) 1×1 (cm$^2$)

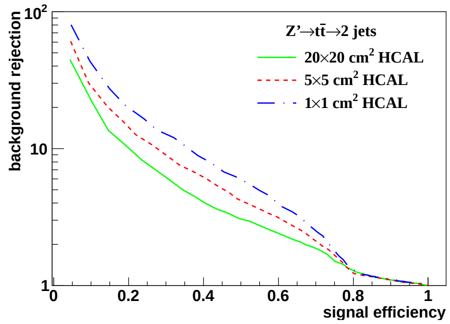
Figure 5: Distributions of soft drop mass for $\beta$=0, with 20 TeV c.m. energies and three different detector cell sizes: 20×20, 5×5, and 1×1 (cm$^2$). The signal (background) process is $Z' \to t\bar{t}$ ($Z' \to q\bar{q}$).



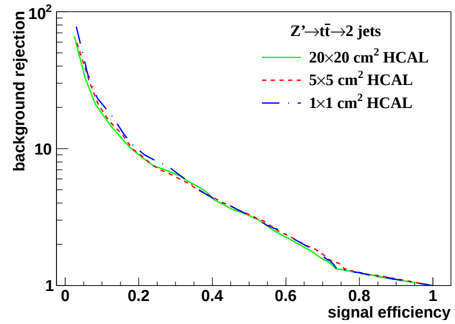

(a) $Z'$ (5 TeV)




(b) $Z'$ (10 TeV)

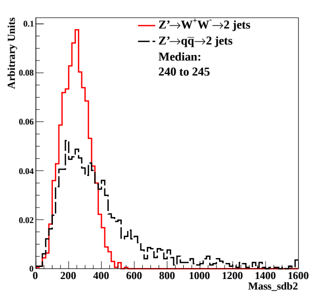



(c) $Z'$ (20 TeV)

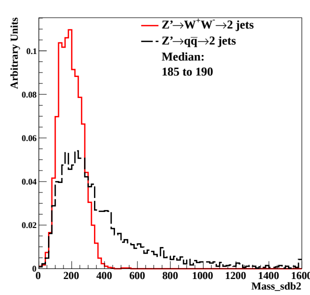



(d) $Z'$ (40 TeV)

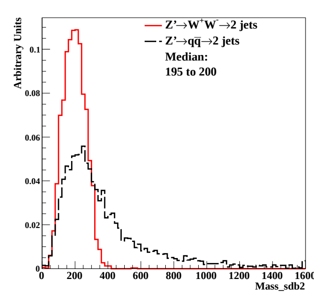
Figure 6: The ROC curves of soft drop mass selection for $\beta$=0 with 5,10, 20, 40 TeV c.m. energies. Three different detector cell sizes are compared: 20×20, 5×5, and 1×1 (cm$^2$). The signal (background) process is $Z' \to t\bar{t}$ ($Z' \to q\bar{q}$).




(a) 20×20 (cm$^2$)

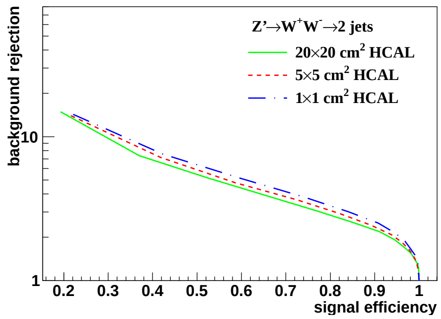



(b) 5×5 (cm$^2$)

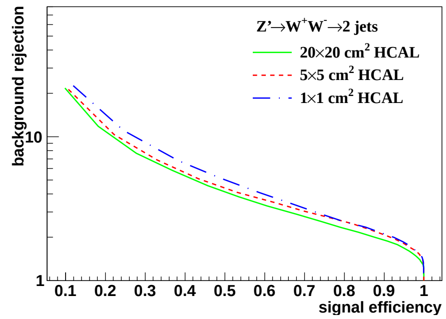



(c) 1×1 (cm$^2$)

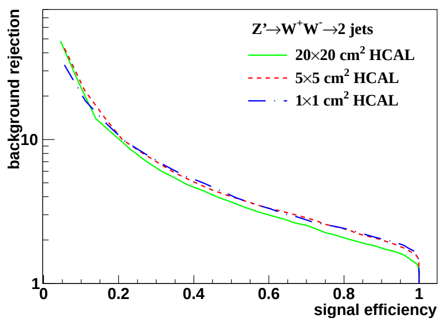
Figure 7: Distributions of soft drop mass for $\beta$=2, with 20 TeV c.m. energies and three different detector cell sizes: 20×20, 5×5, and 1×1 (cm$^2$). The signal (background) process is $Z' \to WW$ ($Z' \to q\bar{q}$).



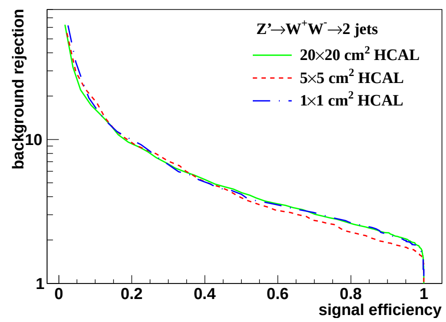

(a) $Z'$ (5 TeV)




(b) $Z'$ (10 TeV)

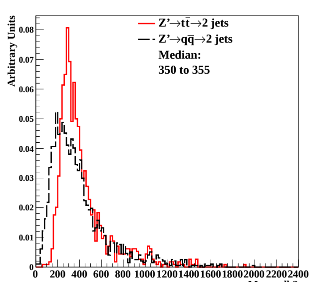



(c) $Z'$ (20 TeV)

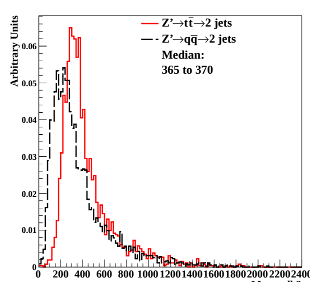



(d) $Z'$ (40 TeV)

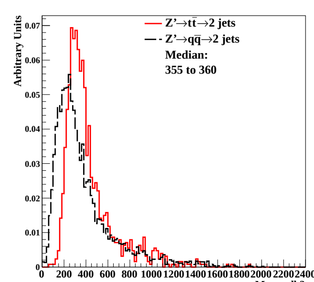
Figure 8: The ROC curves of soft drop mass selection for $\beta$=2 with 5, 10, 20, 40 TeV c.m. energies. Three different detector cell sizes are compared: 20×20, 5×5, and 1×1 (cm$^2$). The signal (background) process is $Z' \to WW$ ($Z' \to q\bar{q}$).




(a) 20×20 (cm$^2$)

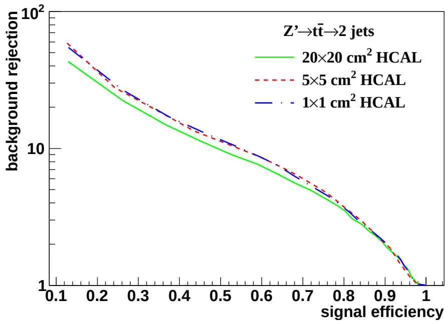



(b) 5×5 (cm$^2$)

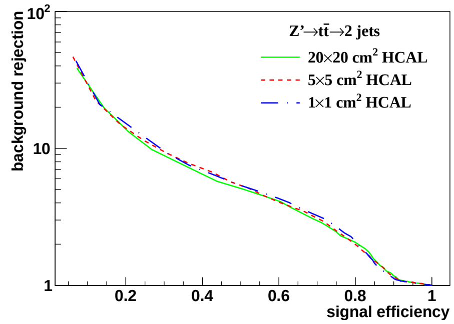



(c) 1×1 (cm$^2$)

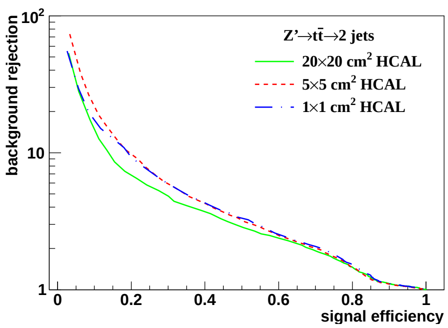
Figure 9: Distributions of soft drop mass for $\beta$=2, with 20 TeV c.m. energies and three different detector cell sizes: 20×20, 5×5, and 1×1 (cm$^2$). The signal (background) process is $Z' \to t\bar{t}$ ($Z' \to q\bar{q}$).



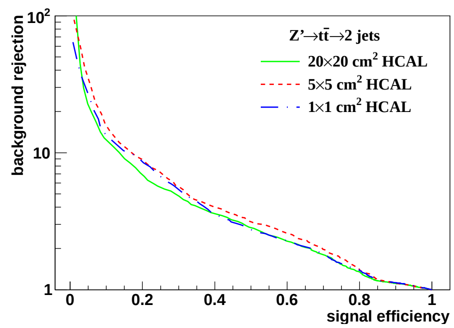

(a) Z'(5 TeV)




(b) Z'(10 TeV)




(c) Z'(20 TeV)




(d) Z'(40 TeV)

Figure 10: The ROC curves of soft drop mass selection for $\beta$=2 with 5, 10, 20, 40 TeV c.m. energies. Three different detector cell sizes are compared: 20×20, 5×5, and 1×1 (cm$^2$). The signal (background) process is $Z' \to t\bar{t}$ ($Z' \to q\bar{q}$).

## 5. Study of detector performance with jet substructure variables

In this section, we use several jet substructure variables to study the performance of detector with various detector cell sizes and c.m. energies.

### 5.1. *N-subjettiness*

The variable $N$-subjettiness [22], denoted by $\tau_N$, is designed to "count" the number of subjet(s) in a large radius jet so to separate signal jets from decays of heavy bosons and background jets from QCD processes. The $\tau_N$ is the $p_{\mathrm{T}}$-weighted angular distance between each jet constituent and the closest subjet axis:

$$\tau_N = \frac{1}{d_0} \sum_k p_{T,k} \min\{\Delta R_{1,k}, \Delta R_{2,k}, .....\Delta R_{N,k}\}, \tag{2}$$

with a normalization factor $d_0$:

$$d_0 = \sum_k p_{T,k} R_0.$$

The $k$ runs over all constituent particles in a given large radius jet, $p_{T,k}$ is the transverse momentum of each individual constituent particle, $\Delta R_{j,k} = \sqrt{(\Delta y)^2 + (\Delta\phi)^2}$ is the distance between the constituent particle $k$ and the candidate subjet axis $j$ in the $y-\phi$ plane. The $R_0$ is the characteristic jet radius used in the anti-$k_t$ jet algorithm.

In this analysis, the anti-$k_t$ algorithm with $R = 0.4$ (AK4) is first employed to reconstruct jets. The subjet axes are obtained by running the exclusive $k_t$ algorithm [23] and reversing the last N clustering steps. Namely, when $\tau_N$ is computed, the $k_t$ algorithm is forced to return exactly $N$ jets. If a large radius jet has $N$ subjet(s), its $\tau_N$ is smaller than $\tau_{N-1}$. Therefore, in our analysis, the ratio of the $\tau_N$ variables, $\tau_{21}$ ($\tau_2/\tau_1$) and $\tau_{32}$ ($\tau_3/\tau_2$), are used to distinguish the one-prong background jets and the two-prong jets from W or the three-prong jets from top.

We use the ROC curves as described in Section 4.2 to analyze the detector performance and determine the cell size that gives the best separation power to distinguish signal from background. Following the suggestion by Ref. [24], requirement on the soft drop mass with $\beta = 0$ is applied before the study of $N$-subjettiness. For each detector configuration and c.m. energy, the soft drop mass selection is determined as follows. First, we look for the median bin of the soft drop mass histogram from simulated signal events as described in Section 4.2. Then, we compare the numbers of events in the bins adjacent to the medium bin (bin $i_{\mathrm{med}} - 1$ and bin $i_{\mathrm{med}} + 1$). The bin with larger number of events is added, in addition to the medium bin, to extend the mass window. The procedure is repeated until the window contains at least 75% of the total number of signal events.

In order to obtain the signal and background efficiencies, various ranges of the $\tau_{21}$ and $\tau_{32}$ are scanned. Since some of the background distributions have long tails and leak into the signal-dominated region, we use the following method as suggested by the Pearson Lemma Method [] to determine the ranges of $\tau$ variables. First, we take the ratio of the signal to background $\tau_{21}$ ($\tau_{32}$) histograms. The boundaries of the bin (seed bin) with maximum signal to background ratio (S/N) give us the first range of $\tau$

selection: $x_{\rm low}^{\rm seedbin} < \tau_{21} < x_{\rm high}^{\rm seedbin}$. Then, we compare the S/N in the bins adjacent to the seed bin. The bin with larger S/N is added, in addition to the seed bin, to extend the $\tau_{21}$ selection window. Every window has its corresponding $\epsilon_{\rm sig}$ and $1/\epsilon_{\rm bkg}$ and an ROC curve is mapped out.

In addition to the ROC curves, we use the so-called "Mann-Whitney" test to quantify the detector performance. The value of Mann-Whitney is related to the integrated area under the ROC curve: if the value is bigger, it indicates the signal and background distributions have similar shapes and can not be well separated from each other. Vice versa, if the value is smaller, we can achieve a better signal and background separation.

Figures 11 and 13 show the distributions of $\tau_{21}$ and $\tau_{32}$ for $\sqrt{s} = 20$ TeV after applying requirement on the soft drop mass. The signals considered are Z'$\to$WW ($\tau_{21}$) and Z'$\to$ $\rm t\bar{t}$ ($\tau_{32}$). Figures 12 and 14 present the ROC curves from different detector cell sizes and c.m. energies, respectively. The smallest detector cell size ($1 \times 1$ cm$^2$) does not have the best separation power. In fact, in some cases, the best separation power comes from detector with bigger cell sizes ($5 \times 5$ cm$^2$ and $20 \times 20$ cm$^2$).

Figures 17 (a) and (b) present the summary plots of $\tau_{21}$ and $\tau_{32}$ with various detector cell sizes and c.m. energies using Mann Whitney U test. For $\tau_{21}$ at smaller c.m. energies, when cell size is smaller, the detector performance improves. However, when c.m. energy increases, no improvement is observed using the smallest detector cell size ($1 \times 1$ cm$^2$). For $\tau_{32}$, the case is similar to $\tau_{21}$. Even worse, with some c.m. energies, the bigger detector cell sizes ($5 \times 5$ cm$^2$ and $20 \times 20$ cm$^2$) have better separation power than the smallest detector size.

### *5.2. Energy correlation function*

The energy correlation function (ECF) [25] is defined as follows:

$$ECF(N,\beta) = \sum_{i_1<i_2<....<i_N\in J} \left(\prod_{a=1}^{N} p_{\mathrm{T}ia}\right) \left(\prod_{b=1}^{N-1}\prod_{c=b+1}^{N} R_{i_b i_c}\right)^{\beta}, \tag{3}$$

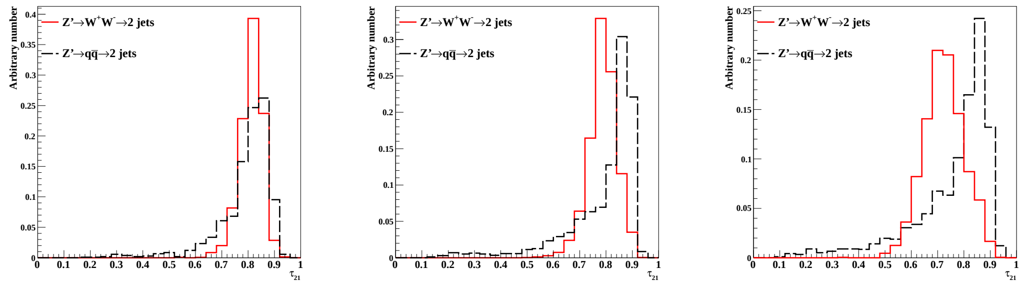
where the sum is looped all particles in the jet $J$, $p_{\rm T}$ is the transverse momentum of each individual particle, and $R$ is the distance between two particles in the $y$-$\phi$ plane. In order to use a dimensionless variable, a parameter $r_N$ is defined:

$$r_N^{(\beta)} \equiv \frac{ECF(N+1,\beta)}{ECF(N,\beta)}. \tag{4}$$

The idea of $r_N$ comes from N-subjettiness $\tau_N$. Both $r_N$ and $\tau_N$ are linear in the energy of the soft radiation for a system of $N$ partons with soft radiation. In general, if the system has N subjets, $ECF(N+1,\beta)$ should be significantly smaller than $ECF(N,\beta)$. Therefore, we can use this feature to distinguish jets with different number of subjets. As in Section 5.1, the ratio $r_N/r_{N-1}$, denoted by $C_N$, (double ratios of ECFs) is used to study the detector performance:

$$C_N^{(\beta)} \equiv \frac{r_N^{(\beta)}}{r_{N-1}^{(\beta)}} = \frac{ECF(N-1,\beta)ECF(N+1,\beta)}{ECF(N,\beta)^2}. \tag{5}$$

In our analysis, we set $N=2$ and $\beta=1$ ($C_2^1$).




(a) 20×20 (cm$^2$)   (b) 5×5 (cm$^2$)   (c) 1×1 (cm$^2$)

Figure 11: Distributions of $\tau_{21}$ in 20 TeV energy collision for different detector sizes. Cell sizes in 20×20, 5×5, and 1×1 cm$^2$ are shown here.

Figure 15 presents the histograms of $C_2^1$ with $\sqrt{s} = 20$ TeV after making requirement on the soft drop mass. The signal considered is Z'→WW. Figure 16 shows the ROC curves from different detector cell sizes for each c.m. energy, respectively. One can see that the smallest detector cell size ($1 \times 1$ cm$^2$) does not have the best signal/background separation power. Figure 17(c) summarizes the result of the Mann Whitney U test for $C_2^1$. When c.m. energy increases, no improvement is observed from detector with the smallest cell size.

## 6. Conclusions

The studies presented in this paper show that the reconstruction of jet substructure variables for future particle colliders will benefit from small cell sizes of the hadronic calorimeters. This conclusion was obtained using the realistic Geant4 simulation of calorimeter responses combined with reconstruction of calorimeter clusters used as inputs for jet reconstruction. Hadronic calorimeters that use the cell sizes of 20×20 cm$^2$ are least performat almost for every substructure variables considered in this analysis for jet transverse momenta between 2.5 – 10 TeV. Such cell sizes are close to the nuclear interaction length of the considered calorimeter, and are similar to those used for the ATLAS and CMS detectors.

It is however interesting to note that for very boosted jets with transverse moment close to 20 TeV, no significant improvement for with decrease of cell sizes was observed. This result still needs to be understood in terms of verious type of simulations and different options for construction of calorimeter clusters.

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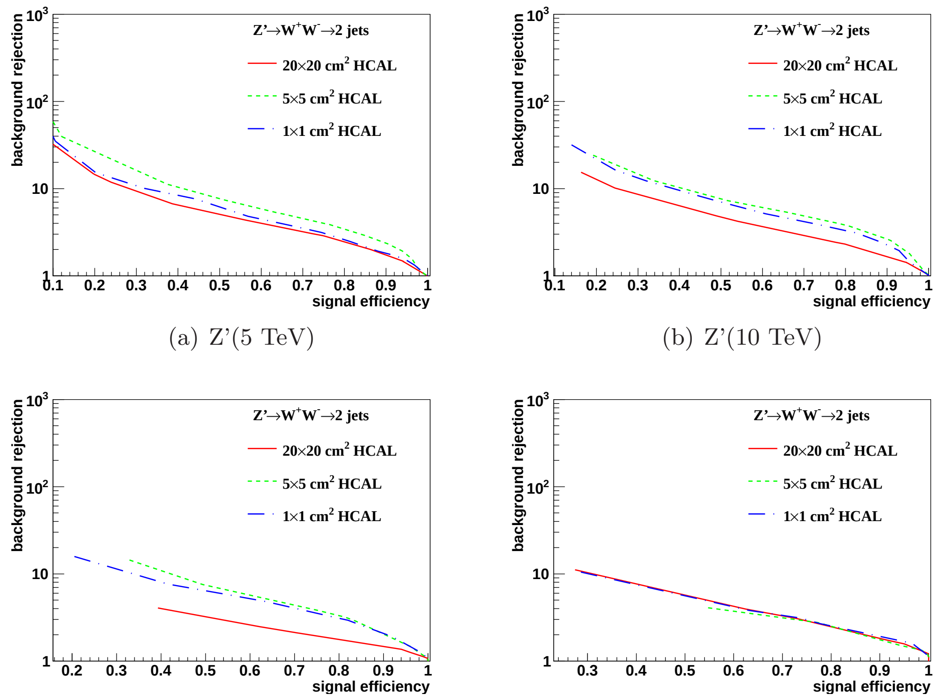


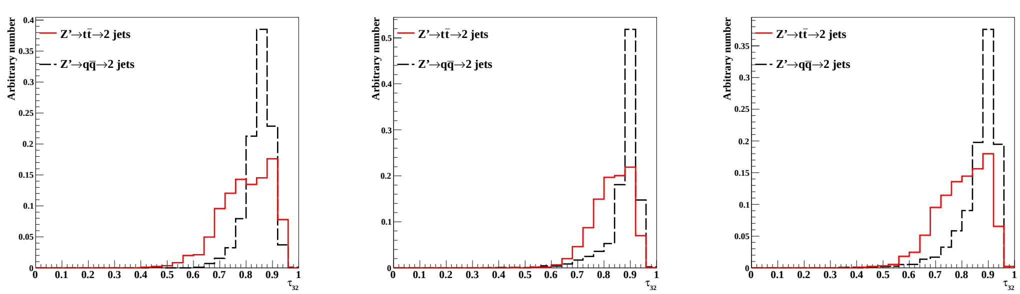

(a) Z'(5 TeV)

(b) Z'(10 TeV)

(c) Z'(20 TeV)

(d) Z'(40 TeV)

Figure 12: Signal efficiency versus background rejection rate using $\tau_{21}$. The energies of collision at (a) 5, (b) 10, (c) 20, and (d) 40 TeV are shown here. In each figure, the three ROC curves correspond to different detector sizes.

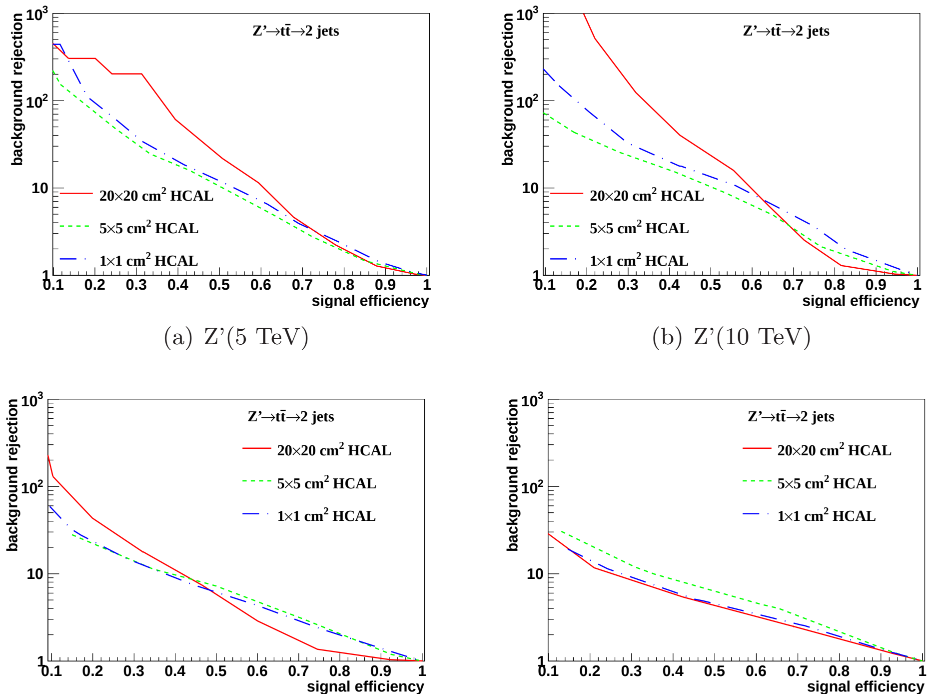



(a) 20×20 $(cm^2)$

(b) 5×5 $(cm^2)$

(c) 1×1 $(cm^2)$

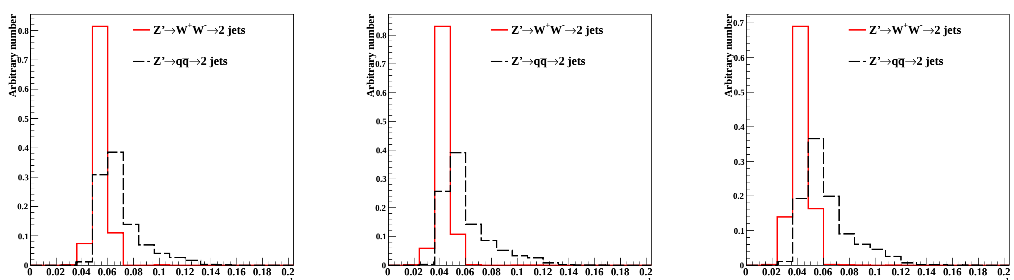
Figure 13: Distributions of $\tau_{32}$ in 20 TeV energy collision for different detector sizes. Cell sizes in 20×20, 5×5, and 1×1 cm$^2$ are shown here.




(a) Z'(5 TeV) (b) Z'(10 TeV)

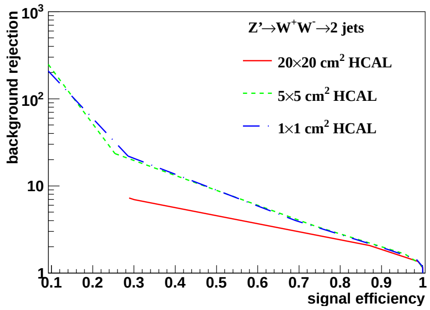
(c) Z'(20 TeV) (d) Z'(40 TeV)

Figure 14: Signal efficiency versus background rejection rate using $\tau_{32}$. The energies of collision at (a) 5, (b) 10, (c) 20, and (d) 40 TeV are shown here. In each figure, the three ROC curves correspond to different detector sizes.

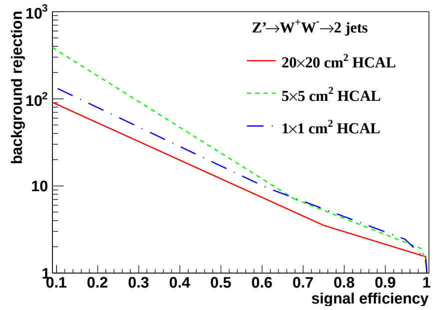



(a) 20×20 ($cm^2$) (b) 5×5 ($cm^2$) (c) 1×1 ($cm^2$)

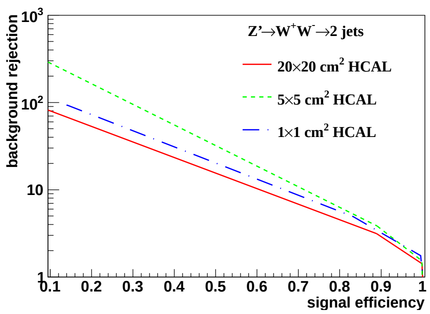
Figure 15: Distributions of $C_2^1$ in 20 TeV energy collision for different detector sizes. Cell sizes in 20×20, 5×5, and 1×1 cm$^2$ are shown here.



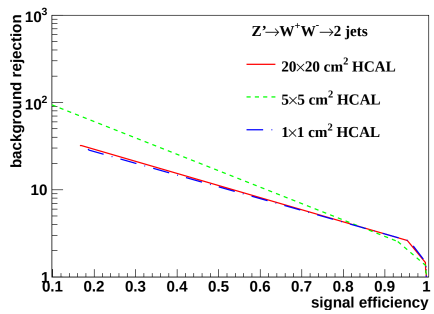

(a) Z'(5 TeV)




(b) Z'(10 TeV)

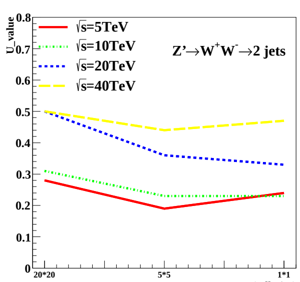



(c) Z'(20 TeV)

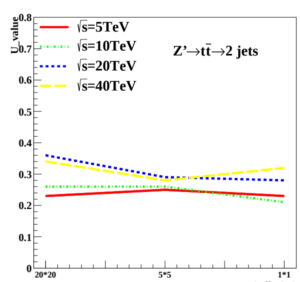



(d) Z'(40 TeV)

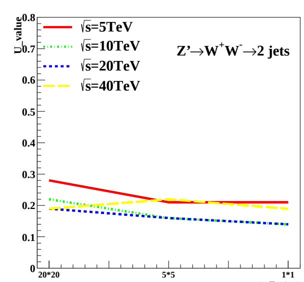
Figure 16: Signal efficiency versus background rejection rate using $C_2^1$. The energies of collision at (a) 5, (b) 10, (c) 20, and (d) 40 TeV are shown here. In each figure, the three ROC curves correspond to different detector sizes.




(a) $\tau_{21}$




(b) $\tau_{32}$




(c) $C_2^{(1)}$

Figure 17: The Mann-Whitney U values for $\tau_{21}$, $\tau_{32}$, and $C_2^{(1)}$ reconstructed with different collision energies and detector cell sizes.

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