This assignment is about fitting mixture of normal densities to data using EM algorithm. It contains four problems.

1. Here, data is one-dimensional. Let $f_i = \mathcal{N}(\mu_i, \sigma_i^2)$, $i = 1, \cdots, 4$. The density model is $f(x) = \sum_{i=1}^{4} \lambda_i f_i(x)$. There are four data sets with different parameter choices.

   - $\mu_1 = 0, \mu_2 = 4, \mu_3 = 8, \mu_4 = 12$, and $\sigma_i^2 = 2, \forall i$.
     a. $\lambda_i = 0.25, \forall i$.
        Dataset is: P1M1L1.txt
     b. $\lambda = (0.1, 0.3, 0.4, 0.2)$.
        Dataset: P1M1L2.txt
   - $\mu_1 = 0, \mu_2 = 2, \mu_3 = 3, \mu_4 = 6$, and $\sigma_i^2 = 2, \forall i$.
     a. $\lambda_i = 0.25, \forall i$.
        Dataset is: P1M2L1.txt
     b. $\lambda = (0.1, 0.3, 0.4, 0.2)$.
        Dataset: P1M2L2.txt

   Learn a mixture density model for the data. In one set of experiments assume you know the exact number of mixture components. In the other set of experiments fit a model where the number of components is set wrongly. Experiment with different initial values for the parameters and with different subsets (of varying sizes) of the data. Tabulate or graph all your experimental results and comment on the results.

2. Here data is 10-dimensional and true density is a mixture of only two normal densities. $\mu_1 \in \Re^{10}$ is a vector of all zeros and $\mu_2 \in \Re^{10}$ has all components equal to 2. We have $\Sigma_i = 2I, i = 1, 2$ where I is the identity matrix. There are two data sets with different choices for mixing coefficients.

   a. $\lambda_i = 0.5,\ i = 1, 2$.
      Dataset is: P2L1.txt
   b. $\lambda = (0.3, 0.7)$.
      Dataset: P2L2.txt

   Once again explore learning a mixture density as in the previous problem.

3. This problem is for learning a classifier when class conditional densities are mixture of normal densities.

   Let $f_1 = \mathcal{N}(-4, 1)$, $f_2 = \mathcal{N}(-1, 1)$, $f_3 = \mathcal{N}(1, 1)$, $f_4 = \mathcal{N}(4, 1)$

   There are two cases of class conditional densities here.

   - Class-1: $0.5f_1 + 0.5f_2$
     Class-2: $0.5f_3 + 0.5f_4$
     Training Data: P3D1train.txt
     Test data: P3D1test.txt
   - Class-1: $0.5f_2 + 0.5f_3$
     Class-2: $0.5f_1 + 0.5f_4$
     Training Data: P3D2train.txt
     Test data: P3D2test.txt

   In each case learn each class conditional density as (i). a single normal density, (ii). as a mixture of two normal densities. Implement Bayes classifier with the learnt densities. Compare the accuracies on the test set of the two classifiers and also that of a nearest neighbour classifier. You can vary any hyper parameters that you think should be explored.

4. This problem is exactly same as the previous problem with just one small change. The details are as below.

   Let $f_1 = \mathcal{N}(-4, 2)$, $f_2 = \mathcal{N}(-1, 2)$, $f_3 = \mathcal{N}(1, 2)$, $f_4 = \mathcal{N}(4, 2)$

   There are two cases of class conditional densities here.

   - Class-1: $0.5f_1 + 0.5f_2$
     Class-2: $0.5f_3 + 0.5f_4$
     Training Data: P4D1train.txt
     Test data: P4D1test.txt
   - Class-1: $0.5f_2 + 0.5f_3$
     Class-2: $0.5f_1 + 0.5f_4$
     Training Data: P4D2train.txt
     Test data: P4D2test.txt

As in the previous problem implement the two Bayes classifiers and the nearest neighbour classifier and compare accuracies on test set. Also comment on the results of this and the previous problem (if you think there is something worth commenting about). Once again you can explore varying any hyper parameters that you want.