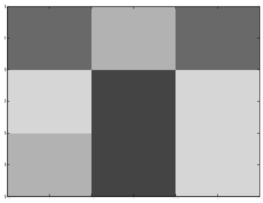
1 Pre-Recorded Tasks

1.1 Interpolation

The following picture shall be enlarged by a factor of 3.

$$\mathbf{x} = \begin{bmatrix} 9 & 15 & 9 \\ 18 & 6 & 18 \\ 15 & 6 & 18 \end{bmatrix}$$



1.1.1 Linear Interpolation

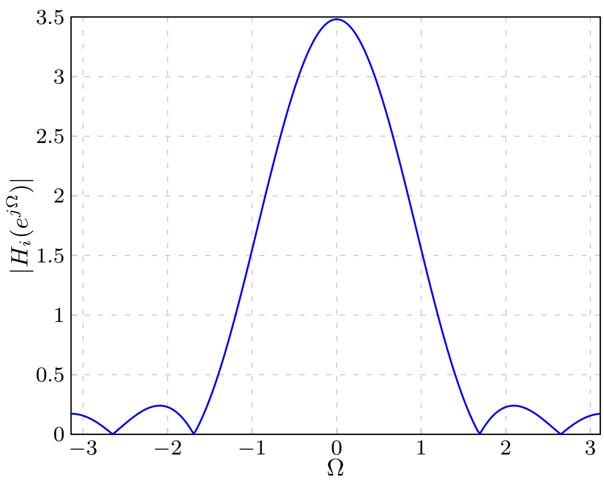
In a first step, a first-order interpolation shall be applied. The picture shall be zero-padded as necessary.

1. Specify the 1-dimensional filter kernel $h[n]$!
2. Calculate the 2-dimensional filter kernel $h[n] * h[n]^T$!
3. Write down the input image after zero-insertion!
4. Calculate the values for $y[4, 4]$, $y[1, 2]$ and $y[3, 3]$!

1.1.2 Ideal Interpolation

Now, instead of the linear interpolation, an ideal interpolation shall be used with $h_{ideal}[n] = sinc(\frac{n}{R})$:

1. What problem are we facing in this case?
2. Calculate a filter $h_i[n]$ with the same number of coefficients that you used for the linear interpolation!
3. Looking at the solution, which problem occurs when you apply this filter?
4. The magnitude frequency response $|H_i(e^{j\Omega})|$ of the DTFT-transformed filter coefficients is as follows:



What do you think about this filter?

2 Self-Study Matlab Tasks

2.1 Spline Interpolation

We now want to use splines for signal interpolation. Copy the files from `~/SHARED_FILES/vcc/Ex6/` to your directory.

1. The function `spline_b0()` calculates values of the zero-order spline φ_0. Plot this spline for $x = -3 : 0.01 : 3$!

2. Construct the higher order splines by numeric convolution with the zero-order spline. Plot the results for up to φ_3. Where are the functions non-zero?

3. Now, we want to use the analytic versions of the splines. The functions to create the splines φ_0, φ_2 and φ_3 are provided in the files `spline_b0.m`, `spline_b2.m` and `spline_b3.m`. Write an analytic version of the spline φ_1. Compare the results of the numeric convolution with the analytic results!

4. You are given the spline coefficients $c = [1\ 9\ 10\ 7\ 8\ 8\ 4\ 7\ 2\ 8]$ defined for the positions $x_{in} = 1 : 10$. Plot each of the weighted splines with a horizontal resolution of $x = -2 : 0.01 : 13$. Use all splines up to φ_3!

5. Next, calculate the interpolated signal. In order to do so, sum up the contributions from all splines. Plot the interpolated signals for all splines up to φ_3!

1 Pre-Recorded Tasks

1.1 Interpolation

1.1.1 Linear Interpolation

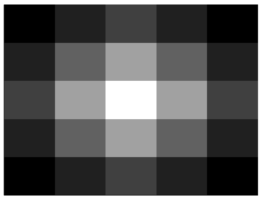
In a first step, a first-order interpolation shall be applied. The picture shall be zero-padded as necessary.

1. Specify the 1-dimensional filter kernel $h[n]$!

$$h[n] = \frac{1}{3}\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \end{bmatrix}, \quad n = -2, ..., 2$$

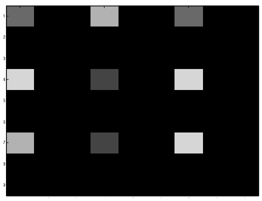
2. Calculate the 2-dimensional filter kernel $h[n] * h[n]^T$!

$$h[n] * h[n]^T = \frac{1}{9}\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 4 & 6 & 4 & 2 \\ 3 & 6 & 9 & 6 & 3 \\ 2 & 4 & 6 & 4 & 2 \\ 1 & 2 & 3 & 2 & 1 \end{bmatrix}$$



3. Write down the input image after zero-insertion!

$$\mathbf{x}_z = \begin{bmatrix} 9 & 0 & 0 & 15 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 18 & 0 & 0 & 6 & 0 & 0 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 15 & 0 & 0 & 6 & 0 & 0 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



4. Calculate the values for $y[4,4]$, $y[1,2]$ and $y[3,3]$!

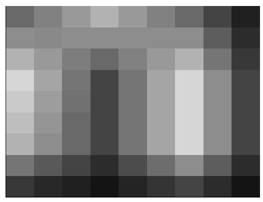
$y[4,4] = \frac{1}{9} \cdot (9 \cdot 6) = 6$

$y[1,2] = \frac{1}{9} \cdot (6 \cdot 9 + 3 \cdot 15) = 11$

$y[3,3] = \frac{1}{9} \cdot (1 \cdot 9 + 2 \cdot 15 + 2 \cdot 18 + 4 \cdot 6) = 11$

The complete picture looks like this:

$$\mathbf{y} = \frac{1}{3}\begin{bmatrix} 27 & 33 & 39 & 45 & 39 & 33 & 27 & 18 & 9 \\ 36 & 36 & 36 & 36 & 36 & 36 & 36 & 24 & 12 \\ 45 & 39 & 33 & 27 & 33 & 39 & 45 & 30 & 15 \\ 54 & 42 & 30 & 18 & 30 & 42 & 54 & 36 & 18 \\ 51 & 40 & 29 & 18 & 30 & 42 & 54 & 36 & 18 \\ 48 & 38 & 28 & 18 & 30 & 42 & 54 & 36 & 18 \\ 45 & 36 & 27 & 18 & 30 & 42 & 54 & 36 & 18 \\ 30 & 24 & 18 & 12 & 20 & 28 & 36 & 24 & 12 \\ 15 & 12 & 9 & 6 & 10 & 14 & 18 & 12 & 6 \end{bmatrix}$$



1.1.2 Ideal Interpolation

Now, instead of the linear interpolation, an ideal interpolation shall be used with $h_{ideal}[n] = sinc(\frac{n}{R})$:

1. What problem are we facing in this case?

   The filter function has infinite length. The picture is very tiny, i. e., pixel values from outside the image will have significant influence.

2. Calculate a filter $h_i[n]$ with the same number of coefficients that you used for the linear interpolation!

   $h_i[n] = \begin{bmatrix} 0.413497 & 0.826993 & 1 & 0.826993 & 0.413497 \end{bmatrix}$

3. Looking at the solution, which problem occurs when you apply this filter?

   Since the filter function was cut off very early the filter coefficients are not normalized (the sum of the coefficients is not 3). Therefore, applying this filter will result in interpolated pixels whose value range is different from the original range. In order to overcome this the filter would have to be scaled. However, the center value needs to stay 1 for preserving the values from the original image.

4. The magnitude frequency response $|H_i(e^{j\Omega})|$ of the DTFT-transformed filter coefficients is as follows:

   What do you think about this filter?

   An ideal (infinite length) sinc filter $h_{ideal}[n]$ would transform to $H_{ideal}(e^{j\Omega}) = 3rect(\frac{3}{2\pi}\Omega)$. The rectangle would cut off at $\frac{\pi}{3}$. The filter $H_i(e^{j\Omega})$, however, is non-zero outside that rectangle and therefore, it will cause aliasing artifacts.

2 Self-Study Matlab Tasks

2.1 Spline Interpolation

We now want to use splines for signal interpolation. Copy the files from `~/SHARED_FILES/vcc/Ex6/` to your directory.

1. The function `spline_b0()` calculates values of the zero-order spline φ_0. Plot this spline for $x = -3 : 0.01 : 3$!

2. Construct the higher order splines by numeric convolution with the zero-order spline. Plot the results for up to φ_3. Where are the functions non-zero?

3. Now, we want to use the analytic versions of the splines. The functions to create the splines φ_0, φ_2 and φ_3 are provided in the files `spline_b0.m`, `spline_b2.m` and `spline_b3.m`. Write an analytic version of the spline φ_1. Compare the results of the numeric convolution with the analytic results!

4. You are given the spline coefficients $c = [1\ 9\ 10\ 7\ 8\ 8\ 4\ 7\ 2\ 8]$ defined for the positions $x_{in} = 1 : 10$. Plot each of the weighted splines with a horizontal resolution of $x = -2 : 0.01 : 13$. Use all splines up to φ_3!

5. Next, calculate the interpolated signal. In order to do so, sum up the contributions from all splines. Plot the interpolated signals for all splines up to φ_3!