Università della Svizzera italiana

Stochastic Methods **Academic Year 2020/2021**

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Assignment 2

Exercise 1: Lagrange Interpolation and Newton's Divided-Differences *(2 Points)*

Consider four points in $\mathbb{R}^2$: $(x_0, y_0) = (0, 2)$, $(x_1, y_1) = (2, 3)$, $(x_2, y_2) = (4, 1)$, $(x_3, y_3) = (6, 2)$.

(a) Compute the third Lagrange interpolating polynomial $P_3(x)$.

(b) Use Newton's divided differences to find the polynomial $P_3(x)$ interpolating the four points.

(c) Compare the results of point (a) and (b). Are the polynomials different? Why?

(d) By using the different algebraic formulation introduced in the tutorial slides, show that the second divided difference $f[x_0, x_1, x_2]$ is equal to the coefficient a_2 obtained by evaluating $P_3(x_2)$.

(e) We now want to include an additional point in our interpolating polynomial. Given the fact that you have access to all the previous results, would you use Lagrange interpolation or Newton's Divided-Differences to compute the new interpolating polynomial? Why? Motivate your answer.

Exercise 2: Implementation of Interpolation *(2.5 Points)*

(a) Write a Matlab function `interpolation()` that takes, as input data, the equidistant nodes $x_0, \ldots, x_n$ and the function values $f(x_0), \ldots, f(x_n)$ to compute the interpolating polynomial $P_n(x)$. The computation of $P_n(x)$ must be done either by using Lagrange method or Newton's Divided-Difference (*hint*: among the input arguments, include a flag accepting the values `'lagrange'` and `'newton'`, in order to allow the user to select the method he prefers).

(b) Write a script `exercise2b.m`, in which you test `interpolation()` on the following function:

$$f(x) = \frac{1}{2 + 3x^2}, \quad x \in [-1, 1], \tag{1}$$

with $n = 2, 3, 4$. Plot all polynomials $P_n(x)$ and the function $f(x)$ in a single picture.

(c) Write a script exercise2c.m, in which you repeat the test defined in point (b) for $x \in [-5, 5]$ and provide a plot of all the polynomials $P_n(x)$ and of $f(x)$ in a single picture. Then, increase the value of n by considering $n = 5, 7, 9, 11, 13$ and produce another plot of these polynomials against $f(x)$. Finally, discuss your results according to the following error estimate:

$$|f(x) - P_n(x)| \leq \frac{M}{(n+1)!} \prod_{i=0}^{n} (x - x_k), \quad M := \sup_{x \in [x_0, x_n]} f^{(n+1)}(x). \tag{2}$$

Exercise 3: Composite Trapezoidal Rule and Composite Simpson's Rule *(2.5 Points)*

Consider the following definite integral:

$$\int_0^1 x^2 e^x \, dx.$$

(a) Approximate the value of the integral by using the Composite Trapezoidal Rule and the Composite Simpson's Rule with $n = 4$ and $n = 8$ (see tutorial notes) and compute the error bound.

(b) Compute the exact value of the integral (*hint:* integration by parts) and check if it is consistent with the approximation and error bounds. Which one of the two methods is more accurate?

Exercise 4: Implementation of Numerical Quadrature *(3 Points)*

(a) Write a Matlab function quadrature() to approximate the value of the definite integral:

$$\int_a^b f(x) \, dx \tag{3}$$

by using Composite Trapezoidal Rule and Composite Simpson's Rule. Use a, b, the number of subintervals n and a *flag* to choose the method as input parameters.

(b) Write a script exercise4b.m, in which you use quadrature() to approximate the integral:

$$\int_{\frac{\pi}{2}}^{3\pi} 4 + 5x \sin(x) \, dx, \tag{4}$$

for $n = 2^j$, where $j = 1, \ldots, 10$. Compute and save the errors of Composite Trapezoidal Rule and of Composite Simpson's Rule for each n. Plot the error vectors in the appropriate scale.

(c) Now we consider the following function:

$$f(x) = \begin{cases} -1, & x \leq 0, \\ +1, & x > 0. \end{cases} \tag{5}$$

Write a script exercise4c.m, in which you use quadrature() to approximate the integral:

$$\int_{-1}^{1} f(x) \, dx, \tag{6}$$

using Composite Simpson's Rule for $n = 2^j$, where $j = 1, \ldots, 10$. Plot the function $f(x)$ and compute analytically the exact value of the integral. Then, compare the approximations with the analytical result and plot the error vector in the appropriate scale. What can you observe with respect to the results obtained in point (b)? What could you do to handle more efficiently the quadrature of functions presenting jumps?

Please write a detailed report with your solutions using the LaTeX template provided on iCorsi.

!!! The code has to be well commented !!!