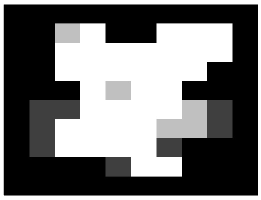
1 Pre-Recorded Tasks

1.1 Edge Detection

The following image section shall be analyzed:

$$\mathbf{X} = \left[\begin{array}{c|cccccccc|c} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 3 & 4 & 0 & 0 & 4 & 4 & 4 & 0 \\ 0 & 0 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 0 \\ 0 & 0 & 4 & 4 & 4 & 4 & 4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 3 & 4 & 4 & 0 & 0 & 0 \\ 0 & 1 & 1 & 4 & 4 & 4 & 4 & 3 & 1 & 0 \\ 0 & 1 & 4 & 4 & 4 & 4 & 3 & 3 & 1 & 0 \\ 0 & 1 & 4 & 4 & 4 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 & 4 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$



1. First, an edge detection with the Prewitt operator shall be calculated. The four filters for the directions 0^o, 45^o, 90^o and 135^o are depicted below:

$$\mathbf{H}_0 = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad \mathbf{H}_{45} = \begin{bmatrix} -1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix},$$

$$\mathbf{H}_{90} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}, \quad \mathbf{H}_{135} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

Which one of the following two matrices is the result of the 2D convolution of the image $\mathbf{X}$ with $\mathbf{H}_{135}$? Give reasons!

$$\mathbf{X}_{135a} = \begin{bmatrix} 0 & 0 & 3 & 7 & 4 & 0 & 4 & 8 & 8 & 4 \\ 0 & -3 & 0 & 11 & 12 & 4 & 4 & 8 & 12 & 8 \\ 0 & -7 & -7 & 4 & 8 & 4 & 0 & 0 & 4 & 4 \\ 0 & -8 & -12 & -4 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -3 & -10 & -6 & 0 & -1 & 4 & 7 & 4 & 1 \\ -1 & 0 & -2 & -2 & 1 & 0 & 4 & 9 & 7 & 2 \\ -2 & -5 & -3 & 0 & 0 & 1 & -1 & -1 & 2 & 1 \\ -2 & -9 & -11 & -8 & -7 & 1 & 6 & 1 & -1 & 0 \\ -1 & -5 & -8 & -9 & -12 & -8 & 3 & 4 & 0 & 0 \\ 0 & 0 & 0 & -1 & -5 & -8 & -4 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{X}_{135b} = \begin{bmatrix} 0 & 0 & -3 & -7 & -4 & 0 & -4 & -8 & -8 & -4 \\ 0 & 3 & 0 & -11 & -12 & -4 & -4 & -8 & -12 & -8 \\ 0 & 7 & 7 & -4 & -8 & -4 & 0 & 0 & -4 & -4 \\ 0 & 8 & 12 & 4 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 10 & 6 & 0 & 1 & -4 & -7 & -4 & -1 \\ 1 & 0 & 2 & 2 & -1 & 0 & -4 & -9 & -7 & -2 \\ 2 & 5 & 3 & 0 & 0 & -1 & 1 & 1 & -2 & -1 \\ 2 & 9 & 11 & 8 & 7 & -1 & -6 & -1 & 1 & 0 \\ 1 & 5 & 8 & 9 & 12 & 8 & -3 & -4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 & 8 & 4 & 0 & 0 & 0 \end{bmatrix}$$

2. Now, generate the resulting edge detection image $\mathbf{Y}_{max}$ by searching for the absolute maximum of the 4 directions at each pixel location!

$$\mathbf{X} * *\mathbf{H}_0 = \begin{bmatrix} 0 & -3 & -7 & -7 & -4 & -4 & -8 & -12 & -8 & -4 \\ 0 & -4 & -8 & -12 & -12 & -12 & -12 & -12 & -8 & -4 \\ 0 & -1 & -1 & -5 & -8 & -8 & -4 & 4 & 4 & 4 \\ 0 & 4 & 4 & 5 & 1 & 1 & 4 & 8 & 8 & 4 \\ -1 & 2 & 2 & 3 & 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & -5 & -5 & -5 & -1 & 0 & -2 & -3 & -4 & -1 \\ 0 & -3 & -3 & -3 & 0 & 3 & 6 & 7 & 4 & 1 \\ 1 & 5 & 9 & 11 & 7 & 2 & 2 & 3 & 4 & 1 \\ 1 & 5 & 9 & 12 & 12 & 9 & 5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 & 9 & 8 & 4 & 0 & 0 \end{bmatrix}$$

$$\mathbf{X} * *\mathbf{H}_{45} = \begin{bmatrix} 0 & -3 & -7 & -4 & 0 & -4 & -8 & -8 & -4 & 0 \\ 0 & -7 & -12 & -5 & -4 & -12 & -12 & -8 & 0 & 4 \\ 0 & -8 & -9 & -1 & -4 & -8 & -4 & 4 & 12 & 8 \\ 0 & -4 & -4 & 1 & 1 & 0 & 4 & 12 & 12 & 4 \\ -1 & -2 & -5 & -3 & 0 & -1 & 5 & 8 & 3 & 0 \\ -2 & -6 & -11 & -7 & -1 & 0 & 3 & 3 & 2 & 1 \\ -2 & -8 & -9 & -3 & 0 & 4 & 8 & 9 & 7 & 2 \\ -1 & -3 & 2 & 7 & 3 & 3 & 7 & 7 & 4 & 1 \\ 0 & 1 & 5 & 7 & 4 & 5 & 9 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 5 & 8 & 4 & 0 & 0 \end{bmatrix}$$

$$\mathbf{X} * *\mathbf{H}_{90} = \begin{bmatrix} 0 & -3 & -4 & 3 & 4 & -4 & -4 & 0 & 4 & 4 \\ 0 & -7 & -8 & 3 & 4 & -4 & -4 & 0 & 8 & 8 \\ 0 & -11 & -12 & 3 & 4 & -4 & -4 & 4 & 12 & 8 \\ 0 & -8 & -12 & -3 & 0 & -1 & 4 & 8 & 8 & 4 \\ -1 & -5 & -11 & -6 & 0 & -1 & 5 & 11 & 7 & 1 \\ -2 & -5 & -10 & -6 & 0 & 0 & 6 & 9 & 6 & 2 \\ -3 & -9 & -9 & -3 & 0 & 4 & 6 & 6 & 6 & 2 \\ -2 & -8 & -6 & -1 & -4 & 1 & 9 & 7 & 3 & 1 \\ -1 & -4 & -3 & -1 & -4 & 0 & 8 & 5 & 0 & 0 \\ 0 & 0 & 0 & -1 & -4 & -3 & 4 & 4 & 0 & 0 \end{bmatrix}$$

1.2 Edge Detection

An edge detection with the set of Prewitt operators shall be performed.

1. Describe the process of finding the object edges with the set of Prewitt operators!
2. The Prewitt operator $\mathbf{H}_{135}$ shall be applied to the image $x_2[m,n]$.

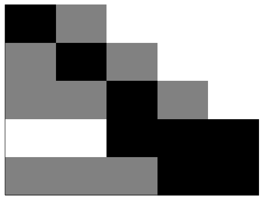
$$x_2[m,n] = \begin{bmatrix} 1 & 0 & 0 & 2 & 1 & 0 \\ 1 & 0 & \boxed{1} & 1 & 2 & 1 \\ 1 & 9 & \boxed{6} & 5 & 7 & 1 \\ 1 & 6 & \boxed{8} & 7 & 5 & 1 \\ 2 & 1 & \boxed{0} & 2 & 1 & 1 \\ 0 & 0 & 2 & 1 & 2 & 0 \end{bmatrix}$$

Calculate the output of the Prewitt filter $y_2[m,n] = x_2[m,n] ** \mathbf{H}_{135}$ for the marked positions ($y_2[2,3]$, $y_2[3,3]$, $y_2[4,3]$ and $y_2[5,3]$)!

1.3 Co-Occurrence Matrix

The following signal is considered:

$$\mathbf{X} = \begin{bmatrix} -1 & 0 & 1 & 1 & 1 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$



1. Calculate the co-occurrence matrices $\mathbf{C}_{10}$, $\mathbf{C}_{01}$ and $\mathbf{C}_{11}$!
2. Write down the normalized matrices $\mathbf{C'}_{10}$, $\mathbf{C'}_{01}$ and $\mathbf{C'}_{11}$!
3. As a next step, calculate the energy of the co-occurrence matrices $E_{k,l} = \sum_{i,j} C'_{k,l}[i,j]^2$ for the shifts (1,0), (0,1) and (1,1)!
4. How large is the homogeneity $h_C = \sum_{i,j} \frac{C'(i,j)}{1+|i-j|}$ in the directions (1,0), (0,1) and (1,1)?

2 Self-Study Matlab Tasks

2.1 Optical Flow

In the following we want to investigate the optical flow implementation from MATLAB. Therefore copy the sequence *Race Horses*, which is provided in two different resolutions, from `~/SHARED_FILES/IVMSP/Ex7/` to your working directory. Load the sequence into your MATLAB workspace. With the function `opticalFlowLK()` you can perform a motion estimation using optical flow as explained in the lecture.
Show the sequence overlain by your estimated motion vectors. Compare the results for both resolutions. What do you observe?

1 Pre-Recorded Tasks

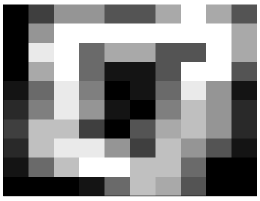
1.1 Edge Detection

1. Which one of the following two matrices is the result of the 2D convolution of the image $\mathbf{X}$ with $\mathbf{H}_{135}$? Give reasons!

   Since the filter kernel is flipped in both dimensions for the 2D convolution, the solution $\mathbf{X}_{135a}$ is correct.

2. Now, generate the resulting edge detection image $\mathbf{Y}_{max}$ by searching for the absolute maximum of the 4 directions at each pixel location!

$$\mathbf{Y}_{max} = \begin{bmatrix} 0 & 3 & 7 & 7 & 4 & 4 & 8 & 12 & 8 & 4 \\ 0 & 7 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 8 \\ 0 & 11 & 12 & 5 & 8 & 8 & 4 & 4 & 12 & 8 \\ 0 & 8 & 12 & 5 & 1 & 1 & 4 & 12 & 12 & 4 \\ 1 & 5 & 11 & 6 & 0 & 1 & 5 & 11 & 7 & 1 \\ 2 & 6 & 11 & 7 & 1 & 0 & 6 & 9 & 7 & 2 \\ 3 & 9 & 9 & 3 & 0 & 4 & 8 & 9 & 7 & 2 \\ 2 & 9 & 11 & 11 & 7 & 3 & 9 & 7 & 4 & 1 \\ 1 & 5 & 9 & 12 & 12 & 9 & 9 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 & 9 & 8 & 4 & 0 & 0 \end{bmatrix}$$



1.2 Edge Detection

1. Describe the process of finding the object edges with the set of Prewitt operators!

   - Filter the input image with all 4 Prewitt filters.
   - Combine the 4 filtered images by finding the absolute maximum for each pixel.
   - Follow the contour along the maximum value.

2. Calculate the output of the Prewitt filter $y_2[m,n] = x_2[m,n] * *\mathbf{H}_{135}$ for the marked positions ($y_2[2,3]$, $y_2[3,3]$, $y_2[4,3]$ and $y_2[5,3]$)!

   For the 2-dimensional convolution the kernel needs to be flipped in horizontal and vertical direction.

$$\mathbf{H}_{135} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}, \quad \text{Kernel after flipping:} \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

   - $y_2[2,3] = 12$
   - $y_2[3,3] = 16$
   - $y_2[4,3] = -11$
   - $y_2[5,3] = -14$

1.3 Co-Occurrence Matrix

1. Calculate the co-occurrence matrices $\mathbf{C}_{10}$, $\mathbf{C}_{01}$ and $\mathbf{C}_{11}$!

$$\mathbf{C}_{10} = \begin{bmatrix} 3 & 3 & 1 \\ 3 & 3 & 0 \\ 0 & 3 & 4 \end{bmatrix}, \quad \mathbf{C}_{01} = \begin{bmatrix} 3 & 3 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 3 \end{bmatrix}, \quad \mathbf{C}_{11} = \begin{bmatrix} 5 & 2 & 0 \\ 0 & 3 & 2 \\ 0 & 1 & 3 \end{bmatrix}$$

2. Write down the normalized matrices $\mathbf{C'}_{10}$, $\mathbf{C'}_{01}$ and $\mathbf{C'}_{11}$!

$$\mathbf{C'}_{10} = \frac{1}{25}\mathbf{C}_{10}, \quad \mathbf{C'}_{01} = \frac{1}{25}\mathbf{C}_{01}, \quad \mathbf{C'}_{11} = \frac{1}{25}\mathbf{C}_{11}$$

3. As a next step, calculate the energy of the co-occurrence matrices $E_{k,l} = \sum_{i,j} C'_{k,l}[i,j]^2$ for the shifts (1,0), (0,1) and (1,1)!

   $E_{10} = \frac{1}{625} \cdot (9+9+1+9+9+0+0+9+16) = \frac{62}{625} = 0.0992$
   $E_{01} = \frac{1}{625} \cdot (9+9+1+9+1+16+0+4+9) = \frac{58}{625} = 0.0928$
   $E_{11} = \frac{1}{625} \cdot (25+4+0+0+9+4+0+1+9) = \frac{52}{625} = 0.0832$

4. How large is the homogeneity $h_C = \sum_{i,j} \frac{C'(i,j)}{1+|i-j|}$ in the directions (1,0), (0,1) and (1,1)?

   $h_{C_{10}} = \frac{1}{25} \cdot (3+\frac{3}{2}+\frac{1}{3}+\frac{3}{2}+3+0+0+\frac{3}{2}+4) = \frac{89}{150} \approx 0.5933$
   $h_{C_{01}} = \frac{1}{25} \cdot (3+\frac{3}{2}+\frac{1}{3}+\frac{3}{2}+1+\frac{4}{2}+0+1+3) = \frac{8}{15} \approx 0.5333$
   $h_{C_{11}} = \frac{1}{25} \cdot (5+1+0+0+3+1+0+\frac{1}{2}+3) = \frac{27}{50} = 0.54$

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2.1 Optical Flow

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