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Stochastic Methods **Academic Year 2020/2021**

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Due date: Tuesday, March 16, 2021, 11.59pm

Assignment 1 - Solution [Please insert your name]

Exercise 1: Mean, Covariance and Correlation in Matlab

(a) Elapsed time is 4.418048 seconds.

Elapsed time is 0.022190 seconds.

(b) Figure inside folder figures in the submission.

Exercise 2: Application of the Central Limit Theorem

(a) Let S_n be the number of heads in 800 tosses.

Sn is the sum of 800 random variables X_i with head in i-th toss

E[X_i] = 1/2; Var[X_i] = 1/4.

Then by central limit theorem-

P(S_n > 415) = P($\sum_{i=1}^{800}$X$_i$ > 415)

$\Rightarrow 1 - P(0 <= \sum_{i=1}^{800} \mathrm{X}_i <= 415)$

$\Rightarrow 1 - P((0 - 800 * E[\mathrm{X}_i])/\sqrt{(Var}[\mathrm{X}_i]*800) <= \sum_{i=1}^{800} X_i - 800 * E[\mathrm{X}_i]/\sqrt{(Var}[\mathrm{X}_i]*800) <=$ 415 - 800*E[X_i]/$\sqrt{(Var}$[X$_i$]*800))

$\Rightarrow 1 - P(-400/\sqrt{200} <= \mathrm{Z}_n <= 15/\sqrt{200})$

(b) P(230 $<= S_n <=$ 255) = P((230 - 800*E[X_i])/$\sqrt{(Var}$[X$_i$]*800) $<= Z_n <=$ 255 - 800*E[X_i]/$\sqrt{Var}$[X$_i$]*800

$\Rightarrow P(230 - 400/\sqrt{200}$ <=Z$_n$ <= 255 - 400/$\sqrt{200}$)

$\Rightarrow P(-170/\sqrt{200}$ <=Z$_n$ <= -145/$\sqrt{200}$)

$\Rightarrow P(-12.02 <= Z$n <= -10.25)

$\Rightarrow \varphi(-10.25) - \varphi(-12.02)$

Exercise 3: Law of the Total Probability

P(left at home) = 1/4; P(left at library) = 1/2; P(left at train) = 1/4;

(a) P(found in library) = 1/2 * .9 = 0.45

P(found in train) = 1/4 * .5 = 0.125

(b) P(left at home) = 1/4

(c) P(not found) = P(not found | left at home)P(left at home) + P(not found | left at lib)P(left at lib) + P(not found | left at train)P(left at train)

$\Rightarrow$ P(not found) = P(chances to not find in home)P(left at home) + P(chances to not find in lib)P(left at lib) + P(chances to not find in train)P(left at train)

$\Rightarrow$ P(not found) = P(chances to not find in home)1/2 + 1-0.9*1/2 + 1 - 0.5*1/4

$\Rightarrow$ P(not found) = P(chances to not find in home)1/2 + 0.05 + 0.125

$\Rightarrow$ P(not found) = 1 - P(chances find in home)1/2 + 0.05 + 0.125

$\Rightarrow$ P(not found) = 1 - P(chances find in home)1/2 + 0.05 + 0.125

$\Rightarrow$ P(not found) = 1 - (1 - 1/3 * P(chances to find in lib) - 1/3 * P(chances to find in train))1/2 + 0.05 + 0.125

$\Rightarrow$ P(not found) = 1 - (1 - 1/3 * 0.9 - 1/3 * 0.5)1/2 + 0.05 + 0.125

$\Rightarrow$ P(not found) = 1 - (1 - 0.13)1/2 + 0.05 + 0.125

$\Rightarrow$ P(not found) = 0.067 + 0.05 + 0.125

$\Rightarrow$ P(not found) = 0.242

Exercise 4: Bayes' Theorem

(a) P(infected) = 0.005

P(positive test|infected) = 0.99

P(positive test|not infected) = 0.02

P(infected|positive test) = P(positive test|infected) * P(infected)

$\Rightarrow$ P(infected|positive test) = 0.99 * 0.005 = 00495

(b) P(no symptoms|V1) = 0.95

P(V1) = 0.84

P(V1|symptoms) = P(symptoms|V1) * P(V1)

⇒ P(V1|symptoms) = 1 - P(no symptoms|V1) * P(V1)

⇒ P(V1|symptoms) = 0.05 * 0.84 = 0.042

Exercise 5: Fixed-Point Iteration

(a)

$$f(x) = e^{-x} - 0.5x$$

Choosing $\varphi(x)$ such that:

$$\varphi(x) = 2\lambda e^{-x} \in [0, 1]$$

⇒0 < $\lambda <= 0.5$

Now the following iteration:

$x_{n+1} = \varphi(x_n)$

will bring us to the approximate solution.

(b)

$$||x^* - x_n|| <= q^n/(1-q)||x_1 - x_0||$$

(1)

Let $\lambda = 0.5$

$x_1 = \varphi(x_0) = 2 * 0.5 * e^(- 0.2) = 0.81$

$||\varphi(x_1) - \varphi(x_0)|| <= q||x_1 - x_0||$

⇒0.37 <= q*0.61

⇒0.37/0.61 <= q

⇒0.606 <= q

from (1): $||0.015|| <= q^n/(1-q)||0.81 - 0.2||$

⇒0.015 * 0.394 / 0.61 = q^n

⇒n ≃ 9.25