2 Binary Operations

2.1 Binary Images

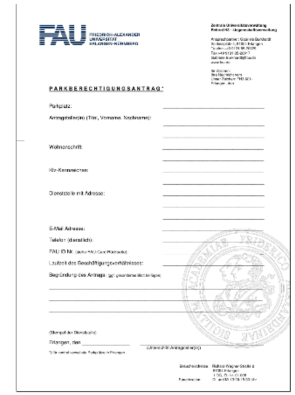
2.2 Morphological Filters

- Dilation
- Erosion

2.3 Opening & Closing

2.4 Morphological Filtering of Gray-Scale Images

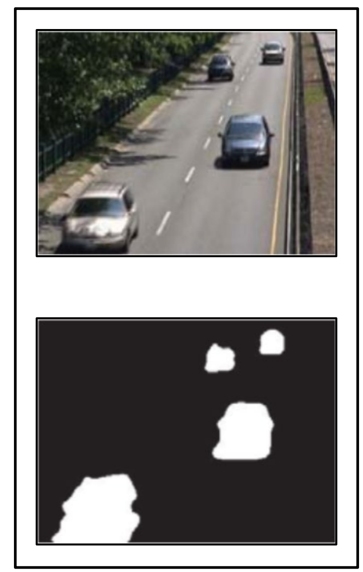
2.1 Binary Images



Binary images are very common

- Document image processing (texts, forms)
- Image analysis
  - Thresholding, segmentation, foreground/background
  - Presence/absence of some image property

Processing by logical functions is simple and fast



Morphological image processing

- Modifies the shape of regions
- It has been extended to gray level images

P. Dong et al., „Foreground Detection with Simultaneous Dictionary Learning and Historical Pixel Maintenance“, Trans. On Imag. Proc., Nov. 2016

2.2 Morphological Filters

Morphology

- Study of forms and structure (originally for animals and plants)

In the context of image processing

- Tool for extracting image components to study structure and shape of objects (regions)

Morphological filtering

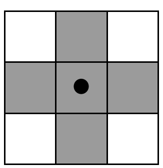
- operations where an object is "hit" with a **structuring element** and thereby reduced/expanded to a more revealing shape
- originally aimed at binary signals; extension for non-binary signals

Structuring Element

Binary structuring element $b[m,n]$ $\hat{=}$ 2D signal

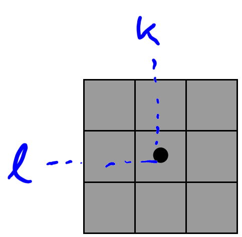
- A mask used to probe ("hit") an image under study for properties of interest
- $b[m-k, n-l]$ denotes translation of $b[m,n]$ so its origin is centered at position $[k,l]$

Examples for $b[m,n]$:



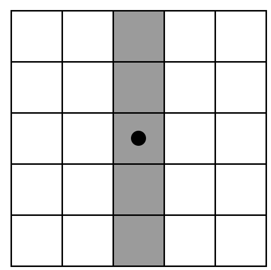
3x3

5 pixels



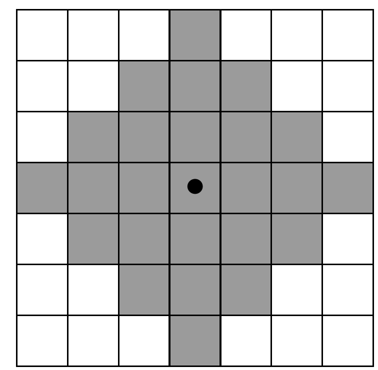
3x3

9 pixels



5x5

5 pixels



7x7

Dilation

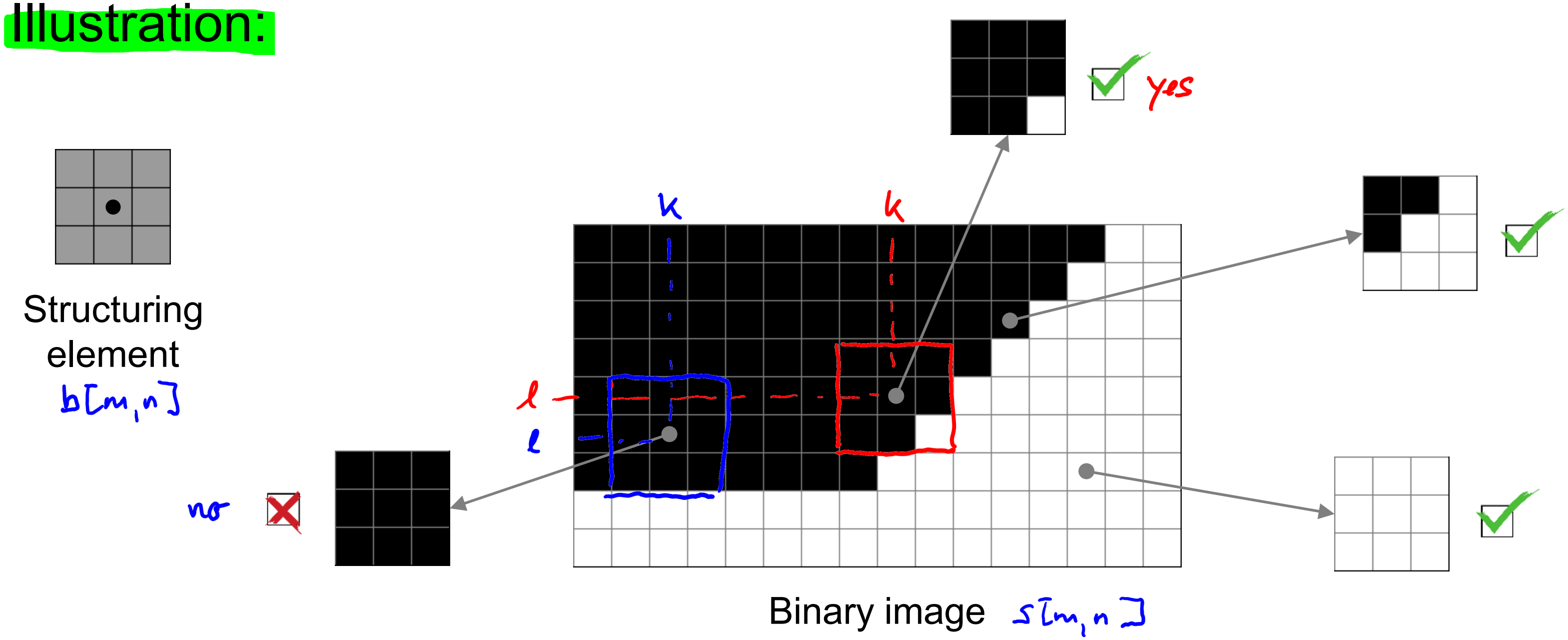
Dilation of binary signal s by b is defined as set of all points (k, l) such that $b[m-k, n-l]$ **hits** $s[m, n]$, i.e. they have non-empty intersection

$$s \oplus b \;:=\; \{(k,l) \mid b[m-k, n-l] \cap s[m,n] \neq \emptyset\}$$

dilated by

intersection

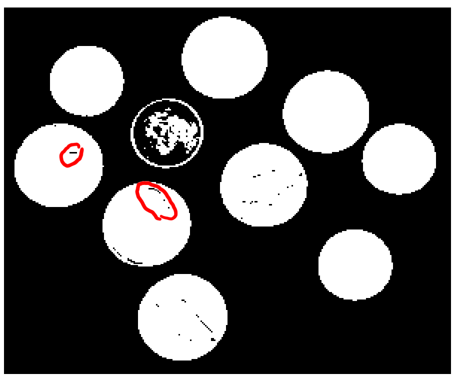
Illustration:



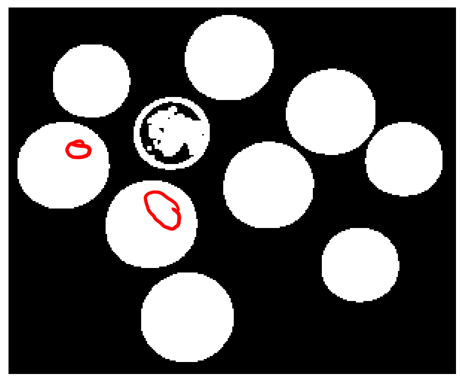
Dilation

Effects of dilation

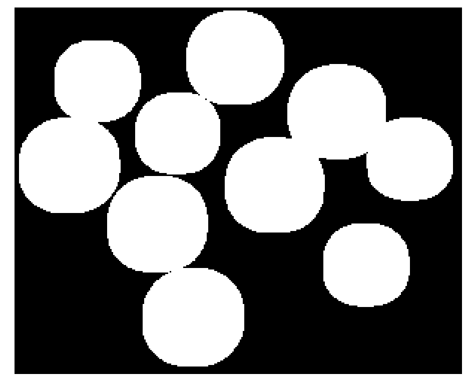
- Expands the size of 1-valued objects
- Smooths object boundaries
- Closes holes and gaps



Original image



Dilation with 3x3 structuring element

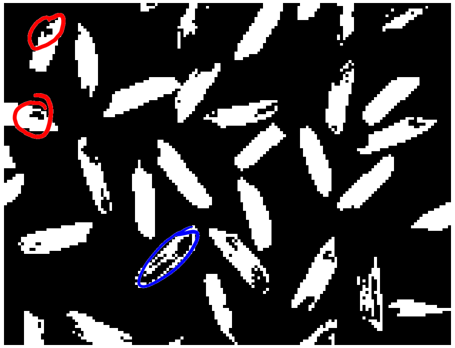


Dilation with 9x9 structuring element

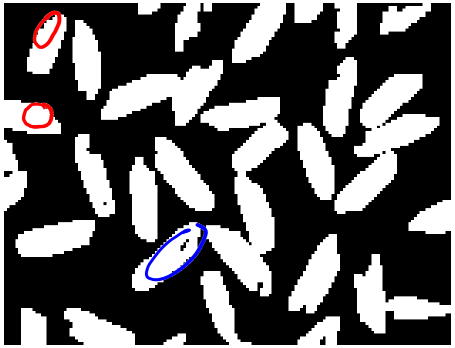
Dilation

Effects of dilation

- Expands the size of 1-valued objects
- Smooths object boundaries
- Closes holes and gaps



Original image



Dilation with 3x3 structuring element

Dilation with 9x9 structuring element

Erosion

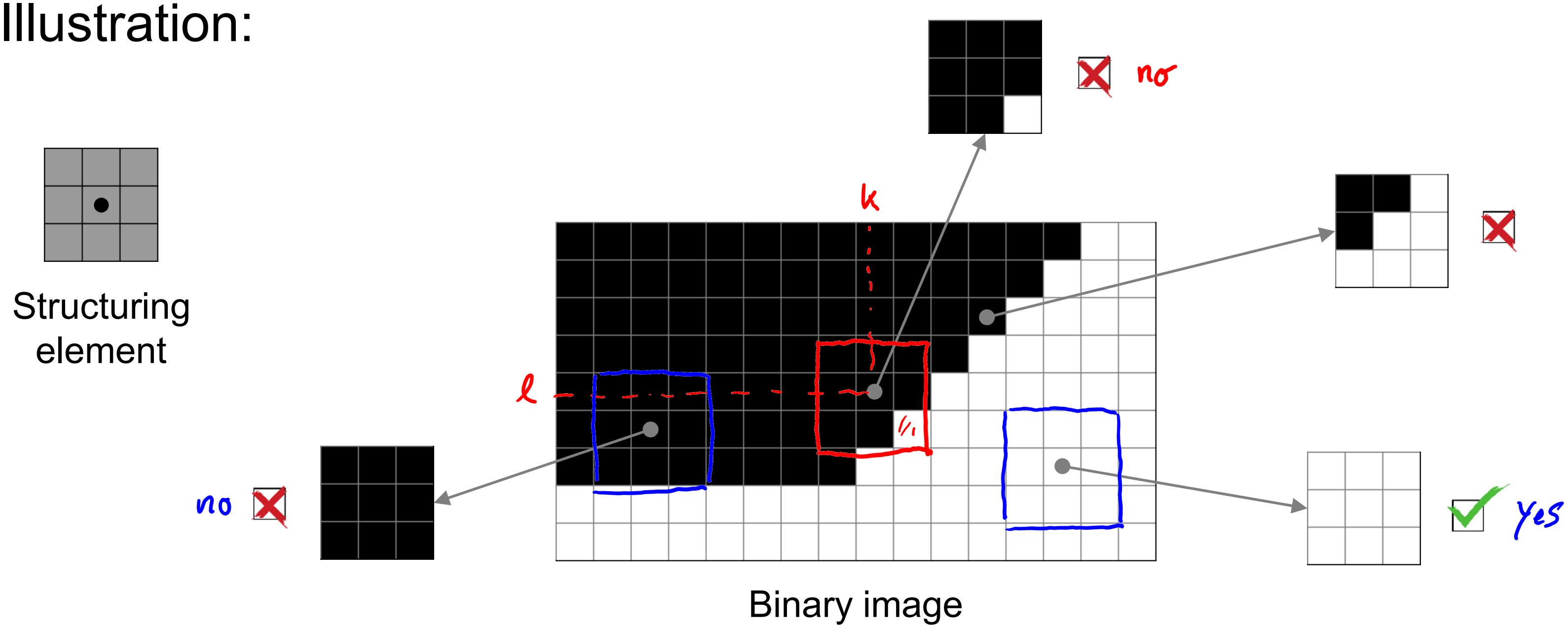
Erosion of binary signal s by b is defined as the set of all points (k, l) such that $b[m-k, n-l]$ is (entirely) **included** in $s[m, n]$

$$s \ominus b \;:=\; \{(k,l) | b[m-k, n-l] \subset s[m,n]\}$$

eroded by

included in

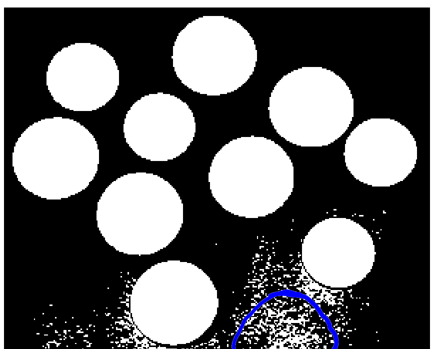
Illustration:



Erosion

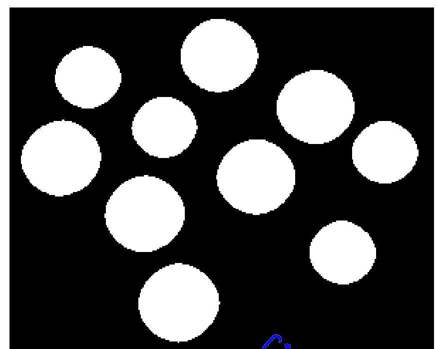
Effects of erosion

- Shrinks the size of 1-valued objects
- Smooths object boundaries
- Removes peninsulas, fingers and small objects ≙ remove noise

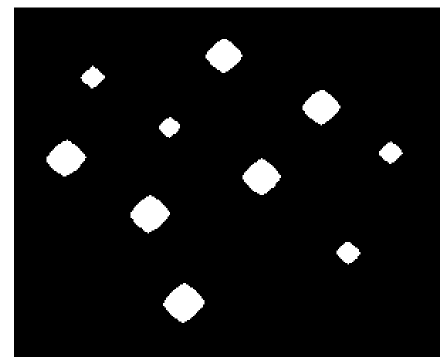


noise

Original image



Erosion with 5x5 structuring element

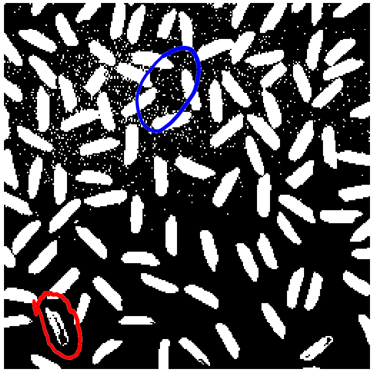


Erosion with 27x27 structuring element

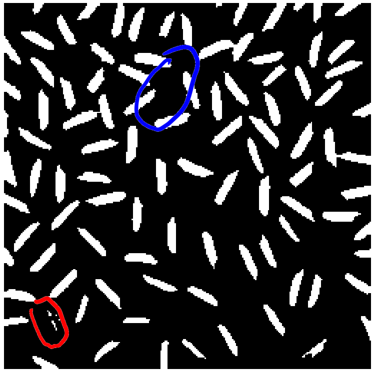
Erosion

Effects of erosion

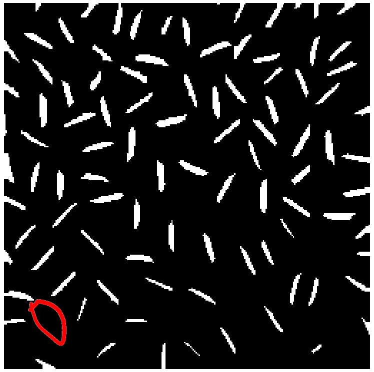
- Shrinks the size of 1-valued objects
- Smooths object boundaries
- Removes peninsulas, fingers and small objects



Original image



Erosion with 3x3 structuring element



Erosion with 5x5 structuring element

Properties of Erosion and Dilation

$s[m,n] \longrightarrow \boxed{b[m,n]} \longrightarrow y[m,n]$

Translational invariance

- Translation of an object causes the same shift in the result

cf. LSI (linear shift invariant) systems => chap. 4

here: not linear but shift invariant

Reversibility

- Erosion and dilation are *not* reversible in general, specifically they are not inverses of each other
- Example: closing holes

Original image | Dilation with 5x5 structuring element <— same —> Erosion with 5x5 structuring element

Properties of Erosion and Dilation

Translational invariance

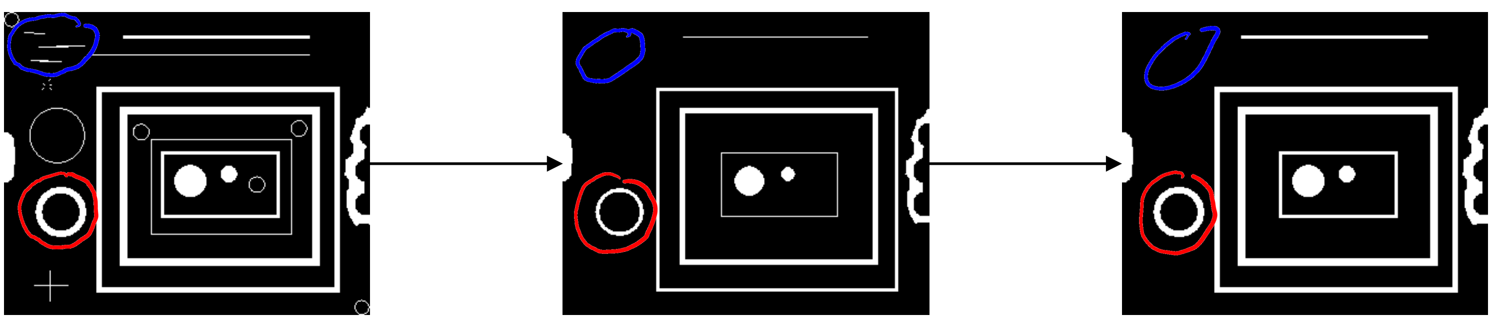
- Translation of an object causes the same shift in the result

Reversibility

- Erosion and dilation are *not* reversible in general, specifically they are not inverses of each other
- Example: erosion of tiny objects/contours

less promi-nent

promi-nent



Original image | Erosion with 3x3 structuring element | Dilation with 3x3 structuring element

Properties of Erosion and Dilation

Distribuitivity

- Consecutive dilation or erosion by different structuring elements b and b'

$$s \oplus (b \cup b') = (s \oplus b) \cup (s \oplus b')$$
$$s \ominus (b \cup b') = (s \ominus b) \cap (s \ominus b')$$

Duality

- Erosion (dilation) of an object by b is equivalent to dilation (erosion) of its background by the reflection of b, i.e.,

"$\hat{b}$" ≙ rotation by 180° around the center pixel

background

$$\overline{s \ominus b} = \bar{s} \oplus \hat{b}$$
$$\overline{s \oplus b} = \bar{s} \ominus \hat{b}$$

- For symmetric structuring elements $b = \hat{b}$

2.3 Opening and Closing

Combination of basic morphological operations (dilation and erosion)

Problem so far: erosion as well as dilation considerably change object size

Goal: smoothing without size change

Opening: *erosion* followed by *dilation*

second

$$s \circ b = (s \ominus b) \oplus b$$

first

- Removes small regions (islands) of s

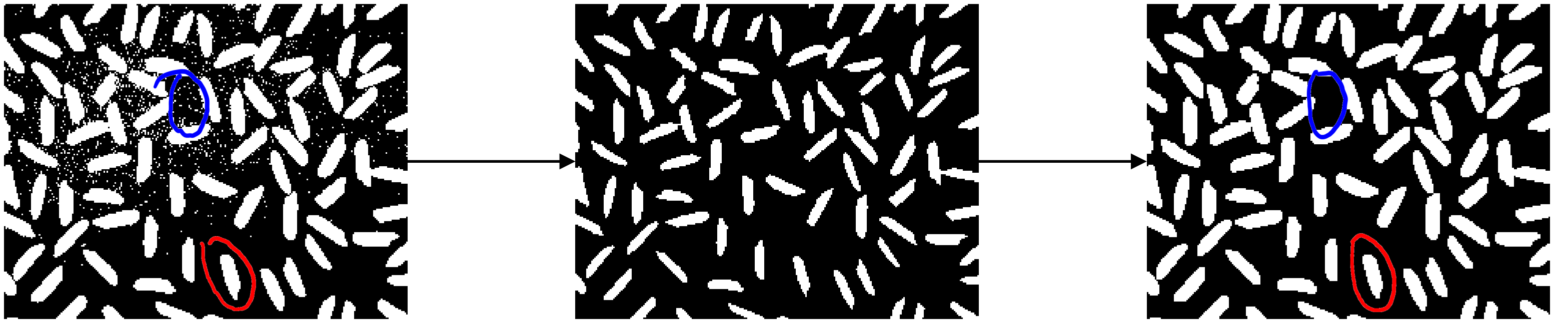
Closing: *dilation* followed by *erosion*

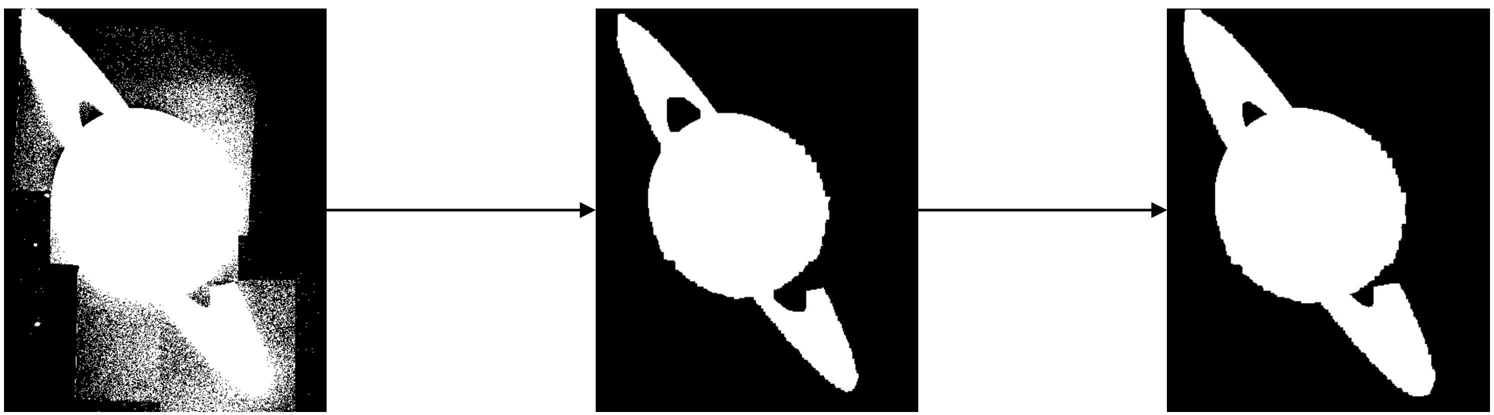
$$s \cdot b = (s \oplus b) \ominus b$$

- Removes small holes and narrow structures within s

Noise Removal by Opening

Original image (256×256) | Erosion with 3x3 structuring element | Dilation with 3x3 structuring element





Original image (1500×1200) | Erosion with 33x33 structuring element | Dilation with 33x33 structuring element

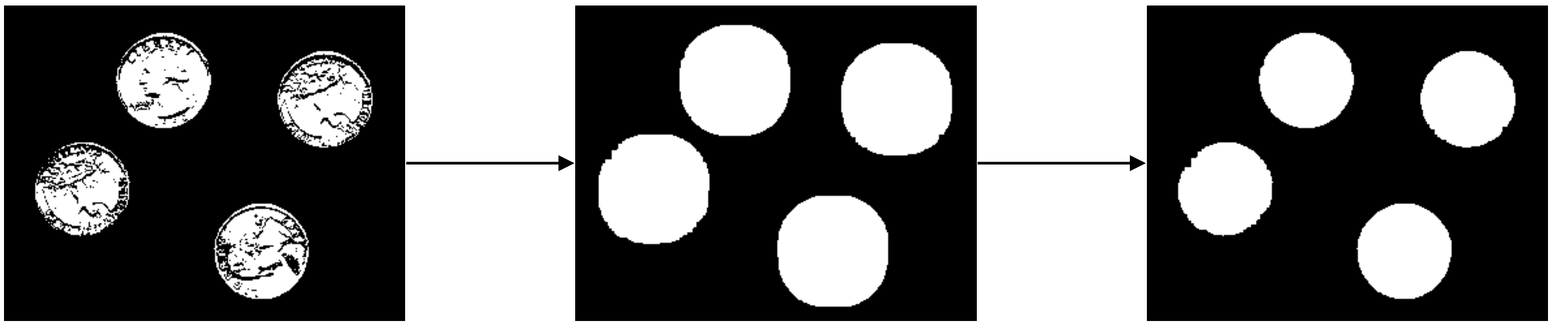
Hole Removal by Closing

Original image (242×308)

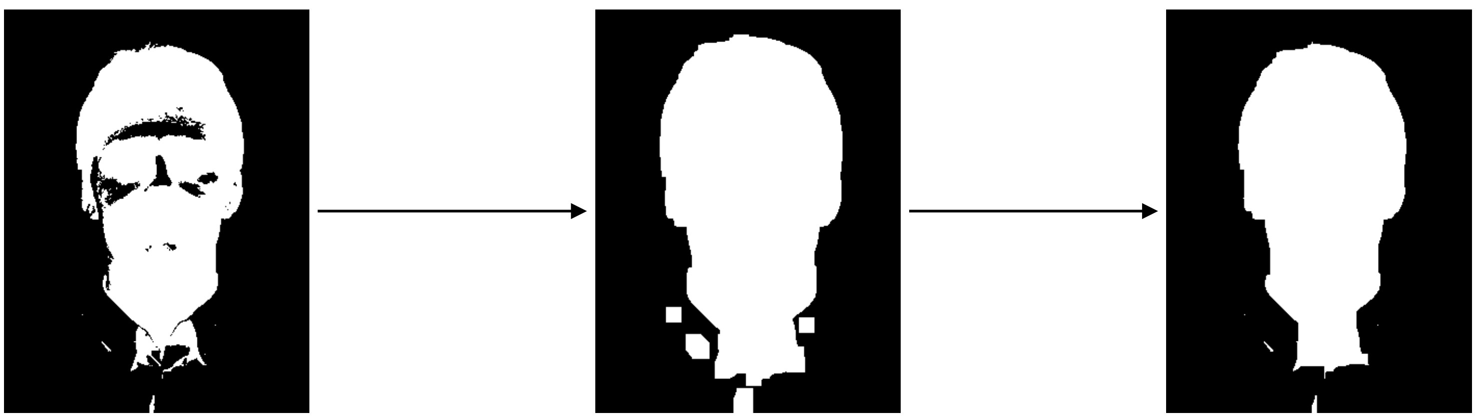
Dilation with 13x13 structuring element

Erosion with 13x13 structuring element

coin counting / detection



face / head detection



Original image (388×294)

Dilation with 15x15 structuring element

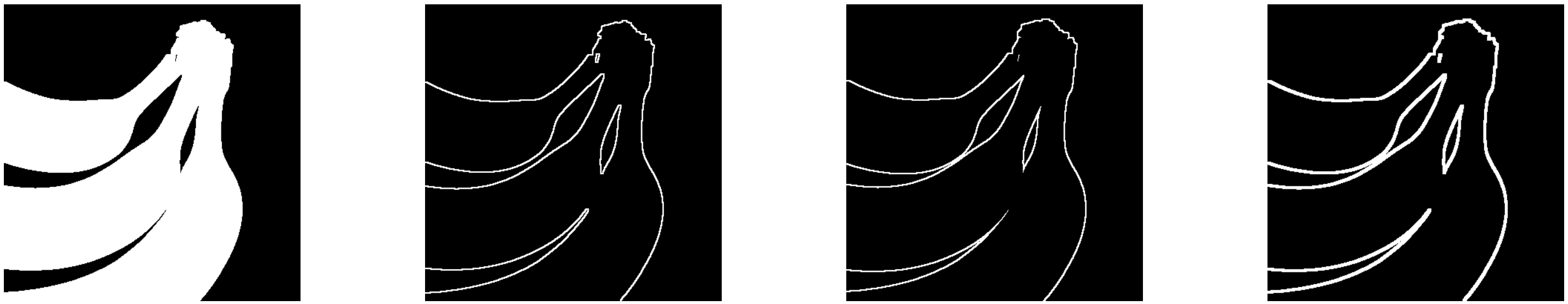
Erosion with 15x15 structuring element

Boundary Extraction

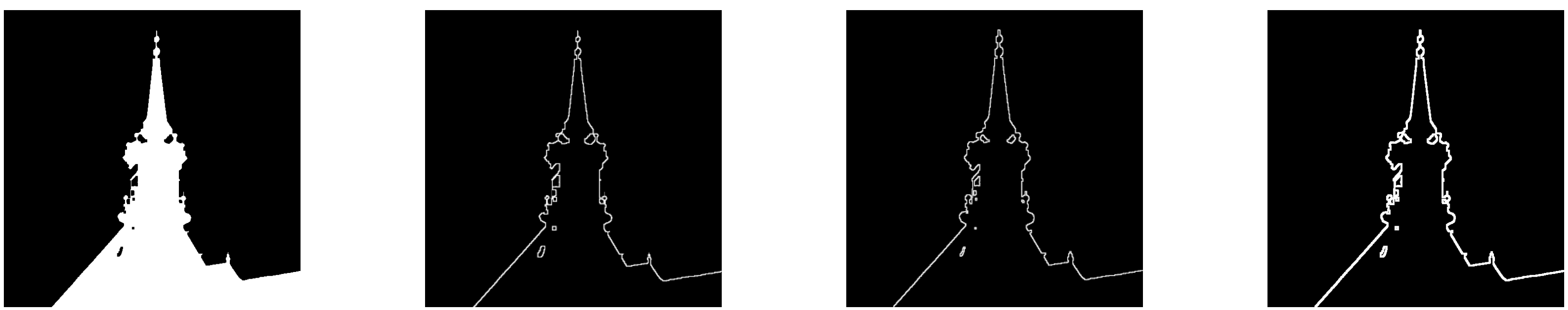
Morphological **edge detectors**

only boundaries remain

more pronounced edges

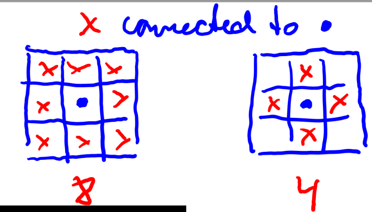


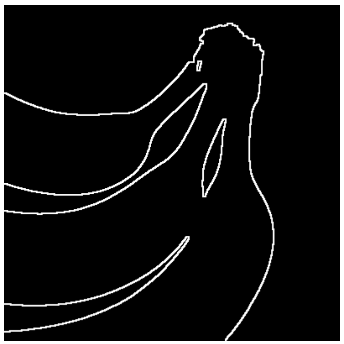
s | $s - (s \ominus b)$ | $(s \oplus b) - s$ | $(s \oplus b) - (s \ominus b)$



Hole Filling

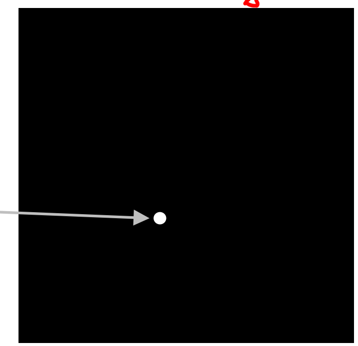
Let r denote an 8-connected boundary of an object





r

Seed



s_0

Iterative **hole filling**:

dilation

$$s_k = (s_{k-1} \oplus b) \underbrace{\cap \, \bar{r}}_{\text{only inside, stop at boundary}} \qquad k = 1, 2, 3, \ldots$$

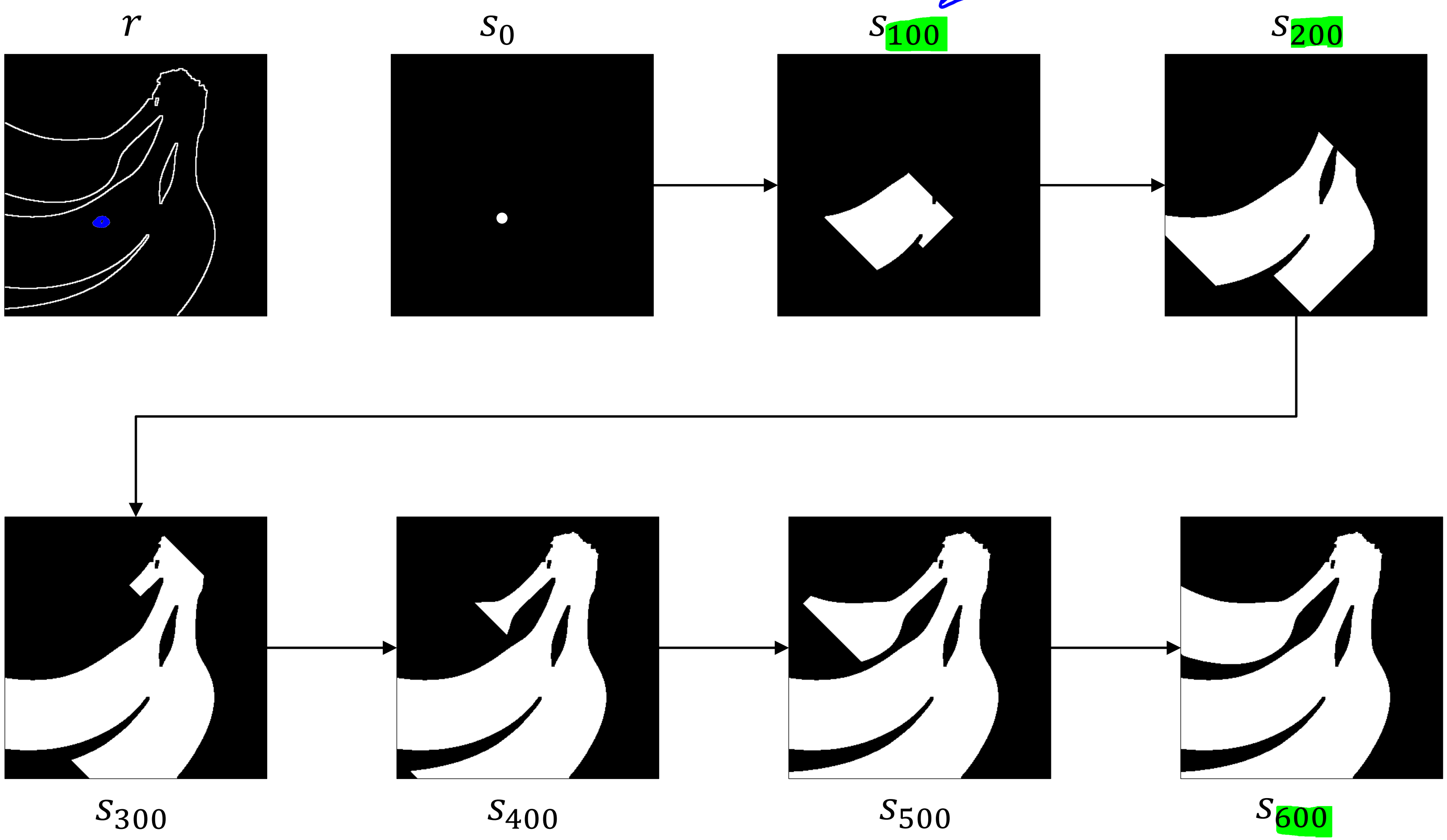
Seed defines the concept of inside (hole) vs. outside

- It is dilated until it hits the boundary: **conditional dilation**

Hole Filling

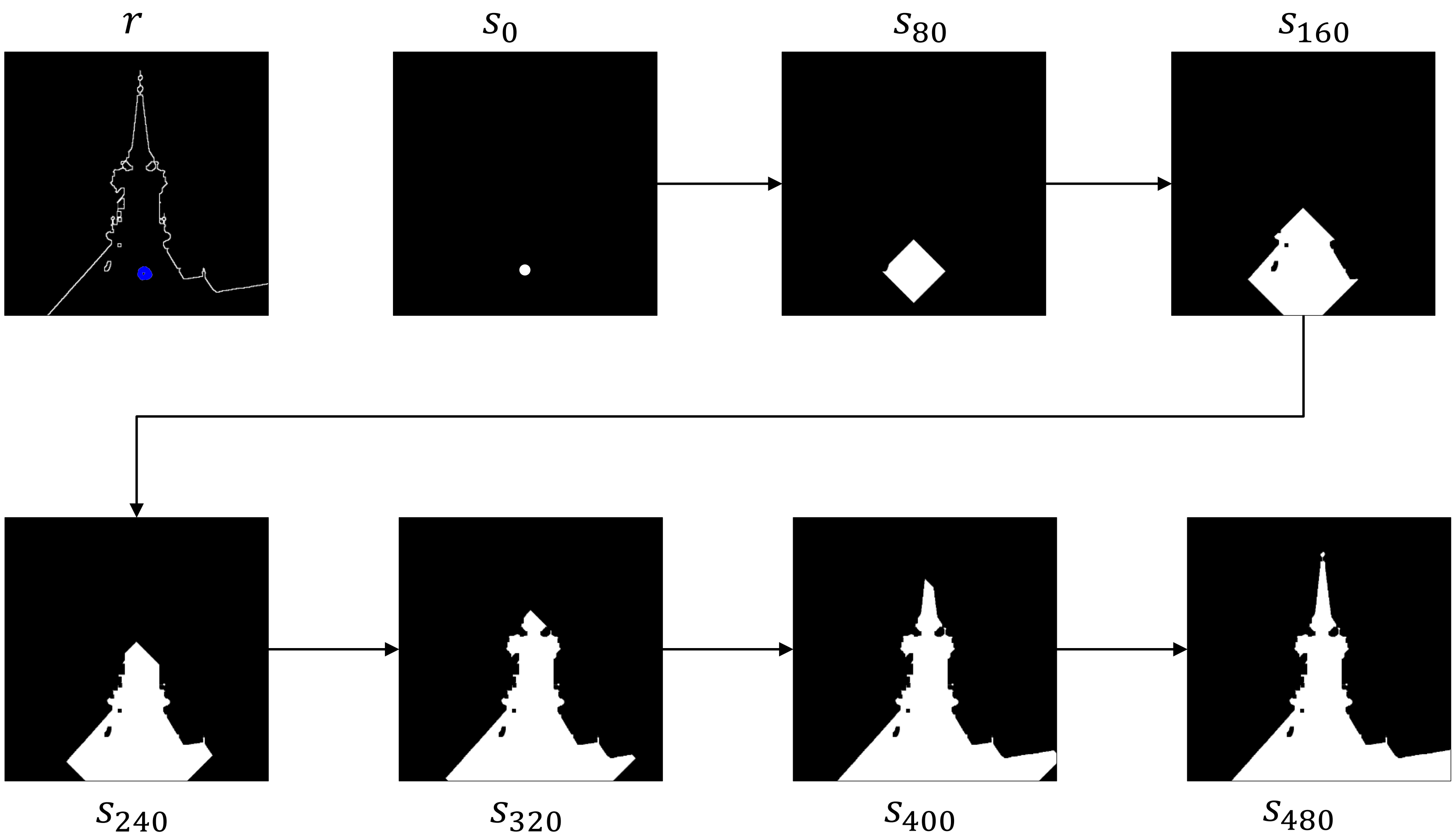
start: boundary

#iterations



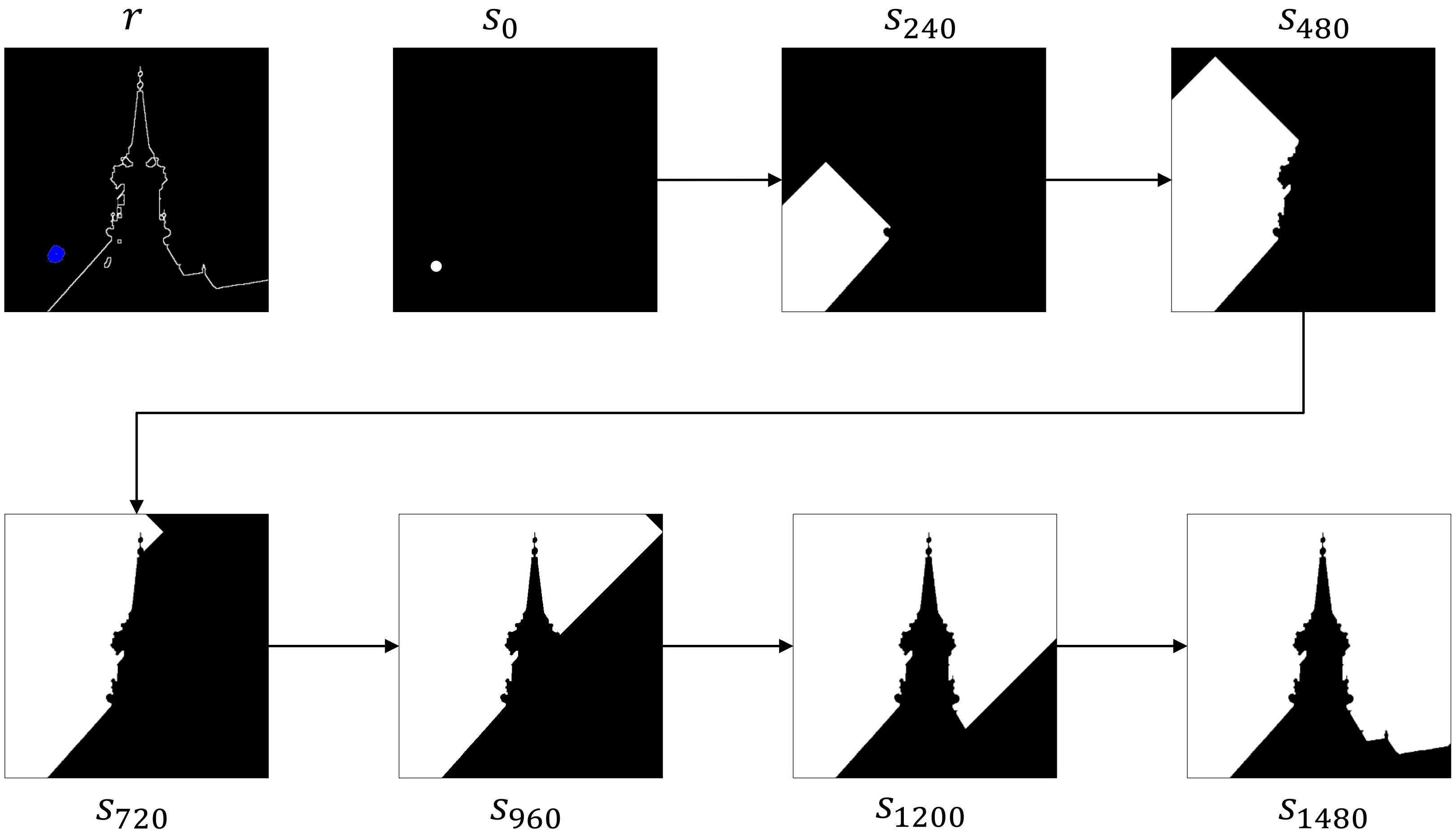
end: object shape

Hole Filling



Hole Filling

Object depends on placement of starting point s_0 => inside / outside



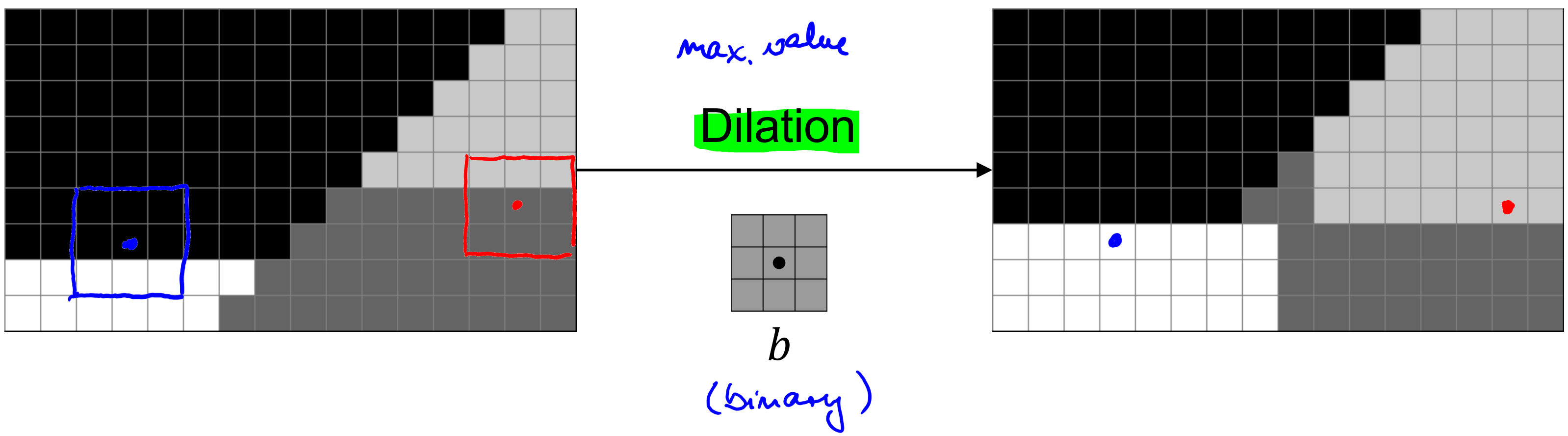
end: background

2.4 Morphological Filtering of Gray-Scale Images

Idea: Extension of morphological operations to gray-scale images using binary (flat) structuring element

- Gray-scale structuring elements are rarely used

Output: Minimum value (erosion) or maximum value (dilation) of gray-scale image within structuring element



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