priors for Bayesian model uncertainty

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Zellner's g-prior

sampling model

$$Y_i \mid \beta_0, \beta_1, \ldots \beta_p, \sigma^2 \overset{\text{ind}}{\sim} \mathsf{N}(\beta_0 + \beta_1(x_{i1} - \bar{x}_1) + \ldots + \beta_p(x_{ip} - \bar{x}_p), \sigma^2)$$

informative conjugate prior

$$\boldsymbol{\beta} \mid \sigma^2 \sim \mathsf{N}\left(b_0, g\,\sigma^2 \mathbf{S}_{\mathbf{XX}}^{-1}\right)$$

- informative prior mean b_0
- scaled variance (and covariances) from OLS
- g controls precision

posterior with Zellner's g-prior

- posterior mean

$$\frac{g}{1+g}\hat{\boldsymbol{\beta}} + \frac{1}{1+g}b_0$$

- posterior variance (given σ^2)

$$\frac{g}{1+g}\,\sigma^2\mathbf{S}_{\mathbf{XX}}^{-1}$$

- posterior distribution

$$\boldsymbol{\beta} \mid \sigma^2, \mathrm{data} \sim \mathsf{N}\left(\frac{g}{1+g}\hat{\beta} + \frac{1}{1+g}b_0, \frac{g}{1+g}\,\sigma^2\mathbf{S}_{\mathbf{XX}}^{-1}\right)$$

Bayes factor

- Zellner's g prior for coefficients in model m with $b_0 = 0$
- reference prior for intercept and variance $p(\beta_0, \sigma^2) \propto 1/\sigma^2$
- Bayes factor of model m to null model $\mathcal{M}_0$

$$BF[\mathcal{M}_m : \mathcal{M}_0] = (1+g)^{(n-p_m-1)/2}(1+g(1-R^2_m))^{-(n-1)/2}$$

$$\frac{p(\mathcal{M}_m|\text{data},g)}{p(\mathcal{M}_0|\text{data},g)} = BF[\mathcal{M}_m : \mathcal{M}_0]\frac{p(\mathcal{M}_m)}{p(\mathcal{M}_0)}$$

choice of g

$$BF[\mathcal{M}_m : \mathcal{M}_0] = (1+g)^{(n-p_m-1)/2}(1+g(1-R^2_m))^{-(n-1)/2}$$

- Bartlett/Lindley paradox $g \to \infty$ then $BF[\mathcal{M}_m : \mathcal{M}_0] \to 0$
- information paradox

  for fixed g as $R^2 \to 1$ Bayes factor is bounded

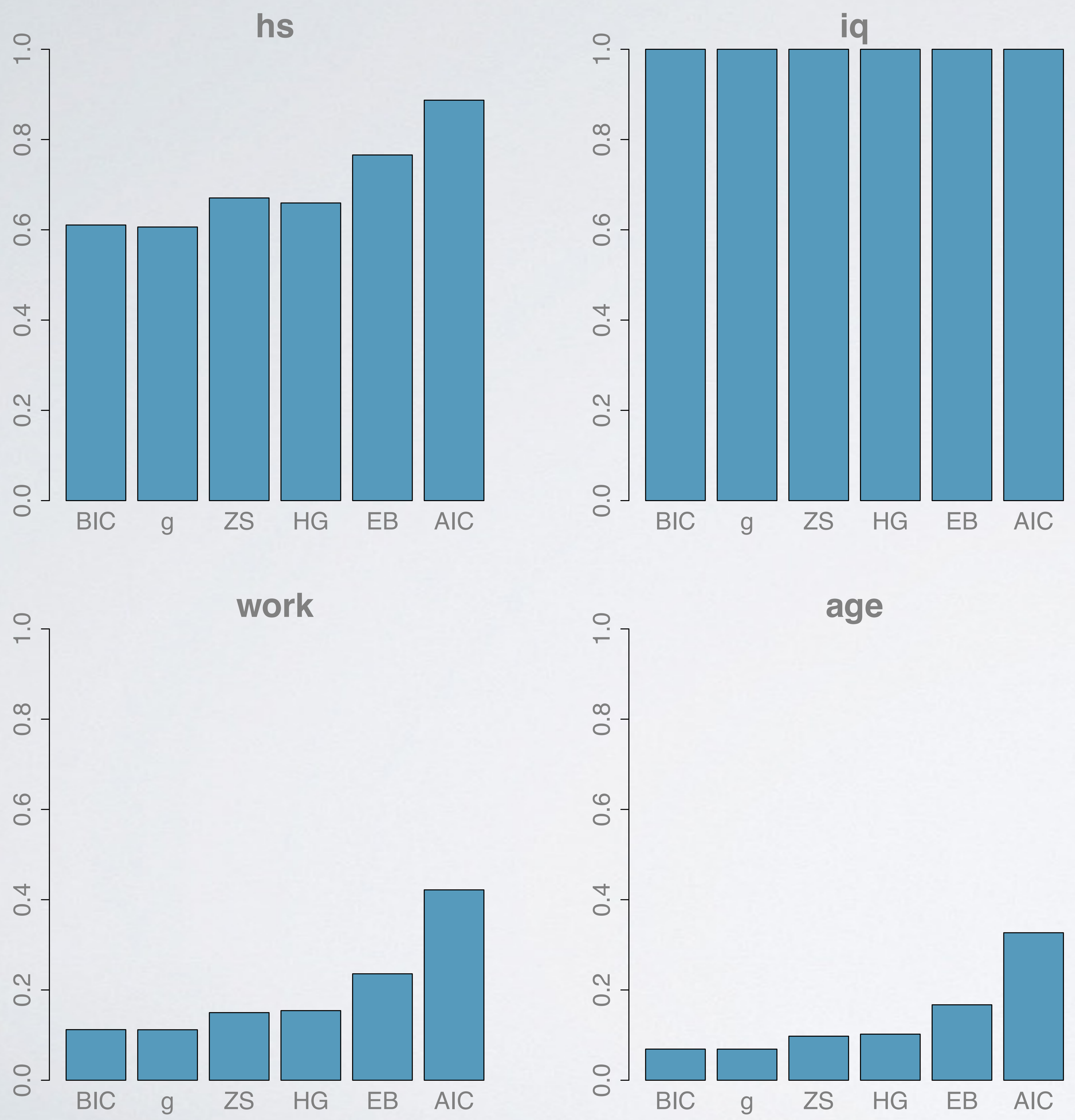
- unintended consequences of choice of g

solutions

let prior depend on *n*

unit information	g = n
Zellner-Siow cauchy	$n/g \sim \mathrm{G}(1/2, 1/2)$
hyper-g/n	$\frac{1}{1+g/n} \sim \mathrm{Beta}(a/2, b/2)$

sensitivity: posterior inclusion probabilities



summary

- introduced Zellner's g prior
- choice of g
- sensitivity of posterior inclusion probabilities

next

- example
- reporting with model certainty