

Water Problem with inequality constraint

$$f(\vec{c}) = -\vec{c}^T \vec{c} \text{ subj to } \vec{M}_2 = \underline{R}_N \vec{c} \text{ and } \vec{c} > \vec{0}$$

let's do this in 2D

$$L_1, L_2) = -(\tau_1^2 + \tau_2^2) \quad \begin{matrix} M_1 = R_{11}\tau_1 + R_{12}\tau_2 \\ M_2 = R_{21}\tau_1 + R_{22}\tau_2 \end{matrix} \quad \begin{matrix} -\tau_1 < 0 \\ -\tau_2 < 0 \end{matrix} \quad \left. \vphantom{\begin{matrix} M_1 \\ M_2 \end{matrix}} \right\} \text{for standard form}$$

$$L = -\vec{c}^T \vec{c} + \vec{\lambda}^T (\vec{M} - \underline{R}_N \vec{c}) + \vec{\mu}^T \vec{c}$$

$$(\tau_1^2 + \tau_2^2) + \lambda_1 (M_1 - R_{11}\tau_1 - R_{12}\tau_2) + \lambda_2 (M_2 - R_{21}\tau_1 - R_{22}\tau_2) + \mu_1 \tau_1 + \mu_2 \tau_2$$

$$\left. \begin{matrix} -2\tau_1 + \lambda_1(-R_{11}) + \lambda_2(-R_{21}) + \mu_1 \\ -2\tau_2 + \lambda_1(-R_{12}) + \lambda_2(-R_{22}) + \mu_2 \end{matrix} \right\} \Rightarrow \boxed{\vec{0} = -2\vec{c} - \underline{R}_N^T \vec{\lambda} + \vec{\mu}}$$

$$M_1 - R_{11}\tau_1 - R_{12}\tau_2 \quad \frac{\partial L}{\partial \lambda_2} = M_2 - R_{21}\tau_1 - R_{22}\tau_2 \Rightarrow \boxed{\vec{M} = \underline{R}_N \vec{c}}$$

$$\tau_1 \quad \frac{\partial L}{\partial \mu_2} = \tau_2 \quad \left. \vphantom{\frac{\partial L}{\partial \mu_2}} \right\} \text{combine} \quad \left\{ \begin{matrix} \tau_1 \\ \tau_2 \end{matrix} \right\} = 0 \quad \frac{\partial L}{\partial \vec{\lambda}} = \vec{c} = 0 \quad \boxed{\vec{c} = 0} \text{ or } \boxed{\vec{\mu} = 0}$$

I'm not sure you can solve this analytically