...ctices

$f(x,y) = 5x - 3y$ subj to $x^2 + y^2 = 136$

$5x - 3y + \lambda(136 - x^2 - y^2)$

$= 5 - 2\lambda x$ $\quad \frac{\partial L}{\partial y} = -3 - 2\lambda y$ $\quad \frac{\partial L}{\partial \lambda} = 136 - x^2 - y^2 = 0$

$= 2\lambda x$ $\quad -3 = 2\lambda y$ $\quad 136 - \left(\frac{5}{2\lambda}\right) - \left(\frac{-3}{2\lambda}\right) = 0$ ← solve for λ

$\boxed{= \frac{5}{2\lambda}}$ $\quad \boxed{y = \frac{-3}{2\lambda}}$

get x & y.

...r Problem in 2D

...ize $f(\vec{\tau}) = -\vec{\tau}^T\vec{\tau} = -(\tau_1^2 + \tau_2^2)$ subject to $\vec{M}_d = \underline{\underline{R}}_N \vec{\tau}$

$$\begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix}$$

$\vec{\tau}^T\vec{\tau} + \vec{\lambda}^T(\vec{M}_d - \underline{\underline{R}}_N \vec{\tau})$

$\vec{\lambda}^T = [\lambda_1 \quad \lambda_2]$

$M_1 = R_{11}\tau_1 + R_{12}\tau_2$

$M_2 = R_{21}\tau_1 + R_{22}\tau_2$

$= -2\tau_1 + \lambda_1(-R_{11}) + \lambda_2(-R_{21})$

$= -2\tau_2 + \lambda_1(-R_{12}) + \lambda_2(-R_{22})$

$\vec{M}_d - \underline{\underline{R}}_N \vec{\tau}$

$$= \begin{Bmatrix} M_1 - R_{11}\tau_1 - R_{12}\tau_2 \\ M_2 - R_{21}\tau_1 - R_{22}\tau_2 \end{Bmatrix}$$

combine eqn's

$$\vec{\lambda}^T(\vec{M}_d - \underline{\underline{R}}_N\vec{\tau}) = \lambda_1(M_1 - R_{11}\tau_1 - R_{12}\tau_2) + \lambda_2(M_2 - R_{21}\tau_1 - R_{22}\tau_2)$$

$$2\begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} + \begin{Bmatrix} \underbrace{[\lambda_1 \quad \lambda_2]}_{1\times 2} \underbrace{\begin{Bmatrix} -R_{11} \\ -R_{21} \end{Bmatrix}}_{2\times 1} \\ [\lambda_1 \quad \lambda_2] \begin{Bmatrix} -R_{12} \\ R_{22} \end{Bmatrix} \end{Bmatrix}$$

$$\vec{\tau} + \begin{bmatrix} -R_{11} & -R_{21} \\ -R_{12} & -R_{22} \end{bmatrix} \begin{Bmatrix} \lambda_1 \\ \lambda_2 \end{Bmatrix}$$

$$\frac{\partial L}{\partial \lambda_1} = M_1 - R_{11}\tau_1 - R_{12}\tau_2$$ Combine

$$\frac{\partial L}{\partial \lambda_2} = M_2 - R_{21}\tau_1 - R_{22}\tau_2$$

$\boxed{\vec{\tau} - \underline{\underline{R}}_N^T \vec{\lambda}}$

$$\begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} - \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} = 0$$

$\boxed{\frac{\underline{\underline{R}}_N^T}{2} \vec{\lambda}}$

$\vec{M}_d = \underline{\underline{R}}_N \vec{\tau}$

$\boxed{\vec{\lambda} = -2(\underline{\underline{R}}_N \underline{\underline{R}}_N^T)^{-1} \vec{M}_d}$

$\vec{M}_d = \underline{\underline{R}}_N \left(\frac{-\underline{\underline{R}}_N^T}{2}\right) \vec{\lambda}$

$\frac{\underline{\underline{R}}_N^T}{\cancel{2}} \left[-\cancel{2} (\underline{\underline{R}}_N \underline{\underline{R}}_N^T)^{-1} \vec{M}_d \right]$

$-2\vec{M}_d = \underline{\underline{R}}_N \underline{\underline{R}}_N^T \vec{\lambda}$