Math 295X: Arithmetic Statistics
Syllabus

Ashvin Swaminathan
swaminathan@math.harvard.edu

Spring 2024

Logistics

We will have lecures each Tuesday and Thursday from 1:30-2:45PM in SC TBA. Office hours will be after each lecture Tuesday from 3-4PM in my office, SC 236. All course materials can be found on the course website: `https://ashvin-swaminathan.github.io/home/Math295X.html`.

Summary

In this course, we will discuss methods for studying the distributions of arithmetic objects, such as polynomials, solutions to Diophantine equations, class groups of number fields, and more. We will focus on the parametrize-and-count approach developed in the work of Bhargava and his collaborators. This approach consists of two steps — an algebraic parametrization step in which one describes the desired arithmetic objects in terms of orbits of a representation — and an analytic counting step in which one determines asymptotic counts of these orbits. Thus, the course will feature techniques from diverse subfields of mathematics.

Prerequisites

In this course we will make use of concepts from the following subjects: linear and abstract algebra, group theory, Galois theory, algebraic number theory, algebraic geometry, representation theory, calculus, and real analysis. It is highly recommended to have taken the equivalent of Math 123 and 129; Math 114, 137, and 222 are helpful but not strictly necessary; and some familiarity with the concepts covered in Math 223 and 229 would not hurt. Undergraduates are welcome.

References

There is no official textbook for this course, but the following collection of papers and notes might prove helpful:

- *Higher Composition Laws*, by Manjul Bhargava. `https://ashvin-swaminathan.github.io/home/BhargavaThesis.pdf`.

- *On the Davenport-Heilbronn theorems and second order terms*, by Manjul Bhargava, Arul Shankar, and Jacob Tsimerman. `https://arxiv.org/pdf/1005.0672.pdf`.
- *Notes on counting extensions of degrees* 2 *and* 3*, following Bhargava,* by Aaron Landesman. `https://people.math.harvard.edu/~landesman/assets/bhargavology-seminar-notes.pdf`.
- *The density of discriminants of quartic rings and fields*, by Manjul Bhargava. `https://annals.math.princeton.edu/wp-content/uploads/annals-v162-n2-p10.pdf`.
- *Asymptotics for number fields and class groups*, by Melanie Matchett Wood. `http://swc.math.arizona.edu/aws/2014/index.html`.

Grading

Weekly homework assignments will make up 60% of the grade. Each week's homework will consist of three problems. Assignments will be posted to the course website on Thursday after lecture and will be due on Tuesday at the start of lecture. If you have any questions, or if you require an extension or any other accommodation, please do not hesitate to email me, and I will be sure to respond promptly.

A final paper will count for the remaining 40% of the grade. This paper should be 7-10 pages in length and should discuss a topic related to the course material but not covered in lecture. The final paper will be due on May 10 at 11:59pm; no extensions will be granted. I will be happy to answer any questions you have about the paper and to provide feedback on rough drafts.