Heuristics Analysis

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Introduction

Many strategies were tested before finalizing on the custom heuristics for Knights isolation. Before we go on to the discussion of the heuristics, let us see how this document is structured.

The results from playing matches with the heuristics are embedded as images with captions on top and bottom. The caption on the top represents the figure number (*Figure x*). The caption on the bottom represents the evaluation function that was used for the different custom heuristics. For example, *Function Mapping: C1 = F1, C2 = F2 and C3 = F3* represents that AB_Custom = F1, AB_Custom_2 = F2, AB_Custom_3 = F3 where F1, F2, F3 are functions given in the evaluation function table in the APPENDIX.

The three heuristic strategies chosen are

1. Max Distance (H1)
2. Weighted Improved (H2)
3. Weighted Penalized (H3)

Strategies

Max Distance

This strategy involves being as distant from the opponent as possible. The rationale behind this strategy is that the opponent would not be able to minimize/block our moves and will give us more opportunities to “escape”.

This strategy uses the common distance between two points formula. For example if the two pieces are at (r1,c1) and (r2, c2) then the eval function would be

Eval Function = $\sqrt[2]{(r1 - r2)^2 + (c1 - c2)^2}$

Figure 1, the AB_Custom heuristic corresponds to this eval function.

Although this heuristic performs well when the number of open moves is more, it suffers significantly as the game progresses. Playing multiple rounds with this heuristic shows that it is able to beat the CPU agents however it does not consistently outperform it the AB_Improved heuristic.

This heuristic was combined with the Open and Improved to determine if there is any improvement in performance, the data shows us that there is no significant performance improvement.

Figure 1

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**************************
     Playing Matches
**************************

Match #    Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                       Won | Lost   Won | Lost   Won | Lost   Won | Lost
    1       Random      9  |  1     10  |  0     10  |  0      9  |  1
    2       MM_Open     9  |  1      8  |  2      7  |  3      8  |  2
    3      MM_Center   10  |  0     10  |  0      9  |  1     10  |  0
    4     MM_Improved   8  |  2      9  |  1      7  |  3      9  |  1
    5       AB_Open     5  |  5      5  |  5      5  |  5      6  |  4
    6      AB_Center    6  |  4      4  |  6      5  |  5      7  |  3
    7     AB_Improved   5  |  5      6  |  4      7  |  3      4  |  6
--------------------------------------------------------------------------
          Win Rate:      74.3%        74.3%        71.4%        75.7%
```

Function Mapping: C1 = H1, C2 = F1, C3 = F2

Weighted Improved

Thinking through Knight's isolation, it made sense that as the game progresses we would not want to minimize opponent moves , rather we would want to choose moves that would maximize our open moves. So a new variable was created called *Board Filled Ratio(BFR)* which is the ratio of the number of open moves to the board size.

$$BoardFilledRatio = \frac{NumberOfMoves}{TotalBoardSize}$$

Figure 2

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**************************
    Playing Matches
**************************
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Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	7	1	8	0	7	1	6	2
2	MM_Open	7	1	6	2	7	1	6	2
3	MM_Center	7	1	7	1	7	1	8	0
4	MM_Improved	8	0	7	1	5	3	6	2
5	AB_Open	2	6	5	3	4	4	5	3
6	AB_Center	4	4	5	3	3	5	8	0
7	AB_Improved	4	4	3	5	5	3	5	3
	Win Rate:	69.6%		73.2%		67.9%		78.6%	

Function Mapping: C1 = H1, C2 = F4, C3 = F1

Initially a heuristic was created where the AB_Improved heuristic would run if the BFR < 0.5 and AB_Open heuristic would run if BFR >= 0.5 (*Open Improved Switch (F4)*). An example game is shown above where AB_Custom_2 runs this heuristic. Running multiple games shows that it does not give improved performance and in fact runs worse than the Max Distance heuristic.

Hence instead of a discrete evaluation function, a continuous evaluation function (Weighted Improved) was tested. The weighted improved heuristic performed very well when compared to Max Distance.

$$\text{Eval Function} = \big(BoardFilledRatio * OwnMoves) + (1 - BoardFilledRatio) * (OwnMoves - OpponentMoves)\big)$$

The rationale behind this strategy is that as there are lesser options for movement on the board we would not want to minimize opponent moves. This strategy had the advantage that the weightage of AB_Improved and AB_Open were balanced based upon the number of open positions on the board.

Figure 3

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*************************
    Playing Matches
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Match #    Opponent     AB_Improved    AB_Custom    AB_Custom_2   AB_Custom_3
                        Won | Lost    Won | Lost    Won | Lost    Won | Lost
    1       Random       8  |  2      8  |  2      10  |  0      9  |  1
    2       MM_Open      8  |  2      5  |  5       7  |  3      7  |  3
    3      MM_Center     8  |  2     10  |  0      10  |  0      9  |  1
    4     MM_Improved    8  |  2      6  |  4       9  |  1      5  |  5
    5       AB_Open      3  |  7      5  |  5       2  |  8      5  |  5
    6      AB_Center     6  |  4      5  |  5       7  |  3      4  |  6
    7     AB_Improved    3  |  7      4  |  6       7  |  3      6  |  4
------------------------------------------------------------------------
           Win Rate:      62.9%        61.4%        74.3%        64.3%
```

Function Mapping: C1 = H1, C2 = H2, C3 = F3

The game above shows the improvement of this heuristic over Max Distance where AB_Custom_2 is the Weighted Improved heuristic. However, this is simply an example of a game where Weighted Improved performed better. Playing multiple games shows that it does not perform consistently better than AB_Improved as shown from game data in the APPENDIX.

Weighted Penalized

The third strategy is a slight modification of the 2nd strategy. In this we penalize/reward the AB_Open/AB_Improved depending on whether over half the board is filled.

If less than 50% of the board is filled then the AB_Improved strategy is awarded and the AB_Open strategy is penalized. If the number of positions filled is 50% or above then it is vice versa.

If $BoardFilledRatio < 50\%$

$$\text{Eval Function} = (BoardFilledRatio * OwnMoves) * 0.5 + (1 - BoardFilledRatio) * (OwnMoves - OpponentMoves) * 2)$$

If $BoardFilledRatio \geq 50\%$

$$\text{Eval Function} = (BoardFilledRatio * OwnMoves) * 2 + (1 - BoardFilledRatio) * (OwnMoves - OpponentMoves) * 0.5)$$

This strategy (Weighted Penalized) was chosen as the final heuristic because the data shows that this heuristic is able to perform at least on par with the AB_Improved heuristic. Playing multiple games shows that it is able to outperform AB_Improved consistently.

Figure 4

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*************************
     Playing Matches
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Match #    Opponent     AB_Improved    AB_Custom    AB_Custom_2    AB_Custom_3
                         Won | Lost    Won | Lost    Won | Lost    Won | Lost
    1        Random       9  |  1      10  |  0      9  |  1      10  |  0
    2        MM_Open      7  |  3       8  |  2      8  |  2       8  |  2
    3       MM_Center    10  |  0      10  |  0     10  |  0       9  |  1
    4     MM_Improved     7  |  3       6  |  4      8  |  2      10  |  0
    5        AB_Open      5  |  5       5  |  5      3  |  7       5  |  5
    6       AB_Center     7  |  3       5  |  5      5  |  5       5  |  5
    7     AB_Improved     5  |  5       4  |  6      5  |  5       4  |  6
--------------------------------------------------------------------------
          Win Rate:      71.4%        68.6%        68.6%        72.9%
```

[Function Mapping: C1 = H1, C2 = H2, C3 = H3]

The table above shows an example game where it is able to outperform the AB_Improved heuristic.

Reasons for choosing strategy

This report recommends that the *Weighted Penalized* evaluation function should be used to play the Knights isolation game.

1. It takes two good strategies, the open and improved heuristic and attempts to use them both to take a decision as to which move must be performed
2. It takes into account the number of positions filled in the board which reduces as the end game approaches.
3. The strategy is simple enough and yields good results as seen from the win percentage table. Intuitively it seems as if the Weighted Improved heuristic must perform as well. However, data shows us that rewarding/penalizing the AB_Improved and AB_Open strategies gives better results.

Win Percentage Table

Win Percentage (Number of games played)	Max Distance	Weighted Improved	Weighted Penalized
AB_Open	48.33% (120)	47.27% (110)	54.54% (110)
AB_Center	58.33% (120)	51.18% (110)	55.45% (110)
AB_Improved	46.54% (550)	47% (520)	51.1% (520)

All the strategies play well against the Random and Minimax agents. The game data in the appendix shows the results of various games.

APPENDIX

Constants and Variables

$$BoardFilledRatio(BFR) = \frac{NumberOfMoves}{TotalBoardSize}$$

$$Reducing\ Factor = \frac{8}{(\sqrt[2]{Width^2 + Height^2})}\ where\ 8 = Max\ Number\ of\ Own\ Moves$$

Table of Evaluation Functions

H1, H2, H3 are the functions chosen as the final Heuristics

$I1, I2$ *are functions that are intermediates and simply used for the purpose of easy representation*

$F1, F2, F3, F4$ *are functions that were tested but not chosen as a heursitic*

Function Id	Function Name	Function Signature
H1	Max Distance	$Eval\ Fn = \sqrt[2]{(r1-r2)^2 + (c1-c2)^2}$
I1	Intermediate Open	$Eval\ Fn = BFR * OwnMoves$
I2	Intermediate Improved	$Eval\ Fn = (1 - BFR) * (OwnMoves - OpponentMoves)$
H2	Weighted Improved	$Eval\ Fn = (BFR * OwnMoves) + (1 - BFR) * (OwnMoves - OpponentMoves))$
H3	Weighted Penalized	$Eval\ Fn = \begin{cases} I1 * 0.5 + I2 * 2, & BFR < 0.5 \\ I1 * 2 + I2 * 0.5, & BFR \geq 0.5 \end{cases}$
F1	Max Distance Ratio	$Eval\ Fn = \begin{cases} OwnMoves - OppMoves, & BFR < 0.3 \\ Max\ Distance, & BFR \geq 0.3 \end{cases}$
F2	Improved Distance	$Eval\ Fn = (Max\ Distance + (OwnMoves - OpponentMoves))$
F3	Improved Distance Weighted	$Eval\ Fn = (Max\ Distance * Reducing\ Factor + Weighted\ Improved)$
F4	Open Improved Switch	$Eval\ Fn = \begin{cases} OwnMoves - OppMoves, & BFR < 0.5 \\ OwnMoves, & BFR \geq 0.5 \end{cases}$

Game Data

Figure 5

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**************************
    Playing Matches
**************************
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Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	8	2	8	2	10	0	9	1
2	MM_Open	8	2	5	5	7	3	7	3
3	MM_Center	8	2	10	0	10	0	9	1
4	MM_Improved	8	2	6	4	9	1	5	5
5	AB_Open	3	7	5	5	2	8	5	5
6	AB_Center	6	4	5	5	7	3	4	6
7	AB_Improved	3	7	4	6	7	3	6	4
	Win Rate:	62.9%		61.4%		74.3%		64.3%	

Function Mapping: C1 = H1, C2 = F1, C3 = F2

Figure 6

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**************************
    Playing Matches
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Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	8	2	9	1	8	2	10	0
2	MM_Open	8	2	6	4	8	2	7	3
3	MM_Center	9	1	10	0	10	0	9	1
4	MM_Improved	8	2	7	3	8	2	8	2
5	AB_Open	6	4	5	5	6	4	5	5
6	AB_Center	6	4	4	6	5	5	7	3
7	AB_Improved	6	4	5	5	6	4	3	7
	Win Rate:	72.9%		65.7%		72.9%		70.0%	

Function Mapping: C1 = H1, C2 = F1, C3 = F2

All of the following game data has AB_Custom as Max Distance, AB_Custom_2 as Weighted Improved and AB_Custom_3 as Weighted Penalized.

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*************************
     Playing Matches
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Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	9	1	8	2	10	0	10	0
2	MM_Open	7	3	6	4	8	2	8	2
3	MM_Center	9	1	10	0	10	0	10	0
4	MM_Improved	9	1	6	4	7	3	9	1
5	AB_Open	4	6	5	5	6	4	6	4
6	AB_Center	6	4	7	3	7	3	7	3
7	AB_Improved	5	5	5	5	4	6	3	7
	Win Rate:	70.0%		67.1%		74.3%		75.7%	

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*************************
     Playing Matches
*************************
```

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	92	8	88	12	93	7	94	6
2	MM_Open	82	18	78	22	79	21	82	18
3	MM_Center	94	6	96	4	99	1	95	5
4	MM_Improved	80	20	81	19	74	26	81	19
5	AB_Open	46	54	48	52	46	54	54	46
6	AB_Center	59	41	61	39	50	50	54	46
7	AB_Improved	53	47	55	45	48	52	48	52
	Win Rate:	72.3%		72.4%		69.9%		72.6%	

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*************************
     Playing Matches
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```

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	AB_Improved	4	6	4	6	5	5	5	5
	Win Rate:	40.0%		40.0%		50.0%		50.0%	

****
Playing Matches
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Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	AB_Improved	3	7	6	4	5	5	5	5
	Win Rate:	30.0%		60.0%		50.0%		50.0%	

****
Playing Matches
****

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	AB_Improved	53	47	47	53	44	56	53	47
	Win Rate:	53.0%		47.0%		44.0%		53.0%	

****
Playing Matches
****

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	AB_Improved	53	47	44	56	43	57	58	42
	Win Rate:	53.0%		44.0%		43.0%		58.0%	

****
Playing Matches
****

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	AB_Improved	54	46	45	55	55	45	47	53
	Win Rate:	54.0%		45.0%		55.0%		47.0%	

```
                    *************************
                         Playing Matches
                    *************************

 Match #    Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                         Won | Lost   Won | Lost   Won | Lost   Won | Lost
    1     AB_Improved    50  |  50    41  |  59    44  |  56    50  |  50
----------------------------------------------------------------------------
           Win Rate:      50.0%        41.0%        44.0%        50.0%
```