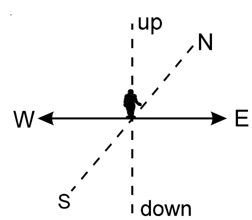
MAGNETIC EFFECT OF CURRENT AND MAGNETIC FORCE ON CHARGE OR CURRENT

1. MAGNET :

Two bodies even after being neutral (showing no electric interaction) may attract / repel strongly if they have a special property. This property is known as magnetism. The force with which they attract or repel is called magnetic force. Those bodies are called magnets. Later on we will see that it is due to circulating currents inside the atoms. Magnets are found in different shape but for many experimental purposes, a bar magnet is frequently used. When a bar magnet is suspended at its middle, as shown, and it is free to rotate in the horizontal plane it always comes to equilibrium in a fixed direction.

One end of the magnet (say A) is directed approximately towards north and the other end (say B) approximately towards south. This observation is made everywhere on the earth. Due to this reason the end A, which points towards north direction is called NORTH POLE and the other end which points towards south direction is called SOUTH POLE. They can be marked as 'N' and 'S' on the magnet. This property can be used to determine the north or south direction anywhere on the earth and indirectly east

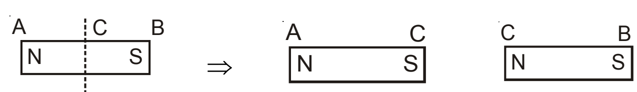


and west also if they are not known by other method (like rising of sun and setting of the sun). This method is used by navigators of ships and aeroplanes. The directions are as shown in the figure. All directions E, W, N, S are in the horizontal plane.
The magnet rotates due to the earths magnetic field about which we will discuss later in this chapter.

1.1 Pole strength magnetic dipole and magnetic dipole moment :

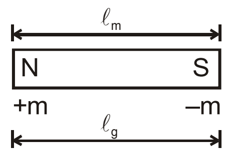
A magnet always has two poles 'N' and 'S' and like poles of two magnets repel each other and the unlike poles of two magnets attract each other they form action reaction pair.

The poles of the same magnet do not come to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. So, 'N' and 'S' always exist together.



They are known as +ve and –ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as –ve pole (or –ve magnetic charge). They are quantitatively represented by their "POLE STRENGTH" +m and –m respectively (just like we have charges +q and –q in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also).
A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges –q and +q). It is called MAGNETIC DIPOLE and it has a

MAGNETIC DIPOLE MOMENT. It is represented by $\vec{M}$. It is a vector quantity. It's direction is from –m to +m that means from 'S' to 'N')



M = m.ℓ_m here ℓ_m = magnetic length of the magnet. ℓ_m is slightly less than ℓ_g (it is geometrical length of the magnet = end to end distance). The 'N' and 'S' are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$ [Actually $\ell_m/\ell_g \simeq 0.84$].
The units of m and M will be mentioned afterwards where you can remember and understand.

1.2 Magnetic field and strength of magnetic field.

The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called MAGNETIC FIELD and that force is called 'MAGNETIC FORCE'. This field is qualitatively represented by 'STRENGTH OF MAGNETIC FIELD' or "MAGNETIC INDUCTION" or "MAGNETIC FLUX DENSITY". It is represented by $\vec{\mathbf{B}}$. It is a vector quantity.

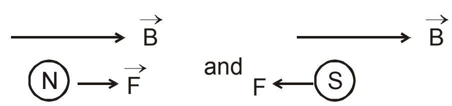
Definition **of** $\vec{\mathbf{B}}$**:** The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

Mathematically, $\vec{\mathbf{B}} = \dfrac{\vec{\mathbf{F}}}{\mathbf{m}}$

Here $\vec{\mathbf{F}}$ = magnetic force on pole of pole strength m. m may be +ve or –ve and of any value.

S.I. unit of $\vec{\mathbf{B}}$ is **Tesla** or **Weber/m²** (abbreviated as T and Wb/m²).

We can also write $\vec{\mathbf{F}} = \mathbf{m}\vec{\mathbf{B}}$. According to this direction of on +ve pole (North pole) will be in the direction of field and on –ve pole (south pole) it will be opposite to the direction of $\vec{\mathbf{B}}$.

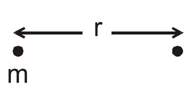


The field generated by sources does not depend on the test pole (for its any value and any sign).

(a) $\vec{B}$ due to various source

(i) Due to a single pole :

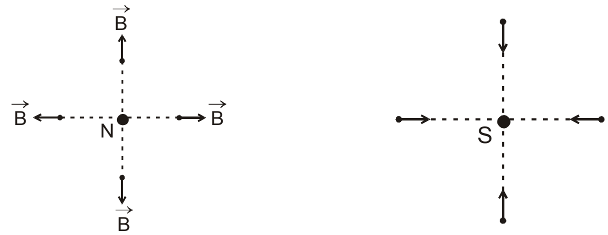
(Similar to the case of a point charge in electrostatics)



$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2}.$$

This is magnitude

Direction of B due to north pole and due to south poles are as shown

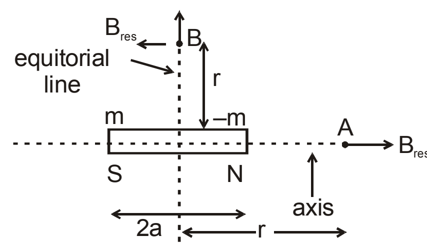


in vector form $\vec{\mathbf{B}} = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^3}\vec{r}$

here m is with sign and $\vec{r}$ = position vector of the test point with respect to the pole.

(ii) Due to a bar magnet :

(Same as the case of electric dipole in electrostatics) Independent case never found. Always 'N' and 'S' exist together as magnet.



at A (on the axis) = $2\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r

at B (on the equatorial) = $-\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r

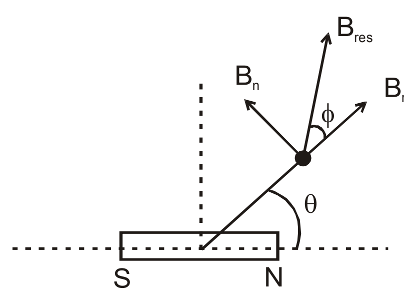
At General point :

$$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$$

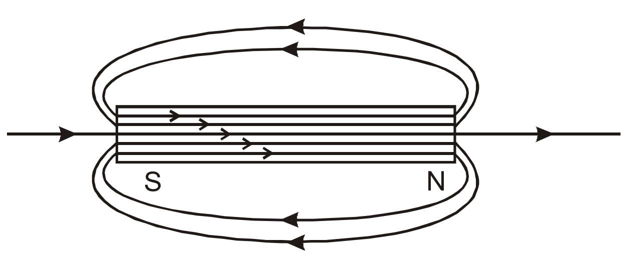
$$B_n = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$$

$$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\phi = \frac{B_n}{B_r} = \frac{\tan\theta}{2}$$



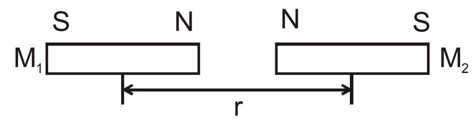
1.3 Magnetic lines of force of a bar magnet :



Solved Examples

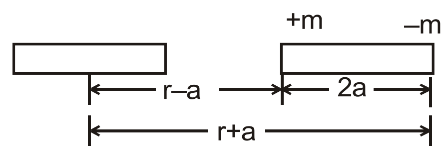
Example 1.

Find the magnetic force on a short magnet of magnetic dipole moment M_2 due to another short magnet of magnetic dipole moment M_1.



Solution :

To find the magnetic force we will use the formula of 'B' due to a magnet. We will also assume m and –m as pole strengths of 'N' and 'S' of M_2. Also length of M_2 as 2a. B_1 and B_2 are the strengths of the magnetic field due to M_1 at +m and –m respectively. They experience magnetic forces F_1 and F_2 as shown.



$$F_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r-a)^3}m$$

and $$F_2 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r+a)^3}m$$

$$\therefore \quad F_{res} = F_1 - F_2 = 2\left(\frac{\mu_0}{4\pi}\right) M_1 m\left[\left(\frac{1}{(r-a)^3}\right)-\left(\frac{1}{(r+a)^3}\right)\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[\left(1-\frac{a}{r}\right)^{-3}-\left(1+\frac{a}{r}\right)^{-3}\right]$$

By using acceleration, Binomial expansion, and neglecting terms of high power we get

$$F_{res} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[1+\frac{3a}{r}-1+\frac{3a}{r}\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\frac{6a}{r}$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 3M_2}{r^4}$$

$$= 6\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 M_2}{r^4}$$

Direction of F_{res} is towards right.

Alternative Method :

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2M_1}{r^3} \quad \Rightarrow \quad \frac{dB}{dr} = -\frac{\mu_0}{4\pi}\times\frac{6M_1}{r^4}$$

$$F = -M_2 \times \frac{dB}{dr} \quad \Rightarrow \quad F = \left(\frac{\mu_0}{4\pi}\right)\frac{6M_1M_2}{r^4}$$

Example 2.

A magnet is 10 cm long and its pole strength is 120 CGS units (1 CGS unit of pole strength = 0.1 A-m). Find the magnitude of the magnetic field B at a point on its axis at a distance 20 cm from it.

Solution :

The pole strength is m = 120 CGS units = 12A-m.
Magnetic length is 2ℓ = 10 cm or ℓ = 0.05 m.
Distance from the magnet is d = 20 cm = 0.2 m. The field B at a point in end-on position is

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-\ell^2)^2}$$

$$= \frac{\mu_0}{4\pi}\frac{4m\ell d}{(d^2-\ell^2)^2}$$

$$= \left(10^{-7}\,\frac{T-m}{A}\right)\frac{4\times(12A-m)\times(0.05m)\times(0.2m)}{[(0.2m)^2-(0.05m)^2]^2}$$

$$= 3.4 \times 10^{-5}\ T.$$

Example 3.

Find the magnetic field due to a dipole of magnetic moment 1.2 A-m^2 at a point 1 m away from it in a direction making an angle of 60º with the dipole-axis.

Solution :

The magnitude of the field is

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

$$= \left(10^{-7}\,\frac{T-m}{A}\right)\frac{1.2\,A-m^2}{1\,m^3}\quad\sqrt{1+3\cos^2 60^\circ}$$

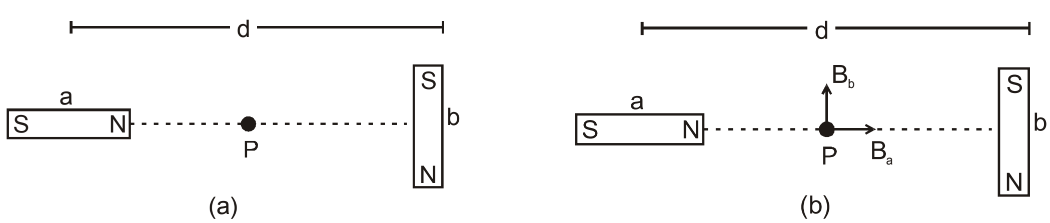
$$= 1.6 \times 10^{-7}\ T.$$

The direction of the field makes an angle α with the radial line where

$$\tan\alpha = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$$

Example 4.

Figure shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.



Solution :

The point P is in end-on position for the dipole (a) and in broadside-on position for the dipole (b).

The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$ along the axis of a, and that due to b is

$B_b = \frac{\mu_0}{4\pi} \frac{M}{(d/2)^3}$ parallel to the axis of b as shown in figure. The resultant field at P is, therefore.

$$B = \sqrt{B_a^2 + B_b^2}$$

$$= \frac{\mu_0 M}{4\pi(d/2)^3} \sqrt{1^2 + 2^2}$$

$$= \frac{2\sqrt{5}\mu_0 M}{\pi d^3}$$

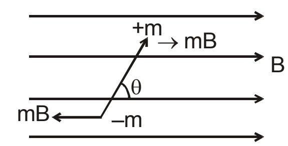
The direction of this field makes an angle α with B_a such that $\tan\alpha = B_b/B_a = 1/2$.

1.4 Magnet in an external uniform magnetic field :

(same as case of electric dipole)

$F_{res} = 0$ (for any angle)

$\tau = MB \sin\theta$



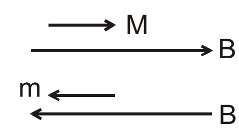
*here θ is angle between $\vec{B}$ and $\vec{M}$

Note :

- $\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$.
- $\vec{\tau}$ is same about every point of the dipole it's potential energy is

$U = -MB\cos\theta$

$= -\vec{M}\cdot\vec{B}$



$\theta = 0°$ is stable equilibrium

$\theta = \pi$ is unstable equilibrium

for small 'θ' the dipole performs SHM about $\theta = 0°$ position

$\tau = -MB\sin\theta$;

$I\alpha = -MB\sin\theta$

for small θ, $\sin\theta \simeq \theta$

$$\alpha = -\left(\frac{MB}{I}\right)\theta$$

Angular frequency of SHM

$$\omega = \sqrt{\frac{MB}{I}}$$

$$= \frac{2\pi}{T}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{MB}}$$

here I $= I_{cm}$ if the dipole is free to rotate

$= I_{hinge}$ if the dipole is hinged

Solved Examples

Example 5.

A bar magnet having a magnetic moment of 1.0×10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60º from the field.

Solution :

The work done by the external agent = change in potential energy

$= (-MB\cos\theta_2) - (-MB\cos\theta_1)$

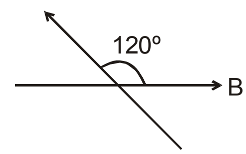
$= -MB(\cos 60^\circ - \cos 0^\circ)$

$= \frac{1}{2}MB$

$= \frac{1}{2} \times (1.0 \times 10^{4}\ \text{J/T})\ (4 \times 10^{-5}\ \text{T}) = 0.2\ \text{J}$

Example 6.

A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in the figure. Find :



(i) Its kinetic energy at $\theta = 90^\circ$

(ii) its maximum kinetic energy during the motion.

(iii) will it perform SHM? oscillation? Periodic motion? What is its amplitude?

Solution :

(i) Apply energy conservation at $\theta = 120^\circ$ and $\theta = 90^\circ$

$- MB\cos 120^\circ + 0$

$= - MB\cos 90^\circ + (\text{K.E.})$

$\text{KE} = \frac{MB}{2}$ **Ans.**

(ii) K.E. will be maximum where P.E. is minimum. P.E. is minimum at $\theta = 0^\circ$. Now apply energy conservation between $\theta = 120^\circ$ and $\theta = 0^\circ$.

$- mB\cos 120^\circ + 0$

$= -mB\cos 0^\circ + (KE)_{max}$

$(KE)_{max} = \frac{3}{2}MB$ **Ans.**

The K.E. is max at $\theta = 0^\circ$ can also be proved by torque method. From $\theta = 120^\circ$ to $\theta = 0^\circ$ the torque always acts on the dipole in the same direction (here it is clockwise) so its K.E. keeps on increases till $\theta = 0^\circ$. Beyond that τ reverses its direction and then K.E. starts decreasing

$\therefore$ $\theta = 0^\circ$ is the orientation of M to here the maximum K.E.

(iii) Since 'θ' is not small.

$\therefore$ the motion is not S.H.M. but it is oscillatory and periodic amplitude is 120º.

Example 7. A bar magnet of mass 100 g, length 7.0 cm, width 1.0 cm and height 0.50 cm takes $\pi/2$ seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of 25μT.

(a) Find the magnetic moment of the magnet.

(b) If the magnet is put in the magnetometer with its 0.50 cm edge horizontal, what would be the time period?

Solution : (a) The moment of inertia of the magnet about the axis of rotation is

$$I = \frac{m'}{12}(L^2 + b^2)$$

$$= \frac{100 \times 10^{-3}}{12}\ [(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2]\ \text{kg-m}^2.$$

$$= \frac{25}{6} \times 10^{-5}\ \text{kg -m}^2.$$

We have, $T = 2\pi\sqrt{\dfrac{I}{MB}}$

or, $$M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2 \times 25 \times 10^{-5} \text{kg/m}^2}{6 \times (25 \times 10^{-6} T) \times \dfrac{\pi^2}{4} s^2}$$

$$= 27\ \text{A-m}^2.$$

(b) In this case the moment of inertia becomes

$$I' = \frac{m'}{12}(L^2 + b^2) \text{ where } b' = 0.5 \text{ cm}.$$

The time period would be

$$T' = \sqrt{\frac{I'}{MB}} \qquad \text{.... (ii)}$$

Dividing by equation (i),

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}} = \frac{\sqrt{\dfrac{m'}{12}(L^2 + b'^2)}}{\sqrt{\dfrac{m'}{12}(L^2 + b^2)}} = \frac{\sqrt{(7\,\text{cm})^2 + (0.5\,\text{cm})^2}}{\sqrt{(7\,\text{cm})^2 + (1.0\,\text{cm})^2}}$$

$$= 0.992$$

or, $$T' = \frac{0.992 \times \pi}{2} s = 0.496\pi\ s.$$

1.5 Magnet in an External Non-uniform Magnetic Field :

No special formulae are applied is such problems. Instead see the force on individual poles and calculate the resistant force torque on the dipole.

Solved Examples

Example 8.

Find the torque on M_1 due to M_2 in Que. 1

Solution :

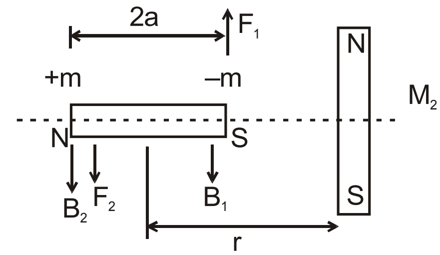
Due to M_2, magnetic fields at 'S' and 'N' of M_1 are B_1 and B_2 respectively. The forces on –m and +m are F_1 and F_2 as shown in the figure. The torque (about the centre of the dipole m_1) will be

$= F_1\, a + F_2 a = (F_1 + F_2)a$

$$= \left[\left(\frac{\mu_0}{4\pi}\right)\frac{M_2}{(r-a)}m + \frac{\mu_0}{4\pi}\frac{M_2}{(r+a)}m\right]a$$

$$\cong \frac{\mu_0}{4\pi}M_2 m\left(\frac{1}{r^3}+\frac{1}{r^3}\right)a \quad \because \ a << r$$

$$= \frac{\mu_0 M_2 m}{4\pi}\ \frac{2}{r^3}a = \frac{\mu_0 M_1 M_2}{4\pi r^3} \qquad \textbf{Ans.}$$

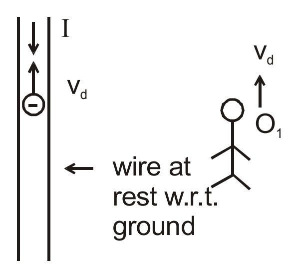


2. MAGNETIC EFFECTS OF CURRENT (AND MOVING CHARGE)

It was observed by **OERSTED** that a current carrying wire produces magnetic field nearly it. It can be tested by placing a magnet in the near by space, it will show some movement (deflection or rotation of displacement). This observation shows that current or moving charge produces magnetic field.

2.1 Frame Dependence of $\vec{B}$.

(a) The motion of anything is a relative term. A charge may appear at rest by an observer (say O_1) and moving at same velocity $\vec{v}_1$ with respect to observer O_2 and at velocity $\vec{v}_2$ with respect to observers O_3 then $\vec{B}$ due to that charge w.r.t. O_1 will be zero and w.r. to O_2 and O_3 it will be $\vec{B}_1$ and $\vec{B}_2$ (that means different).



(b) In a current carrying wire electron move in the opposite direction to that of the current and +ve ions (of the metal) are static w.r.t. the wire. Now if some observer (O_1) moves with velocity V_d in the direction of motion of the electrons then electrons will have zero velocity and +ve ions will have velocity V_d in the downward direction w.r.t. O_1. The density (n) of +ve ions is same as the density of free electrons and their charges are of the same magnitudes

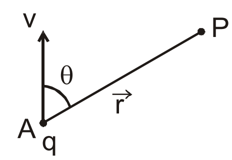
So, w.r.t. O_1 electrons will produce zero magnetic field but +ve ions will produce +ve same $\vec{B}$ due to the current carrying wire does not depend on the reference frame (this is true for any velocity of the observer).

(c) **$\vec{B}$ due to magnet :**

$\vec{B}$ produced by the magnet does not contain the term of velocity

So, we can say that the $\vec{B}$ due magnet does not depend on frame.

2.2 $\vec{B}$ due to a point charge :



A charge particle 'q' has velocity v as shown in the figure. It is at position 'A' at some time. $\vec{r}$ is the position vector of point 'P' w.r. to position of the charge. Then $\vec{B}$ at P due to q is

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{qv\sin\theta}{r^2}$$; here θ = angle between $\vec{v}$ and $\vec{r}$

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q\vec{v}\times\vec{r}}{r^3};$$ with sign

$\vec{B}\perp\vec{v}$ and also $\vec{B}\perp\vec{r}$.

Direction of $\vec{B}$ will be found by using the rules of vector product.

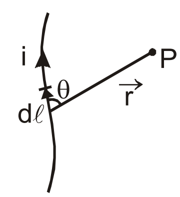
2.3 Biot-savart's law ($\vec{B}$ due to a wire)

It is an experimental law. A current 'i' flows in a wire (may be straight or curved). Due to 'dℓ' length of the wire the magnetic field at 'P' is

$$dB \propto id\ell$$

$$\propto \frac{1}{r^2}$$

$$\propto \sin\theta$$



$$\Rightarrow \quad dB \propto \frac{id\ell\sin\theta}{r^2}$$

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{id\ell\sin\theta}{r^2}$$

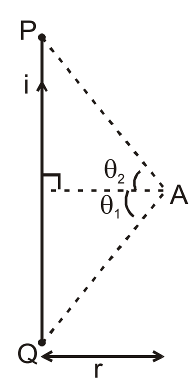
$$\Rightarrow \quad \overrightarrow{dB} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\overrightarrow{d\ell}\times\vec{r}}{r^3}$$

here $\vec{r}$ = position vector of the test point w.r.t. $\overrightarrow{d\ell}$

θ = angle between $\overrightarrow{d\ell}$ and $\vec{r}$. The resultant $\vec{B} = \int\overrightarrow{dB}$

Using this fundamental formula we can derive the expression of $\vec{B}$ due to a long wire.

2.3.1 $\vec{B}$ due to a straight wire :



Due to a straight wire 'PQ' carrying a current 'i' the $\vec{B}$ at A is given by the formula

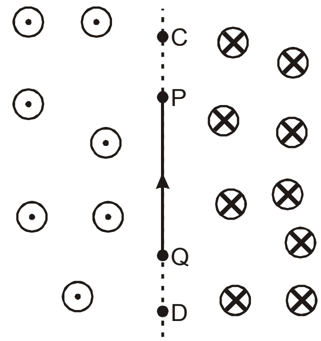
$$B = \frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$$

(Derivation can be seen in a standard text book like your school book or concept of physics of HCV part-II)

Direction :

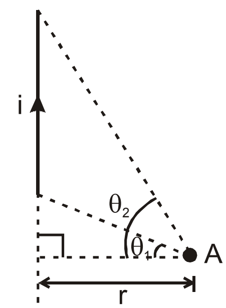
Due to every element of 'PQ' $\vec{B}$ at A is directed in wards. So its resultant is also directed inwards. It is represented by (x)

The direction of $\vec{B}$ at various points is shown in the figure shown.



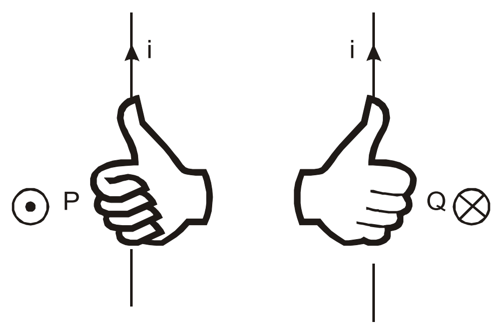
At points 'C' and 'D' $\vec{B}$ = 0 (think how).
For the case shown in figure

B at A = $\frac{\mu_0 i}{4\pi r}$ ($\sin\theta_2 - \sin\theta_1$) $\otimes$

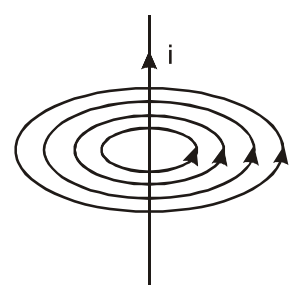


Shortcut for Direction :

The direction of the magnetic field at a point P due to a straight wire can be found by a slight variation in the right-hand thumb rule. If we stretch the thumb of the right hand along the current and curl our fingers to pass through the point P, the direction of the fingers at P gives the direction of the magnetic field there.

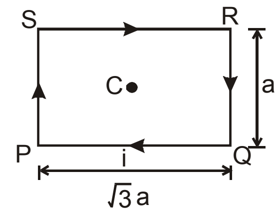


We can draw magnetic field lines on the pattern of electric field lines. A tangent to a magnetic field line given the direction of the magnetic field existing at that point. For a straight wire, the field lines are concentric circles with their centers on the wire and in the plane perpendicular to the wire. There will be infinite number of such lines in the planes parallel to the above mentioned plane.



Example 9.

Find resultant magnetic field at 'C' in the figure shown.



Solution :

It is clear that 'B' at 'C' due all the wires is directed $\otimes$. Also B at 'C' due PQ and SR is same. Also due to QR and PS is same

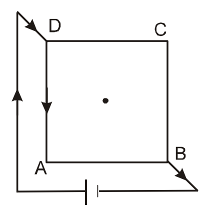
$\therefore \quad B_{res} = 2(B_{PQ} + B_{SP})$

$$B_{PQ} = \frac{\mu_0 i}{4\pi \frac{a}{2}} \quad (\sin 60^\circ + \sin 60^\circ),$$

$$B_{sp} = \frac{\mu_0 i}{4\pi \frac{\sqrt{3}a}{2}} (\sin 30^\circ + \sin 30^\circ) \qquad \Rightarrow \quad B_{res} = 2\left(\frac{\sqrt{3}\,\mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$$

Example 10.

Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points B and D as shown in the figure

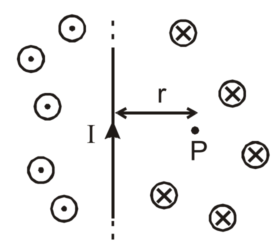


Solution :

The current will be equally divided at D. The fields at the centre due to the currents in the wires DA and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AB and CB will be zero. Hence, the net field at the centre will be zero.

Special case :

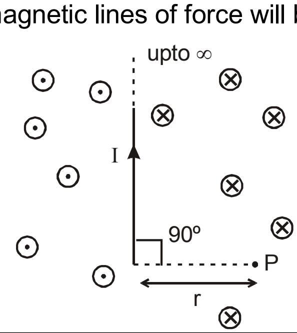
(i) If the wire is infinitely long then the magnetic field at 'P' (as shown in the figure) is given by (using $\theta_1 = \theta_2 = 90^\circ$ and the formula of 'B' due to straight wire)



$$B = \frac{\mu_0 I}{2\pi r} \qquad \Rightarrow \qquad B \propto \frac{I}{r}$$

The direction of $\vec{B}$ at various is as shown in the figure. The magnetic lines of force will be concentric circles around the wire (as shown earlier)

(ii) If the wire is infinitely long but 'P' is as shown in the figure. The direction of $\vec{B}$ at various points is as shown in the figure. At 'P'

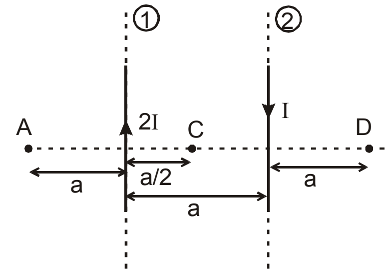


$$B = \frac{\mu_0 I}{4\pi r}$$

Example 11.

In the figure shown there are two parallel long wires (placed in the plane of paper) are carrying currents 2I and I consider points A, C, D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned. Find (i) $\vec{B}$ at A, C, D

(ii) position of point on line A C D where $\vec{B}$ is O.



Solution : (i) Let us call $\vec{B}$ due to (1) and (2) as $\vec{B}_1$ and $\vec{B}_2$ respectively. Then

at A : $\vec{B}_1$ is $\odot$ and $\vec{B}_2$ is $\otimes$

$$B_1 = \frac{\mu_0 2I}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I}{2\pi 2a}$$

$\therefore$ $B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a} \odot$ **Ans.**

at C : $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ also $\otimes$

$\therefore$ $B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi\frac{a}{2}} + \frac{\mu_0 I}{2\pi\frac{a}{2}} = \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a} \otimes$ **Ans.**

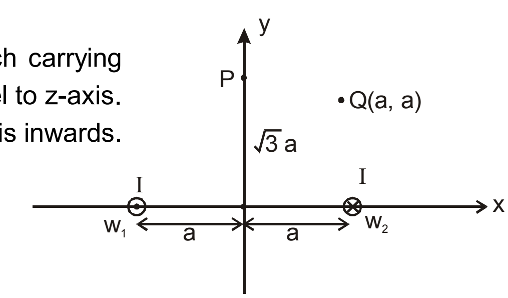
at D : $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ is $\odot$ and both are equal in magnitude.

$\therefore$ $B_{res} = 0$ **Ans.**

(ii) It is clear from the above solution that B = 0 at point 'D'.

Example 12.

In the figure shown two long wires W_1 and W_2 each carrying current I are placed parallel to each other and parallel to z-axis. The direction of current in W_1 is outward and in W_2 it is inwards. Find the $\vec{B}$ at 'P' and 'Q'.



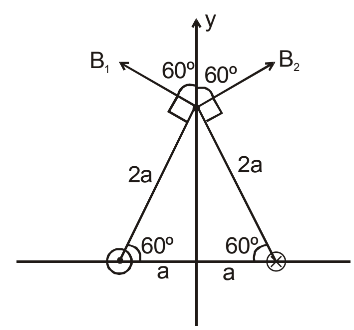
Solution :

Let $\vec{B}$ due to W_1 be $\vec{B}_1$ and due to W_2 be $\vec{B}_2$.

By symmetry $|\vec{B}_1| = |\vec{B}_2| = B$

$$B_p = 2\,B\cos 60^\circ = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$$

$\therefore$ $\overrightarrow{B_p} = \frac{\mu_0 I}{4\pi a}\hat{j}$ **Ans.**

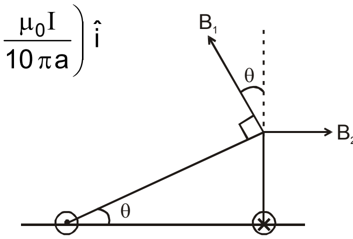


For θ $B_1 = \frac{\mu_0 I}{2\pi\sqrt{5}a}$, $\Rightarrow$ $B_2 = \frac{\mu_0 I}{2\pi a}$

$\tan\theta = \frac{a}{2a} = \frac{1}{2}$ $\Rightarrow$ $\vec{B} = (B_1\cos\theta\,\hat{j}) + (B_2 - B_1\sin\theta)\hat{i}$

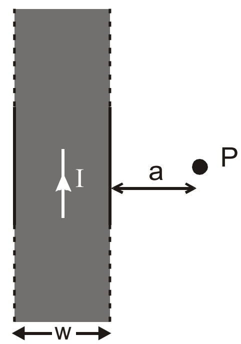
$\sin\theta = \dfrac{1}{\sqrt{5}}$ $\Rightarrow$ $\vec{B} = \dfrac{\mu_0 I}{5\pi a}\,\hat{j} + \left(\dfrac{\mu_0 I}{2\pi a} - \dfrac{\mu_0 I}{10\pi a}\right)\hat{i}$

$\cos\theta = \dfrac{2}{\sqrt{5}}$ $\Rightarrow$ $\vec{B} = \dfrac{2\mu_0 I}{5\pi a}\hat{i} + \dfrac{\mu_0 I}{5\pi a}\hat{j}$



Example 13.

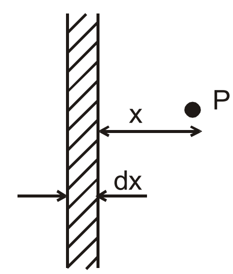
In the figure shown a large metal sheet of width 'w' carries a current I (uniformly distributed in its width 'w'. Find the magnetic field at point 'P' which lies in the plane of the sheet.



Solution :

To find 'B' at 'P' the sheet can be considered as collection of large number of infinitely long wires. Take a long wire distance 'x' from 'P' and of width 'dx'. Due to this the magnetic field at 'P' is 'dB'

$$dB = \frac{\mu_0\left(\frac{I}{w}dx\right)}{2\pi x} \otimes$$



due to each such wire $\vec{B}$ will be directed inwards

$$\therefore \quad B_{res} = \int dB = \frac{\mu_0 I}{2\pi w}\int_{x=a}^{a+w}\frac{dx}{x}\quad \int_{x=a}^{a+w}\frac{dx}{x}$$

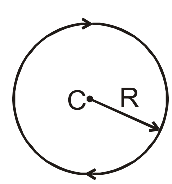
$$= \frac{\mu_0 I}{2\pi w}\,.\,\ln\frac{a+w}{a} \qquad \textbf{Ans.}$$

2.3.2 $\vec{B}$ due to circular loop

(a) **At centre :** Due to each $\vec{d\ell}$ element of the loop $\vec{B}$ at 'c' is inwards (in this case).

$\therefore$ $\overrightarrow{B_{res}}$ at 'c' is $\otimes$.

$B = \dfrac{\mu_0 NI}{2R}$,

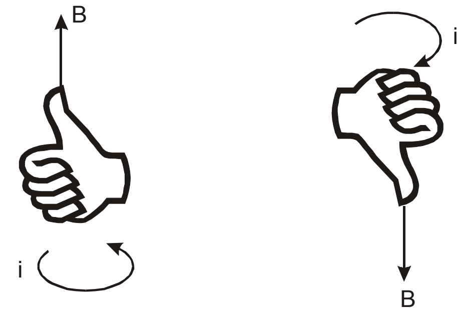


N = No. of turns in the loop.

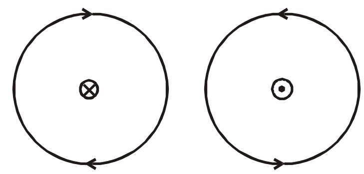
$= \dfrac{\ell}{2\pi R}$; ℓ = length of the loop.

N can be fraction $\left(\dfrac{1}{4}, \dfrac{1}{3}, \dfrac{11}{3} \text{ etc.}\right)$ or integer.

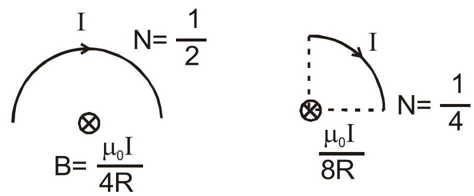
Direction of $\vec{B}$: The direction of the magnetic field at the centre of a circular wire can be obtained using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field (figure).



Another way to find the direction is to look into the loop along its axis. If the current is in anticlockwise direction, the magnetic field is towards the viewer. If the current is in clockwise direction, the field is away from the viewer.

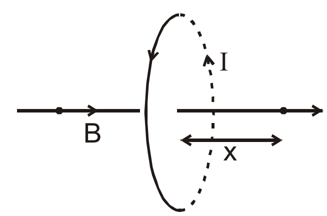


Semicircular and Quarter of a circle :



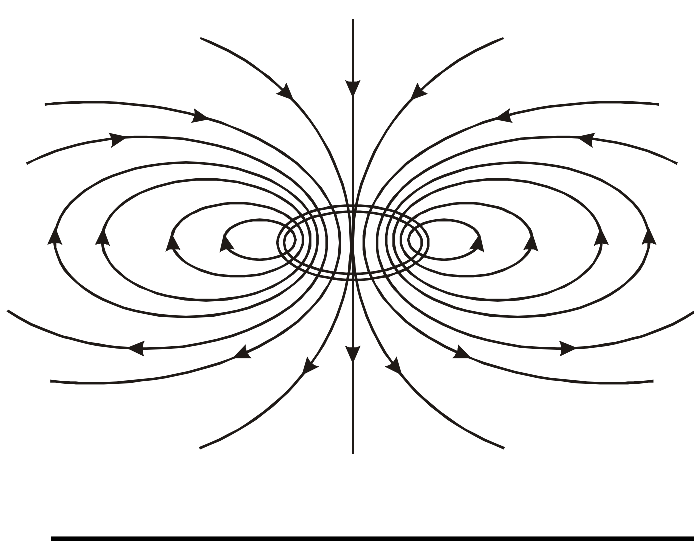
(b) **On the axis of the loop :** $B = \dfrac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}}$

N = No. of turns (integer)



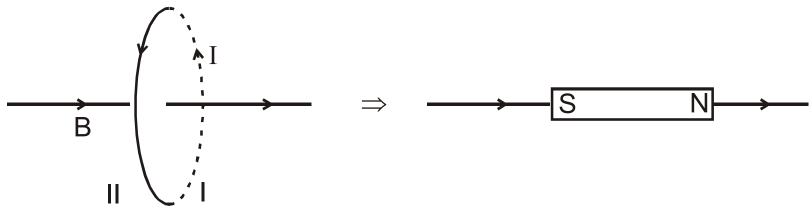
Direction can be obtained by right hand thumb rule. curl your fingers in the direction of the current then the direction of the thumb points in the direction of $\vec{B}$ at the points on the axis.

The magnetic field at a point not on the axis is mathematically difficult to calculate. We show qualitatively in figure the magnetic field lines due to a circular current which will give some idea of the field.

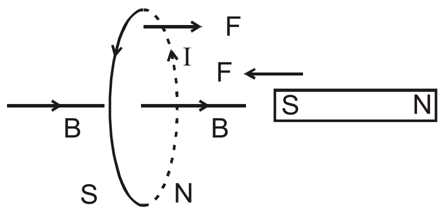


2.3.3 A loop as a magnet :

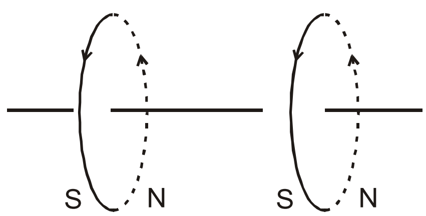
The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.



The side 'I' (the side from which the $\vec{B}$ emerges out) of the loop acts as 'NORTH POLE' and side II (the side in which the $\vec{B}$ enters) acts as the 'SOUTH POLE'. It can be verified by studying force on one loop due to a magnet or a loop.



The loop and the magnet attract each other



The two loops attract each other.

Mathematically

$$B_{axis} = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}} \cong \frac{\mu_0 NIR^2}{2x^3} \quad \text{for } x >> R$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{IN\pi R^2}{x^3}\right)$$

it is similar to B_{axis} due to magnet $= 2\left(\frac{\mu_0}{4\pi}\right)\frac{m}{x^3}$

Magnetic dipole moment of the loop

$M = IN\pi R^2$

$M = INA$ for any other shaped loop.

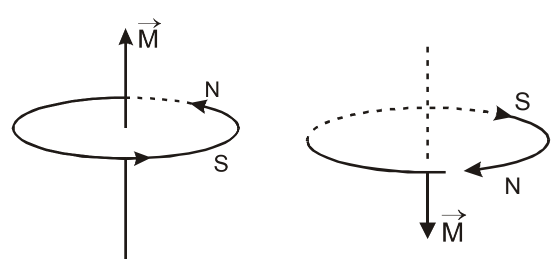
Unit of M is Amp. m^2

Unit of m (pole strength) = Amp. m {$\because$ in magnet $M = m\ell$}

$\vec{M} = IN\vec{A}$,

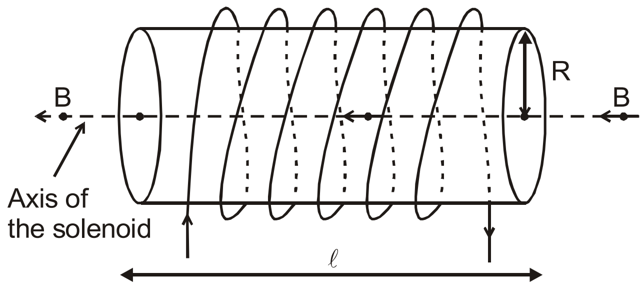
$\vec{A}$ = unit normal vector for the loop.

To be determined by right hand rule which is also used to determine direction of $\vec{B}$ on the axis. It is also from 'S' side to 'N' side of the loop.



2.3.4 Solenoid :

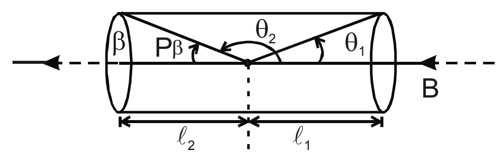
(i) Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (it may be a hollow cylinder or it may be a solid cylinder)



(ii) The winding of the wire is uniform direction of the magnetic field is same at all points of the axis.

(iii) $\vec{B}$ on axis (turns should be very close to each others).

$$B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2)$$



where n : number of turns per unit length.

$$\cos\theta_1 = \frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} \quad ; \quad \cos\beta = \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}} \qquad = -\cos\theta_2$$

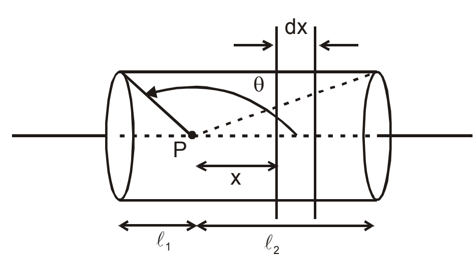
$$B = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}}\right] \quad = \frac{\mu_0 ni}{2}\ (\cos\theta_1 + \cos\beta)$$

Note :

- Use right hand rule for direction (same as the direction due to loop).

Derivation :

Take an element of width dx at a distance x from point P. [point P is the point on axis at which we are going to calculate magnetic field. Total number of turns in the element dn = ndx where n : number of turns per unit length.



$$dB = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}\ (ndx)$$

$$B = \int dB = \int_{-\ell_1}^{\ell_2} \frac{\mu_0 i R^2\, ndx}{2(R^2 + x^2)^{3/2}} \quad = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}}\right]$$

$$= \frac{\mu_0 ni}{2}\left[\cos\theta_1 - \cos\theta_2\right]$$

(iv) For 'Ideal Solenoid' :

***Inside** (at the mid point)

$\ell >> R$ or length is infinite

$\theta_1 \to 0$

$\theta_2 \to \pi$

$$B = \frac{\mu_0 ni}{2}[1 - (-1)]$$

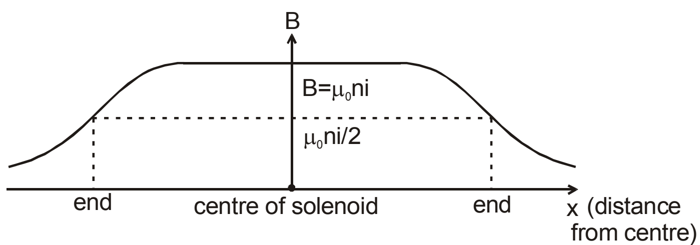
$$\mathbf{B = \mu_0 ni}$$

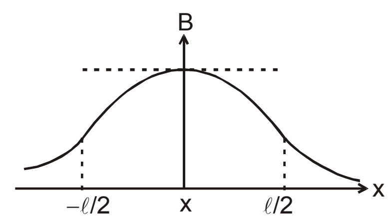
If material of the solid cylinder has relative permeability 'μ_r' then $B = \mu_0\mu_r ni$

At the ends $\mathbf{B} = \frac{\mu_0 ni}{2}$

(v) Comparison between ideal and real solenoid :

(a) Ideal Solenoid **Real Solenoid**





Solved Examples

Example 14.

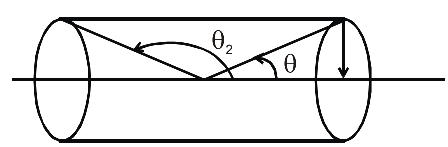
A solenoid of length 0.4 m and diameter 0.6 m consists of a single layer of 1000 turns of fine wire carrying a current of 5.0×10^{-3} ampere. Find the magnetic field on the axis at the middle and at the ends of the solenoid. (Given $\mu_0 = 4\pi \times 10^{-7} \frac{V-s}{A-m}$).

Solution :

$$B = \frac{1}{2}\mu_0 ni[\cos\theta_1 - \cos\theta_2]$$

$\Rightarrow$ $n = \frac{1000}{0.4} = 2500$ per meter

$i = 5 \times 10^{-3}$ A.



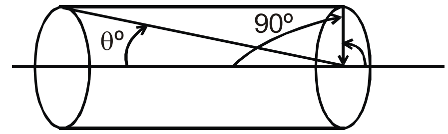
(i) $\cos\theta_1 = \frac{0.2}{\sqrt{(0.3)^2 + (0.2)^2}} = \frac{0.2}{\sqrt{0.13}}$

$\cos\theta_2 = \frac{-0.2}{\sqrt{0.13}}$

$\Rightarrow$ $B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3}\ \frac{2 \times 0.2}{\sqrt{0.13}}$

$= \frac{\pi \times 10^{-5}}{\sqrt{13}} T$

(ii) At the end



$$\cos\theta_1 = \frac{0.4}{\sqrt{(0.3)^2 + (0.4)^2}} = 0.8$$

$\cos\theta_2 = \cos 90^\circ = 0$

$B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \times 0.8$

$\Rightarrow$ $B = 2\pi \times 10^{-6}$ Wb/m²

2.4 AMPERE's circuital law :

The line integral $\oint \vec{B} \cdot \vec{d\ell}$ on a closed curve of any shape is equal to μ_0 (permeability of free space) times the net current I through the area bounded by the curve.

$$\oint \vec{B} \cdot \vec{d\ell} = \mu_0 I$$

Note :

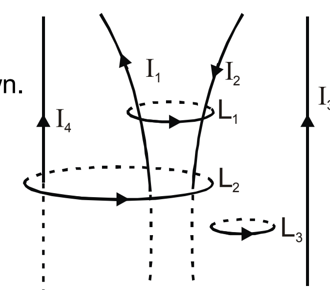
- Line integral is independent of the shape of path and position of wire with in it.
- The statement $\oint \vec{B} \cdot \vec{d\ell} = 0$ does not necessarily mean that $\vec{B} = 0$ everywhere along the path but only that no net current is passing through the path.
- **Sign of current :** The current due to which $\vec{B}$ is produced in the same sense as $\vec{d\ell}$ (i.e. $\vec{B} \cdot \vec{d\ell}$ positive will be taken positive and the current which produces $\vec{B}$ in the sense opposite to $\vec{d\ell}$ will be negative.

Solved Examples

Example 15.

Find the values of $\oint \vec{B} \cdot \vec{d\ell}$ for the loops L_1, L_2, L_3 in the figure shown.

The sense of $\vec{d\ell}$ is mentioned in the figure.



Solution :

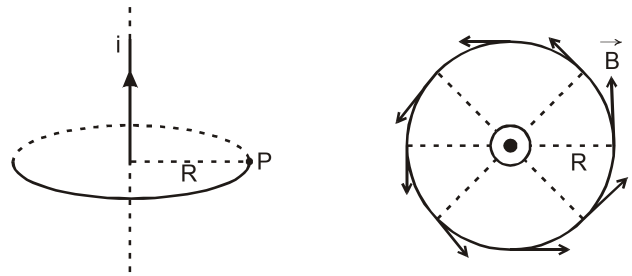
for L_1 $\oint \vec{B} \cdot \vec{d\ell} = \mu_0(I_1 - I_2)$

here I_1 is taken positive because magnetic lines of force produced by I_1 is anti clockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $\vec{d\ell}$ is anticlockwise as seen from top.

for L_2 : $\oint \vec{B} \cdot \vec{d\ell} = \mu_0\ (I_1 - I_2 + I_4)$

for L_3 : $\oint \vec{B} \cdot \vec{d\ell} = 0$

Uses : **2.4.1** To find out magnetic field due to infinite current carrying wire



By B.S.L. $\vec{B}$ will have circular lines. $\overrightarrow{d\ell}$ is also taken tangent to the circle.

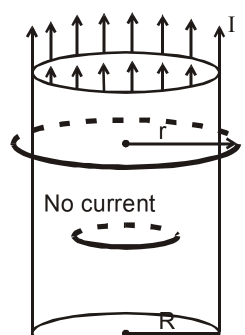
$\oint \vec{B}\cdot\overrightarrow{d\ell} = \oint B.d\ell \quad \because \theta = 0^{o} \;$ so $B\oint d\ell = B\,2\pi R \quad (\because B = \text{const.})$

Now by amperes law :

$$B\,2\pi R = \mu_0 I$$

$$\therefore \qquad B = \frac{\mu_0 i}{2\pi R}$$

2.4.2. Hollow current carrying infinitely long cylinder : (I is uniformly distributed on the whole circumference)



(i) for $r \geq R$

By symmetry the amperian loop is a circle.

$\oint \vec{B}\cdot\overrightarrow{d\ell} = \oint B d\ell \qquad \because \theta = 0$

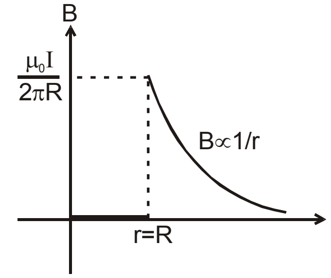
$= B\int_0^{2\pi r} d\ell \qquad \because B = \text{const.} \qquad \Rightarrow \qquad \mathbf{B = \frac{\mu_0 I}{2\pi r}}$

(ii) $r < R$

$= \oint \vec{B}\cdot\overrightarrow{d\ell} \qquad = \oint B d\ell$

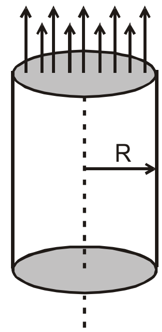
$= B(2\pi\, r) \qquad = 0 \qquad \Rightarrow \qquad \mathbf{B_{in} = 0}$

Graph



2.4.3 Solid infinite current carrying cylinder :

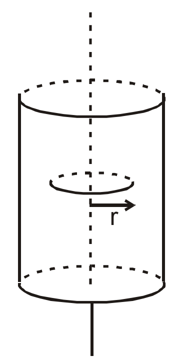
Assume current is uniformly distributed on the whole cross section area



current density $J = \dfrac{I}{\pi R^2}$

Case (I) : $r \le R$

take an amperian loop inside the cylinder. By symmetry it should be a circle whose centre is on the axis of cylinder and its axis also coincides with the cylinder axis on the loop.



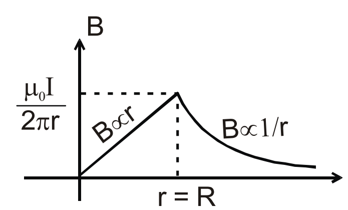
$$\oint \vec{B} \cdot \vec{d\ell} = \oint B \cdot d\ell = B\oint d\ell = B \,.\, 2\pi r = \mu_0 \frac{I}{\pi R^2} \pi r^2$$

$$B = \frac{\mu_0 I r}{2\pi R^2} = \frac{\mu_0 J r}{2} \qquad \Rightarrow \qquad \vec{B} = \frac{\mu_0 \vec{J} \times \vec{r}}{2}$$

Case (II) : $r \ge R$ $\quad \oint \vec{B}.\vec{d\ell} = \oint B d\ell \quad = B\oint d\ell \quad = B \,.\, (2\pi r) \quad = \mu_0 \,.\, I$

$$\Rightarrow \qquad B = \frac{\mu_0 I}{2\pi r} \quad \text{also} \quad \vec{B}\ \frac{\mu_0 I}{2\pi r}\ (\hat{J} \times \hat{r}) \quad = \frac{\mu_0 J \pi R^2}{2\pi r}$$

$$\vec{B} = \frac{\mu_0 R^2}{2r^2}(\vec{J} \times \vec{r})$$

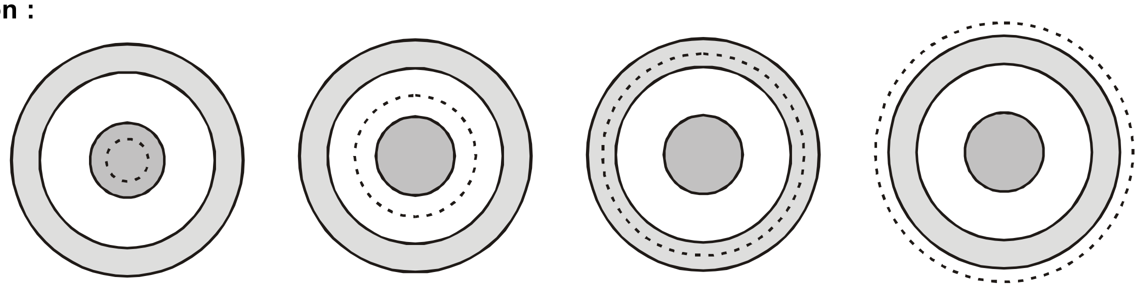


Solved Examples

Example 16.

Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in same direction. Find the magnetic field at a distance x from the axis where (a) x < a, (b) a < x < b (c) b < x < c and (d) x > c. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.

Solution :



(a) (b) (c) (d)

A cross-section of the cable is shown in figure. Draw a circle of radius x with the centre at the axis of the cable. The parts a, b, c and d of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B}.d\vec{\ell} = B2\pi x$$

in each of the four parts of the figure.

(a) The current enclosed within the circle in part b is i_0 so that

$$\frac{i_0}{\pi a^2} \cdot \pi x^2 = \frac{i_0}{a^2}x^2.$$

Ampere's law

$$\oint \vec{B}.d\vec{\ell} = \mu_0 i \text{ gives}$$

$$B.2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \quad \text{or, } B = \frac{\mu_0 i_0 x}{2\pi a^2}.$$

The direction will be along the tangent to the circle.

(b) The current enclosed within the circle in part b is i_0 so that

$$B\ 2\pi x = \mu_0\ i_0 \quad \text{or,} \quad B = \frac{\mu_0 i_0}{2\pi x}.$$

(c) The area of cross-section of the outer shell is $\pi c^2 - \pi b^2$. The area of cross-section of the outer shell within the circle in part c of the figure is $\pi x^2 - \pi b^2$.

Thus, the current through this part is $\frac{i_0(x^2-b^2)}{(c^2-b^2)}$. This is in the same direction to the current i_0 in the inner wire. Thus, the net current enclosed by the circle is

$$i_{net} = i_0 + \frac{i_0(x^2-b^2)}{c^2-b^2} = \frac{i_0(c^2+x^2-2b^2)}{c^2-b^2}.$$

From Ampere's law,

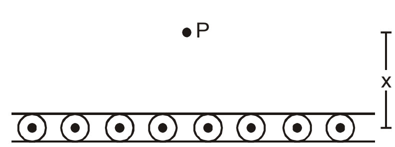
$$B\ 2\pi x = \frac{i_0(c^2+x^2-2b^2)}{c^2-b^2} \quad \text{or,} \quad B = \frac{\mu_0 i_0(c^2+x^2-2b^2)}{2\pi x(c^2-b^2)}$$

(d) The net current enclosed by the circle in part d of the figure is $2i_0$ and hence

$$B\ 2\pi x = \mu_0\ 2i_0 \quad \text{or,} \quad B = \frac{\mu_0 i_0}{\pi x}\text{-}.$$

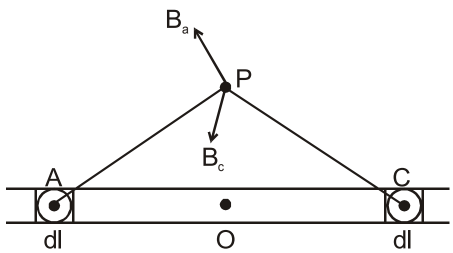
Example 17.

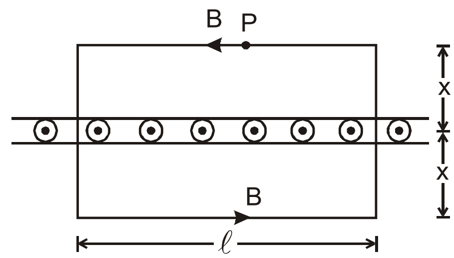
Figure shows a cross-section of a large metal sheet carrying an electric current along its surface. The current in a strip of width dl is λdl where λ is a constant. Find the magnetic field at a point P at a distance y from the metal sheet.



Solution :

Consider two strips A and C of the sheet situated symmetrically on the two sides of P (figure). The magnetic field at P due to the strip A is B_0 perpendicular to AP and that due to the strip C is B_C perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B.





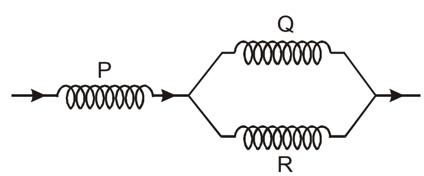
The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying Ampere's law to the rectangle shown in figure.

$$2B\ell = \mu_0 \lambda\ell \qquad \text{or,} \qquad B = \frac{1}{2}\mu_0 \lambda.$$

Note that it is independent of y.

Example 18.

Three identical long solenoids P, Q and R are connected to each other as shown in figure. If the magnetic field at the centre of P is 4 T, what would be the field at the centre of Q? Assume that the field due to any solenoid is confined within the volume of that solenoid only.



Solution :

As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by $B = \mu_0 ni$. Hence the field in Q will be equal to the field in R and will be half the field in P i.e., will be 2 T.

3. MAGNETIC FORCE ON MOVING CHARGE

When a charge q moves with velocity $\vec{v}$, in a magnetic field $\vec{B}$, then the magnetic force experienced by moving charge is given by following formula :

$\vec{F} = q(\vec{v} \times \vec{B})$ Put q with sign.

$\vec{v}$: Instantaneous velocity

$\vec{B}$: Magnetic field at that point.

Note :

- $\vec{F} \perp \vec{v}$ and also $\vec{F} \perp \vec{B}$
- $\because$ $\vec{F} \perp \vec{v}$ $\therefore$ power due to magnetic force on a charged particle is zero. (use the formula of power P = $\vec{F}.\vec{v}$ for its proof).
- Since the $\vec{F} \perp \vec{B}$ so work done by magnetic force is zero in every part of the motion. The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.
- On a stationary charged particle, magnetic force is zero.
- If $\vec{V} \parallel \vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

Solved Examples

Example 19.

A charged particle of mass 5 mg and charge q = +2μC has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$. $\vec{v}$ and $\vec{B}$ are in m/s and Wb/m^2 respectively.

Solution : $\vec{F} = q\vec{v} \times \vec{B}$ = 2 × 10^{-6} $(2\hat{i} - 3\hat{j} \times 4\hat{k})$ × $(3\hat{j} - 2\hat{k})$ = 2 × 10^{-6} [$-6\hat{i}$ + $4\hat{j}$ + $6\hat{k}$] N

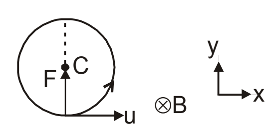
By Newton's Law $\vec{a} = \dfrac{\vec{F}}{m}$ = $\dfrac{2 \times 10^{-6}}{5 \times 10^{-6}}$ $(-6\hat{i} + 4\hat{j} + 6\hat{k})$

= 0.8 $(-3\hat{i} + 2\hat{j} + 3\hat{k})$ m/s^2

3.1 Motion of charged particles under the effect of magnetic force

- Particle released if v = 0 then f_m = 0
  $\therefore$ particle will remain at rest
- $\vec{V} \parallel \vec{B}$ here θ = 0 or θ = 180º

  $\therefore$ F_m = 0 $\therefore$ $\vec{a} = 0$ $\therefore$ $\vec{V}$ = const.
  $\therefore$ particle will move in a straight line with constant velocity



- Initial velocity $\vec{u} \perp \vec{B}$ and $\vec{B}$ = uniform

  In this case $\because$ B is in z direction so the magnetic force in z-direction will be zero $\left(\because \overrightarrow{F_m} \perp \vec{B}\right)$.

  Now there is no initial velocity in z-direction.

  $\therefore$ particle will always move in xy plane.

  $\therefore$ velocity vector is always $\perp \vec{B}$ $\therefore$ F_m = quB = constant

  now quB = $\dfrac{mu^2}{R}$ $\Rightarrow$ R = $\dfrac{mu}{qB}$ = constant.

  The particle moves in a curved path whose radius of curvature is same every where, such curve in a plane is only a circle.

  $\therefore$ path of the particle is circular.

$$R = \frac{mu}{qB} = \frac{p}{qB} = \frac{\sqrt{2mk}}{qB}$$

here p = linear momentum ; k = kinetic energy

now $v = \omega R \Rightarrow \omega = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$

Time period $T = 2\pi m/qB$

frequency $f = qB/2\pi m$

Note :

- ω, f, T are independent of velocity.

Solved Examples

Example 20. A proton (p), α-particle and deuteron (D) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods. (Neglect interaction between particles).

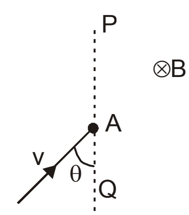
Solution : $R = \frac{\sqrt{2mK}}{qB}$

$\therefore \quad R_p : R_\alpha : R_D = \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2.4mK}}{2qB} : \frac{\sqrt{2.2mK}}{qB} = 1 : 1 : \sqrt{2}$

$T = 2\pi m/qB$

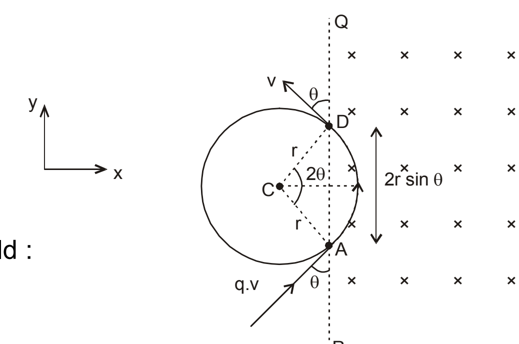
$\therefore \quad T_p : T_\alpha : T_D = \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB} = 1 : 2 : 2$ **Ans.**

Example 21. A positive charge particle of charge q, mass m enters into a uniform magnetic field with velocity v as shown in the figure. There is no magnetic field to the left of PQ.



Find (i) time spent,
(ii) distance travelled in the magnetic field
(iii) impulse of magnetic force.

Solution : The particle will move in the field as shown



Angle subtended by the arc at the centre = 2θ

(i) Time spent by the charge in magnetic field

$\omega t = \theta \Rightarrow \frac{qB}{m} t = \theta \Rightarrow t = \frac{m\theta}{qB}$

(ii) Distance travelled by the charge in magnetic field :

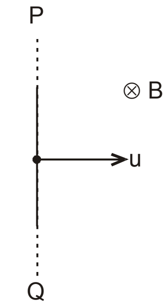
$= r(2\theta) = \frac{mv}{qB} . 2\theta$

(iii) Impulse = change in momentum of the charge

$= (-mv \sin\theta\ \hat{i} + mv \cos\theta\ \hat{j}) - (mv \sin\theta\ \hat{i} + mv \cos\theta\ \hat{j})$

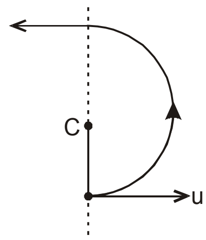
$= -2mv \sin\theta\ \hat{i}$

Example 22.

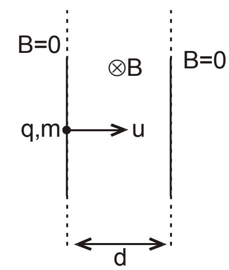


In the figure shown the magnetic field on the left on ‘PQ’ is zero and on the right of ‘PQ’ it is uniform. Find the time spent in the .magnetic field.

Solution : The path will be semicircular time spent = $T/2 = \pi m/qB$



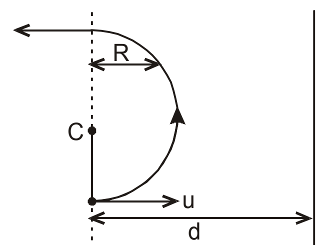
Example 23.



A uniform magnetic field of strength ‘B’ exists in a region of width ‘d’. A particle of charge ‘q’ and mass ‘m’ is shot perpendicularly (as shown in the figure) into the magnetic field. Find the time spend by the particle in the magnetic field if

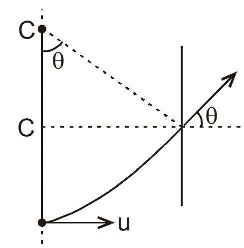
(i) $d > \dfrac{mu}{qB}$ (ii) $d < \dfrac{mu}{qB}$

Solution :



(i) $d > \dfrac{mu}{qB}$ means $d > R$

$$\therefore \quad t = \frac{T}{2} = \frac{\pi m}{qB}$$

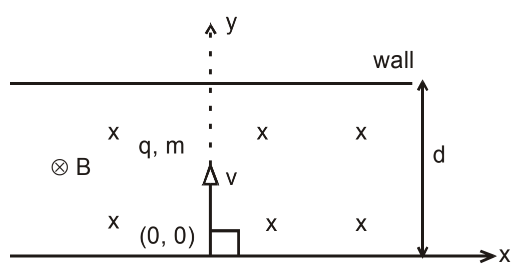


(ii) $\sin\theta = \dfrac{d}{R}$

$$\theta = \sin^{-1}\left(\frac{d}{R}\right)$$

$$\omega t = \theta \quad \Rightarrow \quad t = \frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$$

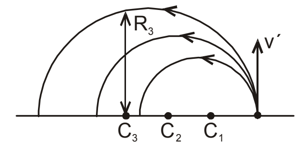
Example 24.



What should be the speed of charged particle so that it can’t collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.

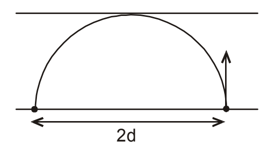
Solution :

(i) The path of the particle will be circular larger the velocity, larger will be the radius.
For particle not to s
strike R < d



$\therefore \quad \dfrac{mv}{qB} < d \quad \Rightarrow \quad v < \dfrac{qBd}{m}.$

(ii) for limiting case $v = \dfrac{qBd}{m}$



$R = d$

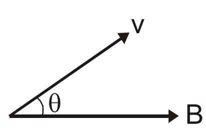
$\therefore$ coordinate = (–2d, 0, 0)

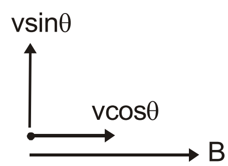
3.2 Helical path :

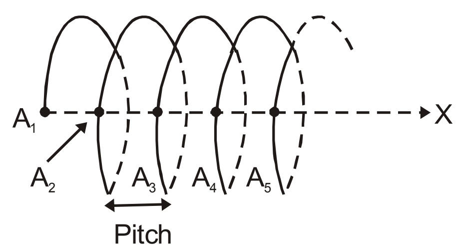
If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components – $v_{||}$, parallel to the field and $v_{\perp}$, perpendicular to the field. The components $v_{||}$ remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \dfrac{mv_{\perp}}{qB}$ as given by equation. The resultant path is helix.

Complete analysis :

Let a particle have initial velocity in the plane of the paper and a constant and uniform magnetic field also in the plane of the paper.







The particle starts from point A_1.
It completes its one revolution at A_2 and 2nd revolution at A_3 and so on. X-axis is the tangent to the helix points

$A_1, A_2, A_3, \ldots\ldots\ldots.$ all are on the x-axis.

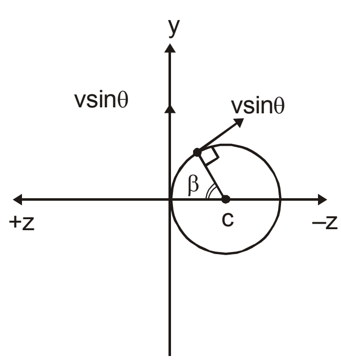
distance $\quad A_1A_2 = A_3A_4 = \ldots\ldots\ldots\ldots\ldots = v\cos\theta.\ T$ = pitch

where $\quad T$ = Time period

Let the initial position of the particle be (0,0,0) and v sin θ in +y direction. Then

in x : $\quad F_x = 0,\ a_x = 0,\ v_x = \text{constant} = v\cos\theta,\ x = (v\cos\theta)t$

In y-z plane :



$\beta = \omega t$

From figure it is clear that

$y = R\sin\beta$, $v_y = v\sin\theta\cos\beta$

$z = -(R - R\cos\beta)$

$v_z = v\sin\theta\sin\beta$

acceleration towards centre = $(v\sin\theta)^2/R = \omega^2 R$

$\therefore \quad a_y = -\omega^2 R\sin\beta$, $a_z = -\omega^2 R\cos\beta$

At any time : the position vector of the particle

(or its displacement w.r.t. initial position)

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, x,y,z already found

velocity $\quad \vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, v_x, v_y, v_z already found

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, a_x, a_y, a_z already found

Radius $\quad q(v\sin\theta)B = \dfrac{m(v\sin\theta)^2}{R} \Rightarrow R = \dfrac{mv\sin\theta}{qB}$

$$\omega = \frac{v\sin\theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f.$$

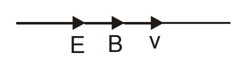
3.3 Charged Particle in $\vec{E}$ & $\vec{B}$

When a charged particle moves with velocity $\vec{V}$ in an electric field $\vec{E}$ and magnetic field $\vec{B}$, then. Net force experienced by it is given by following equation.

$$\vec{F} = q\vec{E} + q(\vec{V} \times \vec{B})$$

Combined force is known as Lorentz force.

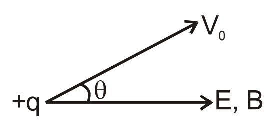
$\vec{E} \,||\, \vec{B} \,||\, \vec{v}$

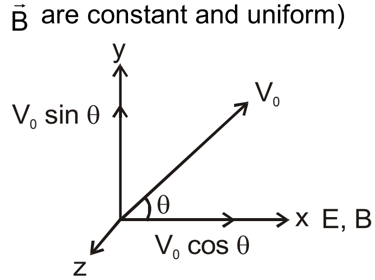


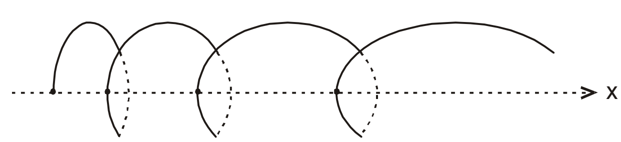
In above situation particle passes undeviated but its velocity will change due to electric field. Magnetic force on it = 0.

Case(i) :

- $\vec{E} \,||\, \vec{B}$ and uniform $\theta \neq 0, 180°$ ($\vec{E}$ and $\vec{B}$ are constant and uniform)

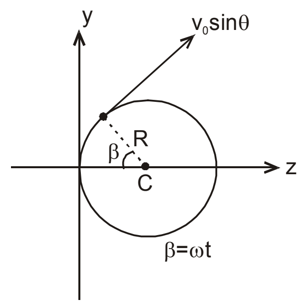






in x : $F_x = qE$, $a_x = \dfrac{qE}{m}$, $v_x = v_0\cos\theta + a_x t$, $x = v_0 t + \dfrac{1}{2}a_x t^2$

in yz plane :



$$qv_0 \sin\theta\, B = m(v_0 \sin\theta)^2 / R$$

$$\Rightarrow \quad R = \frac{mv_0 \sin\theta}{qB},$$

$$\omega = \frac{v_0 \sin\theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$$

$$\vec{r} = \{(V_0 \cos\theta)t + \frac{1}{2}\frac{qE}{m}t^2\}\,\hat{i} + R\sin\omega t\,\hat{j} + (R - R\cos\omega t)\,(-\hat{k})$$

$$\vec{V} = \left(V_0 \cos\theta + \frac{qE}{m}t\right)\hat{i} + (V_0 \sin\theta)\cos\omega t\,\hat{j} + V_0 \sin\theta \sin\omega t\,(-\hat{k})$$

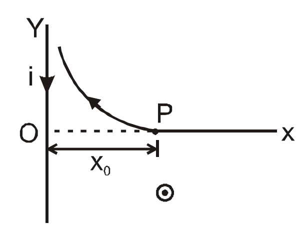
$$\vec{a} = \frac{qE}{m}\,\hat{i} + \omega^2 R[-\sin\beta\,\hat{j} - \cos\beta\,\hat{k}]$$

Solved Examples

Example 25.

A long, straight wire carries a current i. A particle having a positive charge q and mass m kept at a distance x_0 from the wire is projected towards it with a speed v as shown in figure. Find the minimum separation between the wire and the particle

Solution :



Let the particle be initially at P (figure). Take the wire as the Y-axis and the foot of perpendicular from P to the wire as the origin. Take the line OP as the X-axis. We have, OP = x_0. The magnetic field B at any point to the right of the wire is along the positive Z-axis. The magnetic force on the particle is, therefore, in the X–Y plane. As there is no initial velocity along the Z-axis, the motion will be in the X–Y plane. Also, its speed remains unchanged. As the magnetic field is not uniform, the particle does not go along a circle.

The force at time t is $\vec{F} = q\vec{v} \times \vec{B}$

$$= q(\vec{i}\,v_x + \vec{j}\,v_y) \times \left(\frac{\mu_0 i}{2\pi x}\hat{k}\right)$$

$$= -\vec{j}\,qv_x \frac{\mu_0 i}{2\pi x} + \vec{i}\,qv_y \frac{\mu_0 i}{2\pi x}.$$

Thus $\qquad a_x = \dfrac{F_x}{m} = \dfrac{\mu_0 qi}{2\pi m}\dfrac{v_y}{x} = \lambda\,\dfrac{v_y}{x}$ ....(i)

where $\qquad \lambda = \dfrac{\mu_0 qi}{2\pi m}$.

Also, $\qquad a_x = \dfrac{dv_x}{dt} = \dfrac{dv_x}{dx}\dfrac{dx}{dt} = \dfrac{v_x dv_x}{dx}$ ....(ii)

As, $\qquad v_x^2 + v_y^2 = v^2$,

giving $\qquad v_x\, dv_x = -\, v_y\, dv_y$. .....(iii)

From (i), (ii) and (iii),

$$\frac{v_y dv_y}{dx} = \frac{-\lambda v_y}{x}$$

or,
$$\frac{dx}{x} = \frac{-dv_y}{\lambda}$$

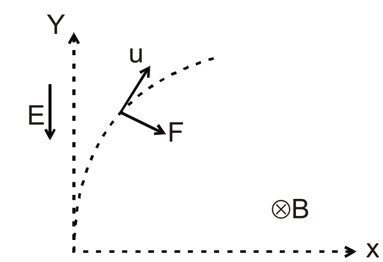
Initially $x = x_0$ and $v_y = 0$. At minimum separation from the wire, $v_x = 0$ so that $v_y = v$.

Thus
$$\int_{x_0}^{x}\frac{dx}{x} = \int_{0}^{v}\frac{dv_y}{-\lambda}$$

or,
$$\ln\frac{x}{x_0} = -\frac{v}{\lambda}$$

or,
$$x = x_0\, e^{v/\lambda} = x_0\, e^{-\frac{2\pi mv}{\mu_0 qi}}$$

Example 26.



An electron is released from the origin at a place where a uniform electric field E and a uniform magnetic field B exist along the negative Y-axis and the negative Z-axis respectively. Find the displacement of the electron along the Y-axis when its velocity becomes perpendicular to the electric field for the first time.

Solution :

Let us take axes as shown in figure. According to the right-handed system, the Z-axis is upward in the figure and hence the magnetic field is shown downwards. At any time, the velocity of the electron may be written as

$$\vec{u} = u_x\vec{i} + u_y\vec{j}$$

The electric and magnetic fields may be written as

$$\vec{E} = -E\vec{j}$$

and $\qquad \vec{B} = -B\vec{k}$

respectively. The force on the electron is

$$\vec{F} = -e(\vec{E} + \vec{u}\times\vec{B})$$

$$= eE\vec{j} + eB(u_y\vec{i} - u_x\vec{j})$$

Thus, $\qquad F_x = eu_y B$

and $\qquad F_y = e(E - u_x B)$.

The components of the acceleration are

$$a_x = \frac{du_x}{dt} = \frac{eB}{m}u_y \qquad \text{....(i)}$$

and $$a_y = \frac{du_y}{dt} = \frac{e}{m}(E - u_x B). \qquad \text{....(ii)}$$

We have,

$$\frac{d^2u_y}{dt^2} = -\frac{eB}{m}\frac{du_x}{dt} = -\frac{eB}{m} \cdot \frac{eB}{m}u_y = -\omega^2 u_y$$

where $$\omega = \frac{eB}{m} \qquad \text{....(iii)}$$

This equation is similar to that for a simple harmonic motion. Thus,

$$u_y = A \sin(\omega t + \delta) \qquad \text{.....(iv)}$$

and hence, $$\frac{du_y}{dt} = A\,\omega \cos(\omega t + \delta) \qquad \text{....(v)}$$

At $t = 0$, $u_y = 0$ and $\frac{du_y}{dt} = \frac{F_y}{dt} = \frac{eE}{m}$.

Putting in (iv) and (v),

$$\delta = 0 \text{ and } A = \frac{eE}{m\omega}\frac{E}{B}.$$

Thus, $$u_y = \frac{E}{B}\sin \omega t.$$

The path of the electron will be perpendicular to the Y-axis when $u_y = 0$. This will be the case for the first time at t where

$$\sin \omega t = 0$$

or, $$\omega t = \pi$$

or, $$t = \frac{\pi}{\omega} = \frac{\pi m}{eB}$$

Also, $$u_y = \frac{dy}{dt} = \frac{E}{B}\sin \omega t$$

or, $$\int_0^y dy = \frac{E}{B}\sin\omega t\ dt$$

or, $$y = \frac{E}{B\omega}(1 - \cos \omega t).$$

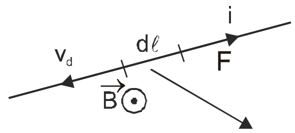
At $$t = \frac{\pi}{\omega},\quad y = \frac{E}{B\omega}(1 - \cos\pi) = \frac{2E}{B\omega}$$

Thus, the displacement along the Y-axis is

$$\frac{2E}{B\omega} = \frac{2Em}{BeB} = \frac{2Em}{eB^2}. \qquad \textbf{Ans.}$$

3.4 Magnetic force on A current carrying wire :

Suppose a conducting wire, carrying a current i, is placed in a magnetic field $\vec{B}$. Consider a small element $d\ell$ of the wire (figure). The free electrons drift with a speed v_d opposite to the direction of the current. The relation between the current i and the drift speed v_d is



$i = jA = nev_dA.$ ....(i)

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average (why average?) magnetic force

$$\vec{f} = -e\vec{v}_d \times \vec{B}$$

The number of free electrons in the small element considered in $nAd\ell$. Thus, the magnetic force on the wire of length $d\ell$ is

$$d\vec{F} = (nAd\ell)(-e\vec{v}_d \times \vec{B})$$

If we denote the length $d\ell$ along the direction of the current by $d\vec{\ell}$, the above equation becomes

$$d\vec{F} = nAev_d\overrightarrow{d\ell} \times \vec{B}.$$

Using (i), $d\vec{F} = id\vec{\ell} \times \vec{B}.$

The quantity $id\vec{\ell}$ is called a *current element*.

$$\overrightarrow{F_{res}} = \int \overrightarrow{dF} = \int id\vec{\ell} \times \vec{B} = i \int \overrightarrow{d\ell} \times \vec{B}$$

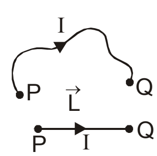
($\because$ i is same at all points of the wire.)

If $\vec{B}$ is uniform then $\vec{F}_{res} = i(\int \overrightarrow{d\ell}) \times \vec{B}$

$$\overrightarrow{F_{res}} = i\vec{L} \times \vec{B}$$

Here $\vec{L} = \int d\vec{\ell}$ = vector length of the wire = vector connecting the end points of the wire.

$\Rightarrow$

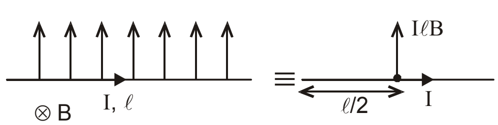


Note :

- If a current loop of any shape is placed in a uniform $\vec{B}$ then $\left.\overrightarrow{F_{res}}\right)_{magnetic}$ on it = 0 ($\because \vec{L} = 0$).

3.5 Point of application of magnetic force :

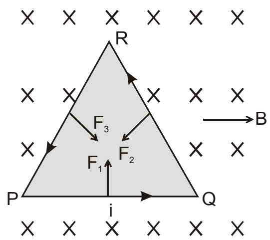
On a straight current carrying wire the magnetic force in a uniform magnetic field can be assumed to be acting at its mid point.



This can be used for calculation of torque.

Example 27.

A wire is bent in the form of an equilateral triangle PQR of side 20 cm and carries a current of 2.5 A. It is placed in a magnetic field B of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.



Solution :

Suppose the field and the current have directions as shown in figure. The force on PQ is

$$\vec{F}_1 = i\vec{\ell} \times \vec{B}$$

or, F_1 = 2.5 A × 20 cm × 2.0 T = 1.0 N

The rule of vector product shows that the force F_1 is perpendicular to PQ and is directed towards the inside of the triangle.

The forces $\vec{F}_2$ and $\vec{F}_3$ on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and towards the inside of the triangle.

The three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalised. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

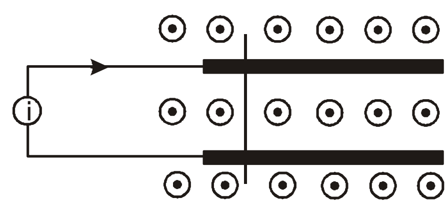
Example 28.

Figure shows two long metal rails placed horizontally and parallel to each other at a separation y. A uniform magnetic field B exists in the vertically upward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.

(a) What soluble the minimum value of μ which can prevent the wire from sliding on the rails?

(b) Describe the motion of the wire if the value of μ is half the

value found in the previous part



Sol. (a) The force on the wire due to the magnetic field is

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

or, F = iyB

It acts towards right tin the given figure. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to μ_0 mg. Thus,

$$\mu_0 mg = iyB$$

or, $$\mu_0 = \frac{iyB}{mg}$$

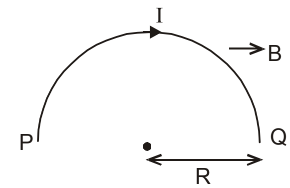
(b) If the friction coefficient is $\mu = \frac{\mu_0}{2} = \frac{iyB}{2mg}$, the wire will slide towards right. The frictional force by the rails is

$$f = \mu mg = \frac{iyB}{2} \text{ towards left.}$$

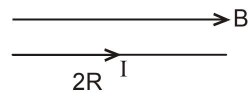
The resultant force is $iyB - \frac{iyB}{2} = \frac{iyB}{2}$ towards right. The acceleration will be $a = \frac{iyB}{2m}$. The wire will slide towards right with this acceleration.

Example 29.

In the figure shown a semicircular wire is placed in a uniform $\vec{B}$ directed toward right. Find the resultant magnetic force and torque on it.



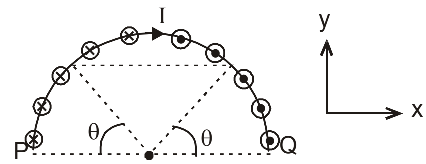
Solution : The wire is equivalent to



$\because \quad \theta = 0$

$\therefore \quad F_{res} = 0$ **Ans.**

forces on individual parts are marked in the figure by ⊗ and ⦿.
By symmetry their will be pair of forces forming couples.



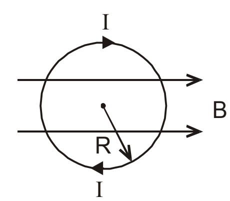
$$\tau = \int_0^{\pi/2} i(Rd\theta)B\sin(90-\theta).2R\cos\theta$$

$$\tau = \frac{i\pi R^2}{2}B$$

$$\Rightarrow \quad \vec{\tau} = \frac{i\pi R^2}{2}B(-\hat{j})$$ **Ans.**

Example 30.

Find the resultant magnetic force and torque on the loop.



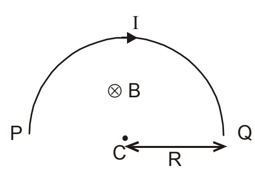
Solution :

$\overrightarrow{F_{res}} = 0$, ($\because$ loop)

and $\vec{\tau} = i\pi R^2 B(-\hat{j})$ using the above method

Example 31

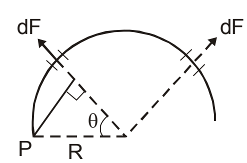
In the figure shown find the resultant magnetic force and torque about 'C', and 'P'.



Solution : $\vec{F}_{nett}$ = I . 2R . B

$\because$ wire is equivalent to

Force on each element is radially outward : $\tau_c = 0$
point about

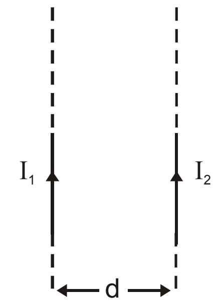


$$P = \int_0^{\pi} [i(Rd\theta)B\sin 90^0]\ R\sin\theta$$

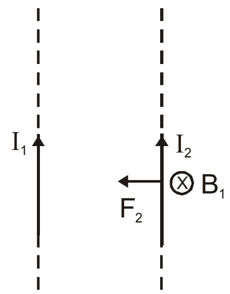
$= 2IBR^2$ **Ans.**

Example 32.

Prove that magnetic force per unit length on each of the infinitely long wire due to each other is $\mu_0 I_1 I_2/2\pi d$. Here it is attractive also.



Solution :



On (2), B due to (i) is = $\frac{\mu_0 I_1}{2\pi d} \otimes$

$\therefore$ F on (2) on 1m length

$= I_2 . \frac{\mu_0 I_1}{2\pi d} . 1$ towards left it is attractive

$= \frac{\mu_0 I_1 I_2}{2\pi d}$ (hence proved)

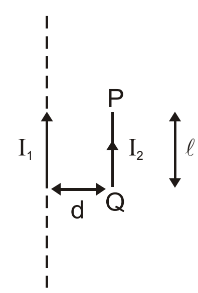
Similarly on the other wire also.

Note :

- Definition of ampere (fundamental unit of current) using the above formula.
  If $I_1 = I_2 = 1A$, d = 1m then $F = 2 \times 10^{-7}$ N
  $\therefore$ "When two very long wires carrying equal currents and separated by 1m distance exert on each other a magnetic force of 2×10^{-7} N on 1m length then the current is 1 ampere."

- The above formula can also be applied if to one wire is infinitely long and the other is of finite length. In this case the force per unit length on each wire will not be same.

  Force per unit length on PQ = $\frac{\mu_0 I_1 I_2}{2\pi d}$ (attractive)

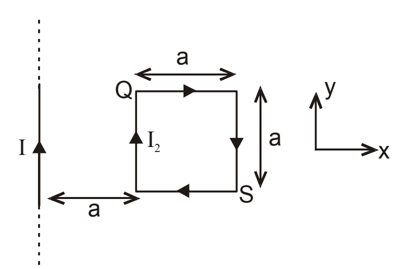


(3) If the currents are in the opposite direction then the magnetic force on the wires will be repulsive.

Example 33.

Find the magnetic force on the loop 'PQRS' due to the loop wire.

Solution :

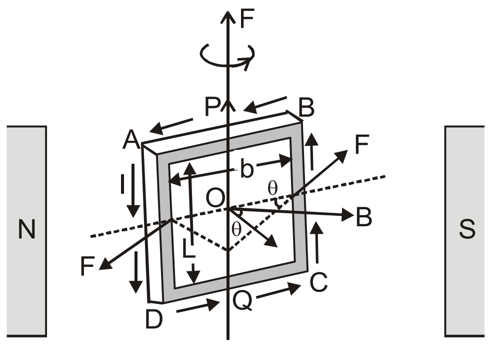


$F_{res} = \frac{\mu_0 I_1 I_2}{2\pi a}\, a\, (-\hat{i}) + \frac{\mu_0 I_1 I_2}{2\pi (2a)} a(\hat{i})$

$= \frac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$

4. TORQUE ON A CURRENT LOOP :

When a current-carrying coil is placed in a uniform magnetic field the net force on it is always zero. However, as its different parts experience forces in different directions so the loop may experience a torque (or couple) depending on the orientation of the loop and the axis of rotation. For this, consider a rectangular coil in a uniform field B which is free to rotate about a vertical axis PQ and normal to the plane of the coil making an angle θ with the field direction as shown in figure (A).



Axis of rotation
(A)

The arms AB and CD will experience forces B(NI)b vertically up and down respectively. These two forces together will give zero net force and zero torque (as are collinear with axis of rotation), so will have no effect on the motion of the coil.

Now the forces on the arms AC and BD will be BINL in the direction out of the page and into the page respectively, resulting in zero net force, but an anticlockwise couple of value

τ = F × Arm = BINL × (b sinθ)

i.e. τ = BIA sinθ with A = NLb ....(i)

Now treating the current–carrying coil as a dipole of moment $\vec{M} = I\vec{A}$ Eqn. (i) can be written in vector form as

$\vec{\tau} = \vec{M} \times \vec{B}$ [with $\vec{M} = I\vec{A} = NIA\vec{n}$ ....(ii)]

This is the required result and from this it is clear that :

(1) Torque will be minimum (= 0) when sinθ = min = 0, i.e., θ = 0º, i.e. 180º i.e., the plane of the coil is perpendicular to magnetic field i.e. normal to the coil is collinear with the field [fig. (A) and (C)]

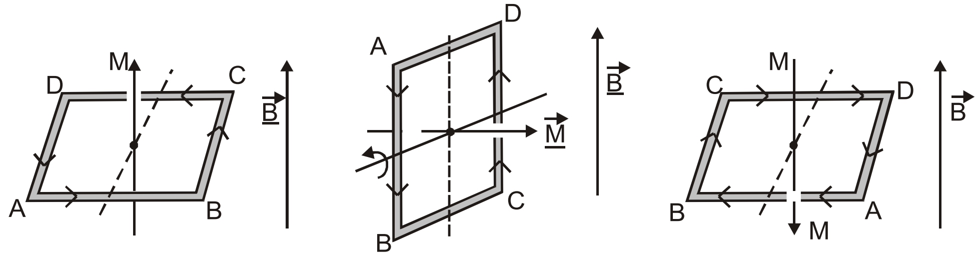
(2) Torque will be maximum (= BINA) when sinθ = max = 1, i.e., θ = 90º i.e. the plane of the coil is parallel to the field i.e. normal to the coil is perpendicular to the field. [fig.(B)].

(3) By analogy with dielectric or magnetic dipole in a field, in case of current–carrying in a field.

$U = -\vec{M} \bullet \vec{B}$ with $F = -\dfrac{dU}{dr}$

and W = MB(1 – cosθ)

The values of U and W for different orientations of the coil in the field are shown in fig.



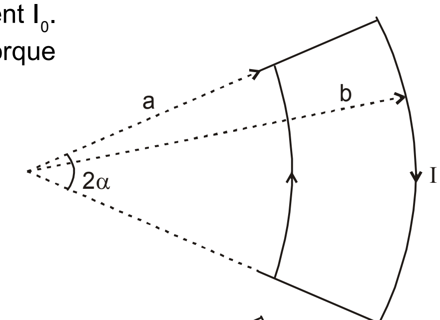
$\theta = 0^\circ$ ($\vec{M}$ is parallel to $\vec{B}$)	$\theta = 90^\circ$ ($\vec{M}$ is $\perp$ to $\vec{B}$)	$\theta = 180^\circ$ ($\vec{M}$ is antiparallel to $\vec{B}$)
$\tau = 0 = \text{min}$	$\tau = MB = \text{max}$	$\tau = 0$
$W = 0 = \text{min}$	$W = MB$	$W = 2MB = \text{max}$
$U = -MB = \text{min}$	$U = 0$	$U = MB = \text{max}$
Stable equilibrium	No equilibrium	Unstable equilibrium
(A)	(B)	(C)

(4) Instruments such as electric motor, moving coil galvanometer and tangent galvanometers etc. are based on the fact that a current–carrying coil in a uniform magnetic field experiences a torque (or couple).

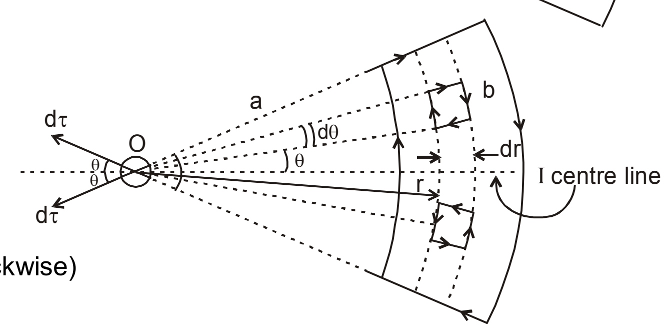
Solved Examples

Example 34.

A loop with current I is in the field of a long straight wire with current I_0. The plane of the loop is perpendicular to the straight wire. Find torque acting on the loop.



Solution :



$d\vec{s} = (r d\theta\, dr)$ (inwards)

$d\vec{M} = (rI\, d\theta\, dr)$ (inwards)

$\vec{B} = \dfrac{\mu_0 I_0}{2\pi r}$ (tangential clockwise)

$d\tau = |\, d\vec{M} \times \vec{B}\,| = \dfrac{\mu_0 I I_0\, d\theta\, dr}{2\pi}$ (towards centre)

$\therefore \quad \tau = \int_{-\alpha}^{\alpha} \int_a^b d\tau \cos\theta$

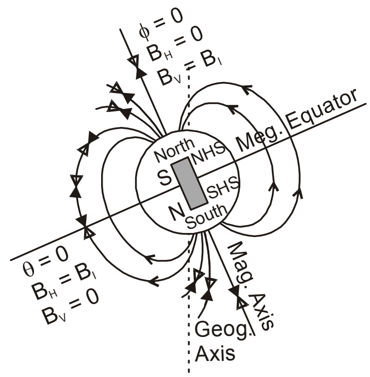
$= \dfrac{\mu_0 I I_0}{2\pi} \int_{-\alpha}^{\alpha} \int_a^b \cos\theta\, d\theta\, dr$

$= \dfrac{\mu_0 I I_0 (b-a) \sin\alpha}{\pi}$ (to the left) **Ans.**

5. TERRESTRIAL MAGNETISM (EARTH'S MAGNETISM) :

5.1 Introduction :

The idea that earth is magnetised was first suggested towards the end of the sixteen'th century by Dr William Gilbert. The origin of earth's magnetism is still a mater of conjecture among scientists but it is agreed upon that the earth behaves as a magnetic dipole inclined at a small angle (11.5º) to the earth's axis of rotation with its south pole pointing north. The lines of force of earth's magnetic field are shown in figure which are parallel to the earth's surface near the equator and perpendicular to it near the poles. While discussing magnetism of the earth one should keep in mind that:



(a) The **magnetic meridian** at a place is not a line but a vertical plane passing through the axis of a freely suspended magnet, i.e., it is a plane which contains the place and the magnetic axis.

(b) The **geographical meridian** at a place is a vertical plane which passes through the line joining the geographical north and south, i.e., it is a plane which contains the place and earth's axis of rotation, i.e., geographical axis.

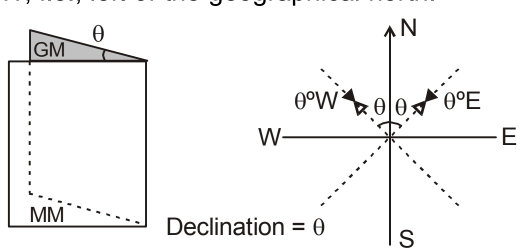
(c) The **magnetic Equator** is a great circle (a circle with the centre at earth's centre) on earth's surface which is perpendicular to the magnetic axis. The magnetic equator passing through Trivandrum in South India divides the earth into two hemispheres. The hemisphere containing south polarity of earth's magnetism is called the northern hemisphere (NHS) while the other, the southern hemisphere (SHS).

(d) The magnetic field of earth is not constant and changes irregularly from place to place on the surface of the earth and even at a given place it varies with time too.

5.2 Elements of the Earth's Magnetism :

The magnetism of earth is completely specified by the following three parameters called elements of earth's magnetism :

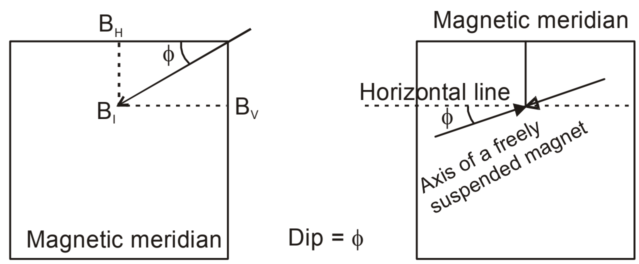
(a) Variation or Declination θ : At a given place the angle between the geographical meridian and the magnetic meridian is called declination, i.e., at a given place it is the angle between the geographical north-south direction and the direction indicated by a magnetic compass needle, Declination at a place is expressed at θº E or θº W depending upon whether the north pole of the compass needle lies to the east (right) or to the west (left) of the geographical north-south direction. The declination at London is 10ºW means that at London the north pole of a compass needle points 10ºW, i.e., left of the geographical north.



(A) (B)

(b) Inclination or Angle of Dip ϕ : It is the angle which the direction of resultant intensity of earth's magnetic field subtends with horizontal line in magnetic meridian at the given place. Actually it is the angle which the axis of a freely suspended magnet (up or down) subtends with the horizontal in magnetic meridian at a given place.

Here, it is worthy to note that as the northern hemisphere contains south polarity of earth's magnetism, in it the north pole of a freely suspended magnet (or pivoted compass needle) will dip downwards, i.e., towards the earth while the opposite will take place in the southern hemisphere.



(A) (B)

Angle of dip at a place is measured by the instrument called Dip-Circle in which a magnetic needle is free to rotate in a vertical plane which can be set in any vertical direction. Angle of dip at Delhi is 42º.

(c) Horizontal Component of Earth's Magnetic Field B_H : At a given place it is defined as the component of earth's magnetic field along the horizontal in the magnetic meridian. It is represented by B_H and is measured with the help of a **vibration** or **deflection magnetometer**. At Delhi the horizontal component of the earth's magnetic field is 35 μT, i.e., 0.35 G.

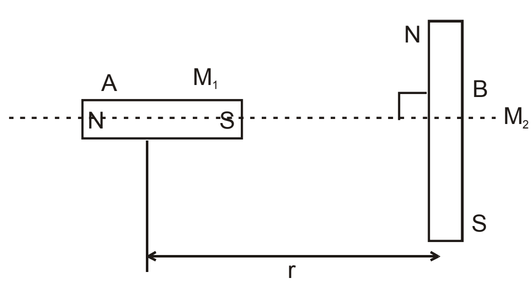
If at a place magnetic field of earth is B_I and angle of dip ϕ, then in accordance with figure (a).

$$B_H = B_I \cos\phi \quad \text{and} \quad B_v = B_I \sin\phi \qquad \text{....(1)}$$

so that, $$\tan\phi = \frac{B_V}{B_H} \quad \text{and} \quad I = \sqrt{B_H^2 + B_V^2} \qquad \text{....(2)}$$

Solved Miscellaneous Problems

Problem 1. Two short magnet A and B of magnetic dipole moments M_1 and M_2 respectively are placed as shown. The axis of 'A' and the equatorial line of 'B' are the same. Find the magnetic force on one magnet due to the other.



Answer : $F = 3\left(\frac{\mu_0}{4\pi}\right)\frac{M_2M_1}{r^4}$ upwards on M_1

down wards on M_2

Solution : Magnetic field due to magnet B :

$$B = \frac{\mu_0}{4\pi}.\frac{M_2}{r^3}$$

Magnetic force acting on magnet A :

$$F = M_1\frac{dB}{dr} = -3\left(\frac{\mu_0}{4\pi}\right)\frac{M_1M_2}{r^4}$$

Problem 2. A bar magnet has a pole strength of 3.6 A-m and magnetic length 8 cm. Find the magnetic field at (a) a point on the axis at a distance of 6 cm from the centre towards the north pole and (b) a point on the perpendicular bisector at the same distance.

Answer : (a) 8.6×10^{-4} T ; (b) 7.7×10^{-5} T.

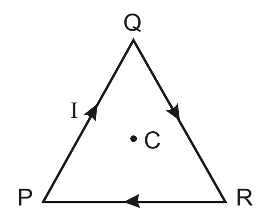
Solution :

$M = 3.6 \times 8 \times 10^{2}$ A.m^2

(a) $B = \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - a^2)^2} = 8.6 \times 10^{-4}$ T.

(b) $B = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + a^2)^{3/2}} = 7.7 \times 10^{-5}$ T

Problem 3. A loop in the shape of an equilateral triangle of side 'a' carries a current I as shown in the figure. Find out the magnetic field at the centre 'C' of the triangle.



Answer : $\frac{9\mu_0 i}{2\pi a}$

Solution : $B = B_1 + B_2 + B_3 = 3B_1$

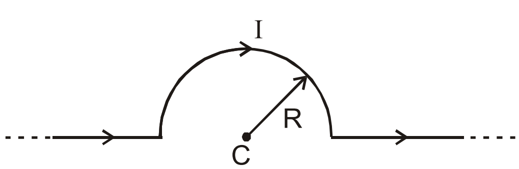
$$= 3\,\frac{\mu_0}{4\pi} \times \frac{i}{\left(\frac{a}{2\sqrt{3}}\right)} \times (\sin 60^\circ + \sin 60^\circ) = \frac{9\mu_0 i}{2\pi a}$$

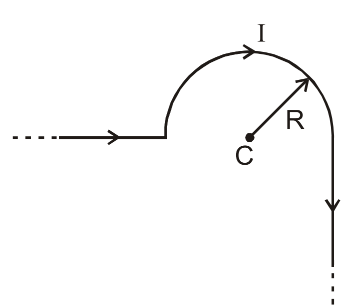
Problem 4. Two long wires are kept along x and y axes they carry currents I & I respectively in +ve x and +ve y directions respectively. Find $\vec{B}$ at a point (0, 0, d).

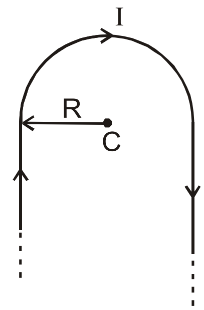
Answer : $\frac{\mu_0 I}{2\pi d}(\hat{i} - \hat{j})$

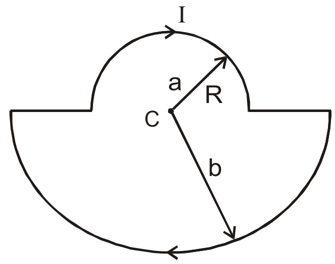
Solution : $\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{2\pi}\,\frac{i}{d}\,((-\hat{j}) + \frac{\mu_0}{2\pi}\frac{i}{d}\,(\hat{i}) = \frac{\mu_0 I}{2\pi d}\,(\hat{i} - \hat{j})$

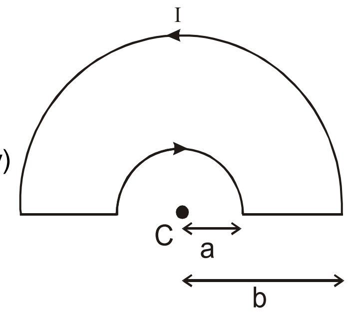
Problem 5. Find 'B' at centre 'C' in the following cases :

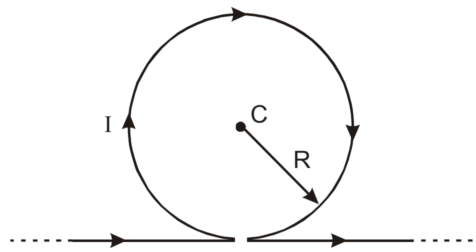
(i) 

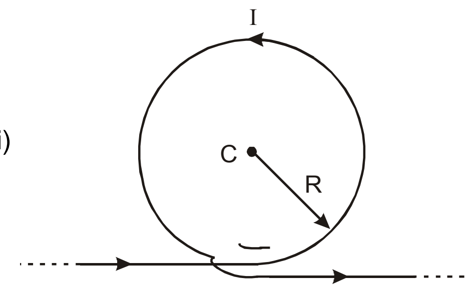
(ii) 

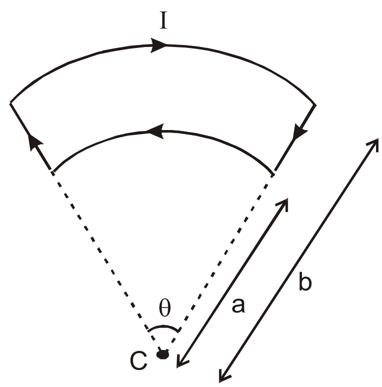
(iii) 

(iv) 

(v) 

(vi) 

(vii) 

(viii) 

Answer : (i) $\frac{\mu_0 I}{4R}\otimes$ (ii) $\frac{\mu_0 I}{4R}\left(1+\frac{1}{\pi}\right)\otimes$ (iii) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right)\otimes$ (iv) $\frac{\mu_0 I}{4}\left(\frac{1}{a}+\frac{1}{b}\right)\otimes$

(v) $\frac{\mu_0 I}{4}\left(\frac{1}{a}-\frac{1}{b}\right)\otimes$ (vi) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)\otimes$ (vii) $\frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)\odot$ (viii) $\frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)\odot$

Solution : (i) $B = \frac{\mu_0 I}{2R} \times \frac{1}{2} = \frac{\mu_0 I}{4R}$

(ii) $B = B_1 + B_2 = \left(\frac{\mu_0 I}{2R} \times \frac{1}{2}\right) + \left(\frac{\mu_0}{4\pi}.\frac{I}{R}\right) = \frac{\mu_0 I}{4R}\left(1+\frac{1}{\pi}\right)$

(iii) $B = B_1 + B_2 + B_3 = 2B_1 + B_2 = \left(2\times\frac{\mu_0}{4\pi}\frac{I}{R}\right) + \left(\frac{\mu_0 I}{2R}\times\frac{1}{2}\right) = \frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right)$

(iv) $B = B_1 + B_2 = \frac{\mu_0}{2a}\times\frac{1}{2}+\frac{\mu_0}{2b}\times\frac{1}{2} = \frac{\mu_0 I}{4}\left(\frac{1}{a}+\frac{1}{b}\right)$

(v) $B = B_1 - B_2 = \left(\frac{\mu_0 I}{2a}\times\frac{1}{2}-\frac{\mu_0 I}{2b}\times\frac{1}{2}\right) = \frac{\mu_0 I}{4}\left(\frac{1}{a}-\frac{1}{b}\right)$

(vi) $B = B_1 - B_2 = \frac{\mu_0 I}{2R} - \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R}\left(1 - \frac{1}{\pi}\right)$

(vii) $B = B_1 + B_2 = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R}\left(1 + \frac{1}{\pi}\right)$

(viii) $B = B_1 - B_2 = \frac{\mu_0 I}{2a} - \frac{\mu_0 I}{2b} \times \frac{\theta}{2\pi} = \frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$

Problem 6. A thin solenoid of length 0.4 m and having 500 turns of wire carries a current 1A; then find the magnetic field on the axis inside the solenoid.

Answer : $5\pi \times 10^{-4}$ T.

Solution : $B = \mu_0 ni = \frac{\mu_0 Ni}{\ell} = 5\pi \times 10^{-4}$ T.

Problem 7. A charged particle of charge 2C thrown vertically upwards with velocity 10 m/s. Find the magnetic force on this charge due to earth's magnetic field. Given vertical component of the earth = 3μT and angle of dip = 37º.

Answer : $2 \times 10 \times 4 \times 10^{-6} = 8 \times 10^{-5}$ N towards west.

Solution : $\tan 37^\circ = \frac{B_V}{B_H} \Rightarrow B_H = \frac{4}{3} \times 3 \times 10^{-6}$ T

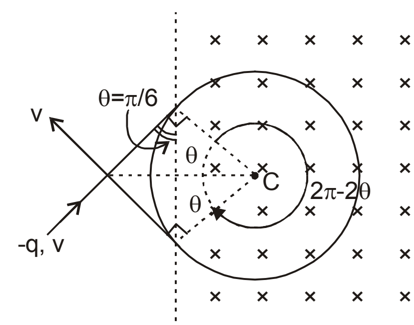
$F = q\, v\, B_H = 8 \times 10^{-5}$ N

Problem 8. A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.

Solution : $\because \vec{F} \perp \vec{B} \quad \therefore \quad \vec{a} \perp \vec{B} \quad \therefore \quad \vec{a}\cdot\vec{B} = 0$

$\therefore (2\hat{i} + x\hat{j}) \cdot (-3\hat{i} + 2\hat{j} - 4\hat{k}) = 0 \Rightarrow -6 + 2x = 0 \Rightarrow x = 3.$

Problem 9. Repeat above question if the charge is –ve and the angle made by the boundary with the velocity is $\frac{\pi}{6}$.



Solution : (i) $2\pi - 2\theta = 2\pi - 2.\frac{\pi}{6} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$= \omega t = \frac{qBt}{m} \Rightarrow t = \frac{5\pi m}{3qB}$

(ii) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$

(iii) Impulse = charge in linear momentum

$= m(-v \sin\theta\,\hat{i} + v\cos\theta\,\hat{j}) - m(v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j})$

$= -2mv\sin\theta\,\hat{i} = -2mv\sin\frac{\pi}{6}\,\hat{i} = -mv\,\hat{i}$

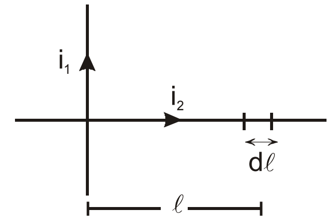
Problem 10. A particle of charge q and mass m is projected in a uniform and constant magnetic field of strength B. The initial velocity vector $\vec{v}$ makes angle 'θ' with the $\vec{B}$. Find the distance travelled by the particle in time 't'.

Answer : vt

Solution : Speed of the particle does not change there fore distance covered by the particle is
s = vt

Problem 11.

Two long wires, carrying currents i_1 and i_2, are placed perpendicular to each other in such a way that they just avoid a contact. Find the magnetic force on a small length $d\ell$ of the second wire situated at a distance ℓ from the first wire.



Solution :

The situation is shown in figure. The magnetic field at the site of $d\ell$, due to the first wire is ,

$$B = \frac{\mu_0 i_1}{2\pi\ell}$$

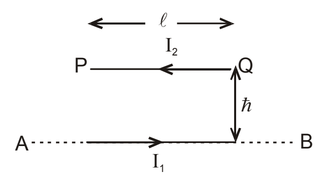
This field is perpendicular to the plane of the figure going into it. The magnetic force on the length $d\ell$ is,

$$dF = i_2\, d\ell\, B \sin 90^\circ \quad = \frac{\mu_0 i_1 i_2 d\ell}{2\pi\ell}$$

This force is parallel to the current i_1.

Problem 12.

In the figure shown the wires AB and PQ carry constant currents I_1 and I_2 respectively. PQ is of uniformly distributed mass 'm' and length 'ℓ'. AB and PQ are both horizontal and kept in the same vertical plane. The PQ is
in equilibrium at height 'h'. Find



(i) 'h' in terms of I_1, I_2, ℓ, m, g and other standard constants.

(ii) If the wire PQ is displaced vertically by small distance prove that it performs SHM. Find its time period in terms of h and g.

Solution : (i) Magnetic repulsive force balances the weight.

$$\frac{\mu_0 I_1 I_2}{2\pi h}\,\ell\ \text{mg} \qquad \Rightarrow \qquad h = \frac{\mu_0 I_1 I_2 \ell}{2\pi mg}$$

(ii) Let the wire be displaced downward by distance x(<< h).
Magnetic force on it will increase, so it goes back towards its equilibrium position. Hence it performs oscillations.

$$F_{res} = \frac{\mu_0 I_1 I_2}{2\pi(h-x)}\ell - mg$$

$$= \frac{mgh}{h-x} - mg = \frac{mg(h-h+x)}{h-x}$$

$$= \frac{mg}{h-x}\,x \cong \frac{mg}{h}x \text{ for } x << h$$

$$\therefore \qquad T = 2\pi\sqrt{\frac{m}{mg/h}} = 2\pi\sqrt{\frac{h}{g}}$$ **Ans.**