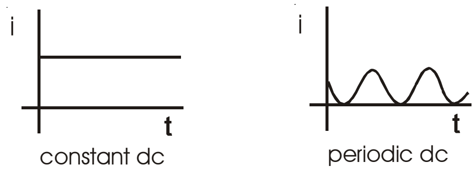
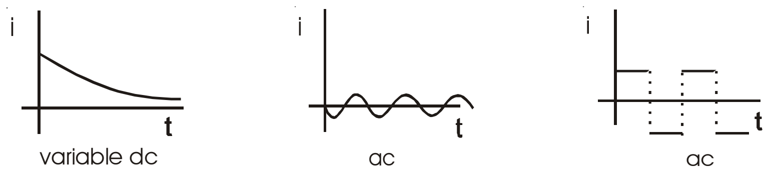
ALTERNATING CURRENT

1. AC AND DC CURRENT :

A current that changes its direction periodically is called alternating current (AC). If a current maintains its direction constant it is called direct current (DC).





If a function suppose current, varies with time as $i = I_m \sin(\omega t+\phi)$, it is called sinusoidally varying function. Here I_m is the peak current or maximum current and i is the instantaneous current. The factor $(\omega t+\phi)$ is called phase. ω is called the angular frequency, its unit rad/s. Also $\omega = 2\pi f$ where f is called the frequency, its unit s^{-1} or Hz. Also frequency f = 1/T where T is called the time period.

2. AVERAGE VALUE :

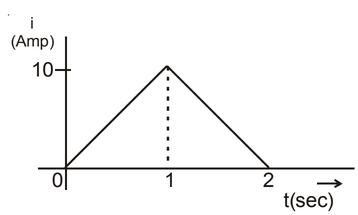
Average value of a function, from t_1 to t_2, is defined as $\langle f\rangle = \dfrac{\int_{t_1}^{t_2} f\,dt}{t_2 - t_1}$. We can find the value of $\int_{t_1}^{t_2} f\,dt$ graphically if the graph is simple. It is the area of f-t graph from t_1 to t_2.

(A) Average value of sin function is zero in a time period or integral multiple of time period. If $i = I_m \sin \omega t$ then average value of i in a time period is zero.

(B) Average value of square of sin function is $\frac{1}{2}$ in a time period or integral multiple of time period. If $i = I_m \sin^2 \omega t$ then average value is $\frac{1}{2}$.

Solved Examples

Exercise 1. Find the average value of current shown graphically, from t = 0 to t = 2 sec.



Solution : From the i – t graph, area from t = 0 to t = 2 sec

$= \frac{1}{2} \times 2 \times 10 = 10$ Amp. sec.

$\therefore$ Average Current $= \frac{10}{2} = 5$ Amp.

3. ROOT MEAN SQUARE VALUE :

Root Mean Square Value of a function, from t_1 to t_2, is defined as $f_{rms} = \sqrt{\dfrac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}}$.

(a) If the current varies as $i = I_m \sin \omega t$ then root mean square value of current is $\frac{1}{\sqrt{2}}$ times of maximum current.

$$i_{rms} = \frac{I_m}{\sqrt{2}}$$

(b) The r m s values for one cycle and half cycle (either positive half cycle or negative half cycle) is same.

Solved Examples

Example 2. Find the effective value of current $i = 2 \sin 100 \pi t + 2 \cos (100 \pi t + 30°)$.

Solution : The equation can be written as $i = 2 \sin 100 \pi t + 2 \sin (100 \pi t + 120°)$
so phase difference $\phi = 120°$

$$I_m)_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$$

$$= \sqrt{4 + 4 + 2 \times 2 \times 2\left(-\frac{1}{2}\right)}$$ = 2, so effective value or rms value = $2/\sqrt{2} = \sqrt{2}$ A

4. AC SINUSOIDAL SOURCE :

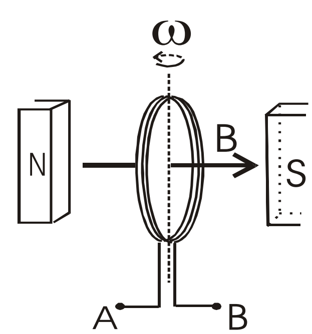
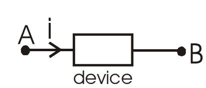


Figure shows a coil rotating in a magnetic field. The flux in the coil changes as φ = NBA cos $(\omega t + \phi)$. Emf induced in the coil, from Faraday's law is $\frac{-d\varphi}{dt}$ = N B A ω sin $(\omega t + \phi)$. Thus the emf between the points A and B will vary as $E = E_0 \sin (\omega t + \phi)$ .The potential difference between the points A and B will also vary as $V = V_0 \sin (\omega t + \phi)$. The symbolic notation of the above arrangement is A•—(~)—•B. We do not put any + or – sign on the AC source.

5. POWER CONSUMED OR SUPPLIED IN AN AC CIRCUIT:



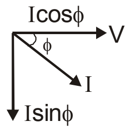
Consider an electrical device which may be a source, a capacitor, a resistor, an inductor or any combination of these. Let the potential difference be $v = V_A - V_B = V_m \sin\omega t$. Let the current through it be $i = I \sin(\omega t + \phi)$. Instantaneous power P consumed by the device $= v\, i = (V_m \sin \omega t)\,(I_m \sin(\omega t + \phi))$

$$\text{Average power consumed in a cycle} = \frac{\int_0^{2\pi/\omega} P dt}{\frac{2\pi}{\omega}} = \frac{1}{2} V_m I_m \cos \phi$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos \phi = V_{rms} I_{rms} \cos \phi.$$

Here $\cos \phi$ is called **power factor**.

Note : I sinϕ is called "wattless current".



5.1 POWER FACTOR

- The factor cos ϕ present in the relation for average power of an ac circuit is called power factor

so $\quad \cos\phi = \dfrac{P_{ac}}{E_{rms}I_{rms}} = \dfrac{P_{av}}{P_v}$

Thus, ratio of average power and virtual power in the circuit is equal to power factor.

- Power factor is also equal to the ratio of the resistance and the impedance of the ac circuit.

Thus, $\quad \cos\phi = \dfrac{R}{Z}$

- Power factor depends upon the nature of the components used in the circuit.
- If a pure resistor is connected in the ac circuit then

$\phi = 0$, $\cos\phi = 1$

$\therefore \quad P_{av} = \dfrac{E_0 I_0}{2} = \dfrac{E_0^2}{2R} = E_{rms}I_{rms}$

Thus the power loss is maximum and electrical energy is converted in the form of heat.

- If a pure inductor or a capacitor are connected in the ac circuit, then

$\phi = \pm 90^\circ$, $\cos\phi = 0$

$\therefore \quad P_{av} = 0$ (minimum)

Thus there is no loss of power.

- If a resistor and an inductor or a capacitor are connected in an ac circuit, then

$\phi \neq 0$ or $\pm 90^\circ$

Thus ϕ is in between 0 & 90º.

- If the components L, C and R are connected in series in an ac circuit, then

$$\tan\phi = \frac{X}{R} = \frac{(\omega L - 1/\omega C)}{R}$$

and $\quad \cos\phi = \dfrac{R}{Z}$

$$= \frac{R}{[R^2 + (\omega L - 1/\omega C)^2]^{1/2}}$$

$\therefore$ Power factor

$$\cos\phi = \frac{R}{Z}$$

- Power factor is a unitless quantity .
- If there is only inductance coil in the circuit, there will be no loss of power and energy will be stored in the magnetic field.
- If a capacitor is only connected in the crcuit, even then there will be no loss of power and energy will be stored in the electrostatic field.

- In reality an inductor and a capacitor do have some resistance, so there is always some loss of power.
- In the state of resonance the power factor is one.

Solved Examples

Example 3. When a voltage $v_s = 200\sqrt{2}$ sin (ωt + 15º) is applied to an AC circuit the current in the circuit is found to be i = 2 sin (ωt + π/4) then average power concsumed in the circuit is

(1) 200 watt (2) 400 $\sqrt{2}$ watt (3) 100 $\sqrt{6}$ watt (4) 200 $\sqrt{2}$ watt

Solution : $P_{av} = v_{rms}\ I_{rms} \cos\phi$

$$= \frac{200\sqrt{2}}{\sqrt{2}}\ \frac{2}{\sqrt{2}} . \cos(30º) = 100\sqrt{6} \text{ watt}$$

6. SOME DEFINITIONS :

The factor **cos ϕ** is called **Power factor**.

I_m sin ϕ is called **wattless current**.

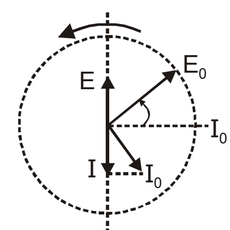
Impedance Z is defined as $Z = \frac{V_m}{I_m} = \frac{V_{rms}}{I_{rms}}$

ωL is called **inductive reactance** and is denoted by X_L.

$\frac{1}{\omega C}$ is called **capacitive reactance** and is denoted by X_C.

7. PHASOR DIAGRAM

It is a diagram in which AC voltages and current are represented by rotating vectors. The phasor represented by a vector of magnitude proportional to the peak value rotate counter clockwise with an angular frequency ω about the origin. The projection of the phasor on vertical axis gives the instantaneous value of the alternating quantity involved. For fig.



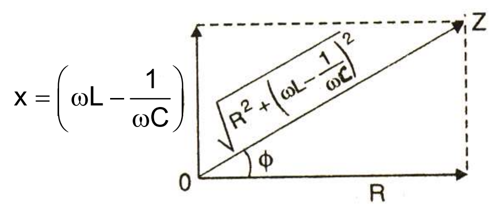
$E = E_0 \sin \omega t$

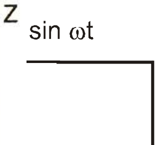
$I = I_0 \sin(\omega t - \pi/2)$

$= -I_0 \cos\omega t$

8. PURELY RESISTIVE CIRCUIT:

Writing KVL along the circuit,





$V_s - IR = 0$

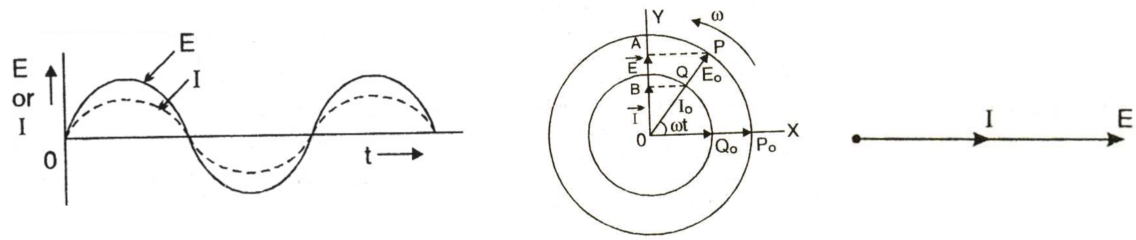
or $I = \frac{V_s}{R} = \frac{V_m \sin\omega t}{R} = I_m \sin \omega t$

$\Rightarrow$ We see that the phase difference between potential difference across resistance, V_R and I_R is 0.

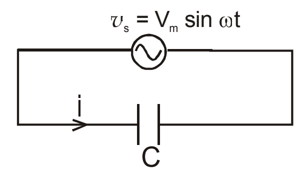
$I_m = \frac{V_m}{R}$ $\Rightarrow$ $I_{rms} = \frac{V_{rms}}{R}$

$\langle P \rangle = V_{rms} I_{rms} \cos\phi = \frac{V_{rms}^2}{R}$

Graphical and vector representations of E and I are shown below :



9. PURELY CAPACITIVE CIRCUIT:

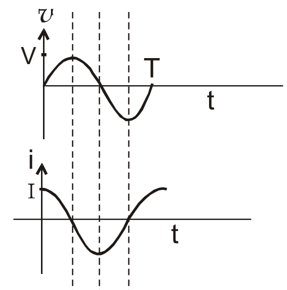


Writing KVL along the circuit,

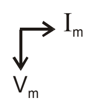
$v_s - \frac{q}{C} = 0$

or $i = \frac{dq}{dt} = \frac{d(Cv)}{dt} = \frac{d(CV_m \sin\omega t)}{dt} = CV_m\omega\cos\omega t = \frac{V_m}{1/\omega C}\cos\omega t = \frac{V_m}{X_C}\cos\omega t = I_m\cos\omega t.$

$X_C = \frac{1}{\omega C}$ and is called capacitive reactance. Its unit is ohm Ω.

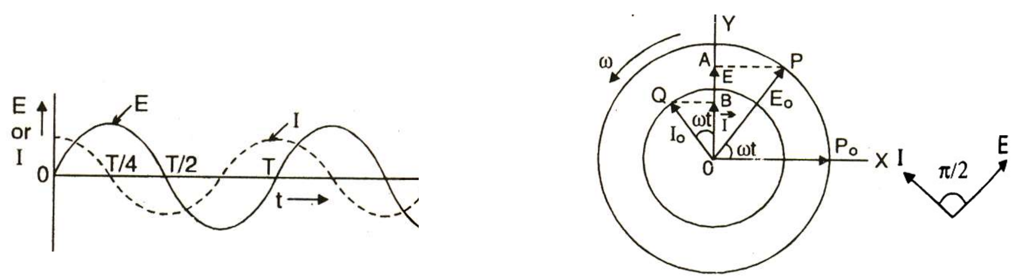


From the graph of current versus time and voltage versus time , it is clear that current attains its peak value at a time $\frac{T}{4}$ before the time at which voltage attains its peak value. Corresponding to $\frac{T}{4}$ the phase difference

$= \omega\Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$. i_C leads v_C by π/2 Diagrammatically (phasor diagram) it is represented as  .

Since $\phi = 90^o$, $\langle P \rangle = V_{rms} I_{rms} \cos\phi = 0$

The graphical and vector representations of E and I are shown in the following figures :



Solved Examples

Example 4. An alternating voltage E = 200 $\sqrt{2}$ sin (100 t) V is connected to a 1μF capacitor through an ac ammeter (it reads rms value). What will be the reading of the ammeter?

Solution : Comparing E = 200 $\sqrt{2}$ sin (100 t) with E = E_0 sin ωt we find that,

$E_0 = 200\sqrt{2}$ V and ω = 100 (rad/s)

So, $X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4\,\Omega$

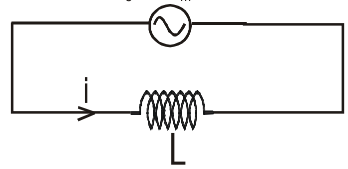
And as ac instruments reads rms value, the reading of ammeter will be,

$$I_{rms} = \frac{E_{rms}}{X_C} = \frac{E_0}{\sqrt{2}X_C} \qquad \left[\text{as } E_{rms} = \frac{E_0}{\sqrt{2}}\right]$$

i.e. $$I_{rms} = \frac{200\sqrt{2}}{\sqrt{2}\times 10^4} = 20\text{mA}$$ **Ans**

10. PURELY INDUCTIVE CIRCUIT:

Writing KVL along the circuit,

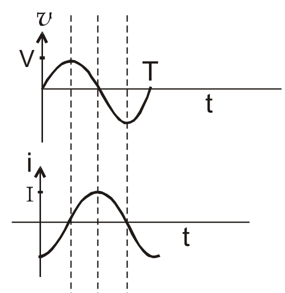


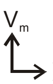
$$v_s - L\frac{di}{dt} = 0 \quad \Rightarrow \quad L\frac{di}{dt} = V_m \sin \omega t$$

$$\int L\,di = \int V_m \sin\omega t\, dt \qquad i = -\frac{V_m}{\omega L}\cos \omega t + C$$

$$\langle i \rangle = 0 \quad \Rightarrow \quad C = 0$$

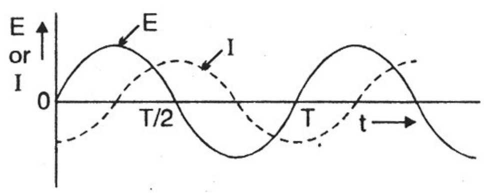
$$\therefore\ i = -\frac{V_m}{\omega L}\cos\omega t \quad \Rightarrow \quad I_m = \frac{V_m}{X_L}$$

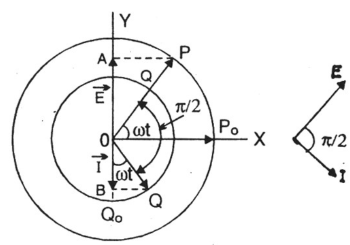


From the graph of current versus time and voltage versus time , it is clear that voltage attains its peak value at a time $\frac{T}{4}$ before the time at which current attains its peak value. Corresponding to $\frac{T}{4}$ the phase difference = $\omega\Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$. Diagrammatically (phasor diagram) it is represented as  . i_L lags behind v_L by $\pi/2$.

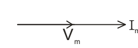
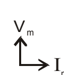
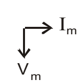
Since $\phi = 90^o$, $\langle P \rangle = V_{rms}I_{rms}\cos \phi = 0$

Graphical and vector respresentations of E and I are shown in the following figures :

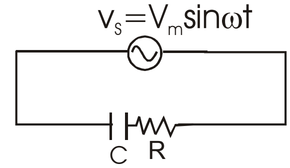




Summary :

AC source connected with	ϕ		Z	Phasor Diagram
Pure Resistor	0	v_R is in same phase with i_R	R	
Pure Inductor	$\pi/2$	v_L leads i_L	X_L	
Pure Capacitor	$\pi/2$	v_C lags i_C	X_C	

11. RC SERIES CIRCUIT WITH AN AC SOURCE :



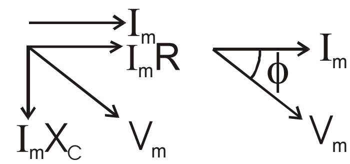
Let $i = I_m \sin(\omega t + \phi)$ $\Rightarrow$ $v_R = iR = I_m R \sin(\omega t + \phi)$

$v_C = I_m X_C \sin(\omega t + \phi - \frac{\pi}{2})$ $\Rightarrow$ $v_S = v_R + v_C$

or $V_m \sin(\omega t + \phi) = I_m R \sin(\omega t + \phi) + I_m X_C \sin(\omega t + \phi - \frac{\pi}{2})$

$$V_m = \sqrt{(I_m R)^2 + (I_m X_C)^2 + 2(I_m R)(I_m X_C)\cos\frac{\pi}{2}}$$

OR $I_m = \dfrac{V_m}{\sqrt{R^2 + X_C^2}}$ $\Rightarrow$ $Z = \sqrt{R^2 + X_C^2}$

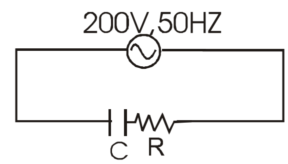


Using phasor diagram also we can find the above result.

$\tan\phi = \dfrac{I_m X_C}{I_m R} = \dfrac{X_C}{R}$, $X_C = \dfrac{1}{\omega C}$.

Solved Examples

Example 5. In an RC series circuit, the rms voltage of source is 200V and its frequency is 50 Hz.If R =100 Ω and C= $\frac{100}{\pi}$ μF, find



(i) Impedance of the circuit	(ii) Power factor angle
(iii) Power factor	(iv) Current
(v) Maximum current	(vi) voltage across R
(vii) voltage across C	(viii) max voltage across R
(ix) max voltage across C	(x) $< P >$
(xi) $< P_R >$	(xii) $< P_C >$

Solution : $X_C = \dfrac{10^6}{\frac{100}{\pi}(2\pi 50)} = 100\ \Omega$

(i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + (100)^2} = 100\sqrt{2}\ \Omega$

(ii) $\tan\phi = \dfrac{X_C}{R} = 1$ $\therefore \phi = 45^\circ$

(iii) Power factor= $\cos\phi = \dfrac{1}{\sqrt{2}}$

(iv) Current $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{200}{100\sqrt{2}} = \sqrt{2}$ A

(v) Maximum current $= I_{rms}\sqrt{2} = 2$A

(vi) voltage across R=$V_{R,rms} = I_{rms}R = \sqrt{2} \times 100$ Volt

(vii) voltage across C=$V_{C,rms} = I_{rms}X_C = \sqrt{2} \times 100$ Volt

(viii) max voltage across R= $\sqrt{2}\ V_{R,rms}$ = 200 Volt

(ix) max voltage across C= $\sqrt{2}\ V_{C,rms}$ = 200 Volt

(x) $<P> = V_{rms} I_{rms} \cos\phi = 200 \times \sqrt{2} \times \dfrac{1}{\sqrt{2}} = 200$ Watt

(xi) $<P_R> = I_{rms}^2 R$ = 200 W

(x) $<P_C>$ = 0

Example 6. In the above question if $v_s(t) = 200\sqrt{2}\ \sin(2\pi\, 50\, t)$, find (a) i (t), (b) v_R and (c) v_C (t)

Solution :
(a) $i(t) = I_m \sin(\omega t + \phi)$ $= 2 \sin(2\pi\, 50\, t + 45^o)$
(b) $v_R = i_R \cdot R$ $= i(t)\, R$ $= 2 \times 100 \sin(100\, \pi t + 45^o)$
(c) $v_C\ (t) = i_C X_C$ (with a phase lag of 90º) $= 2 \times 100 \sin(100\, \pi t + 45 - 90)$

Example 7. An ac source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to ω/3 (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency ω.

Solution : According to given problem,

$$I = \frac{V}{Z} = \frac{V}{[R^2 + (1/C\omega)^2]^{1/2}} \qquad ...\ (1)$$

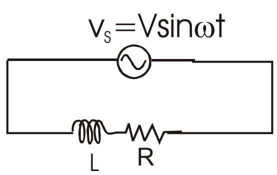
and, $$\frac{I}{2} = \frac{V}{[R^2 + (3/C\omega)^2]^{1/2}} \qquad ...(2)$$

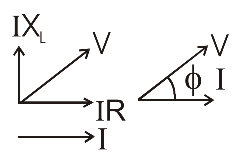
Substituting the value of I from Equation (1) in (2),

$$4\left(R^2 + \frac{1}{C^2\omega^2}\right) = R^2 + \frac{9}{C^2\omega^2}\ .\ \text{ i.e., } \frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So that, $$\frac{X}{R} = \frac{(1/C\omega)}{R} = \frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R} = \sqrt{\frac{3}{5}} \qquad \textbf{Ans.}$$

12. LR SERIES CIRCUIT WITH AN AC SOURCE :





From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R)^2 + (X_L)^2} = I\,Z \quad \Rightarrow \quad \tan\phi = \frac{IX_L}{IR} = \frac{X_L}{R}$$

Solved Examples

Example 8. A $\frac{9}{100\pi}$ H inductor and a 12 ohm resistance are connected in series to a 225 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage.

Solution : Here $X_L = \omega L = 2\pi\, f\, L = 2\pi \times 50 \times \frac{9}{100\pi} = 9\ \Omega$

So, $Z = \sqrt{R^2 + X_L^{\ 2}} = \sqrt{12^{\ 2} + 9^{\ 2}} = 15\ \Omega$

So (a) $I = \frac{V}{Z} = \frac{225}{15} = 15$ A **Ans**

and (b) $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{9}{12}\right)$

$= \tan^{-1} 3/4 = 37^o$

i.e., the current will lag the applied voltage by 37º in phase. **Ans**

Example 9. When an inductor coil is connected to an ideal battery of emf 10 V, a constant current 2.5 A flows. When the same inductor coil is connected to an AC source of 10 V and 50 Hz then the current is 2A. Find out inductance of the coil .

Solution : When the coil is connected to dc source, the final current is decided by the resistance of the coil .

$\therefore \quad r = \frac{10}{2.5} = 4\ \Omega$

When the coil is connected to ac source, the final current is decided by the impedance of the coil .

$\therefore \quad Z = \frac{10}{2} = 5\ \Omega$

But $\quad Z = \sqrt{(r)^2 + (X_L)^2} \qquad X_L^{\,2} = 5^2 - 4^2 = 9$

$X_L = 3\ \Omega$

$\therefore \quad \omega L = 2\pi fL = 3$

$\therefore \quad 2\pi\, 50\, L = 3$

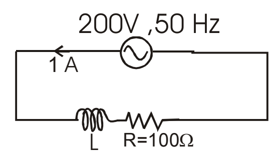
$\therefore \quad L = 3/100\pi$ Henry

Example 10. A bulb is rated at 100 V,100 W , it can be treated as a resistor .Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.

Solution : From the rating of the bulb , the resistance of the bulb is $R = \frac{V_{rms}{}^2}{P} = 100\ \Omega$

For the bulb to be operated at its rated value the rms current through it should be 1A

Also, $\quad I_{rms} = \frac{V_{rms}}{Z}$



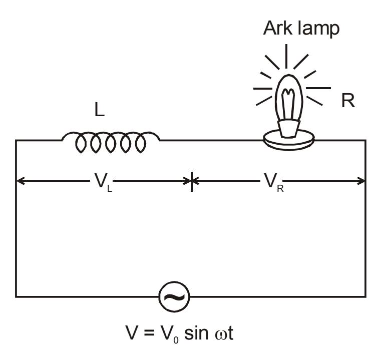
$\therefore \quad 1 = \frac{200}{\sqrt{100^2 + (2\pi 50L)^2}} \qquad L = \frac{\sqrt{3}}{\pi}$ H

Example 11. A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The arc lamp has an effective resistance of 5 Ω when running of 10 A (rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (dc), what additional resistance is required? Compare the power losses in both cases.

Solution : As for lamp $V_R = IR = 10 \times 5 = 50$ V, so when it is connected to 160 V ac source through a choke in series,

$V^2 = V_R^{\,2} + V_L^{\,2}, \qquad V_L = \sqrt{160^2 - 50^2} = 152$ V

and as, $\qquad V_L = IX_L = I\omega L = 2\pi fLI$



So, $\quad L = \frac{V_L}{2\pi fI} = \frac{152}{2 \times \pi \times 50 \times 10} = 4.84 \times 10^{-2}$ H **Ans.**

Now the lamp is to be operated at 160 V dc; instead of choke if additional resistance r is put in series with it,

$V = I(R + r)$, i.e., $160 = 10(5 + r)$

i.e., $\quad r = 11\ \Omega$ **Ans.**

In case of ac, as choke has no resistance, power loss in the choke will be zero while the bulb will consume,

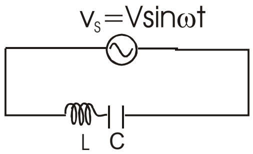
$P = I^2 R = 10^2 \times 5 = 500$ W

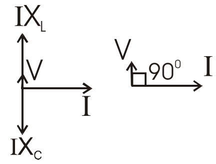
However, in case of dc as resistance r is to be used instead of choke, the power loss in the resistance r will be.

$P_L = 10^2 \times 11 = 1100$ W

while the bulb will still consume 500 W, i.e., when the lamp is run on resistance r instead of choke more than double the power consumed by the lamp is wasted by the resistance r.

13. LC SERIES CIRCUIT WITH AN AC SOURCE :

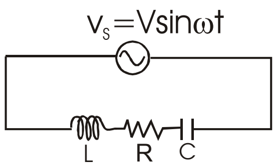


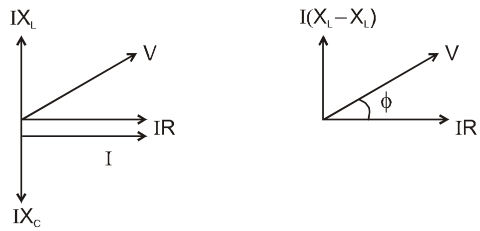


From the phasor diagram

$V = I|(X_L - X_C)| = I\,Z$ $\qquad \phi = 90°$

14. RLC SERIES CIRCUIT WITH AN AC SOURCE :





From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2} = I\,Z \qquad Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

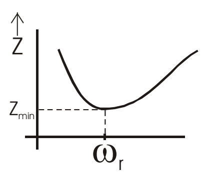
$$\tan\phi = \frac{I(X_L - X_C)}{IR} = \frac{(X_L - X_C)}{R}$$

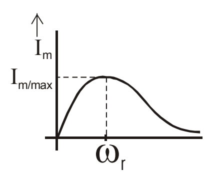
14.1 Resonance :

Amplitude of current (and therefore I_{rms} also) in an RLC series circuit is maximum for a given value of V_m and R , if the impedance of the circuit is minimum, which will be when X_L-X_C =0. This condition is called **resonance**.

So at resonance: X_L-X_C =0.

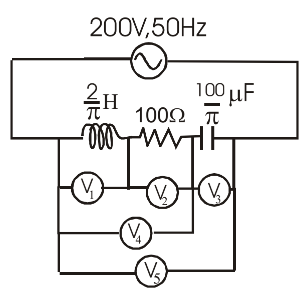
or $\omega L = \frac{1}{\omega C}$ or $\omega = \frac{1}{\sqrt{LC}}$. Let us denote this ω as ω_r.





Solved Examples

Example 12. In the circuit shown in the figure , find
(a) the reactance of the circuit .
(b) impedance of the circuit
(c) the current
(d) readings of the ideal AC voltmeters (these are hot wire instruments and read rms values, they act on heating effect).



Solution : (a) $X_L = 2\pi f L = 2\pi \times 50 \times \frac{2}{\pi} = 200\ \Omega$

$$X_C = \frac{1}{2\pi 50 \frac{100}{\pi} \times 10^{-6}} = 100\ \Omega$$

∴ The reactance of the circuit X = X_L–X_C = 200-100 = 100 Ω

Since X_L > X_C ,the circuit is called inductive.

(b) impedance of the circuit $Z = \sqrt{R^2 + X^2} = \sqrt{100^2 + 100^2} = 100\sqrt{2}\ \Omega$

(c) the current $I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{100\sqrt{2}} = \sqrt{2}$ A

(d) readings of the ideal voltmeter

V_1: $I_{rms}X_L = 200\sqrt{2}$ Volt

V_2: $I_{rms}R = 100\sqrt{2}$ Volt

V_3: $I_{rms}X_c = 100\sqrt{2}$ Volt

V_4: $I_{rms}\sqrt{R^2 + X_L^2} = 100\sqrt{10}$ Volt

V_5: $I_{rms}Z = 200$ Volt ,which also happens to be the voltage of source.

15. ADMITTANCE, SUSCEPTANCE AND CONDUCTANCE

- **Admittance :**

(a) The reciprocal of the impedance of an ac circuit is called admittance. It is represented by Y .

$\therefore$ Admittance = $\frac{1}{\text{Impedance}}$

$Y = \frac{1}{Z}$

(b) The unit of admittance is $(\text{ohm})^{-1}$ or mho.

- **Susceptance :**

(a) The reciprocal of the reactance of an ac circuit is called susceptance. It is represented by S.

$\therefore$ Susceptance = $\frac{1}{\text{Reactance}}$

or $S = \frac{1}{X}$

(b) The unit of susceptance is $(\text{ohm})^{-1}$ or mho.

(c) The susceptance of a coil of inductance L is called inductive susceptance. It is equal to the reciprocal of inductive reactance.

$\therefore$ Inductive susceptance = $\frac{1}{\text{Inductive reactance}}$

(d) The susceptance of a capacitor of capacitance C is called capacitive susceptance. It is equal to the reciprocal of capacitive reactance.

$\therefore$ Capacitive susceptance = $\frac{1}{\text{Capacitive reactance}}$

$$S_C = \frac{1}{X_C} = \frac{1}{1/\omega C} = \omega C \text{ mho}$$

- **Conductance :**

(a) The reciprocal of resistance of a circuit is called conductance. It is represented by G.

$\therefore$ Conductance = $\frac{1}{\text{Resistance}}$

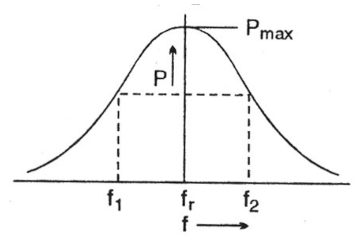
or $G = \frac{1}{R}$

(b) The unit of conductivity is also $(\text{ohm})^{-1}$ or mho.

In the circuit in which different components are connected in parallel and same emf is applied on them its analysis in terms of admittance, susceptance and conductance becomes simpler because current in a component = voltage/(Impedance or Reactance or Resistance) = Voltage × (Admittance or Susceptance or Conductance)

16. HALF-POWER POINTS OR FREQUENCIES, BAND WIDTH & QUALITY FACTOR OF A SERIES RESONANT CIRCUIT

(A) Half power frequencies



- The frequencies at which the power in the circuit is half of the maximum power (the power at resonance), are called half-power frequencies. Thus at these freuencies

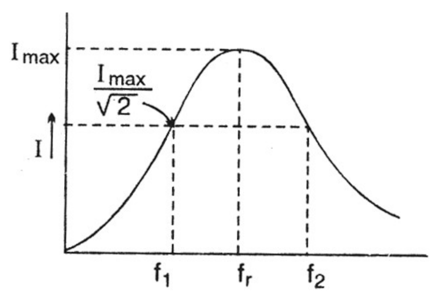
$$P = \frac{P_{max}}{2}$$

- The current in the circuit at half-power frequencies is $\frac{1}{\sqrt{2}}$ or 0.707 or 70.7% of the maximum current I_{max} (current at resonance).

Thus $I = \frac{I_{max}}{\sqrt{2}} = 0.707\ I_{max}$

- There are two half power frequencies f_1 and f_2 :

(a) Lower half power frequency (f_1) :
This half power frequency is less than the resonant frequency. At this frequency the circuit is capacitive.



(b) Upper half power frequency (f_2) :
This half-power frequency is greater than the resonant frequency. At this frequency the circuit is inductive.

(B) Band width (Δf) :

- The difference of half-power frequencies f_1 and f_2 is called band-width (Δf)
- Band width $\Delta f = (f_2 - f_1)$
- For series resonant circuit :

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

(C) Quality factor (Q) :

- In an ac circuit Q is defined by the following ratio :

$$Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipation per cycle}}$$

$$= \frac{2\pi}{T} \times \frac{\text{Maximum energy stored}}{\text{Mean power dissipated}}$$

- For an L–C–R series resonant circuit :

$$Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r CR} = \frac{\omega_0}{\Delta\omega} = \frac{2\pi f_0}{(f_2 - f_1)2\pi}$$

- Quality factor in terms of band-width :

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{2\pi f_r}{2\pi(f_2 - f_1)}$$

$$= \frac{f_r}{(f_2 - f_1)} = \frac{f_r}{\Delta f}$$

- Quality factor = $\frac{\text{Resonant frequency}}{\text{Band width}}$

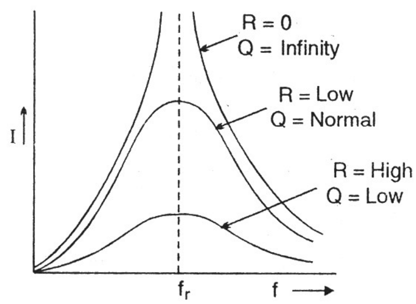
  Thus the ratio of the resonant frequency and the band-width is equal to the quality factor of the circuit.

- In the state of resonance the voltage across the resistor R will be equal to the applied voltage E. The magnitudes of voltage across the inductor and the capacitor will be equal and their values will be equal QE. Thus

$$\therefore \quad V_L = I\omega L = \frac{E}{R}\omega L = EQ$$

and $$V_C = I\left(\frac{I}{\omega C}\right) = \frac{E}{\omega CR} = EQ$$

(D) Sharpness of resonance :



- For an ac circuit Q measures the sharpness of resonance.
- When Q is large, the resonance is sharp and when Q is small, the resonance is flat.
- The sharpness of resonance is inversely proportional to the band-width and the resistance R.
- For resonace to be sharp the resistance of the circuit should be small.

17. FORM FACTOR

- Form factor for a sinusoidal current is defined as :

$$\text{Form factor} = \frac{\text{rms value of ac}}{\text{Average value of positive half cycle}} = \frac{I_{rms}}{2I_0/\pi}$$

$$= \frac{I_0}{\sqrt{2}} \cdot \frac{\pi}{2I_0} = \frac{\pi}{2\sqrt{2}}$$

- Similarly form factor for a sinusoidal voltage :

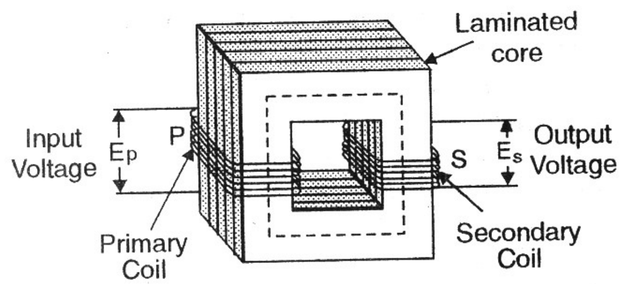
$$F = \frac{\text{rms value of alternating voltage}}{\text{Average value of positive half cycle}} = \frac{\pi}{2\sqrt{2}}$$

18. TRANSFORMER

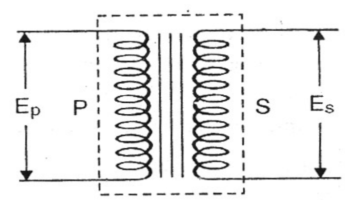
- It is an instrument which changes the magnitude of alternating voltage or current.
- The magnitude of D.C. voltage or current cannot be changed by it.
- It works with alternating current but not with direct current.
- It converts magnetic energy into electrical energy.
- It works on the principle of electro-magnetic induction.
- It consists of two coils :

  (a) Primary coil : in which input voltage is applied.

  (b) Secondary coil : from which output voltage is obtained.



- The frequency of the output voltage produced by the transformer is same as that of input voltage, i.e., frequency remains unchanged.



- Transformer core is laminated and is made of soft iron.
- Let the number of turns in the primary coil be n_p and voltage applied to it be E_p and the number of turns in the secondary coil be n_s and voltage output be E_s, then

$$\frac{E_s}{E_p} = \frac{n_s}{n_p} = K$$

Thus the ratio of voltage obtained in the secondary coil to the voltage applied in the primary coil is equal to the ratio of number of turns of respective coils. This ratio is represented by K and it called transformer ratio.

- If $n_s > n_p$, then $E_s > E_p$ and $K > 1$. The transformer is called step-up transformer.
- If $n_s < n_p$, then $E_s < E_p$ and $K < 1$. The transformer is called step-down transformer.
- In ideal transformer

  Input power = output power

  $E_pI_p = E_sI_s$

  where i_p – current in primary coil

  I_s – current in secondary coil

  or $\frac{I_p}{I_s} = \frac{E_s}{E_p} = \frac{n_s}{n_p} = K$ or $\frac{I_s}{I_p} = \frac{E_p}{E_s} = \frac{1}{K}$

Thus the ratio of currents in the secondary coil and the primary coil is inverse of the ratio of respective voltages.

- As the voltage changes by the transformer, the current changes in the same ratio but in opposite sense, i.e., the current decreases with the increase of voltage and similarly the current increases with the decrease of voltage. Due to this reason the coil in which voltage is lesser, the current will be higher and therefore this coil is thicker in comparison to the other coil so that it can bear the heat due to flow of high current.

- In step-up transformer

  $n_s > n_p$, $K > 1$ $\therefore$ $E_s > E_p$ and $I_s < I_p$

  and in step down transformer

  $n_s < n_p$, $K < 1$ $\therefore$ $E_s < E_p$ and $I_s > I_p$

- If Z_p and Z_s are impedances of primary and secondary coils respectively, then

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{n_s}{n_p} = \sqrt{\frac{Z_s}{Z_p}}$$

- Law of conservation of energy is applicable in the transformer.

- Efficiency of tranformer $\frac{\text{Power obtained from secondary coil}}{\text{Power applied in primary coil}} \times 100\%$

  Generally the efficiency of transformers is found in between 90% to 100%.

- Energy losses in transformers : Losses of energy are due to following reasons :
  (a) Copper losses due to resistance of coils
  (b) Eddy current losses in core.
  (c) Hysteresis losses in core.
  (d) Flux leakage due to poor linking of magnetic flux.

- **Uses of transformer** :
  (a) Step down and step up transformer are used in electrical power distribution.
  (b) Audio frequency transformer are used in radiography, television, radio, telephone etc.
  (c) Ratio frequency transformer are used in radio communication.
  (d) Transformers are also used in impedance matching.

Solved Examples

Example 13.

A 50 Hz a.c. current of crest value 1A flows through the primary of a transformer. If the mutual inductance between the primary and secondary be 1.5 H, the crest voltage induced in secondary is

(1) 75V (2) 150V (3) 225V (4) 300V

Sol. The crest value is attained in T/4 time where T is the time period of A.C.

Thus dI = 1A in dt = T/4 sec.

$T = \frac{1}{50}$ or $dt = \frac{1}{200}$

The induced emf is $|E_2| = M\frac{dI_1}{dt}$

$= 1.5 \times \frac{1}{(1/200)} = 1.5 \times 200 = 300$ V

The correct answer is (4)

Problem 1. The peak voltage in a 220 V AC source is

(1) 220 V (2) about 160 V

(3) about 310 V (4) 440 V

Solution : $V_0 = \sqrt{2}\ V_{rms} = \sqrt{2} \times 220 \simeq 310$ V

Ans is (3)

Problem 2. An AC source is rated 220 V, 50 Hz. The average voltage is calculated in a time interval of 0.01 s. It

(1) must be zero (2) may be zero

(3) is never zero (4) is $(220/\sqrt{2})$V

Solution : May be zero

Ans. is (2)

Problem 3. Find the effective value of current $i = 2 + 4 \cos 100\pi t$.

Solution : $I_{rms} = \left[\int_0^T \frac{(2+4\cos 100\pi t)^2 dt}{T}\right]^{1/2} = 2\sqrt{3}$

Problem 4. The peak value of an alternating current is 5 A and its frequency is 60 Hz. Find its rms value. How long will the current take to reach the peak value starting from zero?

Solution : $I_{RMS} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}$ A, $t = \frac{T}{4} = \frac{1}{240}$ s

Problem 5. An alternating current having peak value 14 A is used to heat a metal wire. To produce the same heating effect, a constant current i can be used where i is

(1) 14 A (2) about 20 A

(3) 7 A (4) about 10 A

Solution : $I_{RMS} = \frac{I_0}{\sqrt{2}} = \frac{14}{\sqrt{2}} \simeq 10$ Ans. is (4)

Problem 6. Find the average power concumed in the circuit if a voltage $v_s = 200\sqrt{2}$ sin ωt is applied to an AC circuit and the current in the circuit is found to be $i = 2 \sin(\omega t + \pi/4)$.

Solution : $P = V_{RMS} I_{RMS} \cos\phi = \frac{200\sqrt{2}}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \times \cos\frac{\pi}{4} = 200$ W

Problem 7. A capacitor acts as an infinite resistance for

(1) DC (2) AC

(3) DC as well as AC (4) neither AC nor DC

Solution : $x_C = \frac{1}{\omega C}$ for DC $\omega = 0$. so, $x_C = \infty$

Ans. is (1)

Problem 8. A 10 μF capacitor is connected with an ac source $E = 200\sqrt{2}\sin(100t)$ V through an ac ammeter (it reads rms value). What will be the reading of the ammeter?

Solution : $I_0 = \frac{V_0}{x_C} = \frac{200\sqrt{2}}{1/\omega C}$; $I_{RMS} = \frac{I_0}{\sqrt{2}} = 200$ mA

Problem 9. Find the reactance of a capacitor (C = 200 μF) when it is connected to (a) 10 Hz AC source, (b) a 50 Hz AC source and (c) a 500 Hz AC source.

Solution : (a) $x_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} \simeq 80\ \Omega$ for f = 10 Hz AC source,

(b) $x_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} \simeq 16\ \Omega$ for f = 50 Hz and

(c) $x_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} \simeq 1.6\ \Omega$ for f= 500 Hz.

Problem 10. An inductor (L = 200 mH) is connected to an AC source of peak current. What is the instantaneous voltage of the source when the current is at its peak value?

Solution : Because phase difference between voltage and current is $\pi/2$ for pure inductor.

So, Ans. is zero

Problem 11. An AC source producing emf $\xi = \xi_0[\cos(100\ \pi\ s^{-1})t + \cos(500\ \pi\ s^{-1})t]$is connected in series with a capacitor and a resistor. The current in the circuit is found to be $i = i_1 \cos[(100\ \pi\ s^{-1})t + \varphi_1]+i_2 \cos[(500\ \pi\ s^{-1})t+ \varphi_1]$

(1) $i_1 > i_2$ (2) $i_1 = i_2$

(3) $i_1 < i_2$ (4) the information is insufficient to find the relation between i_1 and i_2

Solution : Impedence z is given by $z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$

For higher ω, z will be lower so current will be higher

Ans is (3)

Problem 12. An alternating voltage of 220 volt r.m.s. at a frequency of 40 cycles/sec is supplied to a circuit containing a pure inductance of 0.01 H and a pure resistance of 6 ohms in series. Calculate (i) the current, (ii) potential difference across the resistance, (iii) potential difference across the inductance, (iv) the time lag, (v) power factor.

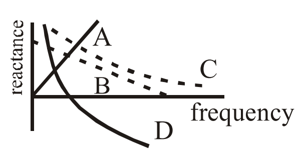
Solution : (i) $z = \sqrt{(\omega L)^2 + R^2} = \sqrt{(2\pi \times 40 \times 0.01^2)^2 + 6^2} = \sqrt{(42.4)}$

$I_{RMS} = \frac{220}{z} = 33.83$ amp.

(ii) $V_{RMS} = I_{RMS} \times R = 202.98$ volts (iii) $\omega L \times I_{RMS} = 96.83$ volts

(iv) $t = T\frac{\phi}{2\pi} = 0.01579$ sec (v) $\cos\phi = \frac{R}{Z} = 0.92$

Problem 13. Which of the following plots may represent the reactance of a series LC combination ?



Ans. D

Problem 14. A series AC circuit has resistance of 4 Ω and a reactance of 3 Ω. the impedance of the circuit is

(1) 5 Ω (2) 7 Ω

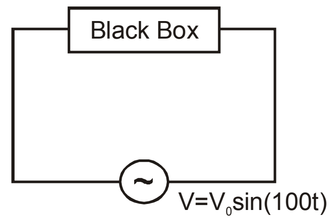
(3) 12/7 Ω (4) 7/12 Ω

Solution : $Z = \sqrt{4^2 + 3^2} = 5\ \Omega$ Ans. is (A)

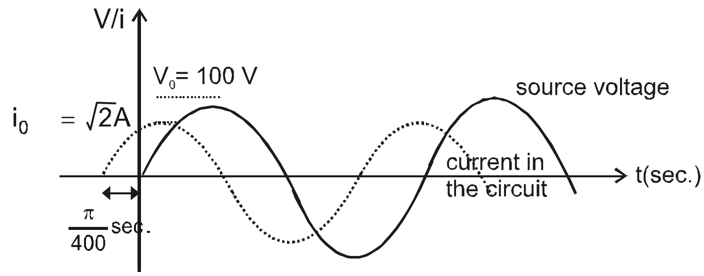
Problem 15.

Comprehension - 1

A voltage source $V = V_0 \sin(100\,t)$ is connected to a black box in which there can be either one element out of L, C, R or any two of them connected in series.



At steady state. the variation of current in the circuit and the source voltage are plotted together with time, using an oscilloscope, as shown



1. The element(s) present in black box is/are :

(1) only C (2) L C (3) L and R (4*) R and C

Sol. As current is leading the source voltage, so circuit should be capacitive in nature and as phase difference is not $\frac{\pi}{2}$, it must contain resistor also.

2. Values of the parameters of the elements, present in the black box are -

(1*) R = 50Ω , C = 200 μf (2) R = 50Ω , L = 2mμ

(3) R = 400 Ω , C = 50 μ f (4) None of these

Sol. Time delay = $\frac{\phi}{\omega} = \frac{\pi}{400} \Rightarrow \phi = \frac{\pi}{4}$

$$\tan^{-1}\left(\frac{1}{R\omega C}\right) = \frac{\pi}{4} \Rightarrow \frac{1}{\omega C} = R$$

$$i_0 = \frac{V_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

$$\sqrt{2} = \frac{100}{\sqrt{R^2 + R^2}} \rightarrow R = 50\ \Omega$$

$$\text{and } C = \frac{1}{50 \times 100} = 200\ \mu F$$

3. If AC source is removed, the circuit is shorted for some time so that capacitor is fully discharged and then a battery of constant EMF is connected across the black box. At t = 0, the current in the circuit will -

(1) increase exponentially with time constant = 0.02 sec.

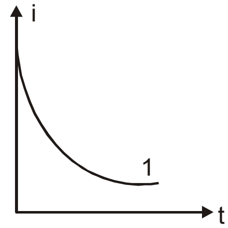
(2*) decrease exponentially with time constant = 0.01 sec.

(3) oscillate with angular frequency 20 rad/sec

(4) first increase and then decrease

Sol. For DC circuit

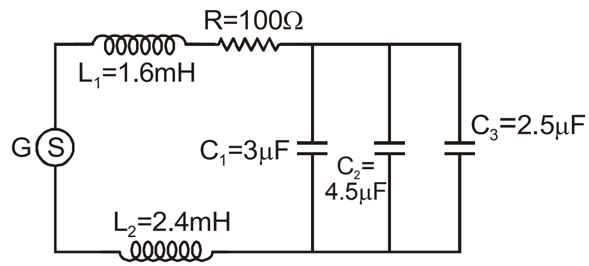
$i = i_0 e^{-\frac{t}{RC}}$ and RC = 0.01 sec.



Problem 16.

Comprehension-2

An ac generator G with an adjustable frequency of oscillation is used in the circuit, as shown.



1. Current drawn from the ac source will be maximum if its angular frequency is -

(1) 10^5 rad/s (2) 10^4 rad/s (3*) 5000 rad/s (4) 500 rad/s

Sol. Current drawn is maximum at resonant angular frequency

L_{eq} = 4 mH C_{eq} = 10 μF

$$\omega = \frac{1}{\sqrt{LC}} = 5000 \text{ rad/s}$$

2. To increase resonant frequency of the circuit, some of the changes in the circuit are carried out. Which change(s) would certainly result in the increase in resonant frequency ?

(1) R is increased.

(2) L_1 is increased and C_1 is decreased.

(3) L_2 is decreased and C_2 is increased.

(4*) C_3 is removed from the circuit.

Sol. (4) C_{eq} decreases thereby increasing resonant frequency.

3. If the ac source G is of 100 V rating at resonant frequency of the circuit, then average power supplied by the source is -

(1) 50 W (2*) 100 W (3) 500 W (4) 1000 W

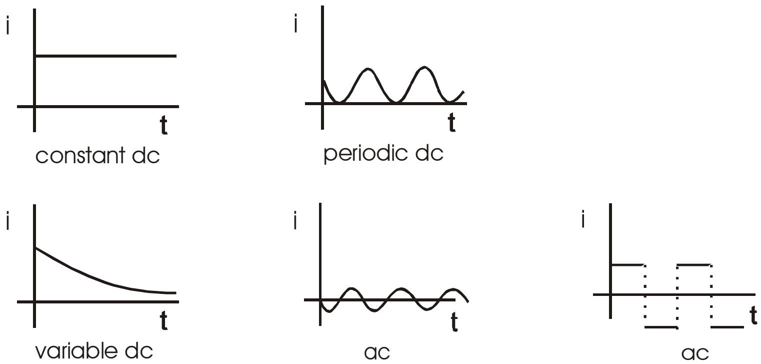
Sol. At resonance $i_{rms} = \frac{100}{100} = 1A$

Power supplied = $V_{rms} I_{rms} \cos\phi$ (ϕ = 0 at resonance) P = 100 W

KEY CONCEPT

☞ **AC and DC Current :**

A current that changes its direction periodically is called alternating current (AC). If a current maintains its direction constant it is called direct current (DC).



☞ **Average Value** :

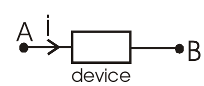
Average value of a function, from t_1 to t_2, is defined as <f> = $\dfrac{\int_{t_1}^{t_2} f.dt}{t_2 - t_1}$.

☞ **Root Mean Square Value:**

Root Mean Square Value of a function, from t_1 to t_2, is defined as $f_{rms} = \sqrt{\dfrac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}}$.

☞ **Power Consumed or Supplied in an ac Circuit**:

Instantaneous power P consumed by the device $= V\,i = (V_m \sin \omega t)\,(I_m \sin(\omega t + \phi))$



Average power consumed in a cycle $= \dfrac{\int_0^{\frac{2\pi}{\omega}} P dt}{\frac{2\pi}{\omega}} = \dfrac{1}{2} V_m I_m \cos \phi$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos \phi = V_{rms} I_{rms} \cos \phi.$$

Here cos ϕ is called **power factor**.

☞ **Some Definitions:**

The factor **cos ϕ** is called **Power factor**.

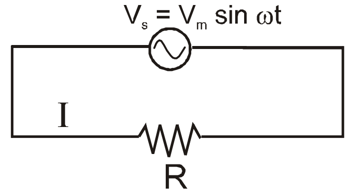
I_m **sin ϕ** is called **wattless current**.

Impedance Z is defined as $Z = \dfrac{V_m}{I_m} = \dfrac{V_{rms}}{I_{rms}}$

ωL is called **inductive reactance** and is denoted by X_L.

$\dfrac{1}{\omega C}$ is called **capacitive reactance** and is denoted by X_C.

☞ **Purely Resistive Circuit:**

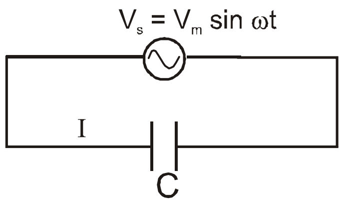


$$I = \frac{V_s}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

$$I_m = \frac{V_m}{R} \qquad \Rightarrow \qquad I_{rms} = \frac{V_{rms}}{R}$$

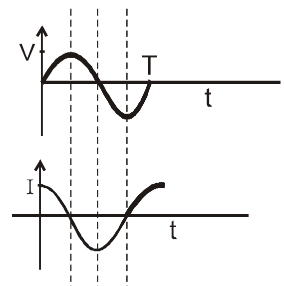
$$\langle P \rangle = V_{rms} I_{rms} \cos \phi = \frac{V_{rms}^{\;2}}{R}$$

☞ **Purely Capacitive Circuit:**

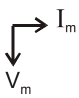


$$I = \frac{dq}{dt} = \frac{d(CV)}{dt} = \frac{d(CV_m \sin \omega t)}{dt} = CV_m \omega \cos \omega t = \frac{V_m}{1/\omega C} \cos \omega t = \frac{V_m}{X_C} \cos \omega t = I_m \cos \omega t.$$

$X_C = \dfrac{1}{\omega C}$ and is called capacitive reactance.



I_C leads V_C by $\pi/2$ Diagrammatically (phasor diagram) it is represented as



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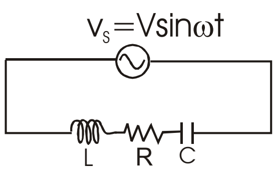
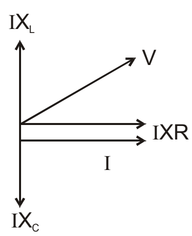
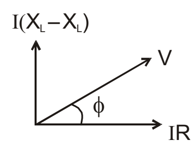
Since $\phi = 90^\circ$, $\langle P \rangle = V_{rms}\, I_{rms} \cos \phi = 0$

☞ **RLC Series Circuit With An ac Source :**







From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2} = I\,Z \qquad Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

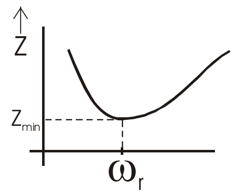
$$\tan \phi = \frac{I(X_L - X_C)}{IR} = \frac{(X_L - X_C)}{R}$$

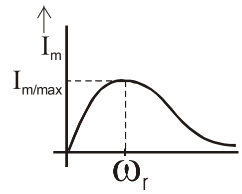
☞ Resonance :

Amplitude of current (and therefore I_{rms} also) in an RLC series circuit is maximum for a given value of V_m and R , if the impedance of the circuit is minimum, which will be when X_L-X_C =0. This condition is called **resonance**.

So at resonance: $X_L - X_C = 0$.

or $\omega L = \frac{1}{\omega C}$ or $\omega = \frac{1}{\sqrt{LC}}$. Let us denote this ω as ω_r.





Quality factor : $Q = \frac{X_L}{R} = \frac{X_C}{R}$

$$Q = \frac{\text{Resonance freq.}}{\text{Band width}} = \frac{\omega_R}{\Delta\omega} = \frac{f_R}{f_2 - f_1}$$

where f_1 & f_2 are half power frequencies.

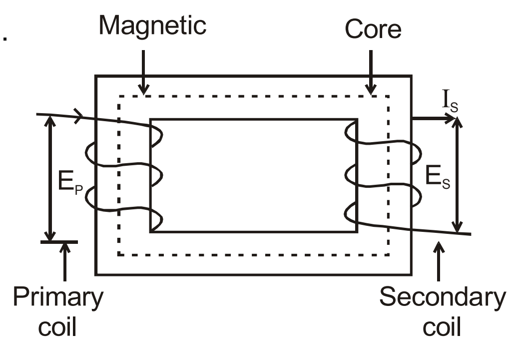
☞ Transformer ;

A transformer changes an alternating potential difference from one value to another of greater or smaller value using the principle of mutual induction . For an ideal tranformer $\frac{E_s}{E_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$, where denotations have their usual meanings.

E_S N and I are the emf, number of turns and current in the coils.

$N_S > N_P \Rightarrow$ $E_S > E_P \rightarrow$ step up transformer.

$N_S < N_P \Rightarrow$ $E_S < E_P \rightarrow$ step down transformer.



Energy Losses In Transformer are due to

1. Resistance of the windings.
2. Eddy Current.
3. Hysteresis.
4. Flux Leakage.