CURRENT ELECTRICITY

1. ELECTRIC CURRENT

(a) Time rate of flow of charge through a cross sectional area is called **Current**.
if Δq charge flows in time interval Δt then average current is given by

$$I_{av} = \frac{\Delta q}{\Delta t} \text{ and}$$

Instantaneous current

$$i =. \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

(b) Direction of current is along the direction of flow of positive charge or opposite to the direction of flow of negative charge. But the current is a scalar quantity.

$\longrightarrow i$ $\qquad\qquad$ $\overset{i}{\longleftarrow}$

$q\oplus \longrightarrow$ velocity $\qquad$ $q\ominus \longrightarrow$ velocity

SI unit of current is ampere and

1 Ampere = 1 coloumb/sec

1 coloumb/sec = 1A

2. CONDUCTOR

In some materials, the outer electrons of each atom or molecule are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons drift in a direction opposite to the field. Such materials are called conductors.

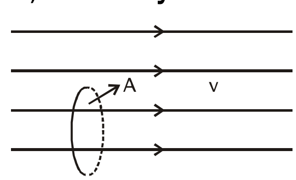
3. INSULATOR

Another class of materials is called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called dielectrics.

4. SEMICONDUCTOR

In semiconductors, the behaviour is like an insulator at low levels of temperature. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A free electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

Current, velocity and current density

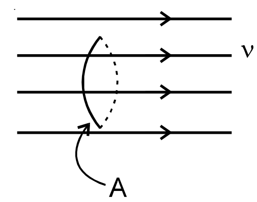


$n \rightarrow$ no. of free charge particles per unit volume
$q \rightarrow$ charge of each free particle
$i \rightarrow$ charge flow per unit time

i = nqvA

Current density, a vector, at a point have magnitude equal to current per unit normal area at that point and direction is along the direction of the current at that point.

$$\vec{J} = \frac{di}{ds}\vec{n}$$



so $\quad di = \vec{J}.d\vec{s}$

Current is flux of current density.

Due to principle of conservation of charge:

Charge entering at one end of a conductor = charge leaving at the other end, so current does not change with change in cross section and conductor remains uncharged when current flows through it.

Solved Examples

Example 1. Find free electrons per unit volume in a metallic wire of density 10^4 kg/m^3, atomic mass number 100 and number of free electron per atom is one.

Solution : Number of free charge particle per unit volume

$$(n) = \frac{\text{total free charge particle}}{\text{total volume}}$$

$\therefore$ Number of free electron per atom means

total free electrons = total number of atoms.

$$= \frac{N_A}{M_W} \times M$$

So $$n = \frac{\frac{N_A}{M_W} \times M}{V} = \frac{N_A}{M_W} \times d = \frac{6.023 \times 10^{23} \times 10^4}{100 \times 10^{-3}}$$

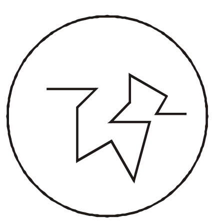
n = 6.023 × 10^{28} m^{-3}

5. MOVEMENT OF ELECTRONS INSIDE CONDUCTOR

All the free electrons are in random motion due to the thermal energy and relationship in given by

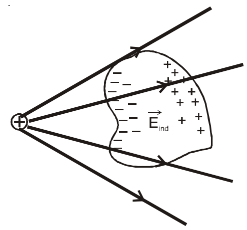
$$\frac{3}{2}KT = \frac{1}{2}mv^2$$

At room temperature its speed is around 10^6 m/sec or 10^3 km/sec

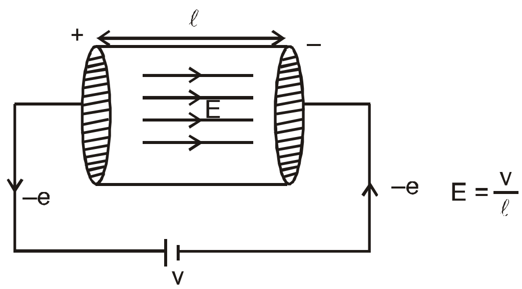


but the average velocity is zero so current in any direction is zero.

When a conductor is placed in an electric field. Then for a small duration electrons, do have an average velocity but its average velocity becomes zero within short interval of time.



When by some means a constant potential difference is applied across the conductor, then the electrons start moving with an acceleration and due to collision with other atoms & electrons, its average velocity becomes nearly constant and is called as drift velocity.



the electric field between the plate = $E = \frac{v}{\ell}$

V_d = drift velocity = average velocity along the wire

hence $i = nAeV_d$

V_d is of the order 10^{-3} m/s

Solved Examples

Example 2. Find the approximate total distance travelled by an electron in the time-interval in which its displacement is one meter along the wire.

Solution : time $= \frac{\text{displacement}}{\text{drift velocity}} = \frac{S}{V_d}$

$\therefore$ $V_d = 1$ mm/s $= 10^{-3}$ m/s (normally the value of drift velocity is 1 mm/s)

$S = 1$ m

time $= \frac{1}{10^{-3}} = 10^3$ s

distance travelled = speed × time

$\therefore$ speed $= 10^6$ m/s

So required distance

$= 10^6 \times 10^3$ m

$= 10^9$ m

6. RELATION BETWEEN I & V IN A CONDUCTOR

In absence of potential difference across a conductor no net current flows through a cross section. When a potential difference is applied across a conductor the charge carriers (electrons in case of metallic conductors) start drifting in a direction opposite to electric field with average drift velocity. If electrons are moving with velocity v_d, A is area of cross section and n is number of free electrons per unit volume then,

$I = nAev_d$.

$v_d = \frac{\lambda}{\tau}$,

$\lambda \rightarrow$ average displacement of electron along the wire between two successive collisions. It is also called **mean free path**.

$\tau \rightarrow$ the time in which the particle does not collide with any other particle and is called as **relaxation time.**

$$\lambda = \frac{1}{2}\left(\frac{eE}{m}\right)\tau^2 = \frac{1}{2}\frac{e\tau^2}{m}.E = \frac{1}{2}\frac{e\tau^2}{m} \times \frac{V}{\ell}$$

$$i = nAe \cdot \frac{1}{2}\frac{e\tau^2}{m} \times \frac{V}{\ell} \times \frac{1}{\tau} = \left(\frac{nAe^2\tau}{2m\ell}\right)V$$

$$i = \frac{nAe^2\tau}{2m\ell}V$$

As temperature (T) $\uparrow$, $\tau \downarrow$

7. ELECTRICAL RESISTANCE

The property of a substance by virtue of which it opposes the flow of electric current through it is termed as electrical resistance. Electrical resistance depends on the size, geometry, temperature and internal structure of the conductor.

We have $\quad i = \dfrac{nAe^2\tau}{2m\ell}V$

Here $\quad i \propto V$

it is known as Ohm's law

$$i = \frac{V}{R}$$

$$R = \frac{2m\ell}{nAe^2\tau}$$

$$V = IR$$

hence $\quad R = \dfrac{2m}{ne^2\tau} \cdot \dfrac{\ell}{A}$

so $\quad$ Here $\quad R = \dfrac{\rho\ell}{A} \quad \Rightarrow \quad V = I \times \dfrac{\rho\ell}{A}$

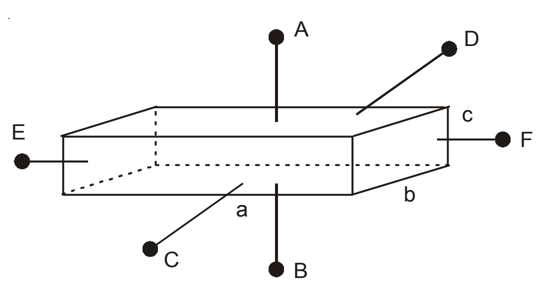
$\Rightarrow \quad \dfrac{V}{\ell} = \dfrac{I}{A}\rho \quad \Rightarrow \quad E = J\rho \quad \Rightarrow \quad J = \dfrac{I}{A}$ = current density

ρ is called resistivity (it is also called specific resistance), and $\rho = \dfrac{2m}{ne^2\tau} = \dfrac{1}{\sigma}$, σ is called conductivity.

Therefore current in conductors is proportional to potential difference applied across its ends. This is **Ohm's Law**. Units: $R \to \text{ohm}(\Omega)$, $\rho \to \text{ohm}-\text{meter}(\Omega - m)$ also called siemens, $\sigma \to \Omega^{-1}m^{-1}$.

Solved Examples

Example 3. The dimensions of a conductor of specific resistance ρ are shown below. Find the resistance of the conductor across AB, CD and EF.



Answer : $R_{AB} = \dfrac{\rho c}{ab}$, $\quad R_{CD} = \dfrac{\rho b}{ac}$, $\quad R_{EF} = \dfrac{\rho a}{bc}$

Solution : For a condition

$$R = \frac{\rho \ell}{A} = \frac{\text{Resistivity} \times \text{length}}{\text{Area of cross section}}$$

$$R_{AB} = \frac{\rho c}{ab}, \; R_{CD} = \frac{\rho b}{ac}, \; R_{EF} = \frac{\rho a}{bc}$$

7.1 Dependence of Resistance on various factors

$$R = \rho \frac{\ell}{A} = \frac{2m}{ne^2 \tau} \cdot \frac{\ell}{A}$$

Therefore R depends as

(1) $\propto \ell$ (2) $\propto \frac{1}{A}$ (3) $\propto \frac{1}{n}$ $\propto \frac{1}{\tau}$

(4) and in metals τ decreases as T increases $\Rightarrow$ R also increases.

Results

(a) On stretching a wire (volume constant)

If length of wire is taken into account then $\frac{R_1}{R_2} = \frac{\ell_1^2}{\ell_2^2}$

If radius of cross section is taken into account then $\frac{R_1}{R_2} = \frac{r_2^4}{r_1^4}$, where R_1 and R_2 are initial and final resistances and ℓ_1, ℓ_2, are initial and final lengths and r_1 and r_2 initial and final radii respectively. (if elasticity of the material is taken into consideration, the variation of area of cross-section is calculated with the help of Young's modulus and Poison's ratio)

(b) Effect of percentage change in length of wire

$$\frac{R_2}{R_1} = \frac{\ell^2 \left[1 + \frac{x}{100}\right]^2}{\ell^2}$$ where ℓ - original length and x- % increment

if x is quite small (say < 5%) then % change in R is

$$\frac{R_2 - R_1}{R_1} \times 100 = \left(\frac{\left(1 + \frac{x}{100}\right)^2 - 1}{1} \right) \times 100 \cong 2x\%$$

Solved Examples

Example 4. If a wire is stretched to double its length, find the new resistance if original resistance of the wire was R.

Solution : As we know that $R = \frac{\rho \ell}{A}$

in case $R' = \frac{\rho\ell'}{A'}$

$\ell' = 2\ell$

$A'\ell' = A\ell$ (volume of the wire remains constant)

$$A' = \frac{A}{2} \Rightarrow \qquad R' = \frac{\rho \times 2\ell}{A/2} = 4\frac{\rho\ell}{A} = 4R$$

Example 5. The wire is stretched to increase the length by 1% find the percentage change in the Resistance.

Solution : As we known that

$$\therefore \qquad R = \frac{\rho\ell}{A}$$

$$\frac{\Delta R}{R} = \frac{\Delta\rho}{\rho} + \frac{\Delta\ell}{\ell} - \frac{\Delta A}{A} \text{ and } \frac{\Delta\ell}{\ell} = -\frac{\Delta A}{A}$$

$$\frac{\Delta R}{R} = O + 1 + 1 = 2$$

Hence percentage increase in the Resistance = 2%

Note :

- Above method is applicable when % change is very small.

Temperature Dependence of Resistivity and Resistance :

The resistivity of a metallic conductor nearly increases with increasing temperature. This is because, with the increase in temperature the ions of the conductor vibrate with greater amplitude, and the collision between electrons and ions become more frequent. Over a small temperature range (upto 100ºC), the resistivity of a metal can be represented approximately by the equation,

$$\rho(T) = \rho_0 [1 + \alpha (T - T_0)] \qquad \text{...(i)}$$

where, ρ_0 is the resistivity at a reference temperature T_0 (often taken as 0ºC or 20ºC) and $\rho(T)$ is the resistivity at temperature T, which may be higher or lower than T_0. The factor α is called the temperature coefficient of resistivity.

The resistance of a given conductor depends on its length and area of cross-section besides the resistivity. As temperature changes, the length and area also change. But these changes are quite small and the factor ℓ/A may be treated as constant.

Then, $\qquad R \propto \rho$

and hence, $\qquad R(T) = R_0 [1 + \alpha(T - T_0)] \qquad$ ...(ii)

In this equation R(T) is the resistance at temperature T and R_0 is the resistance at temperature T_0, often taken to be 0ºC or 20ºC. The temperature coefficient of resistance α is the same constant that appears.

Note :

- The ρ-T equation written above can be derived from the relation,
  α = fractional change in resistivity per unit change in temperature

$$= \frac{d\rho}{\rho dT} = \alpha \qquad \text{or,} \qquad \frac{d\rho}{dT} = \alpha\rho$$

$$\therefore \quad \frac{d\rho}{\rho} = \alpha dT$$ (α can be assumed constant for small temperature variation)

$$\therefore \quad \int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \alpha \int_{T_0}^{T} dT \qquad .....(iii)$$

$$\therefore \quad \ln\left(\frac{\rho}{\rho_0}\right) = \alpha\,(T - T_0)$$

$$\therefore \quad \rho = \rho_0 e^{\alpha(T-T_0)}$$

if $\alpha\,(T - T_0) << 1$ then

$e^{\alpha(T-T_0)}$ can approximately be written as $1 + \alpha(T - T_0)$. Hence,

In the above discussion we have assumed α to be constant. If it is a function of temperature it will come inside the integration in Eq. (iii).

Solved Examples

Example 6. The resistance of a thin silver wire is 1.0 Ω at 20ºC. The wire is placed in liquid bath and its resistance rises to 1.2 Ω. What is the temperature of the bath ? (Here $\alpha = 10^{-2}$ /ºC)

Solution : Here change in resistance is small so we can apply

$$R = R_0(1 + \alpha\Delta\theta)$$

$\Rightarrow \quad 1.2 = 1 \times (1 + 10^{-2}\,\Delta\theta) \qquad \Rightarrow \quad \Delta\theta = 20ºC$

$\Rightarrow \quad \theta - 20 = 20 \qquad \Rightarrow \quad \theta = 40º\ C$ **Ans.**

Example 7. A conductive wire has resistance of 10 ohm at 0ºC, and α is $\frac{1}{273}$ /ºC, then determine its resistance at 273ºC.

Solution : In such a problem, term $\alpha\,\Delta\,T$ will have a larger value so could not be used directly in $R = R_0\,(1 + \alpha\,\Delta\,T)$. We need to go for basics as

As we know that $\alpha = \frac{dR}{RdT}$

$\Rightarrow \quad \int \frac{dR}{R} = \int \alpha dT \qquad \Rightarrow \quad \ell n\frac{R_2}{R_1} = \alpha(T_2 - T_1)$

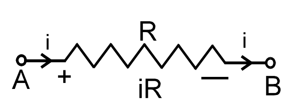
$\Rightarrow \quad R_2 = R_1 e^{\alpha(T_2-T_1)} \qquad \Rightarrow \quad R_2 = 10e^1$

$\Rightarrow \quad R_2 = 10\ e\ \Omega$ **Ans.**

<u>Electric current in resistance</u>

In a resistor current flows from high potential to low potential

High potential is represented by positive (+) sign and low potential



is represented by negative (–) sign.

$V_A - V_B = iR$

If $V_1 > V_2$

then current will flow from A to B

and $i = \dfrac{V_1 - V_2}{R}$

If $V_1 < V_2$

then current will go from B to A and $i = \dfrac{V_2 - V_1}{R}$

Example 8. Calculate current (i) flowing in part of the circuit shown in figure?

Solution : $V_A - V_B = i \times R \;\Rightarrow\; i = \dfrac{6}{2} = 3A$ **Ans.**

8. ELECTRICAL POWER :

Energy liberated per second in a device is called its power. The electrical power P delivered or consumed by an electrical device is given by $P = VI$, where V = Potential difference across the device and

I = Current.

If the current enters the higher potential point of the device then electric power is consumed by it (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e. acts as source).

$$\text{Power} = \frac{V.dq}{dt}$$

$$= V\,I$$

$$\mathbf{P = V\,I}$$

If power is constant then energy = P t

If power is variable then

$$\text{Energy} = \int p\,dt$$

Power consumed by a resistor

$$P = I^2R = VI = \frac{V^2}{R}\ .$$

When a current is passed through a resistor energy is wasted in overcoming the resistance of the wire. This energy is converted into heat.

$$W = VIt = I^2Rt = \frac{V^2}{R}t$$

The heat generated (in joules) when a current of I ampere flows through a resistance of R ohm for t second is given by:

$$H = I^2Rt \text{ Joule} = \frac{I^2Rt}{4.2} \text{ Calorie}$$

1 unit of electrical energy = 1 Kilowatt hour = 1 KWh = 3.6×10^6 Joule.

Example 9. If bulb rating is 100 watt and 220 V then determine

(a) Resistance of filament

(b) Current through filament

(c) If bulb operate at 110 volt power supply then find power consumed by bulb.

Solution : Bulb rating is 100 W and 220 V bulb means when 220 V potential difference is applied between the two ends then the power consumed is 100 W

Here $V = 220$ Volt

$P = 100$ W

$$\frac{V^2}{R} = 100 \qquad \text{So} \qquad R = 484\ \Omega$$

Since Resistance depends only on material hence it is constant for bulb

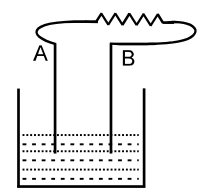
$$I = \frac{V}{R} = \frac{220}{22 \times 22} = \frac{5}{11} \text{ Amp.}$$

power consumed at 110 V

$$\therefore \text{ power consumed} = \frac{110 \times 110}{484} = 25 \text{ W}$$

9. BATTERY (CELL)

A battery is a device which maintains a potential difference across its two terminals A and B. Dry cells, secondary cells, generator and thermocouple are the devices used for producing potential difference in an electric circuit. Arrangement of cell or battery is shown in figure. Electrolyte provides continuity for current.



It is often prepared by putting two rods or plates of different metals in a chemical solution. Some internal mechanism exerts force $(\vec{F}_n)$ on the ions (positive and negative) of the solution. This force drives positive ions towards positive terminal and negative ions towards negative terminal. As positive charge accumulates on anode and negative charge on cathode a potential difference and hence an electric field $\vec{E}$ is developed from anode to cathode. This electric field exerts an electrostatic force $\vec{F} = q\vec{E}$ on the ions. This force is opposite to that of $\vec{F}_n$. In equilibrium (steady state)

$F_n = F_e$ and no further accumulation of charge takes place.

When the terminals of the battery are connected by a conducting wire, an electric field is developed in the wire. The free electrons in the wire move in the opposite direction and enter the battery at positive terminal. Some electrons are withdrawn from the negative terminal. Thus, potential difference and hence, F_e decreases in magnitude while F_n remains the same. Thus, there is a net force on the positive charge towards the positive terminal. With this the positive charge rush towards positive terminal and negative charge rush towards negative terminal. Thus, the potential difference between positive and negative terminal is maintained.

Internal resistance (r) :

The potential difference across a real source in a circuit is not equal to the emf of the cell. The reason is that charge moving through the electrolyte of the cell encounters resistance. We call this the internal resistance of the source.

* The internal resistance of a cell depends on the distance between electrodes ($r \propto d$), area of

electrodes ($r \propto \frac{1}{s}$) and nature, concentration ($r \propto c$) and temperature of electrolyte ($r \propto \frac{1}{\text{Temp.}}$).

Solved Example

Example 10. What is the meaning of 10 Amp. hr ?

Solution : It means if the 10 A current is withdrawn then the battery will work for 1 hour.

10 Amp $\longrightarrow$ 1 hr

1 Amp $\longrightarrow$ 10 hr

½ Amp $\longrightarrow$ 20 hr

10. ELECTROMOTIVE FORCE : (E.M.F.)

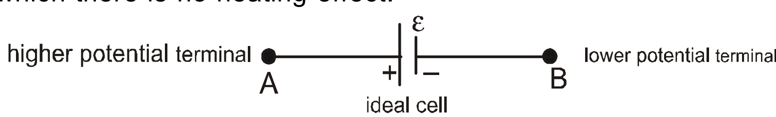
Definition I : Electromotive force is the capability of the system to make the charge flow.

Definition II : It is the work done by the battery for the flow of 1 coloumb charge from lower potential terminal to higher potential terminal inside the battery.

10.1 Representation for battery :

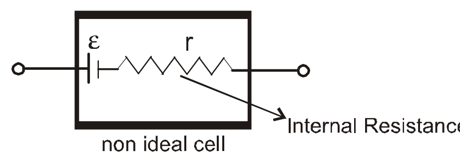
Ideal cell :

Cell in which there is no heating effect.



Non ideal cell :

Cell in which there is heating effect inside due to opposition to the current flow internally



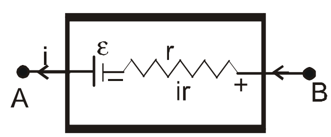
Case I : **Battery acting as a source (or battery is discharging)**

$V_A - V_B = \varepsilon - ir$

$V_A - V_B$

$\Rightarrow$ it is also called terminal voltage.

The rate at which the chemical energy of the cell is consumed = εi



The rate at which heat is generated inside the battery or cell = i^2r

electric power output = $\varepsilon i - i^2 r$

$= (\varepsilon - ir)\, i$

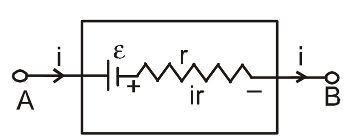
Case II : **Battery acting as a load (or battery charging) :**

$V_A - V_B = \varepsilon + ir$

the rate at which chemical energy stored in the cell = εi

thermal power inside the cell = i^2r

electric power input = $\varepsilon i + i^2 r$ = $(\varepsilon + ir)\, i$ = $(V_A - V_B)\, i$



Definition III :

Electromotive force of a cell is equal to potential difference between its terminals when no current is passing through the circuit.

Case III :

When cell is in open circuit

i = 0 as resistance of open circuit is infinite (∞).

So V = ε, so open circuit terminal voltage difference is equal to emf of the cell.

Case IV :

Short circuiting : Two points in an electric circuit directly connected by a conducting wire are called short circuited, under such condition both points are at same potential.

When cell is short circuited

$i = \frac{\varepsilon}{r}$ and V = 0, short circuit current of a cell is maximum.

Note :

- The potential at all points of a wire of zero resistance will be same.

---

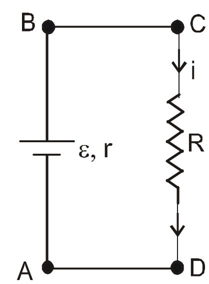
* **Earthing :** If some point of circuit is earthed then its potential is assumed to be zero.

11 RELATIVE POTENTIAL

While solving an electric circuit it is convenient to chose a reference point and assigning its voltage as zero, then all other potentials are measured with respect to this point. This point is also called the common point.

Solved Examples

Example 11. In the given electric circuit find



(a) current

(b) power output

(c) relation between r and R so that the electric power output (that means power given to R) is maximum.

(d) value of maximum power output.

(e) plot graph between power and resistance of load

(f) From graph we see that for a given power output there exists two values of external resistance, prove that the product of these resistances equals r^2.

(g) what is the efficiency of the cell when it is used to supply maximum power.

Solution : (a) In the circuit shown if we assume that potential at A is zero then potential at B is $\varepsilon - ir$. Now since the connecting wires are of zero resistance

$\therefore \quad V_D = V_A = 0 \quad \Rightarrow \quad V_C = V_B = \varepsilon - ir$

Now current through CD is also i

($\because$ it's in series with the cell).

$\therefore \quad i = \frac{V_C - V_D}{R} = \frac{(\varepsilon - ir) - 0}{R}$ Current $i = \frac{\varepsilon}{r + R}$

Note : After learning the concept of series combination we will be able to calculate the current directly

---

(b) Power output $P = i^2 R = \frac{\varepsilon^2}{(r+R)^2} \cdot R$

(c) $\frac{dP}{dR} = \frac{\varepsilon^2}{(r+R)^2} - \frac{2\varepsilon^2 R}{(r+R)^3} = \frac{\varepsilon^2}{(R+r)^3}[R+r-2R]$

for maximum power supply

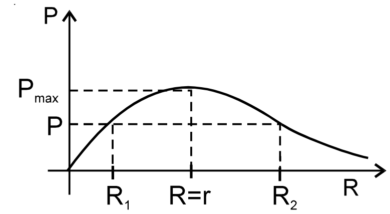
$\frac{dP}{dR} = 0 \quad \Rightarrow \quad r + R - 2R = 0 \Rightarrow \quad$ **r = R**

Here for maximum power output outer resistance should be equal to internal resistance

(d) $P_{max} = \frac{\varepsilon^2}{4r}$

(e) Graph between 'P' and R

maximum power output at R = r



$P_{max} = \frac{\varepsilon^2}{4r} \quad \Rightarrow \quad i = \frac{\varepsilon}{r+R}$

(f) Power output

$$P = \frac{\varepsilon^2 R}{(r+R)^2}$$

$$P\,(r^2 + 2rR + R^2) = \varepsilon^2 R$$

$$R^2 + \left(2r - \frac{\varepsilon^2}{P}\right) R + r^2 = 0$$

above quadratic equation in R has two roots R_1 and R_2 for given values of ε, P and r such that

$\therefore \quad R_1R_2 = r^2 \qquad$ (product of roots)

$\mathbf{r^2 = R_1R_2}$

(g) Power of battery spent

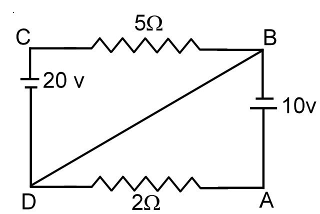
$$= \frac{\varepsilon^2}{(r+r)^2}.2r = \frac{\varepsilon^2}{2r}$$

power (output)

$$= \left(\frac{\varepsilon}{r+r}\right)^2 \times r = \frac{\varepsilon^2}{4r}$$

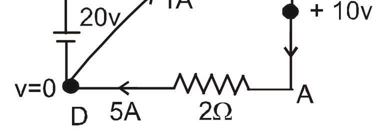
$$\text{Efficiency} = \frac{\text{power output}}{\text{total power spent by cell}} = \frac{\frac{\varepsilon^2}{4r}\times 100}{\frac{\varepsilon^2}{2r}} = \frac{1}{2}\times 100 = 50\%$$

Example 12. In the figure given beside find out the current in the wire BD



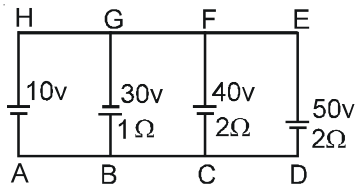
Solution : Let at point D potential = 0 and write the potential of other points then current in wire AD = $\frac{10}{2}$ = 5 A from A to D current

in wire CB = $\frac{20}{5}$ = 4A from C to F



$\therefore$ current in wire BD = 1 A from D to B

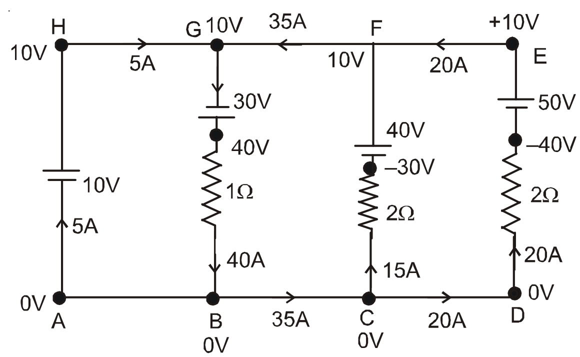
Example 13. Find the current in each wire



Solution : Let potential at point A is 0 volt then potential of other points is shown in figure.

current in BG = $\frac{40-0}{1} = 40$ A from G to B

current in FC = $\frac{0-(-30)}{2} = 15$ A from C to F



current in DE = $\frac{0-(-40)}{2} = 20$ A from D to E

current in wire AH = 40 – 35 = 5 A from A to H

12. KIRCHHOFF'S LAWS

12 1- Kirchhoff's Current Law (Junction law)

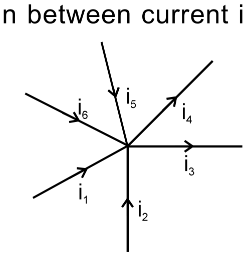
This law is based on law of conservation of charge. It states that "The algebraic sum of the currents meeting at a point of the circuit is zero" or total currents entering a junction equals total current leaving the junction.

$$\Sigma I_{in} = \Sigma I_{out}.$$

It is also known as KCL (Kirchhoff's current law).

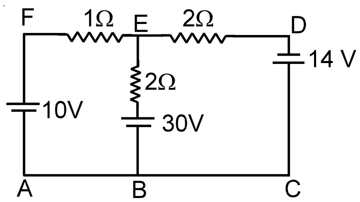
Solved Examples

Example 14. Find relation in between current i_1, i_2, i_3, i_4, i_5 and i_6 .

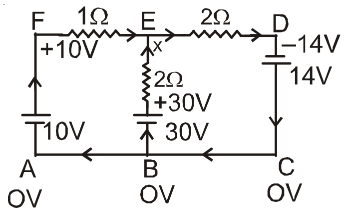


Solution : $i_1 + i_2 - i_3 - i_4 + i_5 + i_6 = 0$

Example 15. Find the current in each wire



Solution :



Let potential at point B = 0. Then potential at other points are mentioned.

$\therefore$ Potential at E is not known numerically.

Let potential at E = x

Now applying kirchhoff's current law at junction E. (This can be applied at any other junction also).

$$\frac{x-10}{1}+\frac{x-30}{2}+\frac{x+14}{2}=0$$

$4x = 36 \quad \Rightarrow \quad x = 9$

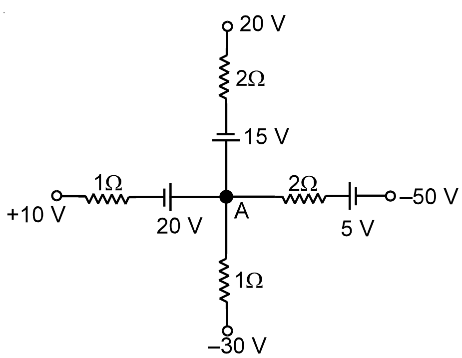
Current in EF = $\frac{10-9}{1}$ = 1 A from F to E

Current in BE = $\frac{30-9}{2}=10.5\,\text{A}$ from B to E

Current in DE = $\frac{9-(-14)}{2}=11.5\,\text{A}$ from E to D

Solved Example

Example 16. Find the potential at point A



Solution : Let potential at A = x, applying kirchhoff current law at juction A

$$\frac{x-20-10}{1}+\frac{x-15-20}{2}+\frac{x+45}{2}+\frac{x+30}{1}=0$$

$$\Rightarrow \quad \frac{2x-60+x-35+x+45+2x+60}{2}=0$$

$\Rightarrow \quad 6x + 10 = 0 \quad \Rightarrow \quad x = -5/3$

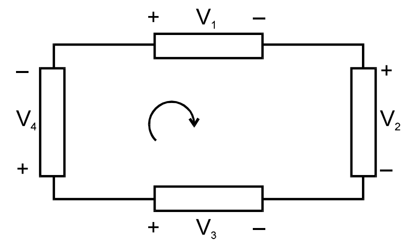
Potential at A = $\frac{-5}{3}$ V

12.2 Kirchhoff's Voltage Law (Loop law) :

"The algebraic sum of all the potential differences along a closed loop is zero.

So $IR + \Sigma\,EMF = 0$".

The closed loop can be traversed in any direction. While traversing a loop if potential increases, put a positive sign in expression and if potential decreases put a negative sign. (Assume sign convention)



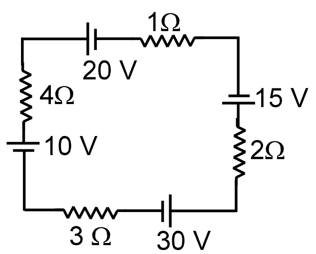
$-V_1 - V_2 + V_3 - V_4 = 0.$

Boxes may contain resistor or battery or any other element (linear or nonlinear).

It is also known as **KVL**

Solved Examples

Example 17. Find current in the circuit



Solution : $\because$ all the elements are connected in series current is all of them will be same

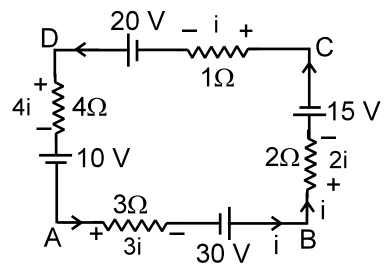
let current = i

Applying kirchhoff voltage law in ABCDA loop

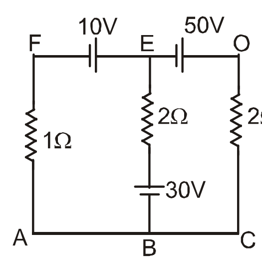
$10 + 4i - 20 + i + 15 + 2i - 30 + 3i = 0$

$10\,i = 25$

$i = 2.5$ A



Example 18. Find the current in each wire applying only kirchhoff voltage law



Solution : Applying kirchhoff voltage law in loop ABEFA

$i_1 + 30 + 2(i_1 + i_2) - 10 = 0$

$3I_1 + 2I_2 + 20 = 0$ -------------- (i)

Applying kirchoff voltage law in BEDCB

$+ 30 + 2(i_1 + i_2) + 50 + 2i_2 = 0$

$4i_2 + 2i_1 + 80 = 0$

$2i_2 + i_1 + 40 = 0$ -------------- (ii)

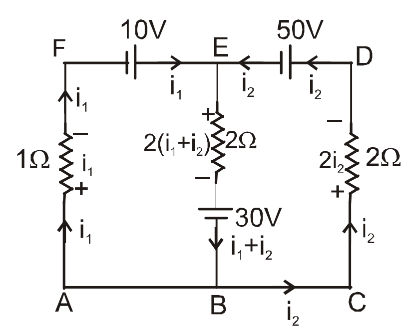
Solving (i) and (ii)

$3\left[-40 - 2i_2\right] + 2i_2 + 20 = 0$

$-120 - 4i_2 + 20 = 0$

$i_2 = -25$ A

and $i_1 = 10$ A



$\therefore \quad i_1 + i_2 = -15$ A

current in wire AF = 10 A from A to F

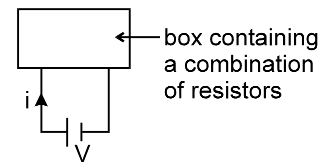
current in wire EB = 15 A from B to E

current in wire DE = 25 A from E to D.

13. COMBINATION OF RESISTANCES :

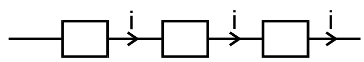
A number of resistances can be connected and all the complicated combinations can be reduced to two different types, namely series and parallel.

The equivalent resistance of a combination is defined as $R_{eq} = \dfrac{V}{i}$



13.1 Resistances in Series:

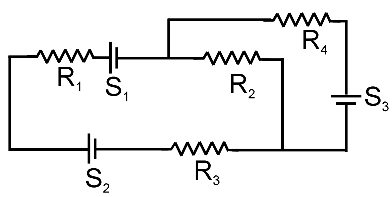
When the resistances (or any type of elements) are connected end to end then they are said to be in series. The current through each element is same.



Resistances in series carry equal current but reverse may not be true.

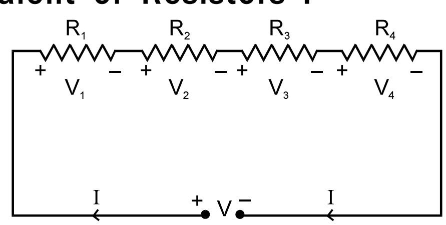
Solved Example

Example 19. Which electrical elements are connected in series.



Solution : Here S_1, S_2, R_1, R_3 connected in one series and R_4, S_3 connected in different series

Equivalent of Resistors :



The effective resistance appearing across the battery (or between the terminals A and B) is

$R = R_1 + R_2 + R_3 + \ldots\ldots\ldots\ldots\ldots + R_n$ (this means R_{eq} is greater then any resistor)

and $V = V_1 + V_2 + V_3 + \ldots\ldots\ldots\ldots\ldots + V_n$

The potential difference across a resistor is proportional to the resistance. Power in each resistor is also proportional to the resistance

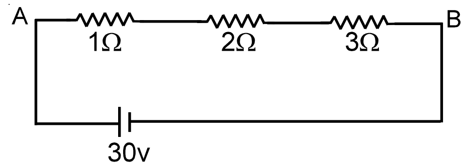
$\because \quad V = IR$ and $P = I^2R$

where I is same through any of the resistor.

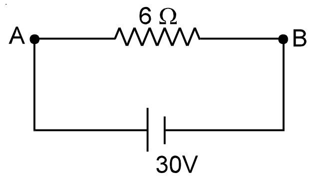
$$V_1 = \frac{R_1}{R_1 + R_2 + \ldots\ldots\ldots + R_n} V \; ; \; V_2 = \frac{R_2}{R_1 + R_2 + \ldots\ldots\ldots + R_n} V \quad ; \quad \text{etc}$$

Solved Examples

Example 20. Find the current in the circuit

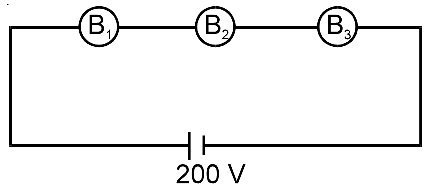


Solution : $R_{eq} = 1 + 2 + 3 = 6\ \Omega$ the given circuit is equivalent to



current $i = \dfrac{v}{R_{eq}} = \dfrac{30}{6}$ = **5 A Ans.**

Example 21. In the figure shown B_1, B_2 and B_3 are three bulbs rated as (200V, 50 W), (200V, 100W) and (200 V, 25W) respectively. Find the current through each bulb and which bulb will give more light?



Solution : $R_1 = \dfrac{(200)^2}{50}$;

$R_2 = \dfrac{(200)^2}{100}$; $R_3 = \dfrac{(200)^2}{25}$

the current following through each bulb is

$$= \frac{200}{R_1 + R_2 + R_3} = \frac{200}{(200)^2\left[\dfrac{2+1+4}{100}\right]} = \frac{100}{200 \times 7} = \frac{1}{14}\text{ A}$$

Since $R_3 > R_1 > R_2$

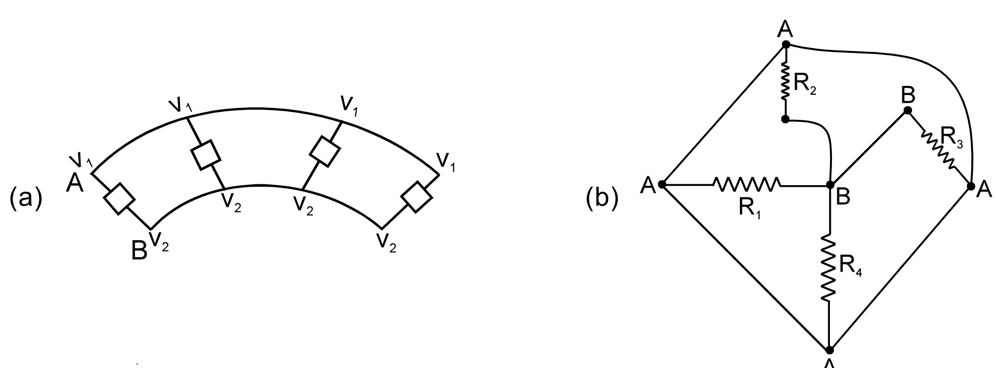
$\therefore$ Power consumed by bulb = i^2R

$\therefore$ if the resistance is of higher value then it will give more light.

$\therefore$ Here Bulb B_3 will give more light.

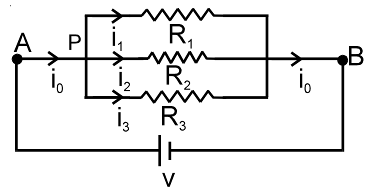
13.2 Resistances in Parallel :

A parallel circuit of resistors is one in which the same voltage is applied across all the components in a parallel grouping of resistors R_1, R_2, R_3,........, R_n.



In the figure (a) and (b) all the resistors are connected between points A and B so they are in parallel.

Equivalent resistance :



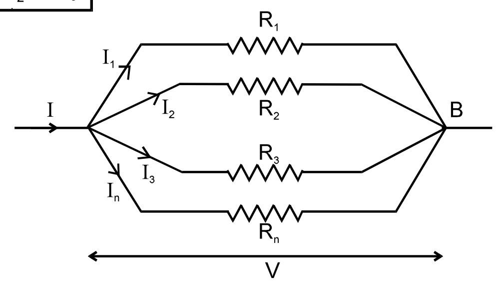
Applying kirchhoff's junction law at point P

$$i_0 = i_1 + i_2 + i_3$$

Therefore, $\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$ $\quad \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

in general,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots\ldots + \frac{1}{R_n}}$$



Conclusions: (about parallel combination)

(a) Potential difference across each resistor is same.

(b) $I = I_1 + I_2 + I_3 + \ldots\ldots\ldots\ldots I_n$.

(c) Effective resistance (R) then $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}$.

(R is less than each resistor).

(d) Current in different resistors is inversely proportional to the resistance.

$$I_1 : I_2 : \ldots\ldots\ldots\ldots I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : \ldots\ldots : \frac{1}{R_n} .$$

$$I_1 = \frac{G_1}{G_1 + G_2 + \ldots\ldots\ldots + G_n} I, \quad I_2 = \frac{G_2}{G_1 + G_2 + \ldots\ldots\ldots + G_n} I, \quad \text{etc.}$$

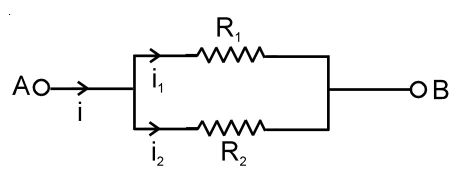
where $G = \frac{1}{R}$ = Conductance of a resistor. [Its unit is Ω^{-1} or ℧ (mho)]

Solved Example

Example 22. When two resistors are in parallel combination then determine i_1 and i_2, if the combination carries a current i ?

Solution : $\therefore \quad i_1R_1 = i_2R_2$

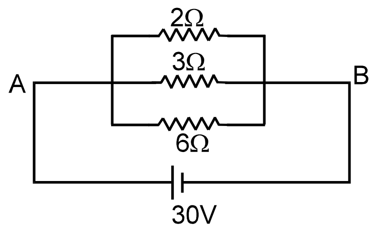
or $\quad \frac{i_1}{i_2} = \frac{R_2}{R_1}$



$$i_1 = \frac{R_2 i}{R_1 + R_2} \quad \Rightarrow \quad i_2 = \frac{R_1 i}{R_1 + R_2} ,$$

Note : Remember this law of $i \propto \frac{1}{R}$ in the resistors connected in parallel. It can be used in problems.

Example 23. Find current passing through the battery and each resistor.



Solution : **Method (I) :**

It is easy to see that potential difference across each resistor is 30 V.

$\therefore$ current is each resistors are $\frac{30}{2}$ = 15 A, $\frac{30}{3} = 10\,A$ and $\frac{30}{6} = 5\,A$

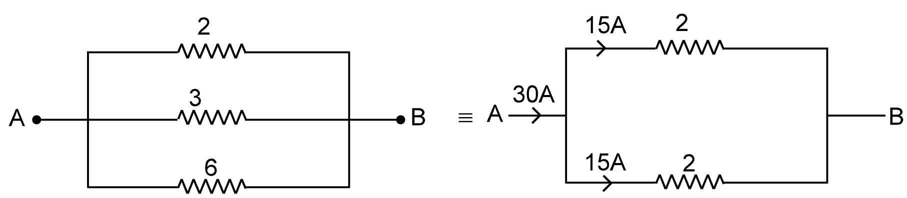
$\therefore$ Current through battery is = 15 + 10 + 5 =30 A.

Method (II) :

By ohm's law $i = \frac{V}{R_{eq}}$ $\Rightarrow$ $\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$ = 1Ω

R_{eq} = 1 Ω $\Rightarrow$ $i = \frac{30}{1}$ = 30 A

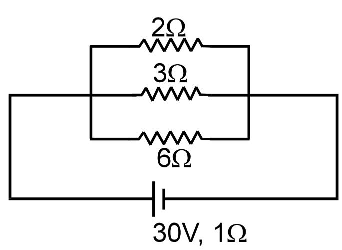
Now distribute this current in the resistors in their inverse ratio.



Current total in 3 Ω and 6 Ω is 15 A it will be divided as 10 A and 5 A.

Note : The method (I) is better. But you will not find such an easy case every where.

Exercise 24. Find current which is passing through battery.



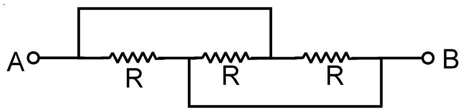
Solution : Here potential difference across each resistor is not 30 V

$\because$ battery has internal resistance. Here the concept of combination of resistors is useful.

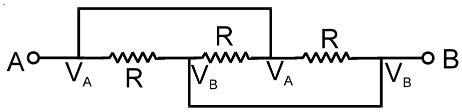
R_{eq} = 1 + 1 = 2 Ω

$i = \frac{30}{2}$ = 15 A.

Example 25. Find equivalent Resistance

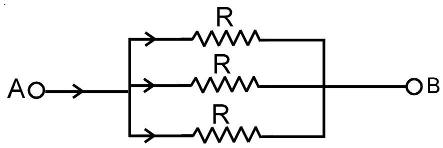


Solution :

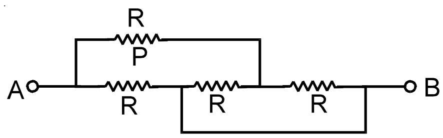


Here all the Resistance are connected between the terminals A and B
Modified circuit is

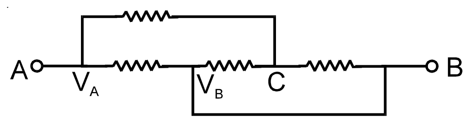
So $\qquad R_{eq} = \dfrac{R}{3}$



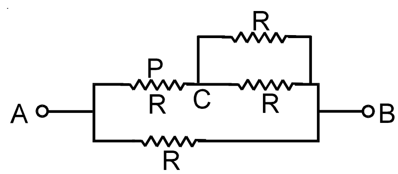
Example 26. Find the current in Resistance P if voltage supply between A and B is V volts



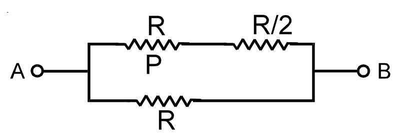
Solution : $\qquad R_{eq} = \dfrac{3R}{5}$



$I = \dfrac{5V}{3R}$ $\qquad$ Modified circuit

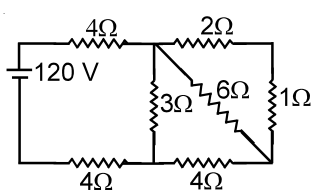


Current in P = $\dfrac{R \times \dfrac{5V}{3R}}{1.5R + R}$



$= \dfrac{2V}{3R}$

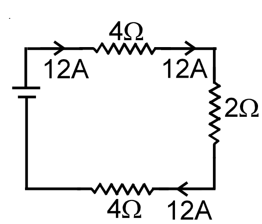
Example 27. Find the current in 2Ω resistance



Solution : 2Ω, 1Ω in series = 3Ω

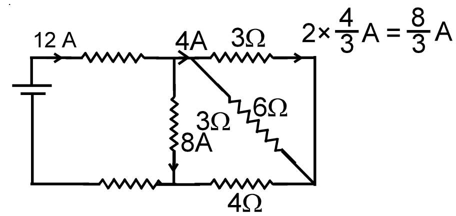
3Ω, 6Ω in parallel = $\dfrac{18}{9}$ =2Ω

2Ω, 4Ω in series = 6Ω
6Ω, 3Ω in parallel = 2Ω
R_{eq} = 4 + 4 + 2 = 10Ω

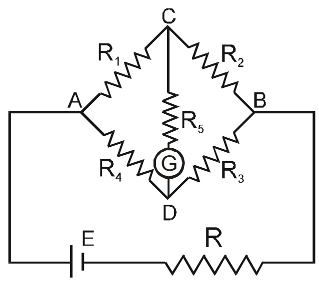


$$i = \frac{120}{10} = 12A$$

So current in 2Ω Resistance $= \frac{8}{3}$ A



14. WHEATSTONE NETWORK : (4 TERMINALNETWORK)



The arrangement as shown in figure, is known as Wheat stone bridge

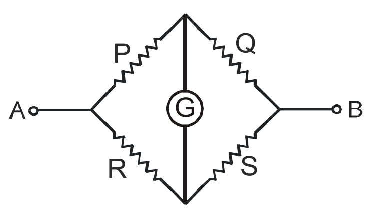
Here there are four terminals in which except two all are connected to each other through resistive elements.

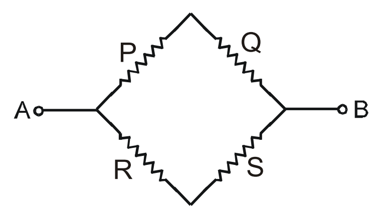
In this circuit if $R_1 R_3 = R_2 R_4$ then $V_C = V_D$ and current in $R_5 = 0$ this is called balance point or null point

When current through the galvanometer is zero (null point or balance point) $\frac{P}{Q} = \frac{R}{S}$, then PS = QR $\Rightarrow$

Here in this case products of opposite arms are equal. Potential difference between C and D at null point is zero. The null point is not affected by resistance R_5, E and R. **It is not affected even if the positions of Galvanometer and battery (E) are interchanged.**

hence, here the circuit can be assumed to be following,

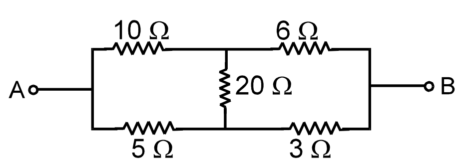




or

Solved Examples

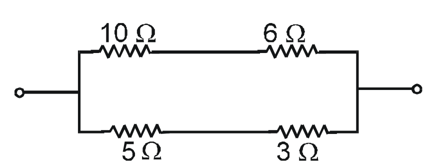
Example 28. Find equivalent resistance of the circuit between the terminals A and B.



Solution : Since the given circuit is wheat stone bridge and it is in balance condition.

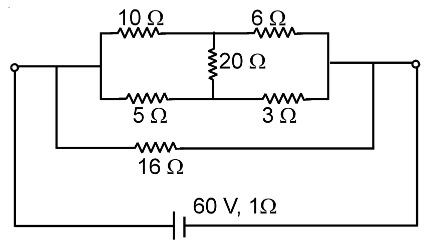
$\because$ $10\times3 = 30 = 6\times5$

hence this is equivalent to



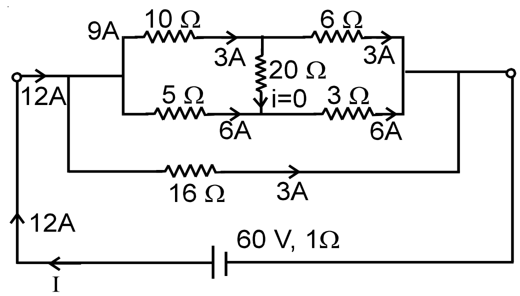
$$R_{eq} = \frac{16\times8}{16+8} = \frac{16}{3}\Omega$$

Example 29.



Find (a) Equivalent resistance (b) and current in each resistance

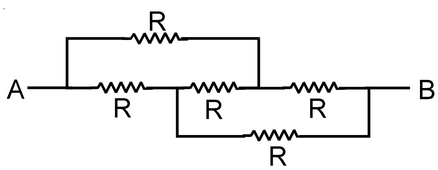
Solution : (a) $R_{eq} = \left(\frac{1}{16}+\frac{1}{8}+\frac{1}{16}\right)^{-1} + 1 = 5\Omega$



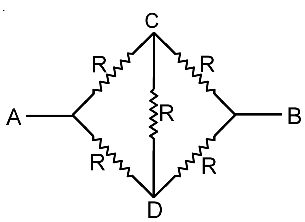
(b) $i = \frac{60}{4+1} = 12$ A

Hence 12 A will flow through the cell.
By using current distribution law.
Current in resistance 10Ω and 6Ω = 3A
Current in resistance 5Ω and 3Ω = 6A
Current in resistance 20Ω = 0
Current in resistance 16Ω = 3A

Example 30. Find the equivalent resistance between A and B

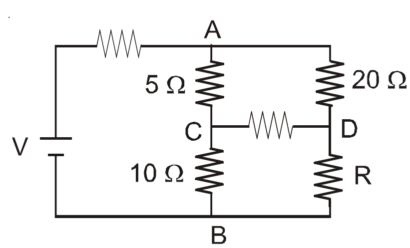


Solution : This arrangement can be modified as shown in figure since it is balanced wheat stone bridge



$$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$$

Example 31. Determine the value of R in the circuit shown in figure, when the current is zero in the branch CD.



Solution : The current in the branch CD is zero, if the potential difference across CD is zero.
That means, voltage at point C = voltage at point D.
Since no current is flowing, the branch CD is open circuited. So the same voltage is applied across ACB and ADB

$V_{10} = V \times \frac{10}{15} \Rightarrow V_R = V \times \frac{R}{20+R}$

$\therefore \quad V_{10} = V_R$ and $V \times \frac{10}{15} = V \times \frac{R}{20+R}$

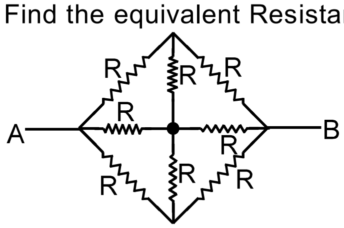
$\therefore \quad R = 40\ \Omega$ **Ans.**

15. SYMMETRICAL CIRCUITS :

Some circuits can be modified to have simpler solution by using symmetry if they are solved by traditional method of KVL and KCL then it would take much time.

Solved Examples

Example 32. Find the equivalent Resistance between A and B

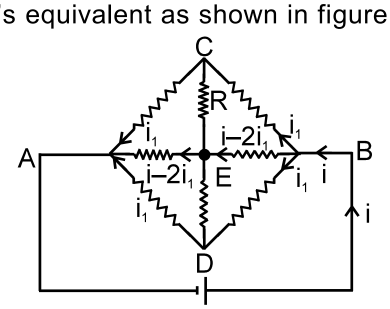


Solution : **I Method :**

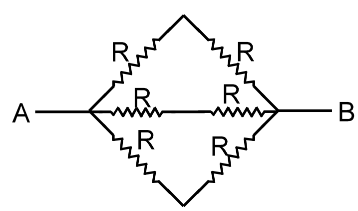
Here no two resistors appear to be in series or parallel no Wheatstone bridge here. This circuit will be solved by using $R_{eq} = \frac{V}{I}$. The branches AC and AD are symmetrical

$\therefore$ current through them will be same.

The circuit is also similar from left side and right side current distribution while entering through B and an exiting from A will be same. Using all these facts the currents are as shown in the figure. It is clear that current in resistor between C and E is 0 and also in ED is 0. It's equivalent as shown in figure (b)



(fig. a)



(fig. b)

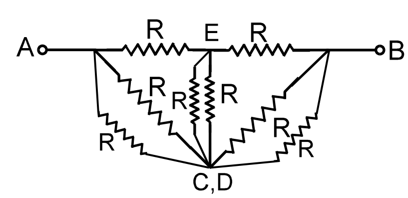
$R_{eq} = \frac{2R}{3}$

II Method

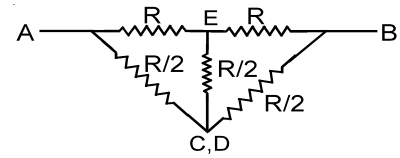
$\because$ The potential difference in R between (B, C) and between (B.D.) is same $V_C = V_D$

Hence the point C and D are same hence circuit can be simplified as this called folding.

Now, it is Balanced Wheatstone bridge



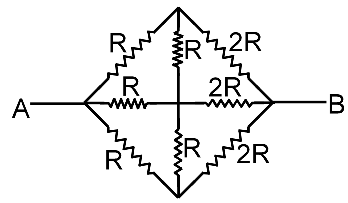
$$R_{eq} = \frac{2R \times R}{2R + R} = \frac{2R}{3}$$



Note : In II Method it is not necessary to know the currents in CA and DA.

Solved Examples

Example 33. Find the equivalent Resistance between A and B



Solution : In this case the circuit has symmetry in the two branches AC and AD at the input

$\therefore$ current in them are same but from input and from exit the circuit is not similar

($\because$ on left R and on right 2R)

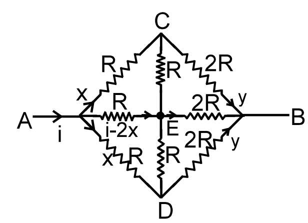
$\therefore$ on both sides the distribution of current will not be similar.

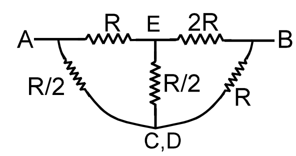
Here $V_c = V_d$

hence C and D are same point

the circuit can be simplified that as shown

Now it is balanced wheat stone bridge

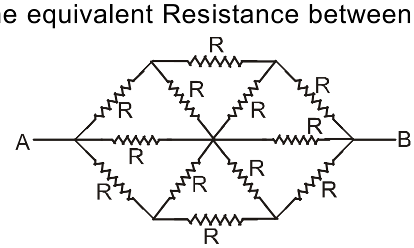




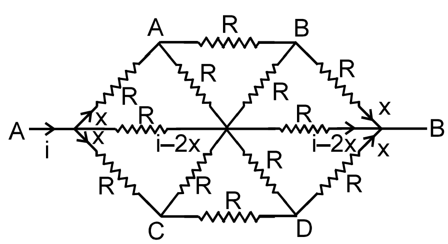
$$R_{eq} = \frac{3R \times \frac{3R}{2}}{3R + \frac{3R}{2}}$$

$$= \frac{\frac{9}{2}R}{\frac{9}{2}} = R.$$

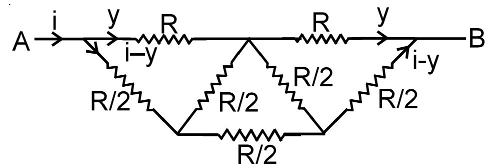
Example 34. Find the equivalent Resistance between A and B



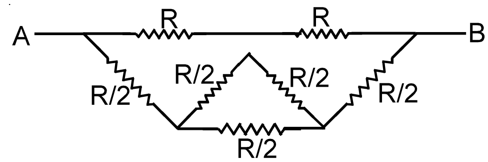
Solution :



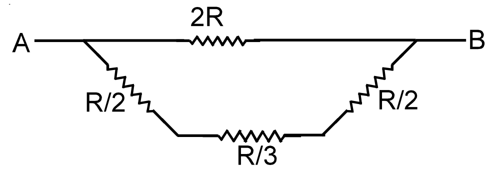
Here $V_A = V_C$ and $V_B = V_D$

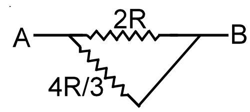


Here the circuit can be simplified as



this circuit can be simplified as

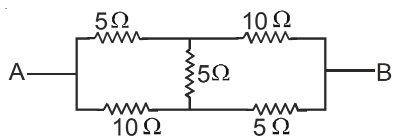




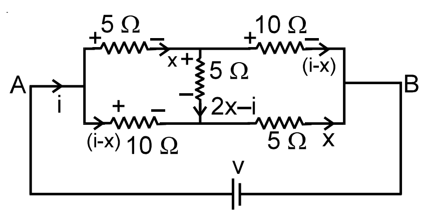
$$R_{eq} = \frac{2R \times \frac{4R}{3}}{\frac{10R}{3}}$$

$$= \frac{4R}{5}$$ **Ans.**

Example 35. Find the equivalent Resistance between A and B



Solution : It is wheat stone bridge but not balanced. No series parallel connections. But similar values on input side and output. Here we see that even after using symmetry the circuit does not reduce to series parallel combination as in previous examples.



$\therefore$ applying kirchoff voltage law

$v - 10(i - x) - 5x = 0$

$v - 10i + 5x = 0$ ...(1)

$10(i - x) - 5(2x - i) - 5x = 0$

$10i - 10x - 10x + 5i - 5x = 0$

$15i - 25x = 0$

$x = \frac{15}{25} i$ $\quad 5x = 3i$ ....(2)

Using (2) and (1)

$\therefore v - 10i + 3i = 0$

$\frac{v}{i} = 7\Omega$

$R_{eq} = 7\Omega$ **Ans.**

16. GROUPING OF CELLS

16.1 Cells in Series :

Equivalent EMF

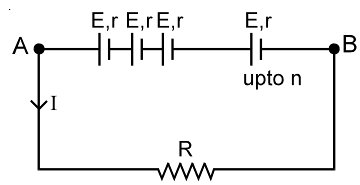
$$E_{eq} = E_1 + E_2 + ... + E_n \qquad \text{[write EMF's with polarity]}$$

Equivalent internal resistance

$$r_{eq} = r_1 + r_2 + r_3 + r_4 + ... + r_n$$

If n cells each of emf E, arranged in series and if r is internal resistance of each cell, then total emf = n E so current in the circuit

$$I = \frac{nE}{R + nr}$$



If nr << R then $I = \frac{nE}{R}$ $\longrightarrow$ Series combination is advantageous.

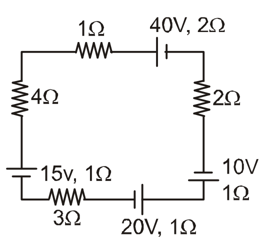
If nr >> R then $I = \frac{E}{r}$ $\longrightarrow$ Series combination is not advantageous.

Note : If polarity of m cells is reversed, then equivalent emf = (n-2m)E while the equivalent resistance is still nr+R, so current in R will be

$$i = \frac{(n-2m)E}{nr + R}.$$

Solved Examples

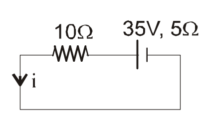
Example 36. Find the current in the loop.



Solution : The given circuit can be simplified as

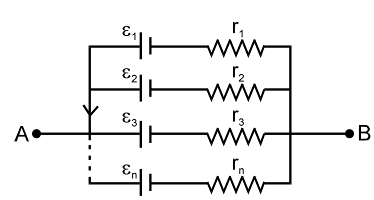
$$i = \frac{35}{10+5} = \frac{35}{15}$$

$$= \frac{7}{3}\text{ A} \quad \Rightarrow \quad I = \frac{7}{3}\text{ A}$$



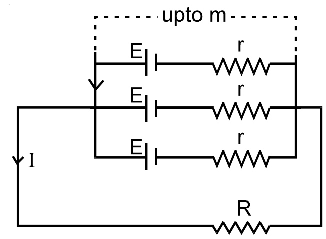
16.2 Cells in Parallel :

$$E_{eq} = \frac{\varepsilon_1/r_1 + \varepsilon_2/r_2 + ... + \varepsilon_n/r_n}{1/r_1 + 1/r_2 + ... + 1/r_n} \qquad \text{[Use emf's with polarity]}$$



$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ... + \frac{1}{r_n}$$

If m cells each of emf E and internal resistance r be connected in parallel and if this combination is connected to an external resistance then equivalent emf of the circuit = E.



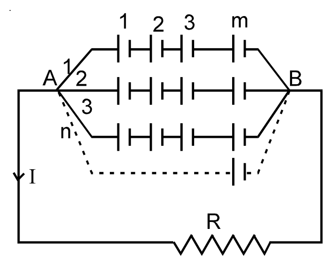
Internal resistance of the circuit = $\frac{r}{m}$.

and $I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$.

If $mR << r$; $I = \frac{mE}{r}$ $\longrightarrow$ Parallel combination is advantageous.

If $mR >> r$; $I = \frac{E}{R}$ $\longrightarrow$ Parallel combination is not advantageous.

16.3 Cells in Multiple Arc :



mn = number of identical cells.
n = number of rows
m = number of cells in each row.
The combination of cells is equivalent to single cell of
emf = mE

and internal resistance = $\frac{mr}{n}$

Current $I = \frac{mE}{R + \frac{mr}{n}}$

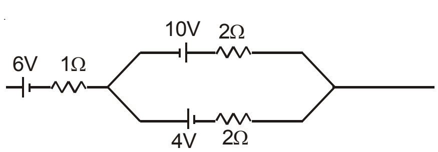
For maximum current $nR = mr$

or $R = \frac{mr}{n}$ = internal resistance of the equivalent battery.

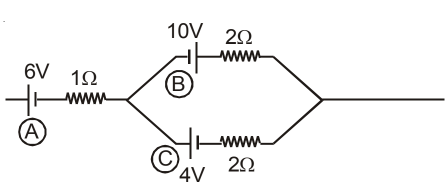
$I_{max} = \frac{nE}{2r} = \frac{mE}{2R}$.

Solved Examples

Example 37. Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



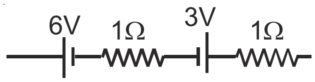
Solution :



Battery (B) and (C) are in parallel combination with opposite polarity. So, their equivalent

$$\varepsilon_{BC} = \frac{\frac{10}{2} + \frac{-4}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{5-2}{1} = 3V \qquad \Rightarrow \qquad r_{BC} = 1\Omega.$$

Now,

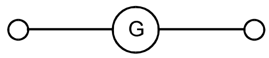


$\varepsilon_{ABC} = 6 - 3 = 3V$

$r_{ABC} = 2\Omega.$ **Ans.**

17. GALVANOMETER

Galvanometer is represented as follow :



It consists of a pivoted coil placed in the magnetic field of a permanent magnet. Attached to the coil is a spring. In the equilibrium position, with no current in the coil, the pointer is at zero and spring is relaxed. When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to current. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement. Thus, the angular deflection of the coil and pointer is directly proportional to the coil current and the device can be calibrated to measure current.

When coil rotates the spring is twisted and it exerts an opposing torque on the coil.

There is a resistive torque also against motion to damp the motion. Finally in equilibrium

$\tau_{\text{magnetic}} = \tau_{\text{spring}} \Rightarrow \quad BINA \sin\theta = C\phi$

But by making the magnetic field radial $\theta = 90°$.

$\therefore \quad BINA = C\phi$

$I \propto \phi$

here B = magnetic field, A = Area of the coil

I = Current, C = torsional constant

N = Number of turns, ϕ = angle rotate by coil.

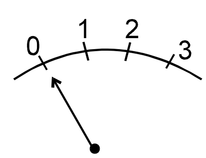
- **Current sensitivity**

  The ratio of deflection to the current i.e. deflection per unit current is called current sensitivity (C.S.) of the galvanometer $CS = \frac{\phi}{I} = \frac{BNA}{C}$

Note :

Shunting a galvanometer decreases its current sensitivity.

A linear scale is obtained. The marking on the galvanometer are proportionate.

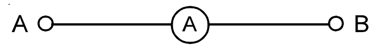


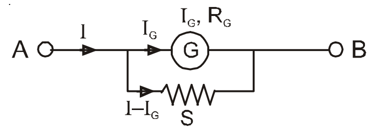
The galvanometer coil has some resistance represented by R_g . It is of the order of few ohms. It also has a maximum capacity to carry a current known as I_g. I_g is also the current required for full scale deflection. This galvanometer is called moving coil galvanometer.

18. AMMETER

A shunt (small resistance) is connected in parallel with galvanometer to convert it into ammeter; An ideal ammeter has zero resistance

Ammeter is represented as follow -





If maximum value of current to be measured by ammeter is I then

$I_G \,.\, R_G = (I - I_G)S$

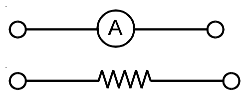
$$S = \frac{I_G . R_G}{I - I_G}$$

$$S = \frac{I_G \times R_G}{I} \quad \text{when} \quad I >> I_G.$$

where I = Maximum current that can be measured using the given ammeter.

For measuring the current the ammeter is connected is series.

In calculation it is simply a resistance



Resistance of ammeter

$$R_A = \frac{R_G . S}{R_G + S}$$

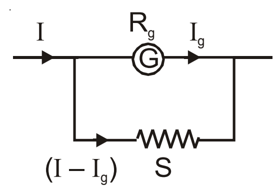
for $\quad S << R_G \quad \Rightarrow \quad R_A = S$

Solved Examples

Example 38. What is the value of shunt which passes 10% of the main current through a galvanometer of 99 ohm ?

Solution :

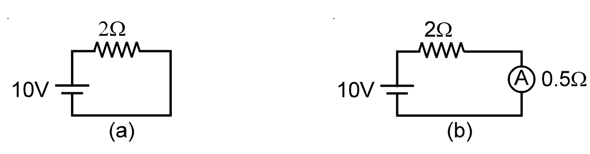
As in figure $\quad R_g I_g = (I - I_g)S$



$$\Rightarrow \quad 99 \times \frac{I}{10} = \left(I - \frac{I}{10}\right) \times S$$

$$\Rightarrow \quad S = 11\ \Omega.$$

Example 39. Find the current in the circuit (a) & (b) and also determine percentage error in measuring the current through an ammeter.



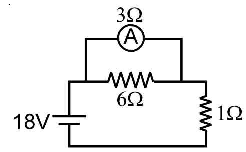
Solution : In A $\quad I = \dfrac{10}{2} = 5A$

In B $\quad I = \dfrac{10}{2.5} = 4A$

Percentage error is = $\dfrac{i - i'}{i} \times 100 = 20\%$ **Ans.**

Here we see that due to ammeter the current has reduced. A good ammeter has very low resistance as compared with other resistors, so that due to its presence in the circuit the current is not affected.

Example 40. Find the reading of ammeter ? Is this the current through 6 Ω ?



Solution : $R_{eq} = \frac{3\times 6}{3+6}+1 = 3\ \Omega$

Current through battery

$$I = \frac{18}{3} = 6\ A$$

So, current through ammeter

$$= 6 \times \frac{6}{9} = 4\ A$$

No, it is not the current through the 6 Ω resistor.

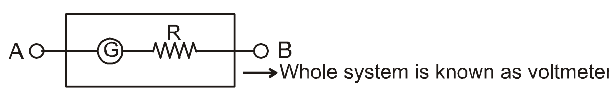
Note :

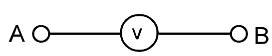
- Ideal ammeter is equivalent to zero resistance wire for calculation potential difference across it is zero.

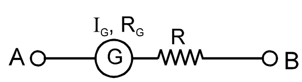
---

19. VOLTMETER

A high resistance is put in series with galvanometer. It is used to measure potential difference across a resistor in a circuit.





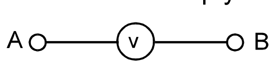


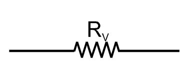
For maximum potential difference

$V = I_G \cdot R + I_G R_G$ $\qquad R = \frac{V}{I_G} - R_G$

If $R_G << R$ $\quad\Rightarrow\quad R_S \approx \frac{V}{I_G}$

For measuring the potential difference a voltmeter is connected across that element. (parallel to that element it measures the potential difference that appears between terminals 'A' and 'B'.)
For calculation it is simply a resistance





Resistance of voltmeter $R_V = R_G + R \approx R$

$I_g = \frac{V_0}{R_g+R}$. $\qquad R \to \infty \Rightarrow$ Ideal voltmeter.

A good voltmeter has high value of resistance.

Ideal voltmeter ⇒ which has high value of resistance.

Note :

- For calculation purposes the current through the ideal voltmeter is zero.
- Percentage error in measuring the potential difference by a voltmeter is = $\frac{V - V'}{V} \times 100$

Solved Example

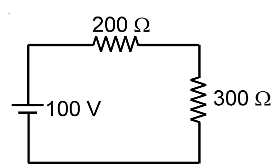
Example 41. A galvanometer has a resistance of G ohm and range of V volt. Calculate the resistance to be used in series with it to extend its range to nV volt.

Solution : Full scale current $i_g = \frac{V}{G}$

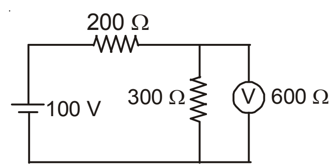
to change its range

$$V_1 = (G + R_s)i_g \Rightarrow \quad nV = (G + R_s)\frac{V}{G} \quad \Rightarrow \quad R_s = G(n-1) \quad \textbf{Ans.}$$

Example 42. Find potential difference across the resistance 300 Ω in A and B.



(A)



(B)

Solution : In (A) : Potential difference = $\frac{100}{200+300} \times 300$ = 60 volt

In (B) : Potential difference = $\frac{100}{200 + \frac{300 \times 600}{300+600}} \times \frac{300 \times 600}{300+600}$ = 50 volt

We see that by connecting voltmeter the voltage which was to be measured has changed. Such voltmeters are not good. If its resistance had been very large than 300 Ω then it would not have affected the voltage by much amount.

Current sensitivity

The ratio of deflection to the current i.e. deflection per unit current is called current sensitivity (C.S.) of the galvanometer CS = $\frac{\theta}{I}$

Note :

- Shunting a galvanometer decreases its current sensitivity.

Solved Examples

Example 43.

A galvanometer with a scale divided into 100 equal divisions, has a current sensitivity of 10 division per mA and voltage sensitivity of 2 division per mV. What adoptions are required to use it (a) to read 5A full scale and (b) 1 division per volt ?

Solution : Full scale deflection current $i_g = \frac{\theta}{cs} = \frac{100}{10}$ mA

$= 10$ mA

Full scale deflection voltage $V_g = \frac{\theta}{vs}$

$= \frac{100}{2}$ mv

$= 50$ mv

So galvanometer resistance $G = \frac{V_g}{i_g} = \frac{50mV}{10mA}$

$= 5\ \Omega$

(a) to convert the galvanometer into an ammeter of range 5A, a resistance of value SΩ is connected in parallel with it such that

$(I - i_g)\,S = i_g\,G$

$(5 - 0.01)\,S = 0.01 \times 5$

$S = \frac{5}{499} \cong 0.01\ \Omega$ **Ans.**

(b) To convert the galvanometer into a voltmeter which reads 1 division per volt, i.e. of range 100 V,

$V = i_g\,(R + G)$

$100 = 10 \times 10^{-3}\,(R + 5)$

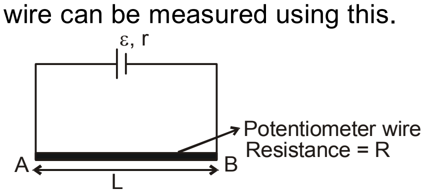
$R = 10000 - 5$

$R = 9995\ \Omega \cong 9.995\ k\Omega$ **Ans.**

20. POTENTIOMETER

A potentiometer is a linear conductor of uniform cross-section with a steady current set up in it. This maintains a uniform potential gradient along the length of the wire. Any potential difference which is less than the potential difference maintained across the potentiometer wire can be measured using this.

The wire should have high resistivity and low expansion coefficient. For example : Manganin or, Constantine wire etc.



$$I = \frac{\varepsilon}{r + R}$$

$$V_A - V_B = \frac{\varepsilon}{R + r}.R$$

Potential gradient (x) $\rightarrow$ Potential difference per unit length of wire

$$x = \frac{V_A - V_B}{L} = \frac{\varepsilon}{R + r} \cdot \frac{R}{L}$$

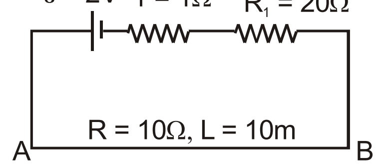
Solved Examples

Example 44. Primary circuit of potentiometer is shown in figure determine :

(A) current in primary circuit

(B) potential drop across potentiometer wire AB

(C) potential gradient (means potential drop per



unit length of potentiometer wire)

(D) maximum potential which we can measure above potentiometer

Solution : (a) $i = \dfrac{\varepsilon}{r+R_1+R} = \dfrac{2}{1+20+10}$ $\Rightarrow$ $i = \dfrac{2}{31}$ A **Ans.**

(b) $V_{AB} = iR = \dfrac{2}{31} \times 10$ $\Rightarrow$ $v_{AB} = \dfrac{20}{31}$ volt **Ans.**

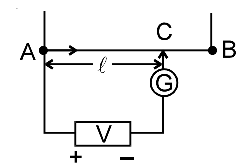
(c) $x = \dfrac{V_{AB}}{L} = \dfrac{2}{31}$ volt/m **Ans.**

(d) Maximum potential which we can measure by it = potential drop across wire AB

$= \dfrac{20}{31}$ volt **Ans.**

Example 45. How to measure an unknown voltage using potentiometer.

Solution : The unknown voltage V is connected across the potentiometer wire as shown in figure. The positive terminal of the unknown voltage is kept on the same side as of the source of the top most battery. When reading of galvanometer is zero then we say that the meter is balanced. In that condition $V = x\,\ell$.



20.1 Application of potentiometer

(a) To find emf of unknown cell and compare emf of two cells.

In case I,

In figure, (2) is joint to (1) then balance length = ℓ_1

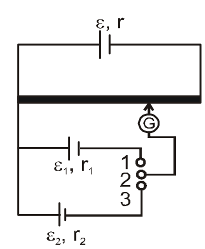
$\varepsilon_1 = x\ell_1$ ....(1)

in case II,

In figure, (3) is joint to (2) then balance length = ℓ_2

$\varepsilon_2 = x\ell_2$ ....(2)

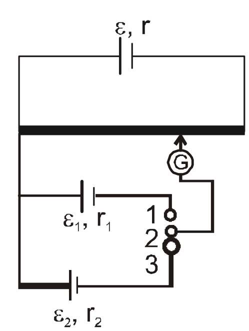
$\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{\ell_1}{\ell_2}$



If any one of ε_1 or ε_2 is known the other can be found. If x is known then both ε_1 and ε_2 can be found

Solved Examples

Example 46. In an experiment to determine the em*f* of an unknown cell, its emf is compared with a standard cell of known emf ε_1 = 1.12 V. The balance point is obtained at 56cm with standard cell and 80 cm with the unknown cell. Determine the emf of the unknown cell.



Solution Here, ε_1 = 1.12 V; ℓ_1 = 56 cm; ℓ_2 = 80 cm

Using equation

$\varepsilon_1 = x\ell_1$ ....(1)

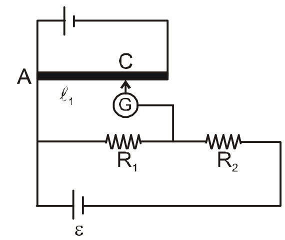
$\varepsilon_2 = x\ell_2$ ....(2)

we get $\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$ $\Rightarrow$ $\varepsilon_2 = \varepsilon_1\left(\frac{\ell_2}{\ell_1}\right)$

or $\varepsilon_2 = 1.12\left(\frac{80}{56}\right) = 1.6$ V Ans

---

(b) To find current if resistance is known



$V_A - V_C = x\ell_1$

$IR_1 = x\ell_1$

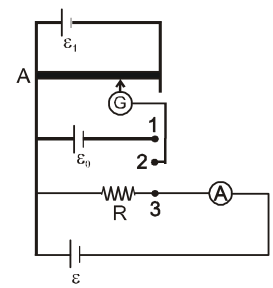
$$I = \frac{x\ell_1}{R_1}$$

Similarly, we can find the value of R_2 also.

Potentiometer is ideal voltmeter because it does not draw any current from circuit, at the balance point.

Solved Examples

Example 47. A standard cell of emf $\varepsilon_0 = 1.11$ V is balanced against 72 cm length of a potentiometer. The same potentiometer is used to measure the potential difference across the standard resistance R = 120 Ω. When the ammeter shows a current of 7.8 mA, a balanced length of 60 cm is obtained on the potentiometer.
(i) Determine the current flowing through the resistor.
(ii) Estimate the error in measurement of the ammeter.



Solution : Here, $\ell_0 = 72$ cm ; $\ell = 60$ cm; R = 120 Ω and $\varepsilon_0 = 1.11$ V

(i) By using equation $\varepsilon_0 = x\,\ell_0$ ....(i)

$V = IR = x\,\ell$ ....(ii)

From equation (i) and (ii)

$$I = \frac{\varepsilon_0}{R}\left(\frac{\ell}{\ell_0}\right) \quad \therefore \quad I = \frac{1.11}{120}\left(\frac{60}{72}\right) = 7.7 \text{ mA}$$

(ii) Since the measured reading 7.8 mA (> 7.7 mA) therefore, the instrument has a positive error.

$\Delta I = 7.8 - 7.7 = 0.1$ mA, $\frac{\Delta I}{I} = \frac{0.1}{7.7} \times 100 = 1.3$ %

(c) To find the internal resistance of cell.

by first arrangement $\varepsilon' = x\ell_1$ ...(1)

by second arrangement $IR = x\ell_2$

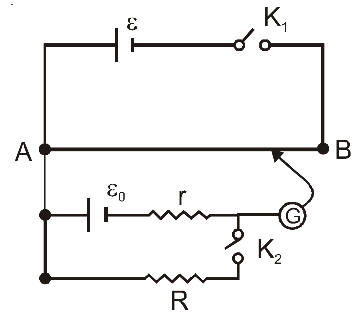
$$I = \frac{x\ell_2}{R},$$

also $\quad I = \dfrac{\varepsilon'}{r'+R} \quad \Rightarrow \quad \dfrac{\varepsilon'}{r'+R} = \dfrac{x\ell_2}{R}$

$\Rightarrow \quad \dfrac{x\ell_1}{r'+R} = \dfrac{x\ell_2}{R} \quad \Rightarrow \quad r' = \left[\dfrac{\ell_1 - \ell_2}{\ell_2}\right]R$

Solved Example

Example 48. The internal resistance of a cell is determined by using a potentiometer. In an experiment, an external resistance of 60Ω is used across the given cell. When the key is closed, the balance length on the potentiometer decreases from 72 cm to 60 cm. calculate the internal resistance of the cell.



Solution : According to equation

$\varepsilon_0 = x\ell_0 \qquad$ ....(i)

$V = IR = x\ell \qquad$ ....(ii)

$I = \dfrac{\varepsilon_0}{R+r} \qquad$ .....(iii)

From equation (i), (ii) and (iii) we get

$$r = R\left(\frac{\ell_0}{\ell} - 1\right)$$

here $\quad \ell_0 = 72$ cm; $\qquad \ell = 60$ cm; $\qquad R = 60\ \Omega$

$\therefore \quad r = (60)\left(\dfrac{72}{60} - 1\right)$

or $\quad r = 12\ \Omega$.

21. METRE BRIDGE (USE TO MEASURE UNKNOWN RESISTANCE)

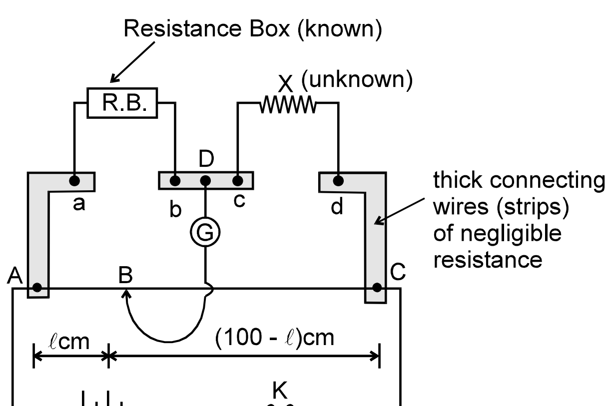
If AB = ℓ cm, then BC = $(100 - \ell)$ cm.

Resistance of the wire between A and B $\quad R \propto \ell$

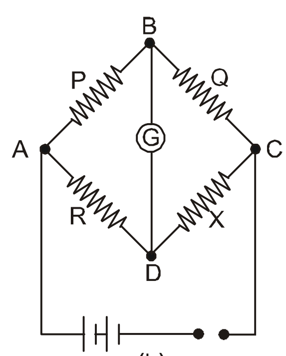
[∵ Specific resistance ρ and cross-sectional area A are same for whole of the wire]

or $\qquad R = \sigma\ell \qquad$ ...(1)

where σ is resistance per cm of wire.



(a)



(b)

Similarly, if Q is resistance of the wire between B and C, then

$$Q \propto 100 - \ell$$

$\therefore \qquad Q = \sigma(100 - \ell) \qquad ....(2)$

Dividing (1) by (2), $\qquad \frac{P}{Q} = \frac{\ell}{100 - \ell}$

Applying the condition for balanced Wheatstone bridge, we get

$$R\ Q = P\ X$$

$\therefore \qquad x = R\ \frac{Q}{P} \qquad$ or $\qquad X = \frac{100 - \ell}{\ell}\ R$

Since R and ℓ are known, therefore, the value of X can be calculated.

Note : For better accuracy, R is so adjusted that ℓ lies between 40 cm and 60 cm.

Solved Example

Example 49. In a meter bridge experiment, the value of unknown resistance is 2Ω. To get the balancing point at 40cm distance from the same end, the resistance in the resistance box will be :

(A) 0.5 Ω (B) 3 Ω (C) 20 Ω (D) 80 Ω

Solution : Apply condition for balance wheat stone bridge,

$$\frac{P}{Q} = \frac{\ell}{100 - \ell} \quad = \quad \frac{P}{2} = \frac{100 - 40}{40}$$

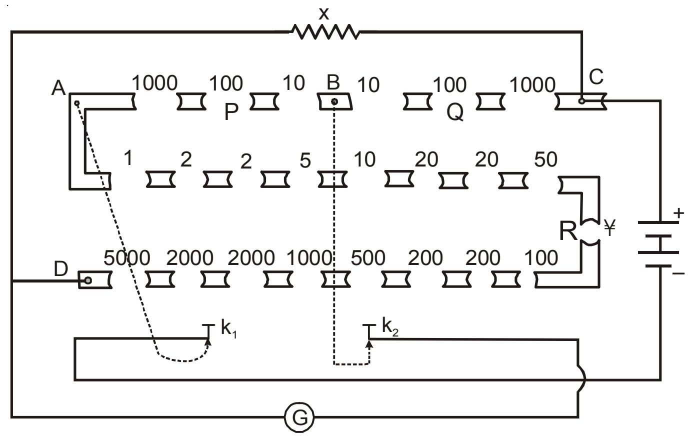
Ans. : P = 3Ω.

21. POST-OFFICE BOX

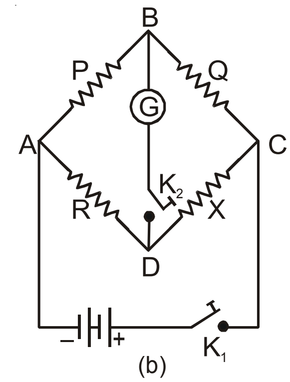
Introduction. It is so named because its shape is like a box and it was originally designed to determine the resistances of electric cables and telegraph wires. It was used in post offices to determine the resistance of transmission lines.

Construction. A post office box is a compact form of Wheatstone bridge with the help of which we can measure the value of the unknown resistance correctly up to 2nd decimal place, i.e., up to 1/100th of an ohm correctly. Two types of post office box are available - plug type and dial type. In the plug-type instrument shown in figure (a), each of the arms AB and BC contains three resistances of 10, 100 and 1000 ohm. These arms are called the ratio arms. While the resistance P can be introduced in the arm AB, the resistance Q can be introduced in the arm BC. The third arm AD, called the resistance arm, is a complete resistance box containing resistances from 1 Ω to 5,000 Ω. In this arm, the resistance R is introduced by taking out plugs of suitable values. The unknown resistance X constitutes the fourth arm CD. Thus, the four arms AB, BC, CD and AD are infect the four arms of the Wheatstone bridge (figure (b)). Two tap keys K_1 and K_2 are also provided. While K_1 is connected internally to the terminal A, K_2 is connected internally to B. These internal connections are shown by dotted lines in figure (a).

A battery is connected between C and key K_1 (battery key). A galvanometer is connected between D and key K_2 (galvanometer key). Thus, the circuit is exactly the same as that shown in figure (b). It is always the battery key which is pressed first and then the galvanometer key. This is because a self-induced current is always set up in the circuit whenever the battery key is pressed or released. If we first press the galvanometer key, the balance point will be disturbed on account of induced current. If the battery key is pressed first, then the induced current becomes zero by the time the galvanometer key is pressed. So, the balance point is not affected.



(a)



(b)

Working : The working of the post office box involves broadly the following four steps :

I. Keeping R zero, each of the resistances P and Q are made equal to 10 ohm by taking out suitable plugs from the arms AB and BC respectively. After pressing the battery key first and then the galvanometer key, the direction of deflection of the galvanometer coil is noted. Now, making R infinity, the direction of deflection is again noted. If the direction is opposite to that in the first case, then the connections are correct.

II. Keeping both P and Q equal to 10Ω, the value of R is adjusted, beginning from 1Ω, till 1 Ω increase reverses the direction of deflection. The 'unknown' resistance clearly lies somewhere between the two final values of R.

$$\left[X = R\frac{Q}{P} = R\frac{10}{10} = R\right]$$

As an illustration, suppose with 3Ω resistance in the arm AD, the deflection is towards left and with 4Ω, it is towards right. The unknown resistance lies between 3Ω and 4Ω.

III. Making P 100 Ω and keeping Q 10 Ω, we again find those values of R between which direction of deflection is reversed. Clearly, the resistance in the arm AD will be 10 times the resistance X of the wire.

$$\left[X = R\frac{Q}{P} = R\frac{10}{100} = \frac{R}{10}\right]$$

In the illustration considered in step II, the resistance in the arm AD will now lie between 30 Ω, and 40 Ω. So, in this step, we have to start adjusting R from 30 Ω onwards. If 32 Ω and 33 Ω are the two values of R which give opposite deflections, then the unknown resistance lies between 3.2 Ω and 3.3 Ω.

IV. Now, P is made 1000 Ω and Q is kept at 10 Ω. The resistance in the arm AD will now be 100 times the 'unknown' resistance.

$$\left[X = R\frac{10}{1000} = \frac{R}{100}\right]$$

In the illustration under consideration, the resistance in the arm AD will lie between 320 Ω and 330Ω. Suppose the deflection is to the right for 326 ohm, towards left for 324 ohm and zero deflection for 325Ω Then, the unknown resistance is 3.25 Ω.

The post office box method is a less accurate method for the determination of unknown resistance as compared to a metre bridge. This is due to the fact that it is not always possible to arrange resistance in the four arms to be of the same order. When the arms ratio is large, large resistance are required to be introduced in the arm R.

Solved Examples

Example 50. The post office box works on the principle of :

(A) Potentiometer (B) Wheatstone bridge

(C) Matter waves (D) Ampere's law **Ans. :** (B)

Example 51. While using a post office box the keys should be switched on in the following order :

(A) first cell key the and then galvanometer key.

(B) first the galvanometer key and then cell key.

(C) both the keys simultaneously.

(D) any key first and then the other key. **Ans. :** (A)

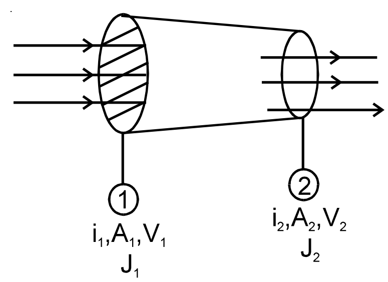
Example 52. In a post office box if the position of the cell and the galvanometer are interchanged, then the :

(A) null point will not change (B) null point will change

(C) post office box will not work (D) Nothing can be said. **Ans. :** (A)

Solved Miscellaneous Problems

Problem 1. Current is flowing from a conductor of non-uniform cross section area if $A_1 > A_2$ then find relation between



(a) i_1 and i_2

(b) j_1 and j_2

(c) v_1 and v_2 (drift velocity)

where i is current, j is current density and V is drift velocity.

Answer : $i_1 = i_2$, $V_1 < V_2$, $J_1 < J_2$

Solution : (a) i = charge flowing through a cross-section per unit time.

$\therefore\ i_1 = i_2$

(b) $j = \dfrac{i}{A}$

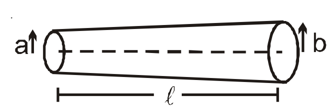
as $A_1 > A_2$ then $j_1 < j_2$

(c) $j = nev_d$

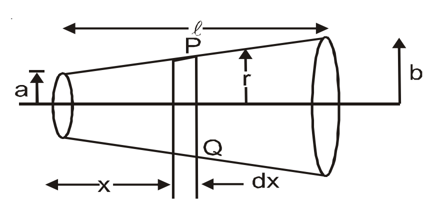
$v_d = \dfrac{j}{ne}$

as $j_1 < j_2$ then, $v_1 < v_2$

Problem 2. Figure shows a conductor of length ℓ having a circular cross-section. The radius of cross-section varies linearly from a to b. The resistivity of the material is ρ. Find the resistance of the conductor.



Solution :



In this problem cross–section area is variable so we can't apply formula $\left(R = \dfrac{\rho\ell}{A}\right)$ directly.

So we assume elementary strip 'PQ' of thickness dx

and radius r resistance of this strip is : $dR = \dfrac{\rho dx}{\pi r^2}$

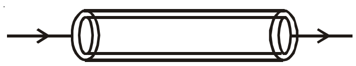
By geometry $= \dfrac{r-a}{x} = \dfrac{b-a}{\ell} \quad \Rightarrow \quad r = \dfrac{b-a}{\ell}x + a$

resistance of conductor is

$$R = \int_0^{\ell} \frac{\rho dx}{\pi\left\{\frac{b-a}{\ell}x + a\right\}^2} \quad \Rightarrow \quad R = \frac{\rho\ell}{\pi ab} \qquad \textbf{Ans.}$$

Problem 3. A cylindrical tube of length ℓ has inner radius a while outer radius b. What is the resistance of the tube between (a) its ends (b) its inner and outer surfaces ? (the resistivity of its material is ρ)

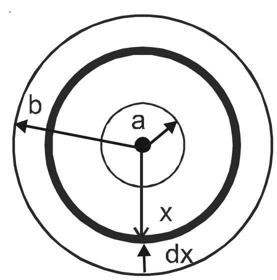
Solution : (a) $A = \pi(b^2 - a^2)$



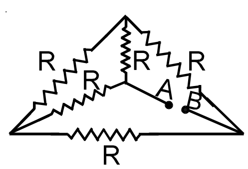
$$R = \frac{\rho\ell}{\pi(b^2 - a^2)}$$

(b) $d_R = \dfrac{\rho dx}{2\pi x\ell}$ for a section taken at distance x from centre.

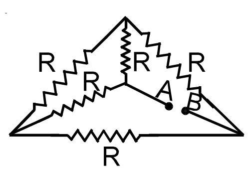
$$d_R = \frac{\rho}{2\pi\ell}\cdot\frac{dx}{x} \quad \Rightarrow \quad R = \int dR = \frac{\rho}{2\pi\ell}\int_a^b \frac{dx}{x} = \frac{\rho}{2\pi\ell}\,\ell n\,\frac{b}{a}$$



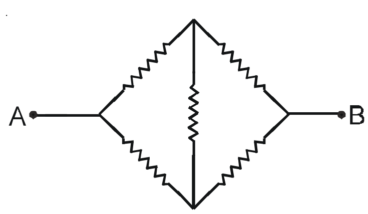
Problem 4. Find the equivalent Resistance between A and B

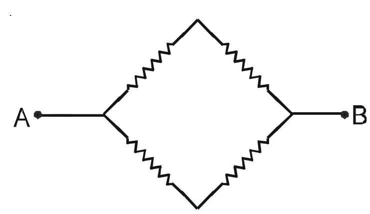


Solution :



Putting A out of the structure in the same plane





$R_{eq} = \dfrac{2R \times 2R}{2R + 2R} = R$ **Ans. :** $\mathbf{R_{eq} = R}$

Problem 5. What shunt resistance is required to convert the 1.0 mA, 20Ω galvanometer into an ammeter with a range of 0 to 50mA ?

Answer : $S = \frac{20}{49} = 0.408\ \Omega$

Solution : $i_g R_g = (i - i_g)S$

$i_g = 1.0 \times 10^{-3}$ A , G = 20Ω
$i = 50 \times 10^{-3}$ A

$$S = \frac{i_g R_g}{i - v_g} = \frac{1\times10^{-3}\times20}{49\times10^{-3}} = 0.408\ \Omega$$

Problem 6. How can we convert a galvanometer with R_g = 20 Ω and i_g = 1.0 mA into a voltmeter with a maximum range of 10 V ?

Answer : A resistance of 9980 Ω is to be connected in series with the galvanometer.

Solution : $v = i_g R_S + i_g R_g$

$10 = 1 \times 10^{-3} \times R_s + 1 \times 10^{-3} \times 20$

$$R_S = \frac{10 - 0.02}{1\times10^{-3}} = \frac{9.98}{10^{-3}} = \mathbf{9980\ \Omega}$$

Problem 7. A Potentiometer wire of 10 m length and having 10 ohm resistance, emf 2 volts and a rheostat. If the potential gradient is 1 micro volt/mm, the value of resistance in rheostat in ohms will be :
(A) 1.99 (B) 19.9 (C) 199 (D) 1990

Solution : d = 10 m , R = 10Ω ,

$E = 2$volts , $\frac{dv}{d\ell} = 1\mu$ v/mm

$$\frac{dv}{d\ell} = \frac{1\times10^{-6}}{1\times10^{-3}}\ \text{v/m} = 1 \times 10^{-3}\ \text{v/m}$$

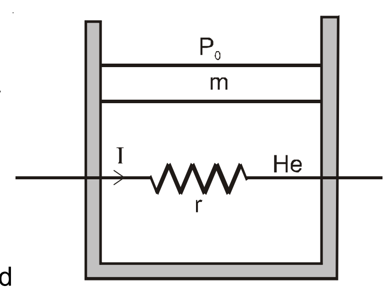
Across wire potential drop ,

$$\frac{dv}{d\ell} \times \ell = 1 \times 10^{-3} \times 10 = 0.01 \text{ volts}$$

$$i = \frac{0.01}{10} = 0.001 = \frac{E}{R + R'}$$ (R′ = resistance of rheostat)

$$R' = \frac{E}{0.001} - R = \frac{2}{0.001} - 10 = 2000 - 10 = 1990\ \Omega$$ **Answer :** (D)

Problem 8. A resistance coil of resistance r connected to an external battery, is placed inside an adiabatic cylinder fitted with a frictionless piston of mass m and same area A. Initially cylinder contains one mole of ideal gas He. A current I flows through the coil such that temperature of gas varies as $T = T_0 + at + bt^2$, keeping pressure constant with time t. Atmosphere pressure above piston is P_0. Find



(a) current I flowing through the coil as function of time and

(b) speed of piston as function of time.

Solution : Heat produced by coil inside the cylinder in time dt is

$dQ = I^2 r\, dt$ ....(i)

(a) As we know

$\Rightarrow$ $dQ = nC_p dT$ ...(ii)