WAVE OPTICS

1. PRINCIPLE OF SUPERPOSITION

When two or more waves simultaneously pass through a point, the disturbance of the point is given by the sum of the disturbances each wave would produce in absence of the other wave(s). In case of wave on string disturbance means displacement, in case of sound wave it means pressure change, in case of Electromagnetic Waves. it is electric field or magnetic field. Superposition of two light travelling in almost same direction results in modification in the distribution of intensity of light in the region of superposition. This phenomenon is called *interference*.

1.1 Superposition of two sinusoidal waves :

Consider superposition of two sinusoidal waves (having same frequency), at a particular point.

Let, $x_1(t) = a_1 \sin \omega t$

and, $x_2(t) = a_2 \sin (\omega t + \phi)$

represent the displacement produced by each of the disturbances. Here we are assuming the displacements to be in the same direction. Now according to superposition principle, the resultant displacement will be given by,

$$x(t) = x_1(t) + x_2(t)$$
$$= a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$
$$= A \sin (\omega t + \phi_0)$$

where $A^2 = a_1^2 + a_2^2 + 2a_1 \cdot a_2 \cos \phi$ .... (1.1)

and $\tan \phi_0 = \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$ ....(1.2)

Solved Examples

Example 1. S_1 and S_2 are two sources of light which produce individually disturbance at point P given by $E_1 = 3\sin \omega t$, $E_2 = 4 \cos \omega t$. Assuming $\vec{E}_1$ & $\vec{E}_2$ to be along the same line, find the resultant after their superposition.

Solution :

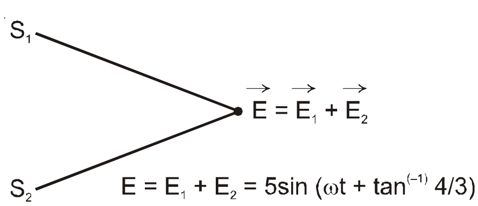


Figure 1.1

$$E = 3 \sin\omega t + 4 \sin(\omega t + \frac{\pi}{2})$$

$$A^2 = 3^2 + 4^2 + 2(3)(4) \cos \frac{\pi}{2} = 5^2$$

$$\tan\phi_0 = \frac{4\sin\frac{\pi}{2}}{3 + 4\cos\frac{\pi}{2}} = \frac{4}{3} \quad \Rightarrow \quad \phi_0 = 53^\circ$$

$$E = 5\sin[\omega t + 53^\circ]$$

1.2 SUPERPOSITION OF PROGRESSIVE WAVES; PATH DIFFERENCE :

Let S_1 and S_2 be two sources producing progressive waves (disturbance travelling in space given by y_1 and y_2)

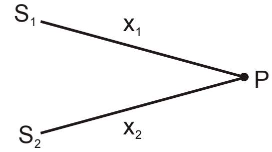


Figure: 1.3

At point P,

$y_1 = a_1 \sin(\omega t - kx_1 + \theta_1)$

$y_2 = a_2 \sin(\omega t - kx_2 + \theta_2)$

$y = y_1 + y_2 = A \sin(\omega t + \Delta\phi)$

Here, the phase difference,

$\Delta\phi = (\omega t - kx_1 + \theta_1) - (\omega t - kx_2 + \theta_2)$

$= k(x_2 - x_1) + (\theta_1 - \theta_2) = k\Delta p - \Delta\theta$ where $\Delta\theta = \theta_2 - \theta_1$

Here $\Delta p = \Delta x$ is the path difference

Clearly, phase difference due to path difference = k (path difference)

where $k = \dfrac{2\pi}{\lambda}$

$$\Rightarrow \quad \Delta\phi = k\Delta p = \frac{2\pi}{\lambda}\Delta x \qquad \text{..... (1.3)}$$

For Constructive Interference :

$\Delta\phi = 2n\pi, \quad n = 0, 1, 2 \ldots\ldots$

or, $\Delta x = n\lambda$

$A_{max} = A_1 + A_2$

Intensity, $\sqrt{I_{max}} = \sqrt{I_1} + \sqrt{I_2} \quad \Rightarrow \quad I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ ... (1.4)

For Destructive interference :

$\Delta\phi = (2n + 1)\pi, \quad n = 0, 1, 2 \ldots\ldots$

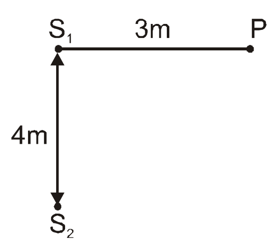
or, $\Delta x = (2n + 1)\lambda/2$

$A_{min} = |A_1 - A_2|$

Intensity, $\sqrt{I_{min}} = \sqrt{I_1} - \sqrt{I_2} \quad \Rightarrow \quad I_{min.} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ ... (1.5)

Solved Examples

Example 2.



S_1 and S_2 are two coherent sources of frequency 'f' each. ($\theta_1 = \theta_2 = 0°$)

$V_{sound} = 330$m/s.

(i) so that constructive interference at 'p'

(ii) so that destructive interference at 'p'

Sol. For constructive interference

$K\Delta x = 2n\pi$

$\dfrac{2\pi}{\lambda} \times 2 = 2n\pi$

$\lambda = \dfrac{2}{n}$

$$V = \lambda f \quad \Rightarrow \quad V = \frac{2}{n} f$$

$$f = \frac{330}{2} \times n$$

For destructive interference

$K\Delta x = (2n + 1)\pi$

$$\frac{2\pi}{\lambda} . 2 = (2n + 1)\pi$$

$$\frac{1}{\lambda} = \frac{(2n+1)}{4}$$

$$f = \frac{V}{\lambda} = \frac{330 \times (2n+1)}{4}$$

Example 3. Light from two sources, each of same frequency and travelling in same direction, but with intensity in the ratio 4 : 1 interfere. Find ratio of maximum to minimum intensity.

Solution :
$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right)^2 = \left(\frac{2+1}{2-1}\right)^2 = 9 : 1.$$

2. WAVEFRONTS

Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillate in phase is an example of a wavefront.

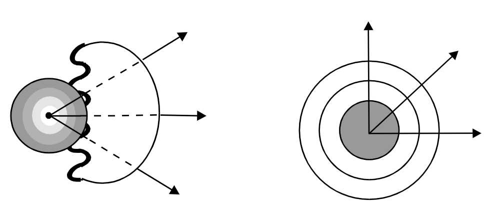
A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the phase speed. The energy of the wave travels in a direction perpendicular to the wavefront. Figure (2.1a) shows light waves from a point source forming a spherical wavefront in three dimensional space. The energy travels outwards along straight lines emerging from the source. i.e.. radii of the spherical wavefront. These lines are the rays. Notice that when we measure the spacing between a pair of wavefronts along any ray, the result is a constant. This example illustrates two important general principles which we will use later:

(i) Rays are perpendicular to wavefronts.

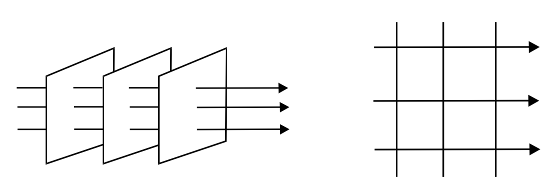
(ii) The time taken by light to travel from one wavefront to another is the same along any ray.

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Figure (2.1b)

A linear source such as a slit illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at larger distance from the source, these wavefronts may be regarded as planar.



(a)



(b)

Figure : 2.1 : Wavefronts and the corresponding rays in two cases: (a) diverging spherical wave. (b) plane wave. The figure on the left shows a wave (e.g.. light) in three dimensions. The figure on the right shows a wave in two dimensions (a water surface).

3. COHERENCE :

Two sources which vibrate with a fixed phase difference between them are said to be coherent. The phase differences between light coming form such sources does not depend on time.

In a conventional light source, however, light comes from a large number of individual atoms, each atom emitting a pulse lasting for about 1 ns. Even if atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases. Consequently light coming from two such sources have a fixed phase relationship for about 1ns, hence interference pattern will keep changing every billionth of a second. The eye can notice intensity changes which lasts at least one tenth of a second. Hence we will observe uniform intensity on the screen which is the sum of the two individual intensities. Such sources are said to be incoherent. Light beam coming from two such independent sources do not have any fixed phase relationship and they do not produce any stationary interference pattern. For such sources, resultant intensity at any point is given by

$$I = I_1 + I_2 \qquad \text{...... (3.1)}$$

4. YOUNG'S DOUBLE SLIT EXPERIMENT (Y.D.S.E.)

In 1802 Thomas Young devised a method to produce a stationary interference pattern. This was based upon division of a single wavefront into two; these two wavefronts acted as if they emanated from two sources having a fixed phase relationship. Hence when they were allowed to interfere, stationary interference pattern was observed.

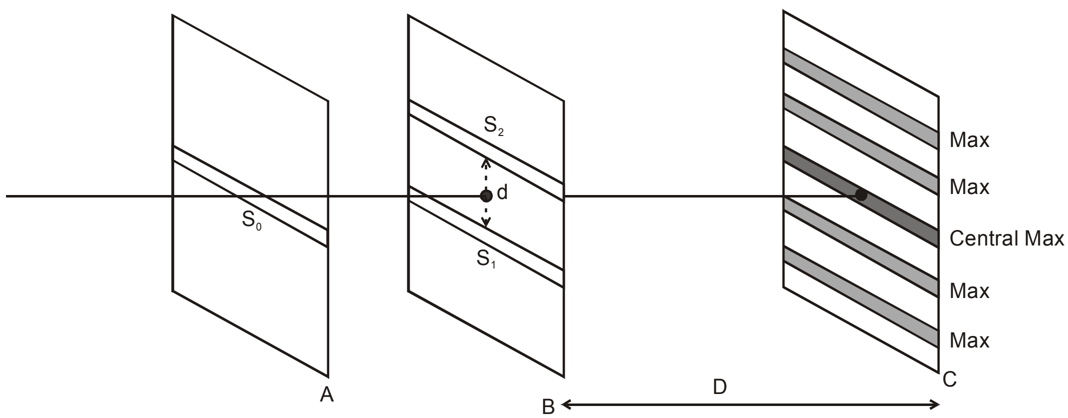


Figure : 4. 1 : *Young's Arrangement to produce stationary interference pattern by division of wave front S_0 into S_1 and S_2*

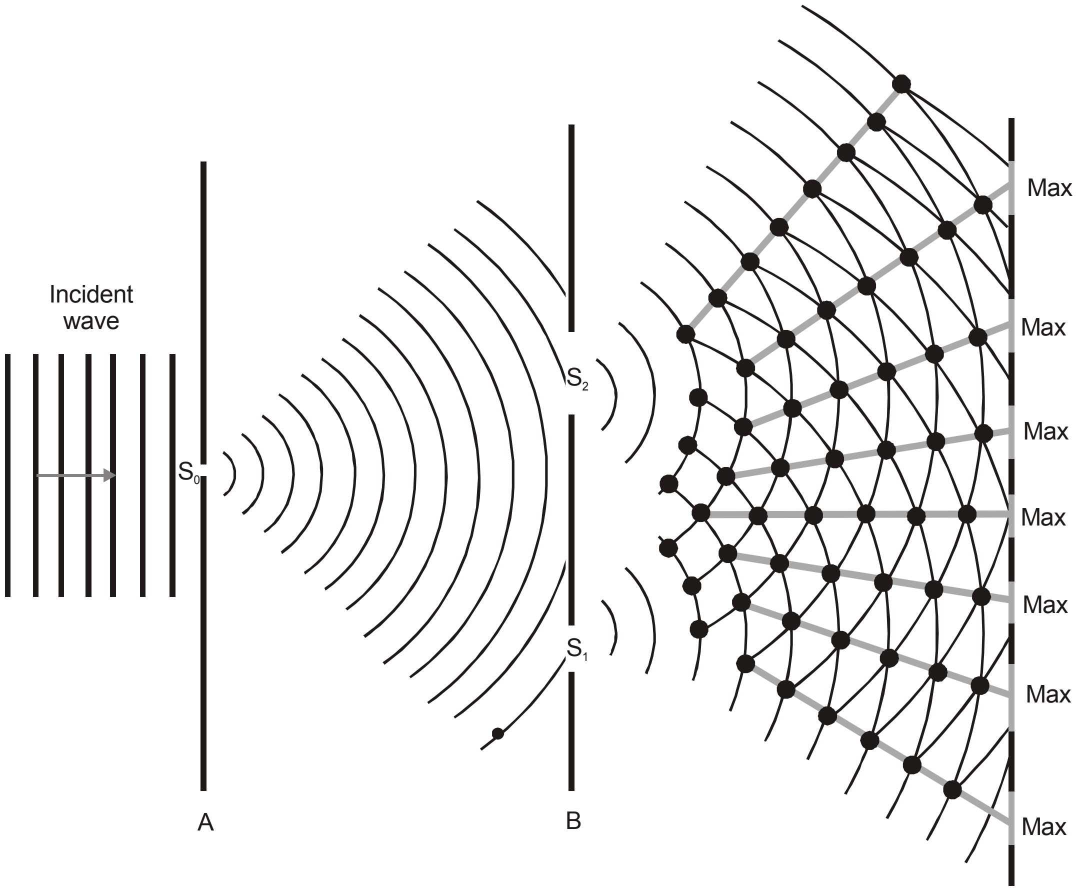


Figure 4.2 : In Young's interference experiment, light diffracted from pinhole S_0 excounters pinholes S1 and S2 in screen B. Light diffracted from these two pinholes overlaps in the region between screen B and viewing screen C, producing an interference pattern on screen C.

4.1 Analysis of Interference Pattern

We have insured in the above arrangement that the light wave passing through S_1 is in phase with that passing through S_2. However the wave reaching P from S_2 may not be in phase with the wave reaching P from S_1, because the latter must travel a longer path to reach P than the former. We have already discussed the phase-difference arising due to path difference. If the path difference is equal to zero or is an integral multiple of wavelengths, the arriving waves are exactly in phase and undergo constructive interference.

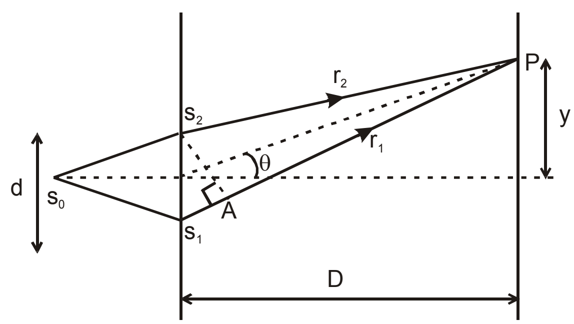


Figure : 4.3

If the path difference is an odd multiple of half a wavelength, the arriving waves are out of phase and undergo fully destructive interference. Thus, it is the path difference Δx, which determines the intensity at a point P.

$$\text{Path difference } \Delta p = S_1P - S_2P = \sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2} \qquad ...(4.1)$$

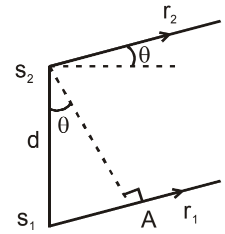


Figure : 4.4

Approximation I :

For D >> d, we can approximate rays $\vec{r}_1$ and $\vec{r}_2$ as being approximately parallel, at angle θ to the principle axis.

Now, $S_1P - S_2P = S_1A = S_1S_2 \sin\theta$

$\Rightarrow$ path difference = $d \sin\theta$ ...(4.2)

Approximation II :

further if θ is small, i.e.y << D, $\sin\theta = \tan\theta = \dfrac{y}{D}$

and hence, path difference = $\dfrac{dy}{D}$ ...(4.3)

for maxima (constructive interference),

$$\Delta p = \frac{d.y}{D} = n\lambda \qquad \Rightarrow \qquad y = \frac{n\lambda D}{d}, \ n = 0, \pm 1, \pm 2, \pm 3 \qquad .....(4.4)$$

Here n = 0 corresponds to the central maxima

n = ±1 correspond to the 1st maxima

n = ±2 correspond to the 2nd maxima and so on.

for minima (destructive interference).

$$\Delta p = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2} \pm\frac{5\lambda}{2}$$

$$\Rightarrow \qquad \Delta p = \begin{cases} (2n-1)\dfrac{\lambda}{2} & n = 1, 2, 3........... \\ (2n+1)\dfrac{\lambda}{2} & n = -1, -2, -3........ \end{cases}$$

$$\text{consequently,} \quad y = \begin{cases} (2n-1)\dfrac{\lambda D}{2d} & n = 1, 2, 3.........\ .... \\ (2n+1)\dfrac{\lambda D}{2d} & n = -1, -2, -3....... \end{cases} \qquad .... (4.5)$$

Here n = ± 1 corresponds to first minima,

n = ± 2 corresponds to second minima and so on.

4.2 Fringe width :

It is the distance between two maxima of successive order on one side of the central maxima. This is also equal to distance between two successive minima.

fringe width $\beta = \dfrac{\lambda D}{d}$ ... (4.6)

- Notice that it is directly proportional to wavelength and inversely proportional to the distance between the two slits.

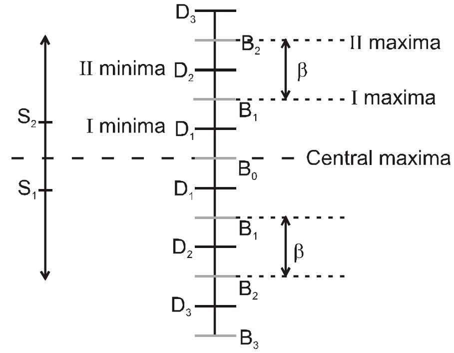


Figure : 4.5 fringe pattern in YDSE

4.3 Maximum order of Interference Fringes :

In section 4.1 we obtained,

$y = \dfrac{n\lambda D}{d}$, n = 0, ± 1, ± 2 ..... for interference maxima, but n cannot take infinitely large values, as that would violate the approximation (II)

i.e., θ is small or $y << D$

$$\Rightarrow \quad \frac{y}{D} = \frac{n\lambda}{d} << 1$$

Hence the above formula (4.4 & 4.5) for interference maxima/minima are applicable when

$$n << \frac{d}{\lambda}$$

when n becomes comparable to $\dfrac{d}{\lambda}$ path difference can no longer be given by equation (4.3) but by (4.2)

Hence for maxima

$$\Delta p = n\lambda \quad \Rightarrow \quad d\sin\theta = n\lambda \quad \Rightarrow \quad n = \frac{d\sin\theta}{\lambda}$$

Hence highest order of interference maxima, $n_{max} = \left[\dfrac{d}{\lambda}\right]$ .... (4.7)

where [ ] represents the greatest integer function.

Similarly highest order of interference minima,

$$n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] \quad .... (4.8)$$

Alter

$\Delta p = S_1P - S_2P$

$\Delta p \le d \quad \Rightarrow \quad \Delta p_{max} = d$

(3rd side of a triangle is always greater than the difference in length of the other two sides)

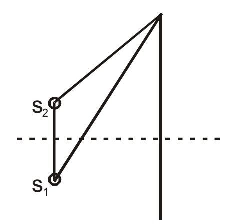


Figure : 4.6

4.4 Intensity :

Suppose the electric field components of the light waves arriving at point P(in the Figure : 4.3) from the two slits S_1 and S_2 vary with time as

$E_1 = E_0 \sin \omega t$

and $E_2 = E_0 \sin (\omega t + \phi)$

Here $\phi = k\Delta x = \dfrac{2\pi}{\lambda}\Delta x$

and we have assumed that intensity of the two slits S_1 and S_2 are same (say I_0); hence waves have same amplitude E_0.

then the resultant electric field at point P is given by,

$E = E_1 + E_2 = E_0 \sin \omega t + E_0 \sin (\omega t + \phi) = E_0' \sin (\omega t + \phi')$

where $E_0'^2 = E_0^2 + E_0^2 + 2E_0 . E_0 \cos \phi = 4 E_0^2 \cos^2 \phi/2$

Hence the resultant intensity at point P,

$$I = 4I_0 \cos^2 \frac{\phi}{2} \quad ....(4.9)$$

$$I_{max} = 4I_0 \text{ when } \frac{\phi}{2} = n\pi, \quad n = 0, \pm1, \pm2,.......,$$

$$I_{min} = 0 \text{ when } \frac{\phi}{2} = \left(n - \frac{1}{2}\right)\pi \quad n = 0, \pm 1, \pm 2 ....$$

Here $\phi = k\Delta x = \frac{2\pi}{\lambda}\Delta x$

If D >> d, $\phi = \frac{2\pi}{\lambda} d \sin\theta$

If D >> d & y << D, $\phi = \frac{2\pi}{\lambda} d\frac{y}{D}$

However if the two slits were of different intensities I_1 and I_2,

say $E_1 = E_{01} \sin \omega t$

and $E_2 = E_{02} \sin (\omega t + \phi)$

then resultant field at point P,

$$E = E_1 + E_2 = E_0 \sin (\omega t + \phi)$$

where $E_0^2 = E_{01}^2 + E_{O2}^2 + 2E_{01} E_{02} \cos \phi$

Hence resultant intensity at point P,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi \qquad \text{............ (4.10)}$$

Solved Examples

Example 4. In a YDSE, D = 1m, d = 1mm and λ = 1/2 mm

(i) Find the distance between the first and central maxima on the screen.

(ii) Find the no of maxima and minima obtained on the screen.

Solution : (i) D >> d

Hence $\Delta P = d \sin\theta$

$$\frac{d}{\lambda} = 2,$$

clearly, $n << \frac{d}{\lambda} = 2$ is not possible for any value of n.

Hence $\Delta p = \frac{dy}{D}$ cannot be used

for Ist maxima,

$\Delta p = d \sin \theta = \lambda$

$\Rightarrow \quad \sin\theta = \frac{\lambda}{d} = \frac{1}{2}$

$\Rightarrow \quad \theta = 30^\circ$

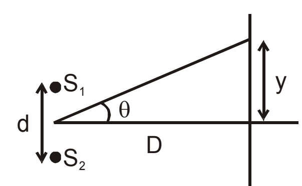


Figure 4.7

Hence, $y = D \tan\theta = \frac{1}{\sqrt{3}}$ meter

(ii) Maximum path difference

$\Delta P_{max} = d = 1$ mm

$\Rightarrow$ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$

and highest order minima $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total no. of maxima = $2n_{max} + 1^* = 5$ *(central maxima).

Total no. of minima = $2n_{min} = 4$

Example 5. Monochromatic light of wavelength 5000 A° is used in Y.D.S.E., with slit-width, d = 1mm, distance between screen and slits, D = 1m. If intensity at the two slits are, $I_1 = 4I_0$, $I_2 = I_0$, find

(i) fringe width β

(ii) distance of 5th minima from the central maxima on the screen

(iii) Intensity at $y = \frac{1}{3}$ mm

(iv) Distance of the 1000th maxima from the central maxima on the screen.

(v) Distance of the 5000th maxima from the central maxima on the screen.

Solution : (i) $\beta = \frac{\lambda D}{d} = \frac{5000 \times 10^{-10} \times 1}{1 \times 10^{-3}} = 0.5$ mm

(ii) $y = (2n - 1)\frac{\lambda D}{2d}$, n = 5

$\Rightarrow \quad y = 2.25$ mm

(iii) At $y = \frac{1}{3}$ mm, $y << D$ Hence $\Delta p = \frac{d.y}{D}$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta p = 2\pi\frac{dy}{\lambda D} = \frac{4\pi}{3}$$

Now resultant intensity

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$$

$$= 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi = 5I_0 + 4I_0\cos\frac{4\pi}{3} = 3I_0$$

(iv) $\frac{d}{\lambda} = \frac{10^{-3}}{0.5 \times 10^{-6}} = 2000$

n = 1000 is not << 2000

Hence now $\Delta p = d \sin\theta$ must be used

Hence, $d\sin\theta = n\lambda = 1000\,\lambda$

$\Rightarrow \quad \sin\theta = 1000\frac{\lambda}{d} = \frac{1}{2} \quad \Rightarrow \quad \theta = 30°$

$y = D\tan\theta = \frac{1}{\sqrt{3}}$ meter

(v) Highest order maxima

$$n_{max} = \left[\frac{d}{\lambda}\right] = 2000$$

Hence, n = 5000 is not possible.

5. SHAPE OF INTERFERENCE FRINGES IN YDSE

We discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.

Fringes are locus of points which move in such a way that its path difference from the two slits remains constant.

$S_2P - S_1P = \Delta$ = constant ....(5.1)

If $\Delta = \pm \frac{\lambda}{2}$, the fringe represents 1st minima.

If $\Delta = \pm \frac{3\lambda}{2}$ it represents 2nd minima

If $\Delta = 0$ it represents central maxima,

If $\Delta = \pm \lambda$, it represents 1st maxima etc.

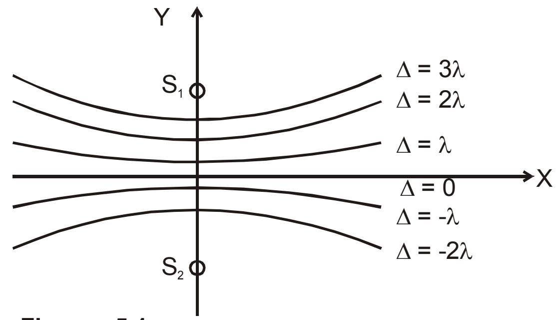


Figure : 5.1

Equation (5.1) represents a hyperbola with its two foci at S_1 and S_2

The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis S_1S_2.

A. If the screen is perpendicular to the X axis, i.e. in the YZ plane, as is generally the case, fringes are hyperbolic with a straight central section.

B. If the screen is in the XY plane, again fringes are hyperbolic.

C. If screen is perpendicular to Y axis (along S_1S_2), ie in the XZ plane, fringes are concentric circles with center on the axis S_1S_2; the central fringe is bright if $S_1S_2 = n\lambda$ and dark if $S_1S_2 = (2n - 1)\frac{\lambda}{2}$.

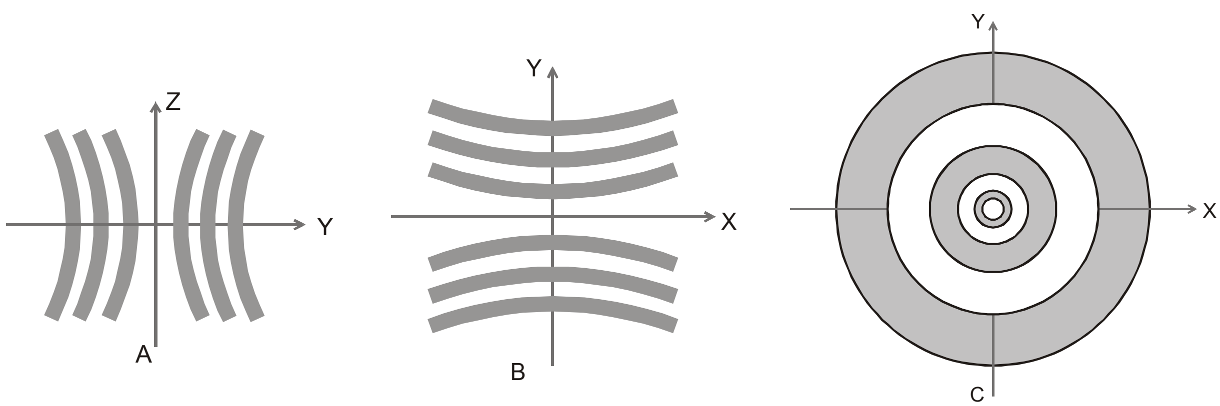


Figure : 5.2

6. YDSE WITH WHITE LIGHT

The central maxima will be white because all wavelengths will constructively interference here. However slightly below (or above) the position of central maxima fringes will be coloured. For example if P is a point on the screen such that

$$S_2P - S_1P = \frac{\lambda_{violet}}{2} = 190 \text{ nm},$$

completely destructive interference will occur for violet light. Hence we will have a line devoid of violet colour that will appear reddish. And if

$$S_2P - S_1P = \frac{\lambda_{red}}{2} \approx 350 \text{ nm},$$

completely destructive interference for red light results and the line at this position will be violet. The coloured fringes disappear at points far away from the central white fringe; for these points there are so many wavelengths which interfere constructively, that we obtain a uniform white illumination. for example if

$$S_2P - S_1 P = 3000 \text{ nm},$$

then constructive interference will occur for wavelengths $\lambda = \frac{3000}{n}$ nm. In the visible region these wavelength are 750 nm (red), 600 nm (yellow), 500 nm (greenish–yellow), 428.6 nm (violet). Clearly such a light will appear white to the unaided eye.

Thus with white light we get a white central fringe at the point of zero path difference, followed by a few coloured fringes on its both sides, the color soon fading off to a uniform white.

In the usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of central maxima in such cases.

Solved Examples

Example 6. A beam of light consisting of wavelengths 6000Å and 4500Å is used in a YDSE with D = 1m and d = 1 mm. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.

Solution: $\beta_1 = \frac{\lambda_1 D}{d} = \frac{6000 \times 10^{-10} \times 1}{10^{-3}} = 0.6$ mm

$\beta_2 = \frac{\lambda_2 D}{d} = 0.45$ mm

Let n_1th maxima of λ_1 and n_2th maxima of λ_2 coincide at a position y.

then, $y = n_1\ \beta_1 = n_2\ \beta_2$ = LCM of β_1 and β_2

$\Rightarrow$ y = LCM of 0.6 mm and 0.45 mm

y = 1.8 mm **Ans.**

At this point 3rd maxima for 6000 Å & 4th maxima for 4500 Å coincide

Example 7. White light is used in a YDSE with D = 1m and d = 0.9 mm. Light reaching the screen at position y = 1 mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.

Solution : $\Delta p = \frac{yd}{D} = 9 \times 10^{-4} \times 1 \times 10^{-3}$ m $= 900$ nm

for minima $\Delta p = (2n - 1)\lambda/2$

$$\Rightarrow \quad \lambda = \frac{2\Delta P}{(2n-1)} = \frac{1800}{(2n-1)}$$

$$= \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{5}, \frac{1800}{7} \ldots\ldots\ldots$$

of these 600 nm and 360 nm lie in the visible range. Hence these will be missing lines in the visible spectrum.

7. GEOMETRICAL PATH & OPTICAL PATH

Actual distance travelled by light in a medium is called geometrical path (Δx). Consider a light wave given by the equation

$$E = E_0 \sin(\omega t - kx + \phi)$$

If the light travels by Δx, its phase changes by $k\Delta x = \frac{\omega}{v}\Delta x$, where ω, the frequency of light does not depend on the medium, but v, the speed of light depends on the medium as $v = \frac{c}{\mu}$.

Consequently, change in phase

$$\Delta\phi = k\Delta x = \frac{\omega}{c}(\mu\Delta x)$$

It is clear that a wave travelling a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum. i.e. a path length of Δx in medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.

The quantity $\mu\Delta x$ is called the optical path length of light, Δx_{opt} . And in terms of optical path length, phase difference would be given by,

$$\Delta\phi = \frac{\omega}{c}\Delta x_{opt} = \frac{2\pi}{\lambda_0}\Delta x_{opt} \qquad .... (7.1)$$

where λ_0 = wavelength of light in vacuum.

However in terms of the geometrical path length Δx,

$$\Delta\phi = \frac{\omega}{c}(\mu\Delta x) = \frac{2\pi}{\lambda}\Delta x \qquad .....(7.2)$$

where λ = wavelength of light in the medium ($\lambda = \frac{\lambda_0}{\mu}$).

7.1 Displacement of fringe on introduction of a glass slab in the path of the light coming out of the slits–

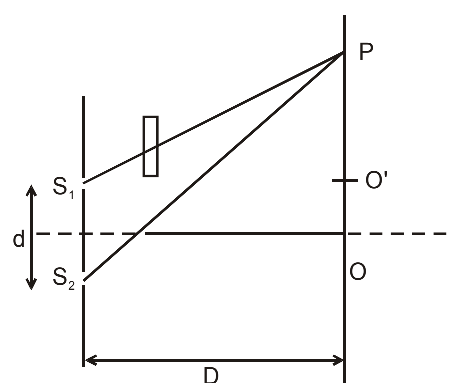


Figure : 7.1

On introduction of the thin glass-slab of thickness t and refractive index μ, the optical path of the ray S_1P increases by $t(\mu - 1)$. Now the path difference between waves coming form S_1 and S_2 at any point P is

$$\Delta p = S_2P - (S_1P + t(\mu - 1))$$
$$= (S_2P - S_1P) - t(\mu - 1)$$

$\Rightarrow \quad \Delta p = d\sin\theta - t(\mu - 1)$ if $d << D$

and $\quad \Delta p = \frac{yd}{D} - t(\mu - 1)$ If $y << D$ as well.

for central bright fringe, $\Delta p = 0$

$\Rightarrow \quad \frac{yd}{D} = t(\mu - 1). \qquad \Rightarrow \quad y = OO' = (\mu - 1)t\,\frac{D}{d} = (\mu - 1)\,t\,.\,\frac{\beta}{\lambda}$

The whole fringe pattern gets shifted by the same distance

$$\Delta = (\mu - 1)\,.\,\frac{D}{d} = (\mu - 1)t\,\frac{B}{\lambda}.$$

* Notice that this shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

Solved Examples

Example 8. In a YDSE with d = 1mm and D = 1m, slabs of (t = 1μm, μ = 3) and (t = 0.5 μm, μ = 2) are introduced in front of upper and lower slit respectively. Find the shift in the fringe pattern.

Solution : Optical path for light coming from upper slit S_1 is

$$S_1P + 1\mu m\,(2 - 1) = S_2P + 0.5\ \mu m$$

Similarly optical path for light coming from S_2 is

$$S_2P + 0.5\ \mu m\,(2 - 1) = S_2P + 0.5\ \mu m$$

Path difference : $\Delta p = (S_2P + 0.5\ \mu m) - (S_1P + 2\mu m) \qquad = (S_2P - S_1P) - 1.5\ \mu m.$

$$= \frac{yd}{D} - 1.5\ \mu m$$

for central bright fringe $\Delta p = 0$

$$\Rightarrow \qquad y = \frac{1.5\mu m}{1mm} \times 1m = 1.5\ mm.$$

The whole pattern is shifted by 1.5 mm upwards. **Ans.**

8. YDSE WITH OBLIQUE INCIDENCE

In YDSE, ray is incident on the slit at an inclination of θ_0 to the axis of symmetry of the experimental set-up

for points above the central point on the screen, (say for P_1)

$$\Delta p = d\sin\theta_0 + (S_2P_1 - S_1P_1)$$

$$\Rightarrow \qquad \Delta p = d\sin\theta_0 + d\sin\theta_1 \qquad (\text{If } d << D)$$

and for points below O on the screen, (say for P_2)

$$\Delta p = |(d\sin\theta_0 + S_2P_2) - S_1P_2|$$

$$= |d\sin\theta_0 - (S_1P_2 - S_2P_2)|$$

$$\Rightarrow \qquad \Delta p = |d\sin\theta_0 - d\sin\theta_2| \qquad (\text{if } d << D)$$

We obtain central maxima at a point where, $\Delta p = 0$.

$$(d\sin\theta_0 - d\sin\theta_2) = 0$$

or $\qquad \theta_2 = \theta_0.$

This corresponds to the point O' in the diagram.

Hence we have finally for path difference.

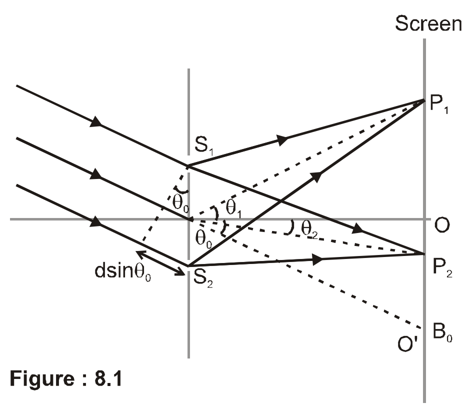


Figure : 8.1

$$\Delta p = \begin{cases} d(\sin\theta_0 + \sin\theta) - \text{for points above O} \\ d(\sin\theta_0 - \sin\theta) - \text{for points between O \& O'} \\ d(\sin\theta - \sin\theta_0) - \text{for points below O'} \end{cases} \qquad \text{... (8.1)}$$

Solved Examples

Example 9. In YDSE with D = 1m, d = 1mm, light of wavelength 500 nm is incident at an angle of 0.57° w.r.t. the axis of symmetry of the experimental set up. If centre of symmetry of screen is O as shown.
(i) find the position of central maxima
(ii) Intensity at point O in terms of intensity of central maxima I_0.
(iii) Number of maxima lying between O and the central maxima.

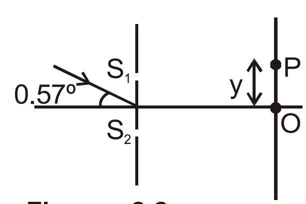


Figure : 8.2

Solution :. (i) $\theta = \theta_0 = 0.57^\circ$

$\Rightarrow \quad y = -D \tan\theta \simeq -D\theta = -1 \text{ meter} \times \left(\frac{0.57}{57}\text{rad}\right)$

$\Rightarrow \quad y = -1\text{cm}.$

(ii) for point 0, $\theta = 0$

Hence, $\Delta p = d \sin\theta_0$; $d\theta_0 = 1 \text{ mm} \times (10^{-2} \text{ rad})$

$= 10{,}000 \text{ nm} = 20 \times (500 \text{ nm})$

$\Rightarrow \quad \Delta p = 20\,\lambda$

Hence point O corresponds to 20th maxima

$\Rightarrow$ intensity at O = I_0

(iii) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima.

9. THIN-FILM INTERFERENCE

In YDSE we obtained two coherent source from a single (incoherent) source by division of wave-front. Here we do the same by division of Amplitude (into reflected and refracted wave).

When a plane wave (parallel rays) is incident normally on a thin film of uniform thickness d then waves reflected from the upper surface interfere with waves reflected from the lower surface.

Clearly the wave reflected from the lower surface travel an extra optical path of $2\mu d$, where μ is refractive index of the film.

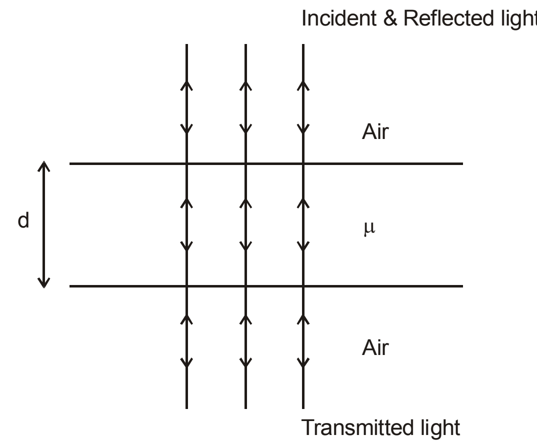


Figure : 9.1

Further if the film is placed in air the wave reflected from the upper surface (from a denser medium) suffers a sudden phase change of π, while the wave reflected from the lower surface (from a rarer medium) suffers no such phase change.

Consequently condition for constructive and destructive interference in the reflected light is given by,

$2\mu d = n\lambda$ for destructive interference

and $2\mu d = (n + \frac{1}{2})\lambda$ for constructive interference ....(9.1)

where $n = 0, 1, 2$ ....,

and λ = wavelength in free space.

Interference will also occur in the transmitted light and here condition of constructive and destructive interference will be the reverse of (9.1)

i.e. $$2\mu d = \begin{cases} n\lambda & \text{for constructive interference} \\ (n + \frac{1}{2}\lambda) & \text{for destructive interference} \end{cases} \quad ....(9.2)$$

This can easily be explained by energy conservation (when intensity is maximum in reflected light it has to be minimum in transmitted light) However the amplitude of the directly transmitted wave and the wave transmitted after one reflection differ substantially and hence the fringe contrast in transmitted light is poor. It is for this reason that thin film interference is generally viewed only in the reflected light.

In deriving equation (9.1) we assumed that the medium surrounding the thin film on both sides is rarer compared to the medium of thin film.

If medium on both sides are denser, then there is no sudden phase change in the wave reflected from the upper surface, but there is a sudden phase change of π in waves reflected from the lower surface. The conditions for constructive and destructive interference in reflected light would still be given by equation 9.1. However if medium on one side of the film in denser and that on the other side is rarer, then either there is no sudden phase in any reflection, or there is a sudden phase change of π in both reflection from upper and lower surface. Now the condition for constructive and destructive interference in the reflected light would be given by equation 9.2 and not equation 9.1.

Solved Examples

Example 10. White light, with a uniform intensity across the visible wavelength range 430–690 nm, is perpendicularly incident on a water film, of index of refraction μ = 1.33 and thickness d = 320 nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

Solution : This situation is like that of Figure (9.1), for which equation (9.1) gives the interference maxima. Solving for λ and inserting the given data, we obtain

$$\lambda = \frac{2\mu d}{m + 1/2} = \frac{(2)(1.33)(320\,\text{nm})}{m + 1/2} = \frac{851\text{nm}}{m + 1/2}$$

for m = 0, this give us λ = 1700 nm, which is in the infrared region. For m = 1, we find λ = 567 nm, which is yellow-green light, near the middle of the visible spectrum. For m = 2, λ = 340 nm, which is in the ultraviolet region. So the wavelength at which the light seen by the observer is brightest is

λ = 567 nm. **Ans.**

Example 11. A glass lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface (figure). The index of refraction of MgF_2 is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum (λ = 550 nm)? Assume the light is approximately perpendicular to the lens surface.

Solution : The situation here differs from figure (9.1) in that $n_3 > n_2 > n_1$. The reflection at point a still introduces a phase difference of π but now the reflection at point b also does the same (see figure 9.2). Unwanted reflections from glass can be, suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of a properly chosen thickness which introduces a phase change of half a wavelength. For this, the path length difference 2L within the film must be equal to an odd number of half wavelengths:

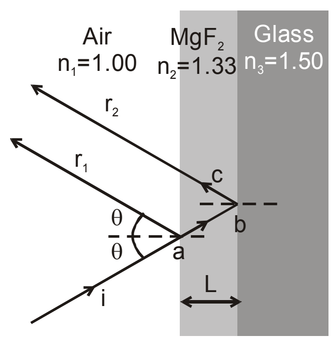


Figure : 9.2

$$2L = (m + 1/2)\lambda_{n2},$$

or, with $\lambda n_2 = \lambda/n_2$,

$$2n_2 L = (m + 1/2)\lambda.$$

We want the least thickness for the coating, that is, the smallest L. Thus we choose m = 0, the smallest value of m. Solving for L and inserting the given data, we obtain

$$L = \frac{\lambda}{4n_2} = \frac{550\,\text{nm}}{(4)(1.38)} = 96.6 \text{ nm}$$ **Ans.**

10 HUYGENS' CONSTRUCTION

Huygens, the Dutch physicist and astronomer of the seventeenth century, gave a beautiful geometrical description of wave propagation. We can guess that he must have seen water waves many times in the canals of his native place Holland. A stick placed in water and oscillated up and down becomes a source of waves. Since the surface of water is two dimensional, the resulting wavefronts would be circles instead of spheres. At each point on such a circle, the water level moves up and down. Huygens' idea is that we can think of every such oscillating point on a wavefront as a new source of waves. According to Huygens' principle, what we observe is the result of adding up the waves from all these different sources. These are called secondary waves or wavelets. Huygens' principle is illustrated in (Figure : 10.1) in the simple case of a plane wave.

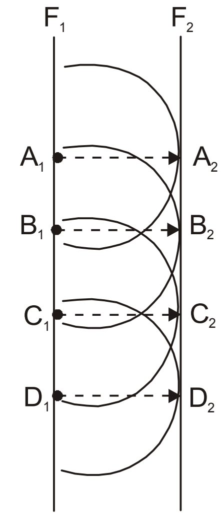


Figure : 10.1

(i) At time t = 0, we have a wavefront F_1, F_1 separates those parts of the medium which are undisturbed from those where the wave has already reached.

(ii) Each point on F_1 acts like a new source and sends out a spherical wave. After a time 't' each of these will have radius vt. These spheres are the secondary wavelets.

(iii) After a time t, the disturbance would now have reached all points within the region covered by all these secondary waves. The boundary of this region is the new wavefront F_2. Notice that F_2 is a surface tangent to all the spheres. It is called the forward envelope of these secondary wavelets.

(iv) The secondary wavelet from the point A_1 on F_1 touches F_2 at A_2. Draw the line connecting any point A_1 on F_1 to the corresponding point A_2 on F_2. According to Huygens, $A_1 A_2$ is a ray. It is perpendicular to the wavefronts F_1 and F_2 and has length vt. This implies that rays are perpendicular to wavefronts. Further, the time taken for light to travel between two wavefronts Is the same along any ray. In our example, the speed 'v' of the wave has been taken to be the same at all points in the medium. In this case, we can say that the distance between two wavefronts is the same measured along any ray.

(v) This geometrical construction can be repeated starting with F_2 to get the next wavefront F_3 a time t later, and so on. This is known as Huygens' construction.

Huygens' construction can be understood physically for waves in a material medium, like the surface of water. Each oscillating particle can set its neighbors into oscillation, and therefore acts as a secondary source. But what if there is no medium, as for light travelling in vacuum? The mathematical theory, which cannot be given here, shows that the same geometrical construction works in this case as well.

10.1 REFLECTION AND REFRACTION.

We can use a modified form of Huygens' construction to understand reflection and refraction of light. Figure (10.2a) shows an incident wavefront which makes an angle 'i' with the surface separating two media, for example, air and water. The phase speeds in the two media are v_1 and v_2. We can see that when the point A on the incident wavefront strikes the surface, the point B still has to travel a distance BC = AC sin *i*, and this takes a time t = BC/v_1 = AC (sin i) / v_1. After a time t, a secondary wavefront of radius v_2t with A as centre would have travelled into medium 2. The secondary wavefront with C as centre would have just started, i.e.. would have zero radius. We also show a secondary wavelet originating from a point D in between A and C. Its radius is less than v_2t. The wavefront in medium 2 is thus a line passing through C and tangent to the circle centred on A. We can see that the angle r' made by this refracted wavefront with the surface is given by AE = v_2t = AC sin r'. Hence, t = AC (sin r'} / v_2. Equating the two expressions for 't' gives us the law of refraction in the form sin i/sin r' = v_1/v_2. A similar picture is drawn in Fig. (10.2 b) for the reflected wave which travels back into medium 1. In this case, we denote the angle made by the reflected wavefront with the surface by r, and we find that i = r. Notice that for both reflection and refraction, we use secondary wavelets starting at different times. Compare this with the earlier application (Fig.10.1) where we start them at the same time.

The preceding argument gives a good physical picture of how the refracted and reflected waves are built up from secondary wavelets. We can also understand the laws of reflection and refraction using the concept that the time taken by light to travel along different rays from one wavefront to another must be the same. (Fig.) Shows the incident and reflected wavefronts when a parallel beam of light falls on a plane surface. One ray POQ is shown normal to both the reflected and incident wavefronts. The angle of incidence i and the angle of reflection r are defined as the angles made by the incident and reflected rays with the normal. As shown in Fig. (c), these are also the angles between the wavefront and the surface.

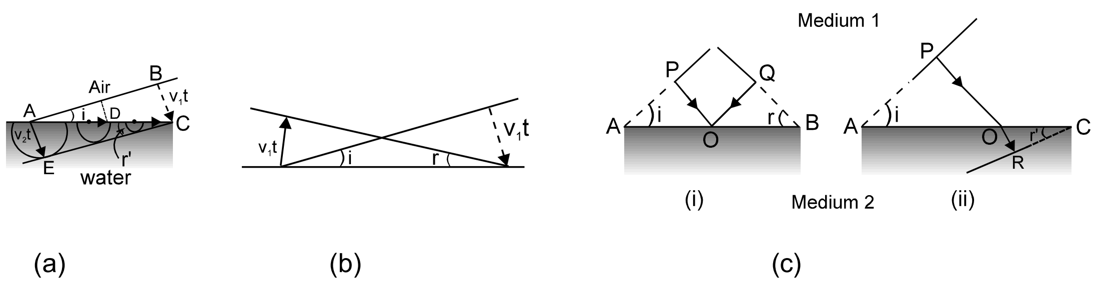


Figure : 10.2

(Fig.) (a) Huygens' construction for the (a) refracted wave. (b) Reflected wave. (c) Calculation of propagation time between wavefronts in (i) reflection and (ii) refraction.

We now calculate the total time to go from one wavefront to another along the rays. From Fig. (c), we have, we have Total time for light to reach from P to Q

$$= \frac{PO}{v_1} + \frac{OQ}{v_1} = \frac{AO \sin i}{v_1} + \frac{OB \sin r}{v_1}$$

$$= \frac{OA \sin i + (AB - OA)\sin r}{v_1} = \frac{AB \sin r + OA(\sin i - \sin r)}{v_1}$$

Different rays normal to the incident wavefront strike the surface at different points O and hence have different values of OA. Since the time should be the same for all the rays, the right side of equation must actually be Independent of OA. The condition, for this to happen is that the coefficient of OA in Eq. (should be zero, i.e., sin i = sin r. We, thus. have the law of reflection, i = r. Figure also shows refraction at a plane surface separating medium 1 (speed of light v_1) from medium 2 (speed of light v_2). The incident and refracted wavefronts are shown, making angles i and r' with the boundary. Angle r' is called the angle of refraction. Rays perpendicular to these are also drawn. As before, let us calculate the time taken to travel between the two wavefronts along any ray.

Time taken from P to R = $\frac{PO}{v_1} + \frac{OR}{v_2}$

$$= \frac{OA \sin i}{v_1} + \frac{(AC - OA)\sin r'}{v_2} = \frac{AC \sin r'}{v_2} + OA\left(\frac{\sin i}{v_1} - \frac{\sin r'}{v_2}\right)$$

This time should again be independent of which ray we consider. The coefficient of OA in Equation is, therefore, zero,. That is, $\frac{\sin i}{\sin r'} = \frac{v_1}{v_2} = n_{21}$

where n_{21} is the refractive index of medium 2 with respect to medium 1. This is the Snell's law of, refraction that we have already dealt with from Eq. n_{21} is the ratio of speed of light in the first medium (v_1) to that in the second medium (v_2). Equation is, known as the Snell's law of refraction. If the first medium is vacuum, we have $\frac{\sin i}{\sin r'} = \frac{c}{v_2} = n_2$

where n_2 is the refractive index of medium 2 with respect to vacuum, also called the absolute refractive index of the medium. A similar equation defines absolute refractive index n_1 of the first medium. From Eq. we then

get $n_{21} = \frac{v_1}{v_2} = \left(\frac{c}{n_1}\right) / \left(\frac{c}{n_2}\right) = \frac{n_2}{n_1}$

The absolute refractive index of air is about 1.0003, quite close to 1. Hence, for all practical purposes, absolute refractive index of a medium may be taken with respect to air. For water, $n_1 = 1.33$, which means $v_1 = \frac{c}{1.33}$, i.e. about 0.75 times the speed of light in vacuum. The measurement of the speed of light in water by Foucault (1850) confirmed this prediction of the wave theory.

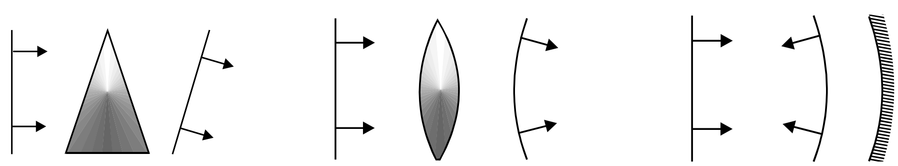
Once we have the laws of reflection and refraction, the behaviour of prisms. lenses, and mirrors can be understood. These topics are discussed in detail in the previous Chapter. Here we just describe the behaviour of the wavefronts in these three cases (Fig.)

(i) Consider a plane wave passing through a thin prism. Clearly, the portion of the incoming wavefront which travels through the greatest thickness of glass has been delayed the most. Since light travels more slowly in glass. This explains the tilt in the emerging wavefront.

(ii) Similarly, the central part of an incident plane wave traverses the thickest portion of a convex lens and is delayed the most. The emerging wavefront has a depression at the centre. It is spherical and converges to a focus,

(iii) A concave mirror produces a similar effect. The centre of the wavefront has to travel a greater distance before and after getting reflected, when compared to the edge. This again produces a converging spherical wavefront.

(iv) Concave lenses and convex mirrors can be understood from time delay arguments in a similar manner. One interesting property which is obvious from the pictures of wavefronts is that the total time taken from a point on the object to the corresponding point on the image is the same measured along any ray (Fig.). For example, when a convex lens focuses light to form a real image, it may seem that rays going through the centre are shorter. But because of the slower speed in glass, the time taken is the same as for rays travelling near the edge of the lens.



(a) (b) (c)

Figure : 10.3