NUCLEAR PHYSICS

It is the branch of physics which deals with the study of nucleus.

1. NUCLEUS :

(a) ***Discoverer*** : Rutherford

(b) ***Constituents*** : neutrons (n) and protons (p) [collectively known as nucleons]

1. **Neutron :** It is a neutral particle. It was discovered by J. Chadwick.
   Mass of neutron, $m_n = 1.6749286 \times 10^{-27}$ kg.

2. **Proton :** It has a charge equal to +e. It was discovered by Goldstein.
   Mass of proton, $m_p = 1.6726231 \times 10^{-27}$ kg

$$m_p \lessapprox m_n$$

(c) ***Representation*** :

$${}_Z X^A \quad \text{or} \quad {}^A_Z X$$

where X ⇒ symbol of the atom
Z ⇒ Atomic number = number of protons
A ⇒ Atomic mass number = total number of nucleons.
= no. of protons + no. of neutrons.

Atomic mass number :
It is the nearest integer value of mass represented in a.m.u. (atomic mass unit).

1 a.m.u. = $\frac{1}{12}$ [mass of one atom of ${}_6C^{12}$ atom at rest and in ground state]

1.6603×10^{-27} kg ; 931.478 MeV/c^2
mass of proton (m_p) = mass of neutron (m_n) = 1 a.m.u.

Some definitions :

(1) ***Isotopes*** :
The nuclei having the same number of protons but different number of neutrons are called isotopes.

(2) ***Isotones :***
Nuclei with the same neutron number N but different atomic number Z are called isotones.

(3) ***Isobars*** :
The nuclei with the same mass number but different atomic number are called isobars.

(d) ***Size*** of nucleus : Order of 10^{-15} m (fermi)
Radius of nucleus ; $R = R_0 A^{1/3}$
where $R_0 = 1.1 \times 10^{-15}$ m (which is an empirical constant)
A = Atomic mass number of atom.

(e) ***Density*** : density = $\frac{\text{mass}}{\text{volume}} \cong \frac{Am_p}{\frac{4}{3}\pi R^3} = \frac{Am_p}{\frac{4}{3}\pi (R_0 A^{1/3})^3} = \frac{3m_p}{4\pi R_0{}^3}$

$$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.1 \times 10^{-15})^3} = 3 \times 10^{17}\ \text{kg/m}^3$$

Nuclei of almost all atoms have almost same density as nuclear density is independent of the mass number (A) and atomic number (Z).

Example 1. Calculate the radius of ^{70}Ge.

Solution : We have,

$R = R_0 A^{1/3} = (1.1\text{ fm})\,(70)^{1/3}$

$= (1.1\text{ fm})\,(4.12) = 4.53\text{ fm}.$

Example 2. Calculate the electric potential energy of interaction due to the electric repulsion between two nuclei of ^{12}C when they 'touch' each other at the surface

Solution : The radius of a ^{12}C nucleus is

$R = R_0 A^{1/3}$

$= (1.1\text{ fm})\,(12)^{1/3} = 2.52\text{ fm}.$

The separation between the centres of the nuclei is 2R = 5.04 fm. The potential energy of the pair is

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

$$= (9 \times 10^9\ \text{N–m}^2/\text{C}^2)\ \frac{(6 \times 1.6 \times 10^{-19}\,\text{C})^2}{5.04 \times 10^{-15}\,\text{m}}$$

$$= 1.64 \times 10^{-12}\ \text{J} = 10.2\ \text{MeV}.$$

2. MASS DEFECT

It has been observed that there is a difference between expected mass and actual mass of a nucleus.

$M_{expected} = Z\,m_p + (A - Z)m_n$

$M_{observed} = M_{atom} - Zm_e$

It is found that

$M_{observed} < M_{expected}$

Hence, mass defect is defined as

Mass defect = $M_{expected} - M_{observed}$

$\Delta m = [Zm_p + (A - Z)m_n] - [M_{atom} - Zm_e]$

3. BINDING ENERGY

It is the minimum energy required to break the nucleus into its constituent particles.

or

Amount of energy released during the formation of nucleus by its constituent particles and bringing them from infinite separation.

Binding Energy (B.E.) = Δmc^2

BE = Δm (in amu) × 931.5 MeV/amu

= Δm × 931.5 MeV

Note : If binding energy per nucleon is more for a nucleus then it is more stable.

For example

If $\left(\frac{B.E_1}{A_1}\right) > \left(\frac{B.E_2}{A_2}\right)$

then nucleus 1 would be more stable.

Example 3. Following data is available about 3 nuclei P, Q & R. Arrange them in decreasing order of stability

	P	Q	R
Atomic mass number (A)	10	5	6
Binding Energy (MeV)	100	60	66

Solution : $\left(\frac{B.E}{A}\right)_P = \frac{100}{10} = 10$

$\left(\frac{BE}{A}\right)_Q = \frac{60}{5} = 12$

$\left(\frac{B.E.}{A}\right)_R = \frac{66}{6} = 11$

$\therefore$ Stability order is Q > R > P.

Example 4. The three stable isotopes of neon: ${}^{20}_{10}Ne$, ${}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51% 0.27% and 9.22% . The atomic masses of three isotopes are 19.99 u. 20.99 u, respectively. Obtain the average atomic mass of neon.

Solution : $m = \frac{90.51 \times 19.99 + 0.27 \times 20.99 + 9.22 \times 22}{100} = 20.18$ u

Example 5 A nuclear reaction is given as

$A + B \rightarrow C + D$

Binding energies of A, B, C and D are given as

B_1, B_2, B_3 and B_4

Find the energy released in the reaction

Solution : $(B_3 + B_4) – (B_1 + B_2)$

Example 6 Calculate the binding energy of an alpha particle from the following data:

mass of ${}^{1}_{1}H$ atom = 1.007826 u

mass of neutron = 1.008665 u

mass of ${}^{4}_{2}He$ atom = 4.00260 u

Take 1 u = 931 MeV/c^2.

Solution : The alpha particle contains 2 protons and 2 neutrons. The binding energy is

$B = (2 \times 1.007826 \text{ u} + 2 \times 1.008665 \text{ u} - 4.00260 \text{ u})c^2$

$= (0.03038 \text{ u})c^2$

$= 0.03038 \times 931 \text{ MeV} = 28.3 \text{ MeV}.$

Example 7. Find the binding energy of ${}^{56}_{26}Fe$. Atomic mass of ${}^{56}Fe$ is 55.9349 u and that of ${}^{1}H$ is 1.00783 u. Mass of neutron = 1.00867 u.

Solution : The number of protons in ${}^{56}_{26}Fe$ = 26 and the number of neutrons = 56 – 26 = 30.

The binding energy of ${}^{56}_{26}Fe$ is

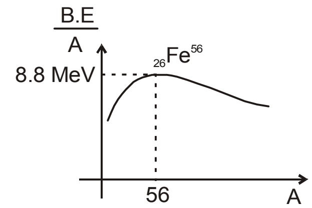
$= [26 \times 1.00783 \text{ u} + 30 \times 1.00867 \text{ u} - 55.9349 \text{ u}]\, c^2$

$= (0.52878 \text{ u})c^2$

$= (0.52878 \text{ u})\,(931 \text{ MeV/u}) = 492 \text{ MeV}.$

3.1 Variation of binding energy per nucleon with mass number :

The binding energy per nucleon first increases on an average and reaches a maximum of about 8.8 MeV for $A \approx 50 \rightarrow 80$. For still heavier nuclei, the binding energy per nucleon slowly decreases as A increases. Binding energy per nucleon is maximum for $_{26}Fe^{56}$, which is equal to 8.8 MeV. Binding energy per nucleon is more for medium nuclei than for heavy nuclei. Hence, medium nuclei are highly stable.



* *The heavier nuclei being unstable have tendency to split into medium nuclei. This process is called **Fission**.*

* *The Lighter nuclei being unstable have tendency to fuse into a medium nucleus. This process is called **Fusion**.*

4. RADIOACTIVITY :

It was discovered by Henry Becquerel.

*Spontaneous emission of radiations (α, β, γ) from unstable nucleus is called **radioactivity**. Substances which shows radioactivity are known as **radioactive substance**.*

Radioactivity was studied in detail by Rutherford.

In radioactive decay, an unstable nucleus emits α particle or β particle. After emission of α or β the remaining nucleus may emit γ-particle, and converts into more stable nucleus.

α-particle :

It is a doubly charged helium nucleus. It contains two protons and two neutrons.

Mass of α-particle = Mass of $_2He^4$ atom $- 2m_e \approx 4\ m_p$

Charge of α-particle = + 2 e

β-particle :

(a) **β^- (electron) :**

Mass = m_e ; Charge = $-e$

(b) **β^+ (positron) :**

Mass = m_e ; Charge = $+e$

positron is an antiparticle of electron.

Antiparticle :

A particle is called antiparticle of other if on collision both can annihilate (destroy completely) and converts into energy. For example : (i) electron ($-$ e, m_e) and positron (+ e, m_e) are anti particles. (ii) neutrino (ν) and antineutrino ($\bar{\nu}$) are anti particles.

γ-particle : They are energetic photons of energy of the order of Mev and having rest mass zero.

5. RADIOACTIVE DECAY (DISPLACEMENT LAW) :

5.1 α-decay :

$$_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 + Q$$

***Q value :** It is defined as energy released during the decay process.*

Q value = rest mass energy of reactants – rest mass energy of products.

This energy is available in the form of increase in K.E. of the products.

Let, M_x = mass of atom $_ZX^A$

M_y = mass of atom $_{Z-2}Y^{A-4}$

M_{He} = mass of atom $_2He^4$.

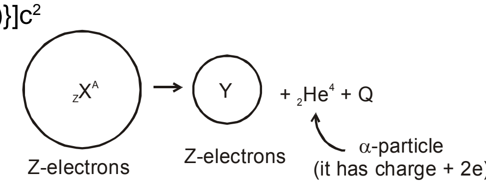
Q value = $[(M_x - Zm_e) - \{(M_y - (Z-2)\ m_e) + (M_{He} - 2m_e)\}]c^2$

$= [M_x - M_y - M_{He}]\ c^2$

Considering actual number of electrons in α-decay

Q value $= [M_x - (M_y + 2m_e) - (M_{He} - 2m_e)]c^2$

$= [M_x - M_y - M_{He}]\ c^2$



Calculation of kinetic energy of final products :

As atom X was initially at rest and no external forces are acting, so final momentum also has to be zero. Hence both Y and α-particle will have same momentum in magnitude but in opposite direction.

$p_\alpha^2 = p_Y^2$ $\qquad$ $2m_\alpha T_\alpha = 2m_Y T_Y$ $\qquad$ (Here we are representing T for kinetic energy)

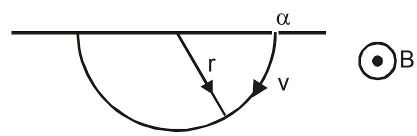
$Q = T_y + T_\alpha$ $\qquad$ $m_\alpha T_\alpha = m_Y T_Y$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y} Q \; ; \qquad T_Y = \frac{m_\alpha}{m_\alpha + m_Y} Q$$

$$T_\alpha = \frac{A-4}{A} Q \quad ; \qquad T_Y = \frac{4}{A} Q$$

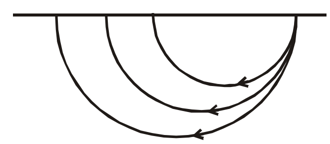
From the above calculation, one can see that all the α-particles emitted should have same kinetic energy. Hence, if they are passed through a region of uniform magnetic field having direction perpendicular to velocity, they should move in a circle of same radius.

$$r = \frac{mv}{qB} = \frac{mv}{2eB} = \frac{\sqrt{2Km}}{2eB}$$



Experimental Observation :

Experimentally it has been observed that all the α-particles do not move in the circle of same radius, but they move in `circles having different radii.



This shows that they have different kinetic energies. But it is also observed that they follow circular paths of some fixed values of radius i.e. yet the energy of emitted α-particles is not same but it is quantized. The reason behind this is that all the daughter nuclei produced are not in their ground state but some of the daughter nuclei may be produced in their excited states and they emits photon to acquire their ground state.

$$X \longrightarrow Y^* (\text{excited state}) + \alpha + Q$$
$$\downarrow$$
$$Y + \text{photon } (\gamma \text{ particle})$$

The only difference between Y and Y* is that Y* is in excited state and Y is in ground state.

Let, $\quad$ the energy of emitted γ-particles be E

$\therefore \quad Q = T_\alpha + T_Y + E$

where $\quad Q = [M_x - M_y - M_{He}]\, c^2$

$T_\alpha + T_Y = Q - E$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y}(Q-E); \quad T_Y = \frac{m_\alpha}{m_\alpha + m_Y}(Q-E)$$

5.2 β^-- decay :

$${}_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0} + Q$$

${}_{-1}e^{0}$ can also be written as ${}_{-1}\beta^{0}$.

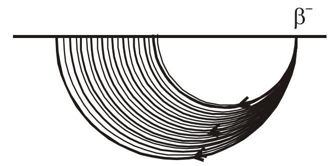
Here also one can see that by momentum and energy conservation, we will get

$$T_e = \frac{m_Y}{m_e + m_Y} Q \; ;$$

$$T_Y = \frac{m_e}{m_e + m_Y} Q$$

as $m_e << m_Y$, we can consider that all the energy is taken away by the electron.

From the above results, we will find that all the β-particles emitted will have same energy and hence they have same radius if passed through a region of perpendicular magnetic field. But, experimental observations were completely different. On passing through a region of uniform magnetic field perpendicular to the velocity, it was observed that β-particles take circular paths of different radius having a continuous spectrum.



To explain this, Paulling has introduced the extra particles called neutrino and antineutrino (antiparticle of neutrino).

$\bar{\nu} \rightarrow$ antineutrino, $\nu \rightarrow$ neutrino

Properties of antineutrino($\bar{\nu}$) & neutrino(n) :

(1) They are like photons having rest mass = 0
speed = c
Energy, $E = mc^2$

(2) They are chargeless (neutral)

(3) They have spin quantum number, $s = \pm \frac{1}{2}$

Considering the emission of antineutrino, the equation of β^- - decay can be written as

$$_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q + \bar{\nu}$$

Production of antineutrino along with the electron helps to explain the continuous spectrum because the energy is distributed randomly between electron and $\bar{\nu}$ and it also helps to explain the spin quantum number balance (p, n and ± e each has spin quantum number ± 1/2).

During β^- - decay, inside the nucleus a neutron is converted to a proton with emission of an electron and antineutrino.

$n \rightarrow p + {}_{-1}e^0 + \bar{\nu}$

Let, M_x = mass of atom $_ZX^A$
M_y = mass of atom $_{Z+1}Y^A$
m_e = mass of electron

Q value = $[(M_X - Zm_e) - \{(M_Y - (z + 1)\, m_e) + m_e\}]\, c^2 \quad = [M_X - M_Y]\, c^2$

Considering actual number of electrons.

Q value = $[M_X - \{(M_Y - m_e) + m_e\}]\, c^2 \quad = [M_X - M_Y]\, c^2$

Solved Examples

Example 8. Consider the beta decay

$^{198}Au \rightarrow {}^{198}Hg^* + \beta^- + \bar{\nu}$

where $^{198}Hg^*$ represents a mercury nucleus in an excited state at energy 1.088 MeV above the ground state. What can be the maximum kinetic energy of the electron emitted? The atomic mass ^{198}Au is 197.968233 u and that of ^{198}Hg is 197.966760 u.

Solution : If the product nucleus ^{198}Hg is formed in its ground state, the kinetic energy available to the electron and the antineutrino is

$$Q = [m(^{198}Au) - m(^{198}Hg)]c^2 .$$

As $^{198}Hg^*$ has energy 1.088 MeV more than ^{198}Hg in ground state, the kinetic energy actually available is

$Q = [m(^{198}Au) - m(^{198}Hg)]c^2 - 1.088$ MeV

$$= (197.968233\text{ u} - 197.966760\text{ u})\left(931\frac{\text{MeV}}{\text{u}}\right) - 1.088\text{ MeV}$$

$$= 1.3686\text{ MeV} - 1.088\text{ MeV} = 0.2806\text{ MeV}.$$

This is also the maximum possible kinetic energy of the electron emitted.

5.3 β^+ - decay :

$_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + \nu + Q$

In β^+ decay, inside a nucleus a proton is converted into a neutron, positron and neutrino.

$p \rightarrow n + {}_{+1}e^0 + \nu$

As mass increases during conversion of proton to a neutron, hence it requires energy for β^+ decay to take place, $\therefore$ β^+ decay is rare process. It can take place in the nucleus where a proton can take energy from the nucleus itself.

Q value $= [(M_X - Zm_e) - \{(M_Y - (Z-1)\, m_e) + m_e\}]\, c^2$

$= [M_X - M_Y - 2m_e]\, c^2$

Considering actual number of electrons.

Q value $= [M_X - \{(M_Y + m_e) + m_e\}]\, c^2$

$= [M_X - M_Y - 2m_e]\, c^2$

Solved Examples

Example 9. Calculate the Q-value in the following decays :

(a) $^{19}O \rightarrow {}^{19}F + e^- + \bar{\nu}$

(b) $^{25}Al \rightarrow {}^{25}Mg + e^+ + \nu$.

The atomic masses needed are as follows:

^{19}O	^{19}F	^{25}Al	^{25}Mg
19.003576 u	18.998403 u	24.990432 u	24.985839 u

Solution : (a) The Q-value of β^- -decay is

$Q = [m(^{19}O) - m(^{19}F)]c^2$

$= [19.003576\text{ u} - 18.998403\text{ u}]\,(931\text{ MeV/u})$

$= 4.816$ MeV

(b) The Q-value of β^+ -decay is

$Q = [m(^{25}Al) - m(^{25}Mg) - 2m_e]c^2$

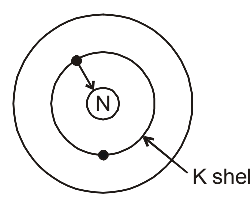
$$= \left[24.99032\,\text{u} - 24.985839\,\text{u} - 2 \times 0.511\frac{\text{MeV}}{c^2}\right]c^2$$

$= (0.004593\text{ u})\,(931\text{ MeV/u}) - 1.022$ MeV

$= 4.276\text{ MeV} - 1.022\text{ MeV} = 3.254$ MeV.

5.4 K capture :

It is a rare process which is found only in few nucleus. In this process the nucleus captures one of the atomic electrons from the K shell. A proton in the nucleus combines with this electron and converts itself into a neutron. A neutrino is also emitted in the process and is emitted from the nucleus.



$p + {}_{-1}e^0 \rightarrow n + \nu$

If X and Y are atoms then reaction is written as :

$_ZX^A \rightarrow {}_{Z-1}Y^A + \nu + Q$ + characteristic x-rays of Y.

If X and Y are taken as nucleus, then reaction is written as :

$_ZX^A + {}_{-1}e^0 \rightarrow {}_{Z-1}Y^A + \nu$

Note : (1) Nuclei having atomic numbers from Z = 84 to 112 shows radioactivity.

(2) Nuclei having Z = 1 to 83 are stable (only few exceptions are there)

(3) Whenever a neutron is produced, a neutrino is also produced.

(4) Whenever a neutron is converted into a proton, a antineutrino is produced.

6. NUCLEAR STABILITY :

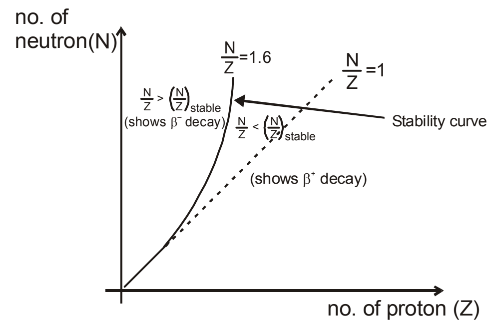


Figure shows a plot of neutron number N versus proton number Z for the nuclides found in nature. The solid line in the figure represents the stable nuclides. For light stable nuclides, the neutron number is equal to the proton number so that ratio N/Z is equal to 1. The ratio N/Z increases for the heavier nuclides and becomes about 1.6 for the heaviest stable nuclides. The points (Z, N) for stable nuclides fall in a rather well-defined narrow region. There are nuclides to the left of the stability belt as well as to the right of it. The nuclides to the left of the stability region have excess neutrons, whereas, those to the right of the stability belt have excess protons. These nuclides are unstable and decay with time according to the laws of radioactive disintegration. Nuclides with excess neutrons (lying above stability belt) show β^- decay while nuclides with excess protons (lying below stability belt) show β^+ decay and K - capture.

7. NUCLEAR FORCE :

(i) Nuclear forces are basically attractive and are responsible for keeping the nucleons bound in a nucleus in spite of repulsion between the positively charge protons.

(ii) It is strongest force with in nuclear dimensions ($F_n \approx 100\ F_e$)

(iii) It is short range force (acts only inside the nucleus)

(iv) It acts only between neutron-neutron, neutron-proton and proton-proton i.e. between nucleons.

(v) It does not depend on the nature of nucleons.

(vi) An important property of nuclear force is that it is not a central force. The force between a pair of nucleons is not solely determined by the distance between the nucleons. For example, the nuclear force depends on the directions of the spins of the nucleons. The force is stronger if the spins of the nucleons are parallel (i.e., both nucleons have $m_s = +1/2$ or $-1/2$) and is weaker if the spins are antiparallel (i.e., one nucleon has $m_s = +1/2$ and the other has $m_s = -1/2$). Here m_s is spin quantum number.

8. RADIOACTIVE DECAY : STATISTICAL LAW :

(Given by Rutherford and Soddy)

Rate of radioactive decay $\propto$ N

where N = number of active nuclei

$= \lambda N$

where λ= decay constant of the radioactive substance.

Decay constant is different for different radioactive substances, but it does not depend on amount of substance and time.

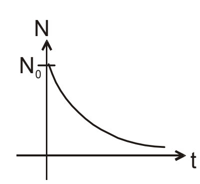
SI unit of λ is s^{-1}

If $\lambda_1 > \lambda_2$ then first substance is more radioactive (less stable) than the second one.

For the case, if A decays to B with decay constant λ

	$A \xrightarrow{\lambda}$	B	
t = 0	N_0	0	where N_0 = number of active nuclei of A at t = 0
t = t	N	N'	where N = number of active nuclei of A at t = t

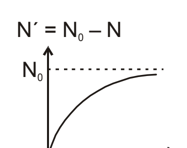
Rate of radioactive decay of A = $-\frac{dN}{dt} = \lambda N$



$$-\int_{N_0}^{N} \frac{dN}{N} = \int_0^t \lambda dt \Rightarrow \quad N = N_0\, e^{-\lambda t} \quad \text{(it is exponential decay)}$$

Number of nuclei decayed (i.e. the number of nuclei of B formed)

$$N' = N_0 - N$$
$$= N_0 - N_0 e^{-\lambda t}$$
$$N' = N_0(1 - e^{-\lambda t})$$



8.1 Half life ($T_{1/2}$) :

It is the time in which number of active nuclei becomes half.

$$N = N_0 e^{-\lambda t}$$

After one half life, $N = \dfrac{N_0}{2}$

$$\frac{N_0}{2} = N_0 e^{-\lambda t} \Rightarrow t = \frac{\ln 2}{\lambda} \Rightarrow \frac{0.693}{\lambda} = t_{1/2}$$

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$ **(to be remembered)**

Number of nuclei present after n half lives i.e. after a time $t = n\, t_{1/2}$

$$N = N_0 e^{-\lambda t} \quad = N_0 e^{-\lambda n t_{1/2}} \quad = N_0\, e^{-\lambda n \frac{\ln 2}{\lambda}}$$

$$= N_0\, e^{\ln 2^{(-n)}} \quad = N_0 (2)^{-n} = N_0 (1/2)^n \quad = \frac{N_0}{2^n}$$

$\{n = \dfrac{t}{t_{1/2}}$. It may be a fraction, need not to be an integer$\}$

or $$N_0 \xrightarrow[\text{half life}]{\text{after 1st}} \frac{N_0}{2} \xrightarrow{2} N_0\left(\frac{1}{2}\right)^2 \xrightarrow{3} N_0\left(\frac{1}{2}\right)^3 \ldots\ldots\ldots\ldots \xrightarrow{n} N_0\left(\frac{1}{2}\right)^n$$

Solved Examples

Example 10. A radioactive sample has 6.0×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives?

Solution : In one half-life the number of active nuclei reduces to half the original number. Thus, in two half lives the number is reduced to $\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore, $6.0 \times 10^{18} \times \left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right) = 1.5 \times 10^{18}$.

Example 11. The number of ^{238}U atoms in an ancient rock equals the number of ^{206}Pb atoms. The half-life of decay of ^{238}U is 4.5×10^{9} y. Estimate the age of the rock assuming that all the ^{206}Pb atoms are formed from the decay of ^{238}U.

Solution : Since the number of ^{206}Pb atoms equals the number of ^{238}U atoms, half of the original ^{238}U atoms have decayed. It takes one half-life to decay half of the active nuclei. Thus, the sample is 4.5×10^9 y old.

8.2 Activity :

Activity is defined as rate of radioactive decay of nuclei

It is denoted by A or R $\quad A = \lambda N$

If a radioactive substance changes only due to decay then

$$A = -\frac{dN}{dt}$$

As in that case, $N = N_0 e^{-\lambda t}$

$$A = \lambda N = \lambda N_0 e^{-\lambda t}$$

$$A = A_0 e^{-\lambda t}$$

SI Unit of activity : becquerel (Bq) which is same as 1 dps (disintegration per second)

The popular unit of activity is curie which is defined as

1 curie = 3.7×10^{10} dps (which is activity of 1 gm Radium)

Solved Examples

Example 12. The decay constant for the radioactive nuclide ^{64}Cu is 1.516×10^{-5} s^{-1}. Find the activity of a sample containing 1 μg of ^{64}Cu. Atomic weight of copper = 63.5 g/mole. Neglect the mass difference between the given radioisotope and normal copper.

Solution : 63.5 g of copper has 6×10^{23} atoms. Thus, the number of atoms in 1 μg of Cu is

$$N = \frac{6 \times 10^{23} \times 1\mu g}{63.5\, g} = 9.45 \times 10^{15}$$

The activity = λN

$$= (1.516 \times 10^{-5}\ s^{-1}) \times (9.45 \times 10^{15}) \quad = 1.43 \times 10^{11} \text{ disintegrations/s}$$

$$= \frac{1.43 \times 10^{11}}{3.7 \times 10^{10}} \text{ Ci} = 3.86 \text{ Ci.}$$

Activity after n half lives : $\mathbf{\frac{A_0}{2^n}}$

Example 13. The half-life of a radioactive nuclide is 20 hours. What fraction of original activity will remain after 40 hours?

Solution : 40 hours means 2 half lives.

Thus, $$A = \frac{A_0}{2^2} = \frac{A_0}{4}$$

or, $$\frac{A}{A_0} = \frac{1}{4}.$$

So one fourth of the original activity will remain after 40 hours.

Specific activity : *The activity per unit mass is called specific activity.*

8.3 Average Life :

$$T_{avg} = \frac{\text{sum of ages of all the nuclei}}{N_0} = \frac{\int_0^{\infty} \lambda N_0 e^{-\lambda t}\, dt.t}{N_0} = \frac{1}{\lambda}$$

Solved Examples

Example 14. The half-life of ^{198}Au is 2.7 days. Calculate (a) the decay constant, (b) the average-life and (c) the activity of 1.00 mg of ^{198}Au. Take atomic weight of ^{198}Au to be 198 g/mol.

Solution : (a) The half-life and the decay constant are related as

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \qquad \text{or,} \qquad \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.7\,\text{days}}$$

$$= \frac{0.693}{2.7 \times 24 \times 3600\,s} = 2.9 \times 10^{-6}\ s^{-1}.$$

(b) The average-life is $t_{av} = \frac{1}{\lambda}$ = 3.9 days.

(c) The activity is $A = \lambda N$. Now, 198 g of ^{198}Au has 6×10^{23} atoms.
The number of atoms in 1.00 mg of ^{198}Au is

$$N = 6 \times 10^{23} \times \frac{1.0\,mg}{198\,g} = 3.03 \times 10^{18}.$$

Thus, $A = \lambda N = (2.9 \times 10^{-6}\ s^{-1})(3.03 \times 10^{18})$

$= 8.8 \times 10^{12}$ disintegrations/s

$$= \frac{8.8 \times 10^{12}}{3.7 \times 10^{10}}\ Ci = 240\ Ci.$$

Example 15. Suppose, the daughter nucleus in a nuclear decay is itself radioactive. Let λ_p and λ_d be the decay constants of the parent and the daughter nuclei. Also, let N_p and N_d be the number of parent and daughter nuclei at time t. Find the condition for which the number of daughter nuclei becomes constant.

Solution : The number of parent nuclei decaying in a short time interval t to t + dt is $\lambda_p N_p dt$. This is also the number of daughter nuclei decaying during the same time interval is $\lambda_d N_d dt$. The number of the daughter nuclei will be constant if

$$\lambda_p N_p dt = \lambda_d N_d dt$$

or, $$\lambda_p N_p = \lambda_d N_d.$$

Example 16. A radioactive sample decays with an average-life of 20 ms. A capacitor of capacitance 100 μF is charged to some potential and then the plates are connected through a resistance R. What should be the value of R so that the ratio of the charge on the capacitor to the activity of the radioactive sample remains constant in time?

Solution : The activity of the sample at time t is given by

$$A = A_0\, e^{-\lambda t}$$

where λ is the decay constant and A_0 is the activity at time t = 0 when the capacitor plates are connected. The charge on the capacitor at time t is given by

$$Q = Q_0\, e^{-t/CR}$$

where Q_0 is the charge at t = 0 and C = 100 μF is the capacitance. Thus, $\frac{Q}{A} = \frac{Q_0}{A_0}\frac{e^{-t/CR}}{e^{-\lambda t}}$.

It is independent of t if $\lambda = \frac{1}{CR}$

or, $$R = \frac{1}{\lambda C} = \frac{t_{av}}{C} = \frac{20 \times 10^{-3}\,s}{100 \times 10^{-6}\,F} = 200\ \Omega.$$

Example 17. A radioactive nucleus can decay by two different processes. The half-life for the first process is t_1 and that for the second process is t_2. Show that the effective half-life t of the nucleus is given by

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}.$$

Solution : The decay constant for the first process is $\lambda_1 = \frac{\ln 2}{t_1}$ and for the second process it is $\lambda_2 = \frac{\ln 2}{t_1}$. The probability that an active nucleus decays by the first process in a time interval dt is $\lambda_1 dt$. Similarly, the probability that it decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is λ, this probability is also equal to λdt. Thus.

$$\lambda dt = \lambda_1 dt + \lambda_2 dt$$

or, $$\boxed{\lambda = \lambda_1 + \lambda_2}$$

or, $$\boxed{\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}.}$$ (**To be remembered**)

Example 18. A factory produces a radioactive substance A at a constant rate R which decays with a decay constant λ to form a stable substance. Find (i) the no. of nuclei of A and (ii) Number of nuclei of B, at any time t assuming the production of A starts at t = 0. (iii) Also find out the maximum number of nuclei of 'A' present at any time during its formation.

Solution : Factory $\xrightarrow[\text{const rate}]{R} A \xrightarrow[\text{decay}]{\lambda} B$

Let N be the number of nuclei of A at any time t

$$\therefore \qquad \frac{dN}{dt} = R - \lambda N \qquad\qquad \int_0^N \frac{dN}{R-\lambda N} = \int_0^t dt$$

On solving we will get

$N = R/\lambda(1 - e^{-\lambda t})$

(ii) Number of nuclei of B at any time t, $N_B = R\,t - N_A = Rt - R/\lambda(1 - e^{-\lambda t}) = R/\lambda\,(\lambda\,t - 1 + e^{-\lambda t})$.

(iii) Maximum number of nuclei of 'A' present at any time during its formation = R/λ.

Example 19. A radioactive substance "A" having N_0 active nuclei at t = 0, decays to another radioactive substance "B" with decay constant λ_1. B further decays to a stable substance "C" with decay constant λ_2. (a) Find the number of nuclei of A, B and C after time t. (b) What would be the answer of part (a) if $\lambda_1 >> \lambda_2$ and $\lambda_1 << \lambda_2$.

Solution : (a) The decay scheme is as shown

	$A \xrightarrow{\lambda_1}$	$B \xrightarrow{\lambda_2}$	C (stable)
t = 0	N_0	0	0
t	N_1	N_2	N_3

Here N_1, N_2 and N_3 represent the nuclei of A, B and C at any time t.

For A, we can write

$$N_1 = N_0\, e^{-\lambda_1 t} \qquad .... (1)$$

For B, we can write

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \qquad .....(2)$$

$$\text{or,} \qquad \frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_1$$

This is a linear differential equation with integrating factor

$$\text{I.F.} = e^{\lambda_2 t}$$

$$e^{\lambda_2 t}\frac{dN_2}{dt} + e^{\lambda_2 t}\lambda_2 N_2 = \lambda_1 N_1\, e^{\lambda_2 t}$$

$$\int d(N_2 e^{\lambda_2 t}) = \int \lambda_1 N_1 e^{\lambda_2 t}\, dt$$

$$N_2\, e^{\lambda_2 t} = \lambda_1 N_0 \int e^{-\lambda_1 t}\, e^{\lambda_2 t} dt \qquad \text{...... using (1)}$$

$$N_2\, e^{\lambda_2 t} = \lambda_1 N_0 \frac{e^{(\lambda_2-\lambda_1)t}}{\lambda_2 - \lambda_1} + C \qquad ......(3)$$

At $\quad t = 0, \quad N_2 = 0 \qquad 0 = \dfrac{\lambda_1 N_0}{\lambda_2 - \lambda_1} + C$

Hence $\quad C = \dfrac{\lambda_1 N_0}{\lambda_1 - \lambda_2}$

Using C in eqn. (3), we get

$$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

and $N_1 + N_2 + N_3 = N_0$

$\therefore$ $N_3 = N_0 - (N_1 + N_2)$

(b) For $\lambda_1 >> \lambda_2$ $\quad N_2 = \dfrac{\lambda_1 N_0}{-\lambda_1}\,(-e^{-\lambda_2 t}) \quad = N_0\, e^{-\lambda_2 t}$

For $\lambda_1 << \lambda_2$ $\quad N_2 = \dfrac{\lambda_1 N_0}{\lambda_2}\,(e^{-\lambda_1 t}) \quad = 0$

Alternate solution of (b) part without use of answer of part (a) :

If $\lambda_1 > \lambda_2$ that means A will decay very fast to 'B' and B will then decay slowly. We can say that practically N_1 vanishes in very short time & B has initial no. of atoms as N_0.

$\therefore$ Now $N_2 = N_0\, e^{-\lambda_2 t}$ & $N_1 = N_0\, e^{-\lambda_1 t}$

If $\lambda_1 << \lambda_2$ then B is highly unstable and it will soon decay into C.

So, it's rate of formation $\approx$ its rate of decay.

$$\therefore \quad \lambda_1 N_1 \approx \lambda_2 N_2 \quad \Rightarrow \quad N_2 = \frac{\lambda_1 N_1}{\lambda_2} = \frac{\lambda_1 N_0}{\lambda_2}\,(e^{-\lambda_1 t})$$

9. NUCLEAR FISSION :

In nuclear fission heavy nuclei of A, above 200, break up into two or more fragments of comparable masses. The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use $_{92}U^{236}$ as the fission material. The technique is to hit a uranium sample by slow-moving neutrons (kinetic energy $\approx$ 0.04 eV, also called thermal neutrons). A $_{92}U^{235}$ nucleus has large probability of absorbing a slow neutron and forming $_{92}U^{236}$ nucleus. This nucleus then fissions into two or more parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236} \rightarrow X + Y + 2\,{}_0n^1,$$

or

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236} \rightarrow X' + Y' + 3\,{}_0n^1$$

and a number of other combinations.

* On an average 2.5 neutrons are emitted in each fission event.
* Mass lost per reaction $\approx$ 0.2 a.m.u.
* In nuclear fission the total B.E. increases and excess energy is released.
* In each fission event, about 200 MeV of energy is released a large part of which appears in the form of kinetic energies of the two fragments. Neutrons take away about 5MeV.

eg. $^{235}_{92}U + {}_0n^1 \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}_0n^1$ + energy

Q value = $[(M_U - 92m_e + m_n) - \{(M_{Ba} - 56\, m_e) + (M_{Kr} - 36m_e) + 3m_n\}]c^2$

= $[(M_U + m_n) - (M_{Ba} + M_{Kr} + 3m_n)]c^2$

* A very important and interesting feature of neutron-induced fission is the chain reaction. For working of nuclear reactor refer your text book.

10. NUCLEAR FUSION (THERMO NUCLEAR REACTION):

(a) Some unstable light nuclei of A below 20, fuse together, the B.E. per nucleon increases and hence the excess energy is released. The easiest thermonuclear reaction that can be handled on earth is the fusion of two deuterons (D–D reaction) or fusion of a deuteron with a triton (D–T reaction).

$_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + {}_0n^1$ + 3.3 MeV (D–D)

Q value = $[2(M_D - m_e) - \{(M_{He^3} - 2m_e) + m_n\}]c^2$

$= [2M_D - (M_{He^3} + m_n)]c^2$

$_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1$ + 4.0 MeV (D–D)

$$Q \text{ value} = [2(M_D - m_e) - \{(M_T - m_e) + (M_H - m_e)\}]c^2$$
$$= [2M_D - (M_T + M_H)]c^2$$
$${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n + 17.6 \text{ MeV } (D - T)$$
$$Q \text{ value} = [\{(M_D - m_e) + (M_T - m_e)\} - \{(M_{He^4} - 2m_e) + m_n\}]c^2$$
$$= [(M_D + M_T) - (M_{He^4} + m_n)]c^2$$

Note : In case of fission and fusion, $\Delta m = \Delta m_{atom} = \Delta m_{nucleus}$.

(b) These reactions take place at ultra high temperature ($\cong 10^7$ to 10^9). At high pressure it can take place at low temperature also. For these reactions to take place nuclei should be brought upto 1 fermi distance which requires very high kinetic energy.

(c) Energy released in fusion exceeds the energy liberated in the fission of heavy nuclei.

Solved Examples

Example 20. Calculate the energy released when three alpha particles combine to form a ^{12}C nucleus. The atomic mass of ${}^4_2He$ is 4.002603 u.

Solution : The mass of a ^{12}C atom is exactly 12 u.

The energy released in the reaction $3\left({}^4_2He\right) \rightarrow {}^{12}_6C$ is

$$[3\ m({}^4_2He) - m({}^{12}_6C)]\ c^2$$
$$= [3 \times 4.002603 \text{ u} - 12 \text{ u}]\ (931 \text{ MeV/u}) = 7.27 \text{ MeV}.$$

Example 21. Consider two deuterons moving towards each other with equal speeds in a deutron gas. What should be their kinetic energies (when they are widely separated) so that the closest separation between them becomes 2fm? Assume that the nuclear force is not effective for separations greater than 2 fm. At what temperature will the deuterons have this kinetic energy on an average?

Solution : As the deuterons move, the Coulomb repulsion will slow them down. The loss in kinetic energy will be equal to the gain in Coulomb potential energy. At the closest separation, the kinetic energy is zero and the potential energy is $\frac{e^2}{4\pi\varepsilon_0 r}$. If the initial kinetic energy of each deuteron is K and the closest separation is 2fm, we shall have

$$2\text{ K} = \frac{e^2}{4\pi\varepsilon_0(2\text{fm})} = \frac{(1.6\times10^{-19}C)^2\times(9\times10^9 N-m^2/C^2)}{2\times10^{-15}m}$$

or, $K = 5.7 \times 10^{-14}$ J.

If the temperature of the gas is T, the average kinetic energy of random motion of each nucleus will be 1.5 kT. The temperature needed for the deuterons to have the average kinetic energy of 5.7×10^{-14} J will be given by

$$1.5\ kT = 5.7 \times 10^{-14}\ J$$

where k = Botzmann constant

or, $$T = \frac{5.7\times10^{-14} J}{1.5\times1.38\times10^{-23} J/K}$$
$$= 2.8 \times 10^9 \text{ K}.$$