1. False.

   Proof:

   We have to show that: $(\exists m \in N)\ (\exists n \in N)\ (3m+5n=12)$ is false.
   We will prove it using case approach. To divide in cases, we can start with m.

   Given N = [1,2, 3……], and 12/3 = 4, possible values for m are 1,2,3,4.

   For m = 1, $3m+5n = 12$
   $3+5n = 12$
   $n = 9/5$, which is not an integer.

   Similarly, we can show for m=2,3.

   For m = 4 possible value for n = 0. But since 0 is not an element of N, this proves that the statement is false.

   Thus, it can be concluded that statement is false.

2. True

   Proof: Let $n \in Z$. We have to show that $[n + (n+1) + (n+2) + (n+3) + (n+4)]$ is divisible by 5.
   Let $m \in Z$, and
   $m = [n + (n+1) + (n+2) + (n+3) + (n+4)]$
   $= (5n + 10)$
   $= 5 * (n+2)$

   Since m is multiple of 5, m is divisible by 5.

   Thus, sum of 5 consecutive terms is divisible by 5.

   Thus, it can be concluded that statement is true.

3. True

   Proof: We have $n \in Z$. We have to show that $(n^2 + n + 1)$ is odd. To prove we can either use induction or we can go with case approach. Here, let's use case approach. There are two possibilities for n, it is either even or odd.

   Case i: n is even, then n^2 is even and n+1 is odd. Since (even + odd) is odd, consequently, (n^2+n+1) is odd.

   Case ii: n is odd, then n^2 is odd, and n+1 is even. Since (even + odd) is odd, consequently, (n^2+n+1) is odd.

   Thus (n2+n+1) is odd.

   Thus, it can be concluded that statement is true.

4. True

Proof: We have $n \in N$. Since every natural number can be expressed in one of the following form: 4n, 4n+1, 4n+2, 4n+3.

Since 4n is even for $\forall n \in N$, 4n+1 is odd, 4n+2 is even and 4n+3 is odd.
Thus of the four possible expressions 4n, 4n+1, 4n+2, 4n+3, covering all natural numbers, two expression covering odd numbers are 4n+1 and 4n+3, these must cover all odd numbers.
Thus, it can be concluded that statement is true.

5. True

Proof: We have to prove that for any integer $n \in Z$, at least one of the integers n, $n+2$, $n+4$ is divisible by 3.
Since n % 3 is one of 0,1,2.

Case i: n%3 = 0
Thus n is divisible by 3

Case ii: n%3 = 1,
For some q,r ∈ Z, n = 3q+1
⇨ (n+2) % 3 = (3q+1+2) % 3
= 0
Thus n+2 is divisible by 3

Case iii: n%3 = 2,

For some q,r ∈ Z, n = 3q+2
⇨ (n+4) % 3 = (3q+2+4) % 3
⇨ = (3q+6) % 3
= 0

Thus, in all the cases, we found that one of the n, (n+1), (n+2) is divisible by 3.

Thus, it can be concluded that statement is true.

6. We have to show that, except (3,5,7), there is not prime triplet.
No even number, being divisible by 2, is prime, except 2 itself, so triplets will consist of three consecutive odd numbers.
If we take three consecutive odd numbers, at least one of them is divisible by 3.
Thus, except first triplet, which has 3 itself, all other triplets have at least one number which is divisible by 3, making at least one of the element of triplet non-prime.

Thus, no prime triplet exists.

7. We have to prove that for any natural number n,
$2+2^2+2^3+\ldots+2^n=2^{n+1}-2$

We will use induction to prove this.
For n =1, left side = 2 = 4 - 2 = 2^2 -2 = right side
Now assuming statement is true for n, we need to prove that it is true for n+1

For (n+1)

LHS = $2+2^2+2^3+\ldots+2^n + 2^{n+1}$

= $2^{n+1}-2$ + 2^{n+1} (Since statement is true for n)

= $(2^* \, 2^{n+1}) - 2$ = $2^{n+2}-2$ = RHS

Thus, statement is true.

8. Since $\{a_n\}^{\infty}_{n=1}$ tends to limit L as $n\to\infty$, we can say:
$(\forall \mathcal{E} > 0)\ (\exists\ n \in N)\ (\forall\ m >= n)\ [\ |a_m\text{-}L| < \mathcal{E}]$
⇨ $(\forall \mathcal{E} > 0)\ (\exists\ n \in N)\ (\forall\ m >= n)\ [\ -\mathcal{E} < a_m\text{-}L < \mathcal{E}]$

Now given that M > 0, multiplying inequality with M does not change it.
So, we can say,
$[\ -\mathcal{E}M < M(a_m\text{-}L) < M\mathcal{E}] = [\ -\mathcal{E}M < (Ma_m\text{-}ML) < M\mathcal{E}]$
Let $\mathcal{E}' = M\mathcal{E}$
⇨ $\mathcal{E} = \mathcal{E}'/M$
⇨ $[\ -\mathcal{E}M < (Ma_m\text{-}ML) < M\mathcal{E}]$
$= [\ -\mathcal{E}' < (Ma_m\text{-}ML) < \mathcal{E}']$

Thus, for all $\mathcal{E}'$, we can choose suitable n', such that

$(\forall \mathcal{E}' > 0)\ (\exists\ n' \in N)\ (\forall\ m' >= n')\ [\ |Ma_m\text{-}ML| < \mathcal{E}']$

Thus, $\{Ma_n\}^{\infty}_{n=1}$ tends to limit ML as $n\to\infty$.

9. Let $A_n = (0, 1/n)$
Then $A_{n+1} = (0, 1/(n+1))$
$A_n \cap A_{n+1} = (\max(0,0)\ ,\ \min(1/n, 1/(n+1))$
$= (0, 1/(n+1))$
$= A_{n+1}$
Thus $A_{n+1} \subset A_n$
Since $\{1/n\}^{\infty}_{n=1}$ tends to limit 0 as $n\to\infty$, and since all subsequent sets are proper subset of previous one, and 0 is not part of any sets,
$\bigcap^{\infty}_{n=1} A_n = \emptyset$

Hence proved.

10. Let $A_n = (-1/n, 1/n)$
Then $A_{n+1} = (-1/(n+1), 1/(n+1))$
$A_n \cap A_{n+1} = (\max(-1/n, -1/(n+1)), \min(1/n, 1/(n+1))$
$= (-1/(n+1), 1/(n+1))$
$= A_{n+1}$
Thus $A_{n+1} \subset A_n$
Since $\{1/n\}^{\infty}_{n=1}$ tends to limit 0 as $n \to \infty$, and since all subsequent sets are proper subset of previous one,
$\bigcap^{\infty}_{n=1} A_n = \{0\}$.

Hence proved.