

Project 1: Conditional Random Fields for Structured Output Prediction

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Maximum Objective Value for 1C	200.33
Average Log Probability for 2A	-31.28
Optimal Objective Value for 2B	3701.15

1a Solution

We can simplify $\log p(y^t, X^t)$ as follows:

$$\log \frac{1}{Z_X} \exp \left(\sum_{s=1}^m \langle \mathbf{w}_{y_s}, \mathbf{x}_s \rangle + \sum_{s=1}^{m-1} T_{y_s, y_{s+1}} \right) = \sum_{s=1}^m \langle \mathbf{w}_{y_s}, \mathbf{x}_s \rangle + \sum_{s=1}^{m-1} T_{y_s, y_{s+1}} - \log Z_X \quad (1)$$

The $\nabla_{\mathbf{w}_y} \sum_{s=1}^m \langle \mathbf{w}_{y_s}, \mathbf{x}_s \rangle + \sum_{s=1}^{m-1} T_{y_s, y_{s+1}}$ is the following:

$$\sum_{s=1}^m \mathbb{I}[y_s = y] x_s^t \quad (2)$$

This is because while taking the derivative of a dot product involving w_y , X_s will only appear whenever $y_s = y$. And, the sum of transitions will disappear because it does not depend on w .

The $\nabla_{\mathbf{w}_y} \log Z_X$ is computed via the chain rule in the following way:

$$\nabla_{\mathbf{w}_y} \log Z_X = \frac{\sum_{\hat{\mathbf{y}} \in \mathcal{Y}^m} \exp \left(\sum_{s=1}^m \langle \mathbf{w}_{\hat{y}_s}, \mathbf{x}_s \rangle + \sum_{s=1}^{m-1} T_{\hat{y}_s, \hat{y}_{s+1}} \right)}{\sum_{\hat{\mathbf{y}} \in \mathcal{Y}^m} \exp \left(\sum_{s=1}^m \langle \mathbf{w}_{\hat{y}_s}, \mathbf{x}_s \rangle + \sum_{s=1}^{m-1} T_{\hat{y}_s, \hat{y}_{s+1}} \right)} * \sum_{s=1}^m \mathbb{I}[y_s = y] x_s^t \quad (3)$$

This can be further reduced, by substituting $p(y|X)$ into equation 3 and rearranging the sums, in the following way:

$$\sum_{\hat{\mathbf{y}} \in \mathcal{Y}^m} p(y|X) \sum_{s=1}^m \mathbb{I}[y_s = y] x_s^t = \sum_{s=1}^m \sum_{\hat{\mathbf{y}} \in \mathcal{Y}^m} p(y|X) \mathbb{I}[y_s = y] x_s^t \quad (4)$$

Finally, we recognize that the inner summation is a marginalization over y except the label we are taking the gradient against. Therefore we can further reduce the equation to:

$$\sum_{s=1}^m p(y_s = y | X^t) x_s^t \quad (5)$$

Using equations 2 and 5, the gradient is

$$\nabla_{\mathbf{w}_y} \log p(\mathbf{y}^t | X^t) = \sum_{s=1}^m (\mathbb{I}[y_s^t = y] - p(y_s = y | X^t)) \mathbf{x}_s^t, \quad (6)$$

Now computing gradient for T

The $\nabla_{T_{ij}} \sum_{s=1}^m \langle \mathbf{w}_{y_s}, \mathbf{x}_s \rangle + \sum_{s=1}^{m-1} T_{y_s, y_{s+1}}$ is the following:

$$\sum_{s=1}^{m-1} \mathbb{I}[y_s = i, y_{s+1} = j] \quad (7)$$

This is because when we differentiate with respect to T_{ij} , the only terms that remain are the ones that specify a transition between i and j . There is only a single weight that lies at this transition point so when we differentiate it, it becomes 1. Also, the dot product goes away because it does not depend on T.

The $\nabla_{T_{ij}} \log Z_X$ is computed via the chain rule in the following way:

$$\nabla_{T_{ij}} \log Z_X = \frac{\sum_{\hat{\mathbf{y}} \in \mathcal{Y}^m} \exp \left(\sum_{s=1}^m \langle \mathbf{w}_{\hat{y}_s}, \mathbf{x}_s \rangle + \sum_{s=1}^{m-1} T_{\hat{y}_s, \hat{y}_{s+1}} \right)}{\sum_{\hat{\mathbf{y}} \in \mathcal{Y}^m} \exp \left(\sum_{s=1}^m \langle \mathbf{w}_{\hat{y}_s}, \mathbf{x}_s \rangle + \sum_{s=1}^{m-1} T_{\hat{y}_s, \hat{y}_{s+1}} \right)} * \sum_{s=1}^{m-1} \mathbb{I}[y_s = i, y_{s+1} = j] \quad (8)$$

This can be further reduced, by substituting $p(y|X)$ into equation 8 and rearranging the sums, in the following way:

$$\sum_{\hat{\mathbf{y}} \in \mathcal{Y}^m} p(y|X) \sum_{s=1}^{m-1} \mathbb{I}[y_s = i, y_{s+1} = j] = \sum_{s=1}^{m-1} \sum_{\hat{\mathbf{y}} \in \mathcal{Y}^m} p(y|X) \mathbb{I}[y_s = i, y_{s+1} = j] \quad (9)$$

Finally, we recognize that the inner summation is a marginalization over y except the label we are taking the gradient against. Therefore we can further reduce the equation to:

$$\sum_{s=1}^{m-1} p(y_s = i, y_{s+1} = j | X^t) \quad (10)$$

Using equations 7 and 9 the gradient is:

$$\sum_{s=1}^{m-1} \llbracket y_s = i, y_{s+1} = j \rrbracket - p(y_s = i, y_{s+1} = j | X^t) \quad (11)$$

1b Solution

The features will take the following form. If they are a feature that depends on the letter's label, they will look as follows:

$$x_i \llbracket y_i = j \rrbracket \quad (12)$$

where i will be between $1 \dots m$ and j will be between $1 \dots 26$. If the feature depends on the transition between two letters then that feature will take the following form.

$$\llbracket y_i = k, y_{i+1} = z \rrbracket \quad (13)$$

where i is between $1 \dots m$ and k, z are between $1 \dots 26$

The gradient of $\log Z_x$ is described in equations 3 and 8. The feature function $\phi(x)$ will "place" the input in specific positions so that the correct dot product and transitions are computed. For example, the feature function enables the selection of w_7 to be used in $\langle x_i, w_7 \rangle$ when $y_i = 7$.

Considering features that depend on the label first. The expectation of some features with respect to $p(\mathbf{y}|X)$ will be:

$$\sum_{s=1}^m p(\mathbf{y}|X) \phi(X) \quad (14)$$

And this is equivalent to:

$$\sum_{s=1}^m p(\mathbf{y}|X^t) x_s \mathbb{I}[y_s = y] \quad (15)$$

The feature function (via the indicator) will only be non-zero for terms s.t. $y_s = y$ and the others will go to zero. Therefore we can say that the above is equivalent to equation 3 because the $p(\mathbf{y}|X)$ term will become $p(y_s = y|X)$ via marginalization, and x_s will be selected by the feature function. The gradient with respect to T has a similar argument. But instead what will be important is the feature $\mathbb{I}[y_s = i, y_{s+1} = j]$ which will marginalize $p(\mathbf{y}|X)$ to $p(y_s = i, y_{s+1} = j)$.