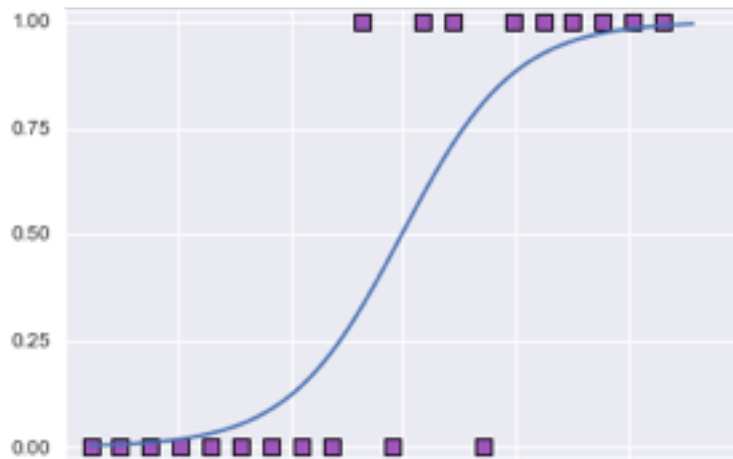


INTRO to DATA SCIENCE

LECTURE 11: BAYESIAN STATISTICS

- I. REGRESSION RECAP**
- II. LOGISTIC REGRESSION**
- III. INTERPRETING RESULTS**
- IV. DECISION BOUNDARIES**
- V. EVALUATING CLASSIFIERS**



any questions?

DATA EXPLORATION

SUPERVISED LEARNING: REGRESSION

SUPERVISED LEARNING: CLASSIFICATION

UNSUPERVISED LEARNING

VARIOUS TOPICS

DATA EXPLORATION

SUPERVISED LEARNING: REGRESSION

SUPERVISED LEARNING: CLASSIFICATION

UNSUPERVISED LEARNING

VARIOUS TOPICS

LOGISTIC REGRESSION

NAIVE BAYES (TODAY)

RANDOM FORESTS

SUPPORT VECTOR MACHINES

COMPETITION

0. DEMO SAMPLE PROJECTS (IDEAS DUE NEXT LESSON)

I. PROBABILITY

II. BAYES' THEOREM

III. EXAMPLE: BAYESIAN COIN FLIPS (OPTIONAL)

IV. NAIVE BAYES

INTRO TO DATA SCIENCE

I. PROBABILITY

Q: What is a probability?

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A: A number between 0 and 1 that characterizes the likelihood that some event will occur.

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$$P(\Omega) = 1$$

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A: A number between 0 and 1 that characterizes the likelihood that some event will occur.

The probability of event A is denoted $P(A)$.

*The space of all possible events is called the **sample space** and denoted by Ω*

$$P(\Omega) = 1$$

$$P(\emptyset) = 0$$

\emptyset is the empty set $\{ \}$

It makes sense to think of events of subsets in the sample space Ω

*Two events A and B are **mutually exclusive** or **disjoint** if they are not overlapping:*

$$A \cap B = \emptyset$$

Their intersection is an empty set

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$$P(\text{🎲} \text{ and } \text{🎲}) = 0$$

you cannot have both 4 and 6 in one throw

It makes sense to think of events of subsets in the sample space Ω

*If two events A and B are **mutually exclusive** or **disjoint**, i.e., if they are not overlapping, then we can add their probabilities*

$$P(A \text{ or } B) = P(A) + P(B)$$

It makes sense to think of events of subsets in the sample space Ω

*If two events A and B are **mutually exclusive** or **disjoint**, i.e., if they are not overlapping, then we can add their probabilities*

$$P(A \text{ or } B) = P(A) + P(B)$$

*If A and B are **not** mutually exclusive, then we have*

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

It makes sense to think of events of subsets in the sample space Ω

*If two events A and B are **mutually exclusive***

$$P(\text{⚡ or ⚡⚡}) = P(\text{⚡}) + P(\text{⚡⚡}) = 1/6 + 1/6 = 1/3$$

It makes sense to think of events of subsets in the sample space Ω

*If two events A and B are **mutually exclusive***

$$P(\text{⚪⚪ or ⚫⚫}) = P(\text{⚪⚪}) + P(\text{⚫⚫}) = 1/6 + 1/6 = 1/3$$

*If A and B are **not** mutually exclusive*

$$\begin{aligned} P(\text{7 or } \spadesuit) &= P(\text{7}) + P(\spadesuit) - P(\text{7 of } \spadesuit) \\ &= 1/13 + 1/4 - 1/52 = 11/26 \end{aligned}$$

*Two events A and B are called **independent** if their joint probability is the product of their individual probabilities:*

$$P(A \text{ and } B) = P(A) P(B)$$

We often write $A \perp B$

*Two events A and B are called **independent** if their joint probability is the product of their individual probabilities:*

$$\text{two throws} \quad P(\underset{\substack{1\text{st} \\ \text{throw}}}{\text{⚡}} \text{ and } \underset{\substack{2\text{nd} \\ \text{throw}}}{\text{⚡}}) = P(\underset{\substack{1\text{st} \\ \text{throw}}}{\text{⚡}}) P(\underset{\substack{2\text{nd} \\ \text{throw}}}{\text{⚡}}) = 1/6 \times 1/6 = 1/36$$

*Q: Suppose event B has occurred. What quantity represents the probability of A **given** this information about B ?*

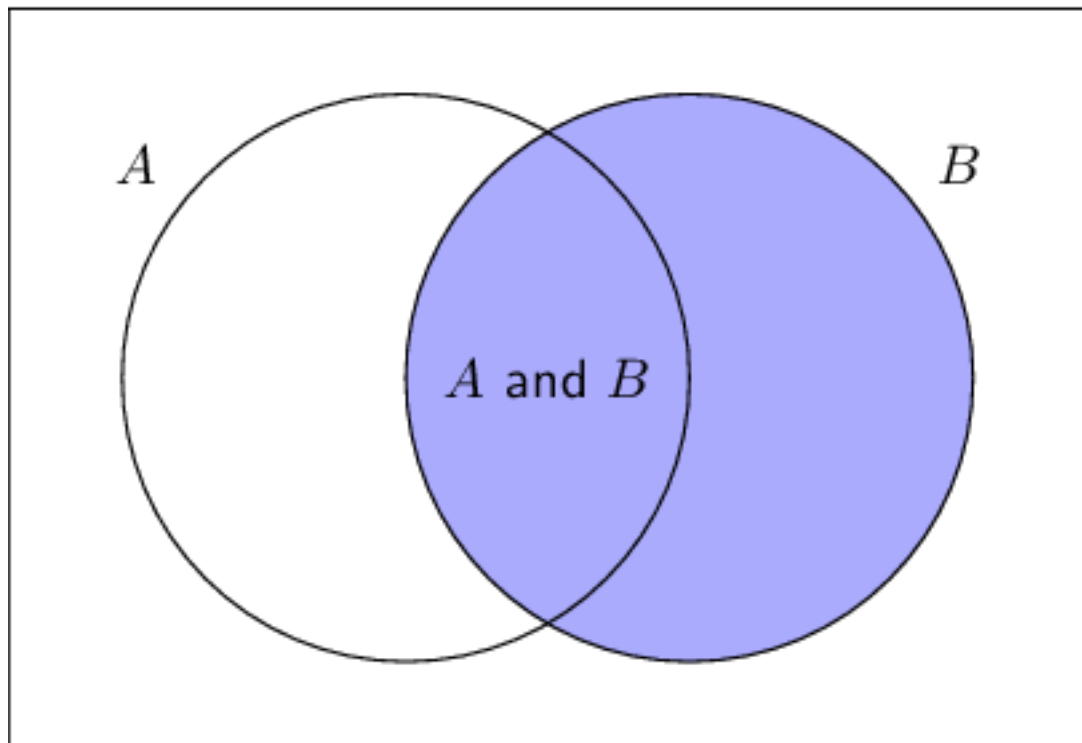
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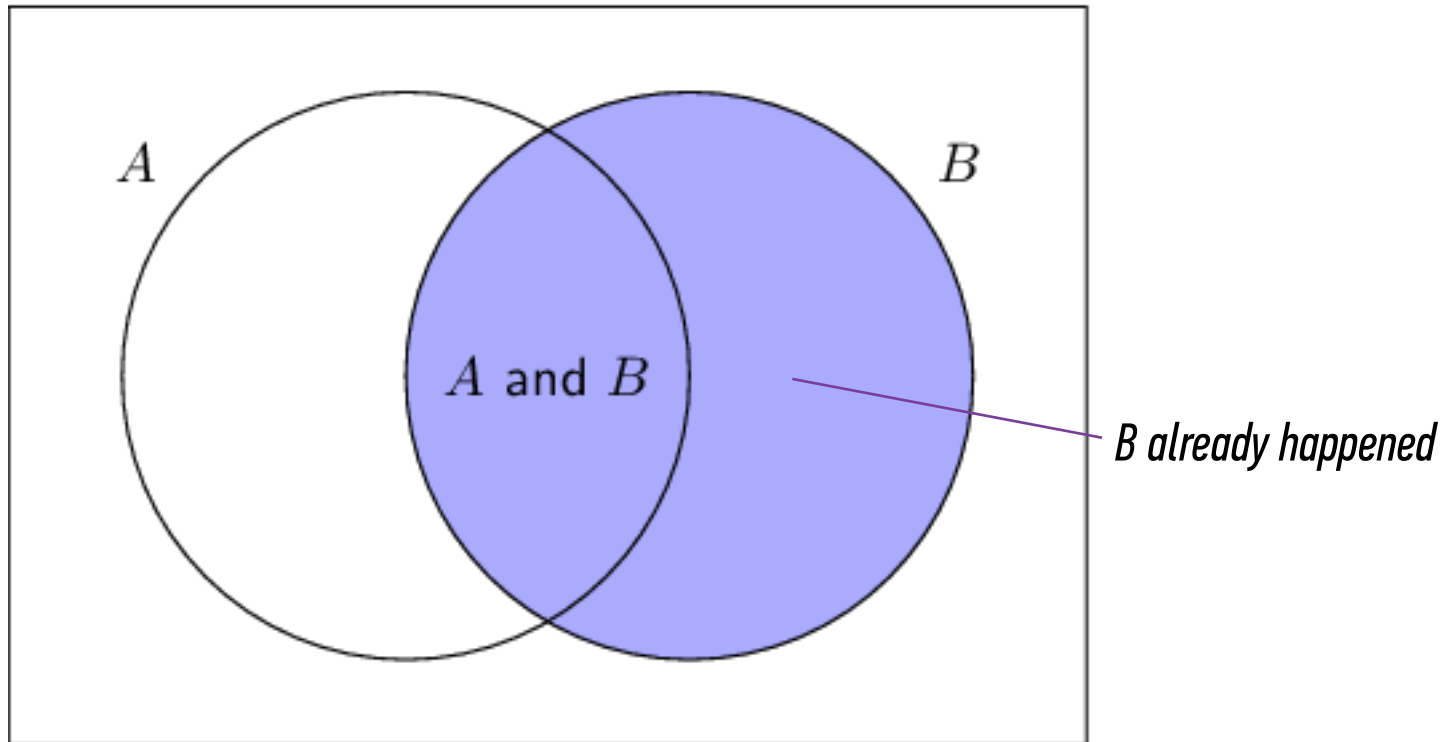
A: The intersection of A & B divided by region B .

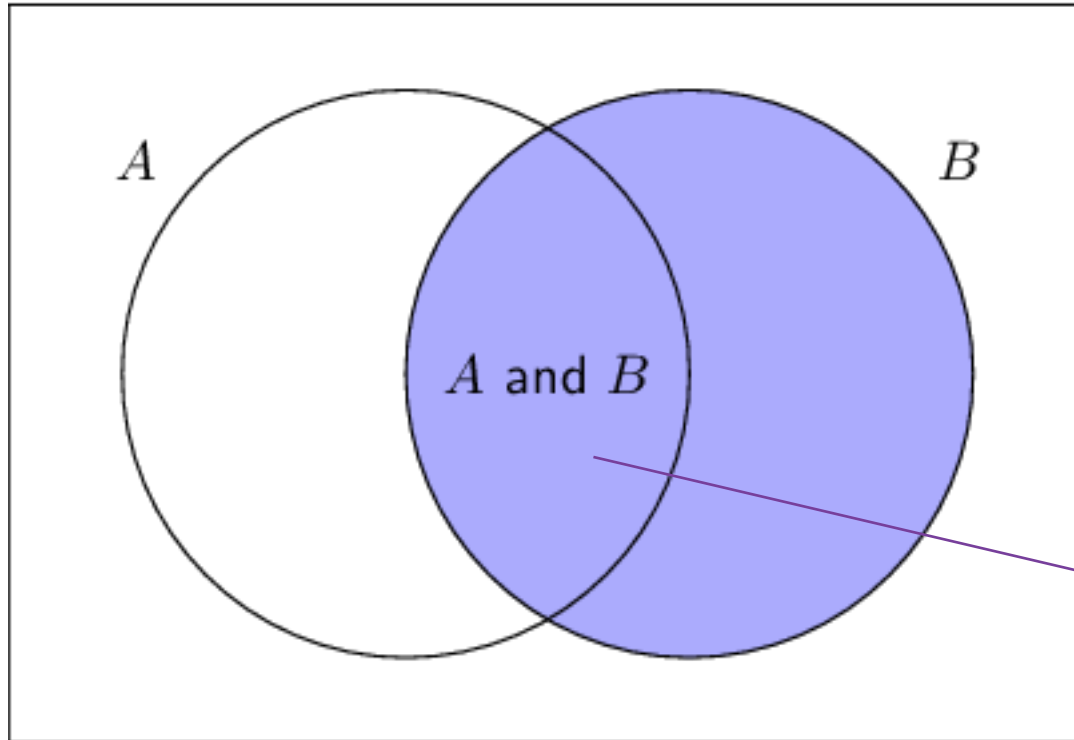
*Q: Suppose event B has occurred. What quantity represents the probability of A **given** this information about B ?*

A: The intersection of A & B divided by region B .

*This is called the **conditional probability** of A given B , written $P(A|B) = P(AB) / P(B)$.*







B already happened

*The probability of A
is now given by $A \cap B$
(or AB)*

*Two **conditional probability** is the probability of some event A, given the occurrence of some other event B.*

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

*Two **conditional probability** is the probability of some event A , given the occurrence of some other event B .*

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

It follows that if $P(A \mid B) = P(A)$ if and only if $A \perp B$

INTRO TO DATA SCIENCE

QUIZ QUESTION

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1) Linda is a bank teller.

2) Linda is a bank teller and active in the feminist movement.

Q: What does it mean for two events to be independent?

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A: Information about one does not affect the probability of the other.

This can be written as $P(A|B) = P(A)$

And we have $P(A \text{ and } B) = P(A) P(B)$

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

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Which is larger?

1) $P(\text{bank teller})$

2) $P(\text{bank teller and feminist movement})$

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is larger?

1) $P(\text{bank teller})$

2) $P(\text{bank teller}) \times P(\text{feminist movement})$

II. BAYES' THEOREM

Recall the **conditional probability**

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

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*Recall the **conditional probability***

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

We can rewrite that as $P(AB) = P(A \mid B) P(B)$

As well as $P(AB) = P(B \mid A) P(A)$

It follows that
$$P(A \mid B) = \frac{P(B \mid A) P(A)}{P(B)}$$

This result is called **Bayes' Theorem**

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

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$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

It means you can swap conditional probabilities

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It means you can swap conditional probabilities

*In a movie it's raining. What's the
chance the movie is shot in Holland?*

*This result is called **Bayes' Theorem***

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

It means you can swap conditional probabilities

*In a movie it's raining. What's the
chance the movie is shot in Holland?*

*B = it is raining
A = it is Holland*

*This result is called **Bayes' Theorem***

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

It means you can swap conditional probabilities

$$\begin{array}{l} \text{In a movie it's raining. What's the} \\ \text{chance the movie is shot in Holland?} \end{array} = \frac{P(\text{raining in Holland}) P(\text{random movie shot in Holland})}{P(\text{raining anywhere in the world})}$$

Each term in this relationship has a name, and each plays a distinct role in any probability calculation (including ours).

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

*This term is the **posterior probability** of A. It's the probability of A after the conditional data is taken into account.*

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

In a movie it's raining. What's the chance the movie is shot in Holland?

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*This term is the **posterior probability** of A. It's the probability of A after the conditional data is taken into account.*

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

The goal of any Bayesian computation is to find (“learn”) the posterior distribution of a particular variable.

*This term is the **prior probability** of A. It's the probability of A before any conditional data is taken into account.*

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

In a movie it's raining. What's the chance the movie is shot in Holland?

$$= \frac{P(\text{raining in Holland}) P(\text{random movie shot in Holland})}{P(\text{raining anywhere in the world})}$$

*This term is the **prior probability** of A. It's the probability of A before any conditional data is taken into account.*

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

The value of the prior is often observed from general knowledge, the actual data, or even some desired scale or distribution.

*This term is the **likelihood** function. This one swaps the conditional probabilities: it's the probability of your condition B, given A*

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

In a movie it's raining. What's the chance the movie is shot in Holland?

$$= \frac{P(\text{raining in Holland}) P(\text{random movie shot in Holland})}{P(\text{raining anywhere in the world})}$$

*This term is the **likelihood** function. This one swaps the conditional probabilities: it's the probability of your condition B, given A*

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

The value of the likelihood function is observed from the actual data.

*This term is a **normalization constant**. It doesn't depend on A, and is generally ignored while optimizing for maximum probabilities.*

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

In a movie it's raining. What's the chance the movie is shot in Holland?

$$= \frac{P(\text{raining in Holland}) P(\text{random movie shot in Holland})}{P(\text{raining anywhere in the world})}$$

*This term is a **normalization constant**. It doesn't depend on A, and is generally ignored while optimizing for maximum probabilities.*

For example, while running through countries to assess their weather and movie business to find the most likely one, the chance of “rain somewhere” is not relevant.

$$\begin{aligned} &\text{In a movie it's raining. What's the} \\ &\text{chance the movie is shot in Holland?} \end{aligned} = \frac{P(\text{raining in Holland}) P(\text{random movie shot in Holland})}{P(\text{raining anywhere in the world})}$$

Many machine learning techniques use Bayesian statistics to estimate the parameters of their model

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Coefficients of regression

Class labels of samples

Student proficiency and question difficulty

Many machine learning techniques use Bayesian statistics to estimate the parameters of their model

$$P(\beta | \text{data}) = \frac{P(\text{data} | \beta) P(\beta)}{P(\text{data})}$$


Data points in Euclidean space

List of labeled samples

Student responses

Starting out with a prior belief of the parameters β ...

$$P(\beta | \text{data}) = \frac{P(\text{data} | \beta) P(\beta)}{P(\text{data})}$$



*What are reasonable coefficients?
What are common class labels?
How are student proficiencies
generally distributed?*

... and updating the likelihood as new data comes in.

$$P(\beta | \text{data}) = \frac{P(\text{data} | \beta) P(\beta)}{P(\text{data})}$$

*Given these parameters, are my data reasonable?
Given these proficiencies and difficulties, how likely
are these seen student responses?*

Now you see why the normalization constant is generally ignored.

$$P(\beta | \text{data}) = \frac{P(\text{data} | \beta) P(\beta)}{P(\text{data})}$$



How likely is this data anyway?

*The idea of Bayesian inference, then, is to **update our beliefs** about the distribution of A using the data (“evidence”) at our disposal*

$$P(\beta | \text{data}) = \frac{P(\text{data} | \beta) P(\beta)}{P(\text{data})}$$

*The **maximum likelihood estimator (MLE)** finds the parameters that make the data most likely*

$$P(\beta | \text{data}) = \frac{P(\text{data} | \beta) P(\beta)}{P(\text{data})}$$

*The **maximum a posteriori estimate (MAP)** finds the parameters that are most likely, given the data and the prior*

$$P(\beta | \text{data}) = \frac{P(\text{data} | \beta) P(\beta)}{P(\text{data})}$$

As a final remark, Bayes' Theorem offers a “wormhole” between two different “interpretations” of probability

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*The **frequentist** interpretation regards an event's probability as its limiting frequency across a very large number of trials*

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*The **frequentist** interpretation regards an event's probability as its limiting frequency across a very large number of trials*

*The **Bayesian** interpretation regards an event's probability as a “degree of belief,” which applies even to events that haven't occurred yet*

If this sounds crazy to you, don't worry...we won't dwell on the theoretical details.

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If this sounds interesting: this a good direction to head if you're serious in becoming a rock star data scientist.

Method

Predictions

*The **frequentist** interpretation*

point estimates

*The **Bayesian** interpretation*

distributions

III. BAYESIAN COIN FLIPS

EXAMPLE (SIT BACK & RELAX)

INTRO TO DATA SCIENCE

(A FREQUENTIST)
QUIZ QUESTION

Problem:

We observe the following coin flips:

HTHH

What is $P(X = \text{Heads})$?

Problem:

We observe the following coin flips:

HTHH

What is $P(X = \text{Heads})$? $3/4$, Why?

Problem:

We observe the following coin flips:

HTHHTHT

What is $P(X = \text{Heads})$?

Problem:

We observe the following coin flips:

HTHHTHT

What is $P(X = \text{Heads})$? $4/7$, Why?

Problem:

We observe the following coin flips:

HTHHTHT

What is $P(X = \text{Heads})$? 4/7, Why?

With the classical method,

$$P(X = \text{head}) = \frac{\# \text{ heads}}{\# \text{ tosses}}$$

Problem:

We observe the following coin flips:

HTHHTHT

What is $P(X = \text{Heads})$? 4/7, Why?

With the classical method,

$$P(X = \text{head}) = \frac{\# \text{ heads}}{\# \text{ tosses}}$$

... which is not so reliable
with little data

Problem:

We observe the following coin flips:

H

What is $P(X = \text{Heads})$?

Problem:

We observe the following coin flips:

H

What is $P(X = \text{Heads})$? Exactly 1.

Why do you care?

Many problems are binary and are estimated using counts...

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Ex. 1: Sample 100 people and ask if they support a politician.

23 say Yes

Why do you care?

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23 say Yes - Is the correct prediction $23/100$?

Why do you care?

Many problems are binary and are estimated using counts...

Ex. 2: Sample 100 people and ask *which* politician they support
3 say Trump

Why do you care?

Many problems are binary and are estimated using counts...

Ex. 2: Sample 100 people and ask *which* politician they support
3 say Trump - Is the correct prediction $P(\text{Trump}) = 3/100$?

For the frequentist method, you need a lot of data to succeed

Let's try the Bayesian approach

Let's try the Bayesian approach



- This is going to be a lot of high-level math to illustrate Bayes*
- *If you'd like to fully understand, see the enclosed notebook*
 - *If you're fine with hand-waiving: sit back and relax*

$$P(P(H) = p | \text{tosses}) = \frac{P(\text{tosses} | P(H) = p) \times P(P(H) = p)}{P(\text{tosses})}$$

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We don't not estimate the probability $P(H)$ directly

$$P(P(H) = p | \text{tosses}) = \frac{P(\text{tosses} | P(H) = p) \times P(P(H) = p)}{P(\text{tosses})}$$

We don't not estimate the probability $P(H)$ directly, but we ask:

Given the observed tosses, what is the chance that this probability $P(H)$ is equal to some value p ?

$$P(P(H) = p | \text{tosses}) = \frac{P(\text{tosses} | P(H) = p) \times P(P(H) = p)}{P(\text{tosses})}$$

We don't not estimate the probability $P(H)$ directly, but we ask:

Given the observed tosses, what is the chance that this probability $P(H)$ is equal to some value p ?

We look for which p the probability of $P(H) = p$ is the most likely.

$$P(P(H) = p | \text{tosses}) = \frac{P(\text{tosses} | P(H) = p) \times P(P(H) = p)}{P(\text{tosses})}$$

$$P(p | D) = \frac{P(D | p) \times P(p)}{P(D)}$$

let's clean up notation

$$P(P(H) = p | \text{tosses}) = \frac{P(\text{tosses} | P(H) = p) \times P(P(H) = p)}{P(\text{tosses})}$$

$$P(p|D) = \frac{P(D|p) \times P(p)}{P(D)}$$

let's clean up notation

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

look for which p the probability of $P(H) = p$ is the **most likely**

*What is the **prior distribution** of p ?*

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

*What is the **prior distribution** of p ?*

Let's pick a simple Beta distribution

$$P(p) = 6 p (1 - p)$$



*What is the **likelihood function** of p ?*

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

*What is the **likelihood function** of p ?*

This is the reversed question: Given any probability p , what is the chance I'd see the observed coin tosses D ?

*What is the **likelihood function** of p ?*

This is the reversed question: Given any probability p , what is the chance I'd see the observed coin tosses D ?

That's the binomial distribution

$$P(D|p) = \binom{N}{n} p^n (1 - p)^{N-n}$$

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

$$\max_p P(p|D) = \max_p P(D|p)P(p) = \max_p \underbrace{\binom{N}{n} p^n (1-p)^{N-n}}_{\text{likelihood function}} \underbrace{6p(1-p)}_{\text{prior belief}}$$

$$\max_p P(p|D) = \max_p P(D|p)P(p) = \max_p \underbrace{\binom{N}{n} p^n (1-p)^{N-n}}_{\text{likelihood function}} \underbrace{6p(1-p)}_{\text{prior belief}}$$

$$\frac{d}{dp} P(p|D) = 0$$

derivative is zero at maximum

$$\max_p P(p|D) = \max_p P(D|p)P(p) = \max_p \underbrace{\binom{N}{n} p^n (1-p)^{N-n}}_{\text{likelihood function}} \underbrace{6p(1-p)}_{\text{prior belief}}$$

$$\frac{d}{dp} P(p|D) = 0$$

derivative is zero at maximum

$$p = \frac{n+1}{N+2}$$

solution follows algebraically

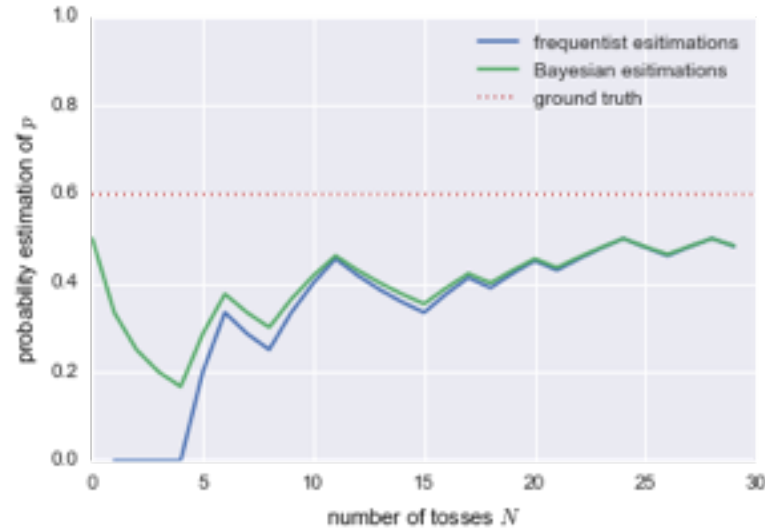
$$p = \frac{n+1}{N+2} = \begin{cases} 1/2 & \text{if no coins have been tossed yet } (N=0) \end{cases}$$

$$p = \frac{n+1}{N+2} = \begin{cases} 1/2 & \text{if no coins have been tossed yet } (N=0) \\ \rightarrow n/N & \text{if many coins have been tossed (i.e., frequentist)} \end{cases}$$

In this case, you could interpret this as the classical approach, after having added two imaginary coin flips at the beginning. We count one more head, and one more tail:

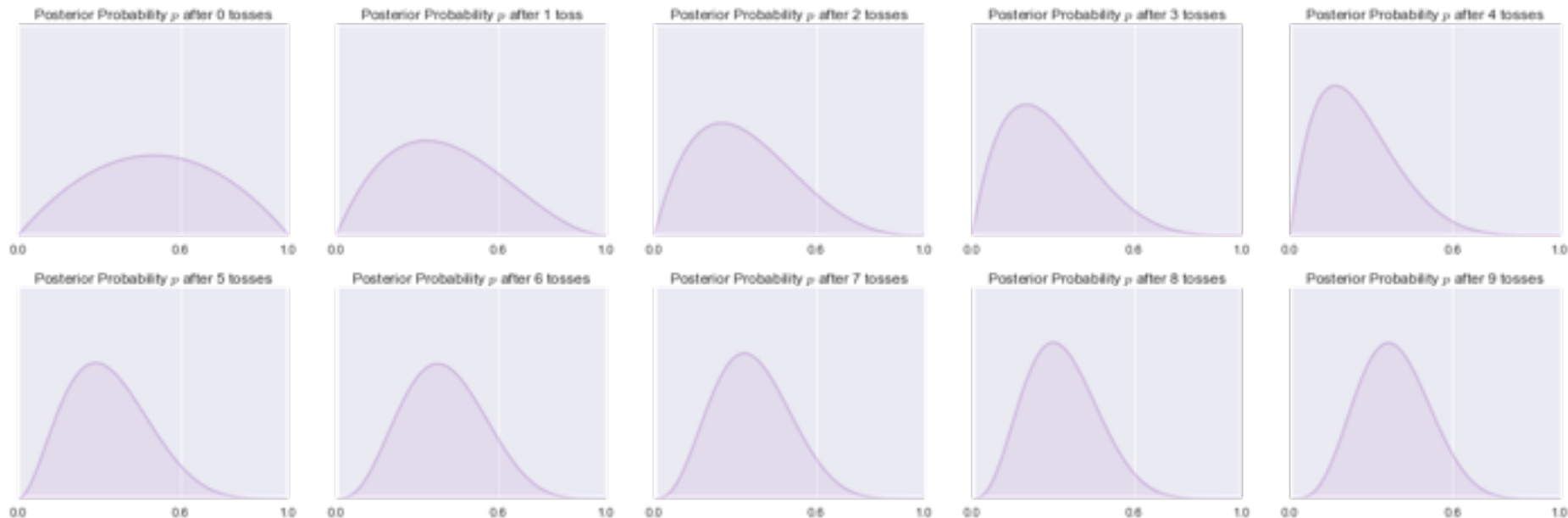
[HT]HTHHTHT

$$p = \frac{n+1}{N+2} = \begin{cases} 1/2 & \text{if no coins have been tossed yet } (N=0) \\ \rightarrow n/N & \text{if many coins have been tossed (i.e., frequentist)} \end{cases}$$



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Bayes provides you distributions, rather than point estimates



INTRO TO DATA SCIENCE

(A BAYESIAN)
QUIZ QUESTION

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HTHH

What is $P(X = \text{Heads})$?

Problem:

We observe the following coin flips:

HTHH

What is $P(X = \text{Heads})$?

Well, it depends on...

... on what?

Problem:

We observe the following coin flips:

HTHH

What is $P(X = \text{Heads})$?

Well, it depends on the prior

Which one shall we take?

Problem:

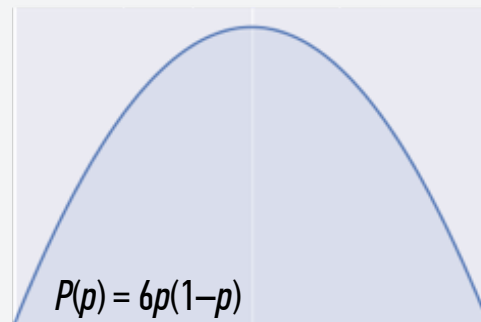
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What is $P(X = \text{Heads})$?

Well, it depends on the prior

- ▶ Let's take the Beta prior



Problem:

We observe the following coin flips:

HTHH

What is $P(X = \text{Heads})$?

Well, it depends on the prior

- ▶ Let's take the Beta prior

$$P(X = \text{head}) = \frac{\# \text{ heads} + 1}{\# \text{ tosses} + 2}$$

Problem:

We observe the following coin flips:

HTHH

What is $P(X = \text{Heads})$? $2/3$

Well, it depends on the prior

- ▶ Let's take the Beta prior

$$P(X = \text{head}) = \frac{\# \text{ heads} + 1}{\# \text{ tosses} + 2}$$

IV. NAIVE BAYES

Confused?

Confused? Relax, it gets easier!

*Suppose we have a dataset with features x_1, \dots, x_n and class labels c .
What can we say about classification using Bayes' theorem?*

*Suppose we have a dataset with features x_1, \dots, x_n and class labels C .
What can we say about classification using Bayes' theorem?*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

What is the chance that these words
(and n-grams) have class label C ?

Suppose we have a dataset with features x_1, \dots, x_n and class labels C . What can we say about classification using Bayes' theorem?

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, given the data we observe.

However, the likelihood function can often be intractably difficult in practice to determine

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

$$P(\{x_i\} \mid C) = P(\{x_1, x_2, \dots, x_n\} \mid C)$$

What is the chance that a random sample from a given class C has exactly all these words (and n-grams)?

So let's make a simplifying assumption. In particular, we assume that the features x_i are conditionally independent from each other:

$$P(\{x_i\}|C) = P(x_1, x_2, \dots, x_n|C) \approx P(x_1|C) * P(x_2|C) * \dots * P(x_n|C)$$



What is the chance that a random word (or n-gram) from a given class C is exactly word x_1 ?

So let's make a simplifying assumption. In particular, we assume that the features x_i are conditionally independent from each other:

$$P(\{x_i\}|C) = P(x_1, x_2, \dots, x_n|C) \approx P(x_1|C) * P(x_2|C) * \dots * P(x_n|C)$$

This “naive” assumption simplifies the likelihood function to make it tractable.

The Naive Bayes algorithm combines the probability of a class C overall with the probabilities of each individual feature appearing in class C

$$P(C|\{x_i\}) \sim P(C) \prod_i P(x_i|C)$$

INTRO TO DATA SCIENCE

DISCUSSION