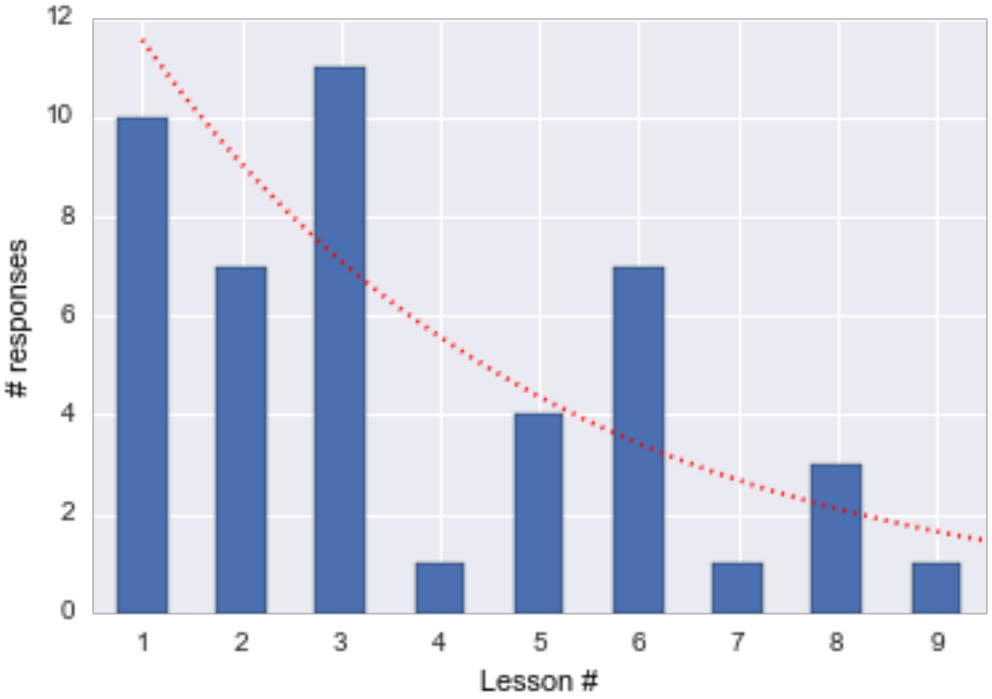
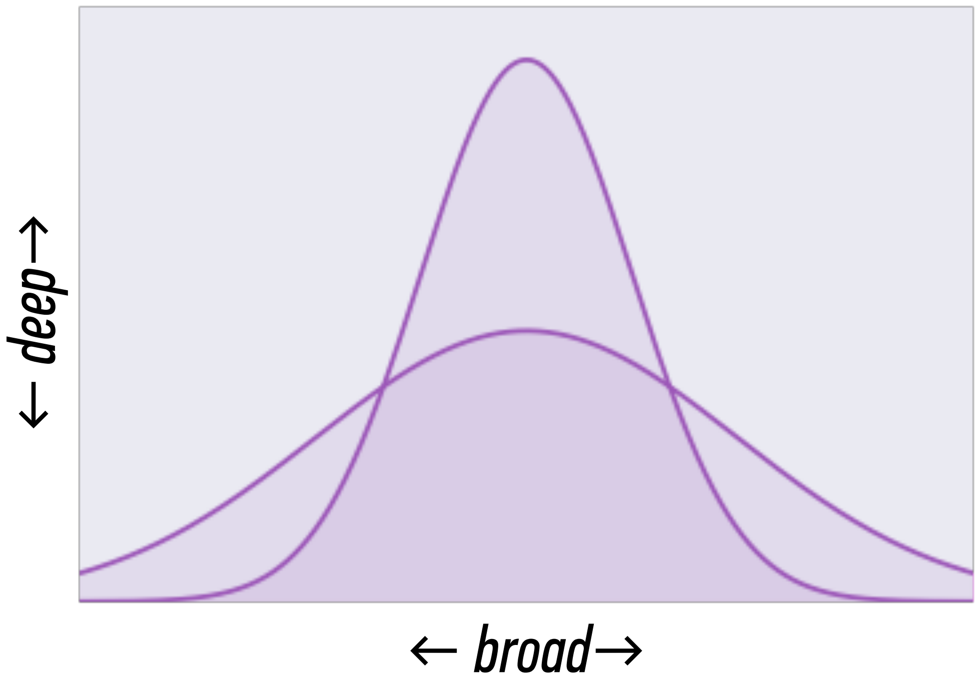
GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

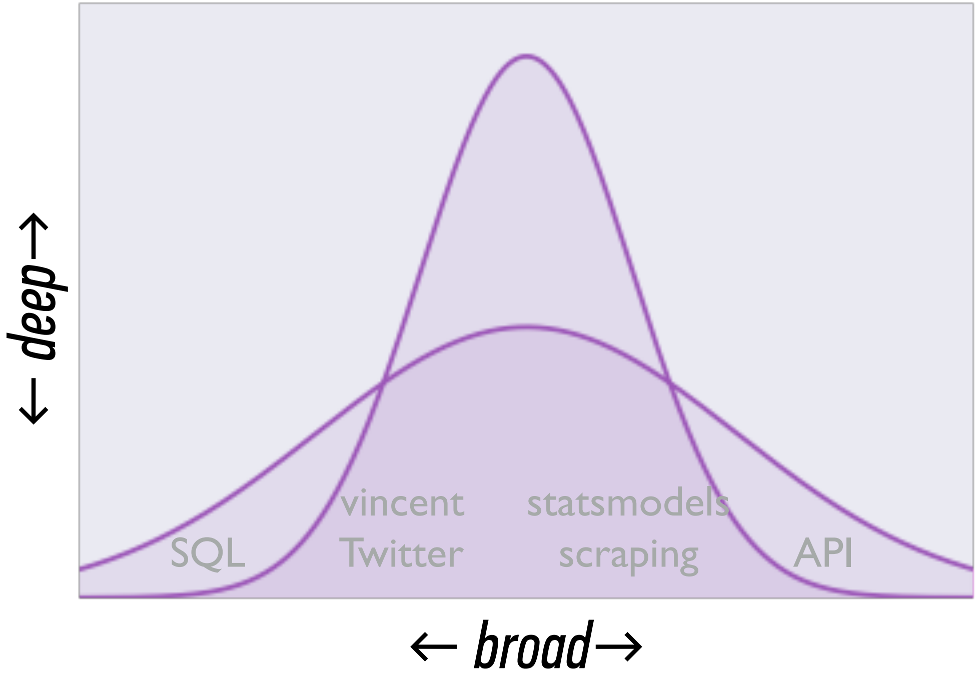
LECTURE 10: BAYESIAN STATISTICS

EXIT TICKETS REVIEW


12
10
8
6
4
2
0
responses
1
2
3
4
5
6
7
8
9
Lesson #

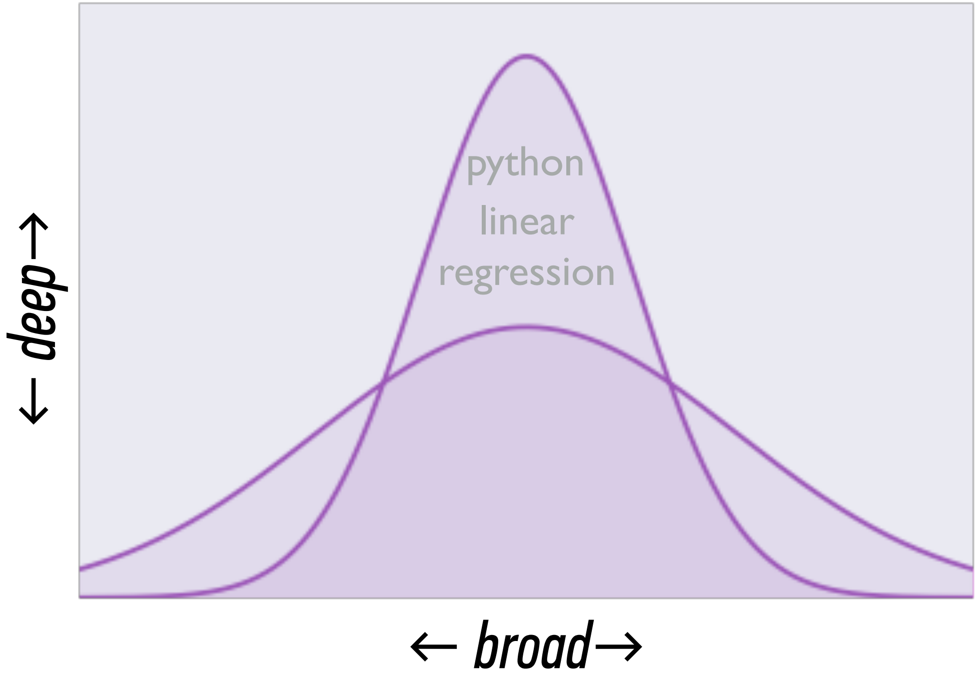


Objective is to be able to apply core techniques in controlled settings (e.g., sklearn in exercises), and know where to look if you need more



Objective is to be able to apply core techniques in controlled settings (e.g., sklearn in exercises), and know where to look if you need more

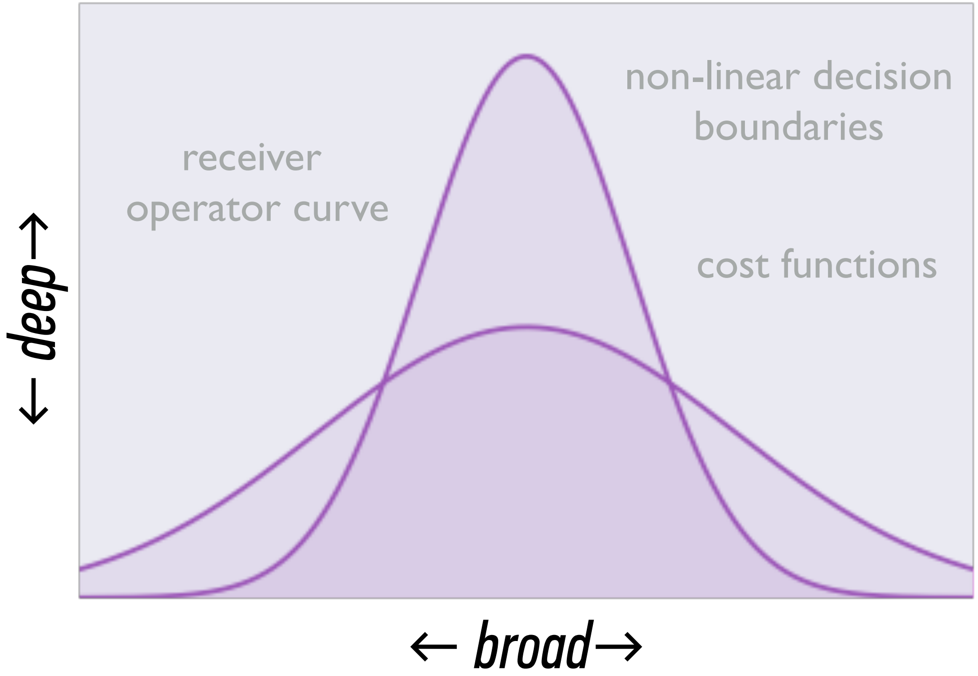
Go quickly through a lot of different tools you might need to use later as a data scientist, so you know where to find them



Objective is to be able to apply core techniques in controlled settings (e.g., sklearn in exercises), and know where to look if you need more

Go quickly through a lot of different tools you might need to use later as a data scientist, so you know where to find them

Spend additional time for essentials you'll need over and over again



Objective is to be able to apply core techniques in controlled settings (e.g., sklearn in exercises), and know where to look if you need more

Go quickly through a lot of different tools you might need to use later as a data scientist, so you know where to find them

Spend additional time for essentials you'll need over and over again

Optionally, provide mathematical foundation when there's interest

EXIT TICKETS – SOME COMMENTS HAVE BEEN INCORPORATED

"Homework should be assigned after each class"

"Class moved at a better pace […] examples and explanations were helpful"

"He talks very fast and it's often difficult to follow"

Source: GADS23 Exit Tickets

"Homework should be assigned after each class"

"Class moved at a better pace [...] examples and explanations were helpful"

"He talks very fast and it's often difficult to follow"

"Concerned that I will be unable to complete the next assignment."

Please reach out and we'll get you up to speed

Source: GADS23 Exit Tickets

EXIT TICKETS – SOME INTERESTS ARE HARD TO UNIFY

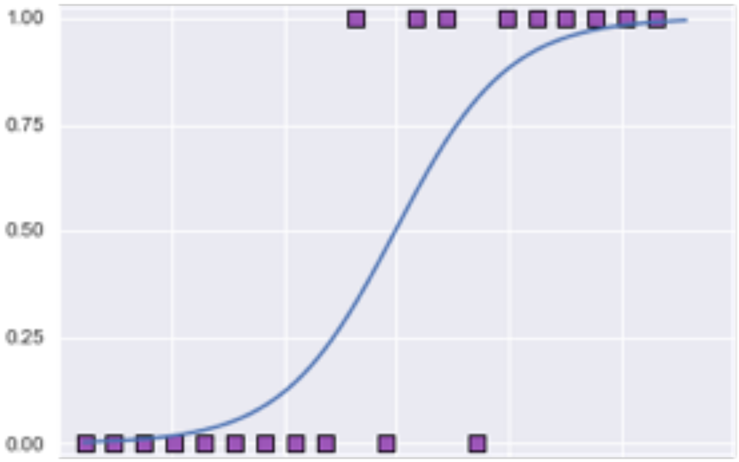
Fast vs slow	Extremely fast-paced, as usual. Hard for much to sink in.	A lot to cover, but I'd rather go fast than slow. Getting 80% of 500 is better than 100% of 100.
	Great pace, liked that there was plenty of time to work on the exercise and ask questions	We can go a little faster in my opinion.
Deep vs broad	I thought some of the points were not covered in depth or in detail.	Same lesson
	[Please] give the reasoning behind why things are rather than just telling us to do something.	Instructor did a good job of explaining things.
Group vs individual	I am looking forward to do more group participation	*Who actually worked together on the second assignment?*

Source: GADS23 Exit Tickets

Please take a few minutes to provide feedback and fill in the exit ticket for last class(es)

BACK TO CLASS…

I. REGRESSION RECAP
II. LOGISTIC REGRESSION
III. INTERPRETING RESULTS
IV. DECISION BOUNDARIES
V. EVALUATING CLASSIFIERS



any questions?

DATA EXPLORATION

SUPERVISED LEARNING: REGRESSION

SUPERVISED LEARNING: CLASSIFICATION

UNSUPERVISED LEARNING

VARIOUS TOPICS

- DATA EXPLORATION
- SUPERVISED LEARNING: REGRESSION
- SUPERVISED LEARNING: CLASSIFICATION
- UNSUPERVISED LEARNING
- VARIOUS TOPICS

- LOGISTIC REGRESSION
- NAIVE BAYES (TODAY)
- RANDOM FORESTS
- SUPPORT VECTOR MACHINES
- COMPETITION

REVIEW ASSIGNMENT #2
GUIDELINES FOR FINAL PROJECT
GUEST SPEAKER: ROHIT ACHARYA, FIRST ACCESS

I. PROBABILITY
II. BAYES' THEOREM
III. EXAMPLE: BAYSAIN COIN FLIPS (OPTIONAL)
IV. NAIVE BAYES

I. PROBABILITY

Q: What is a **probability***?*

Q: What is a **probability***?*

A: A number between 0 and 1 that characterizes the likelihood that some event will occur.

Q: What is a **probability***?*

A: A number between 0 and 1 that characterizes the likelihood that some event will occur.

The probability of event A *is denoted* $P(A)$.

Q: What is a **probability***?*

A: A number between 0 and 1 that characterizes the likelihood that some event will occur.

The probability of event A *is denoted* $P(A)$*.*

The space of all possible events is called the **sample space** *and denoted by* Ω

Q: What is a **probability***?*

A: A number between 0 and 1 that characterizes the likelihood that some event will occur.

The probability of event A *is denoted* $P(A)$.

The space of all possible events is called the **sample space** *and denoted by* Ω

$$P(\Omega) = 1$$

Q: What is a **probability***?*

A: A number between 0 and 1 that characterizes the likelihood that some event will occur.

The probability of event A *is denoted* $P(A)$*.*

The space of all possible events is called the **sample space** *and denoted by* Ω

$$P(\Omega) = 1$$
$$P(\emptyset) = 0$$

$\emptyset$ *is the empty set* { }

It makes sense to think of events of subsets in the sample space Ω

Two events A *and* B *are* **mutually exclusive** or **disjoint**
if they are not overlapping:

$$A \cap B = \varnothing$$

Their intersection is an empty set

It makes sense to think of events of subsets in the sample space Ω

Two events A and B are **mutually exclusive** or **disjoint**
if they are not overlapping:

$$A \cap B = \varnothing$$

Their intersection is an empty set

$P($⚃ and ⚅$) = 0$ *you cannot have both 4 and 6 in one throw*

It makes sense to think of events of subsets in the sample space Ω

If two events A *and* B *are* **mutually exclusive** or **disjoint**, *i.e., if they are not overlapping, then we can add their probabilities*

$$P(A \text{ or } B) = P(A) + P(B)$$

It makes sense to think of events of subsets in the sample space Ω

If two events A and B are **mutually exclusive** or **disjoint**, *i.e., if they are not overlapping, then we can add their probabilities*

$$P(A \text{ or } B) = P(A) + P(B)$$

*If A and B are **not** mutually exclusive, then we have*

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

It makes sense to think of events of subsets in the sample space Ω

If two events A *and* B *are* **mutually exclusive**

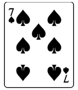
$P($ ⚃ or ⚅ $) = P($⚃$) + P($⚅$) = 1/6 + 1/6 = 1/3$

It makes sense to think of events of subsets in the sample space Ω

If two events A and B are **mutually exclusive**

$$P(⚃ \text{ or } ⚅) = P(⚃) + P(⚅) = 1/6 + 1/6 = 1/3$$

If A and B are ***not*** *mutually exclusive*

$P(7 \text{ or } \spadesuit) = P(7) + P(\spadesuit) - P($  $)$

$$= 1/13 + 1/4 - 1/52 = 11/26$$

Two events A and B are called **independent** *if their joint probability is the product of their individual probabilities:*

$$P(A \; and \; B) = P(A)\,P(B)$$

We often write $A \perp A$

Two events A and B are called **independent** *if their joint probability is the product of their individual probabilities:*

two throws $P($  and  $) = P($  $)\ P($  $) = 1/6 \times 1/6 = 1/36$

Q: Suppose event B has occurred. What quantity represents the probability of A **given** *this information about B?*

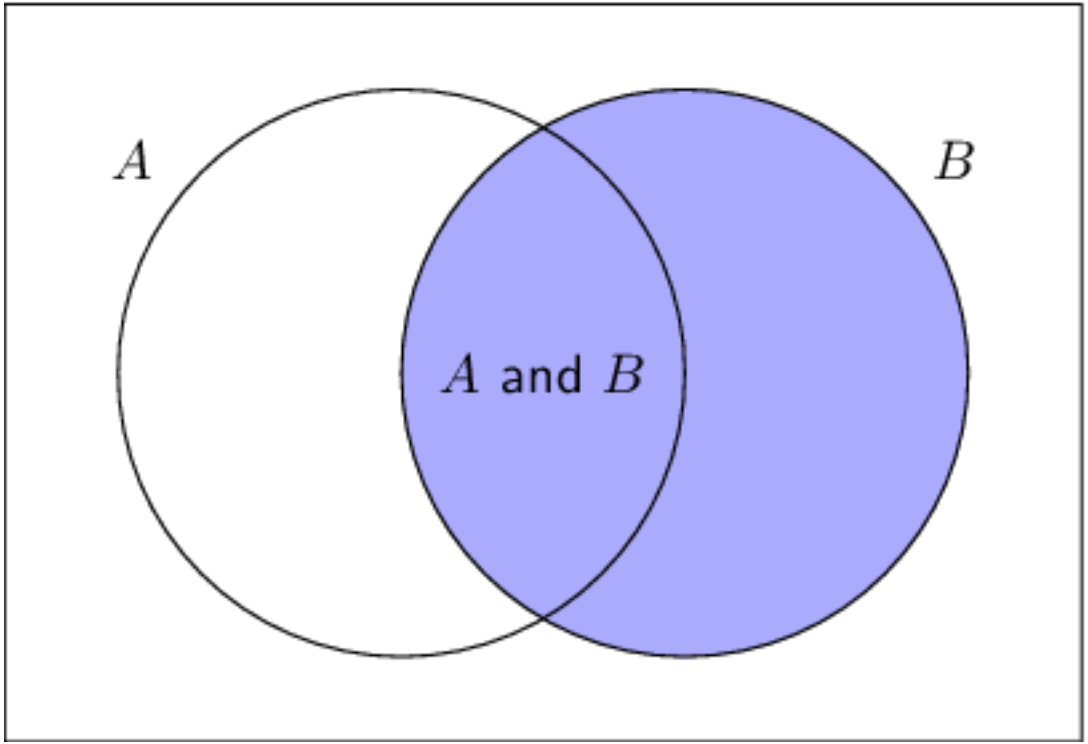
Q: Suppose event B has occurred. What quantity represents the probability of A **given** *this information about B?*

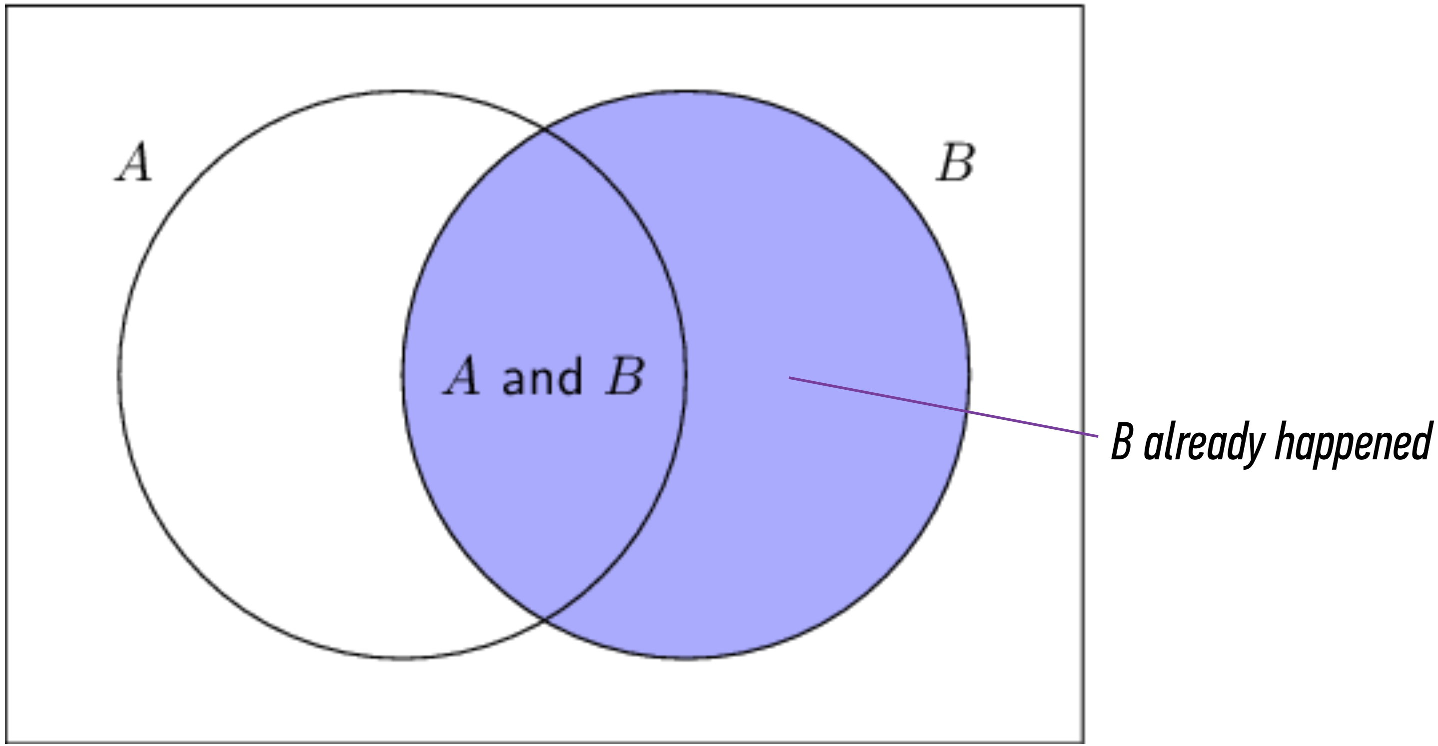
A: The intersection of A & B divided by region B.

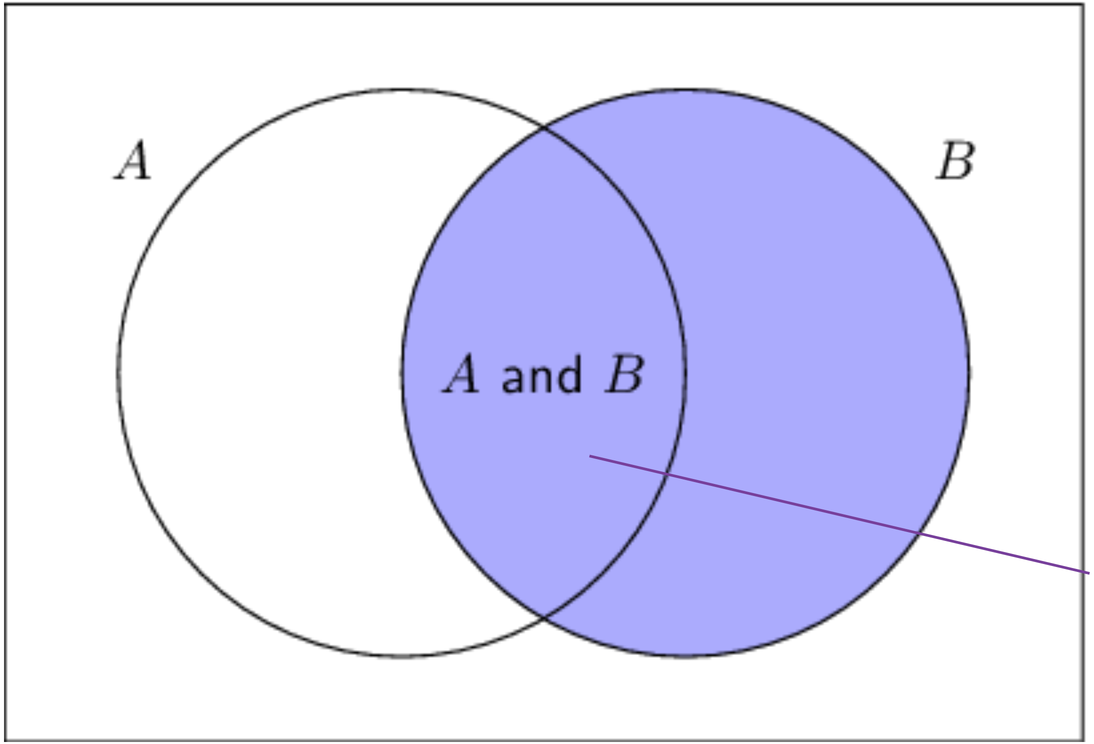
Q: Suppose event B has occurred. What quantity represents the probability of A **given** *this information about B?*

A: The intersection of A & B divided by region B.

This is called the **conditional probability** *of A given B, written $P(A|B) = P(AB) / P(B)$.*


A
B
A and B


A
B
A and B
B already happened



B already happened

The probability of A is now given by A ∩ B (or AB)

Two **conditional probability** *is the probability of some event A, given the occurrence of some other event B.*

$$P(A \mid B) = \frac{P(A \, and \, B)}{P(B)}$$

Two **conditional probability** *is the probability of some event A, given the occurrence of some other event B.*

$$P(A \mid B) = \frac{P(A\, and\, B)}{P(B)}$$

It follows that if $P(A \mid B) = P(A)$ *if and only if* $A \perp B$

QUIZ QUESTION

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1) *Linda is a bank teller.*
2) *Linda is a bank teller and active in the feminist movement.*

Q: What does it mean for two events to be **independent***?*

Q: What does it mean for two events to be **independent***?*

A: Information about one does not affect the probability of the other.

Q: What does it mean for two events to be **independent***?*

A: Information about one does not affect the probability of the other.

This can be written as $P(A|B) = P(A)$

Q: What does it mean for two events to be **independent***?*

A: Information about one does not affect the probability of the other.

This can be written as $P(A|B) = P(A)$

And we have $P(A \; and \; B) = P(A)\,P(B)$

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1) *Linda is a bank teller.*
2) *Linda is a bank teller and active in the feminist movement.*

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is larger?

1) P(bank teller)

2) P(bank teller and feminist movement)

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is larger?

1) $P(\text{bank teller})$
2) $P(\text{bank teller}) \times P(\text{feminist movement})$

II. BAYES' THEOREM

Recall the **conditional probability**

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

Recall the **conditional probability**

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

We can rewrite that as $P(AB) = P(A|B)\,P(B)$

Recall the **conditional probability**

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

We can rewrite that as $P(AB) = P(A|B)\,P(B)$

As well as $P(AB) = P(B|A)\,P(A)$

Recall the **conditional probability**

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

We can rewrite that as $P(AB) = P(A|B)\,P(B)$

As well as $P(AB) = P(B|A)\,P(A)$

It follows that $$P(A \mid B) = \frac{P(B|A)\,P(A)}{P(B)}$$

This result is called **Bayes' Theorem**

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

This result is called **Bayes' Theorem**

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

It means you can swap conditional probabilities

This result is called **Bayes' Theorem**

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

It means you can swap conditional probabilities

In a movie it's raining.What's the chance the movie is shot in Holland?

This result is called **Bayes' Theorem**

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

It means you can swap conditional probabilities

In a movie it's raining.What's the chance the movie is shot in Holland?

$$= \frac{P(\textit{raining in Holland})\ P(\textit{random movie shot in Holland})}{P(\textit{raining anywhere in the world})}$$

Each term in this relationship has a name, and each plays a distinct role in any probability calculation (including ours).

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

This term is the **posterior probability** *of A. It's the probability of A* *after the conditional data is taken into account.*

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

In a movie it's raining.What's the chance the movie is shot in Holland?

$$= \frac{P(\text{raining in Holland})\;P(\text{random movie shot in Holland})}{P(\text{raining anywhere in the world})}$$

*This term is the **posterior probability** of A. It's the probability of A after the conditional data is taken into account.*

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

The goal of any Bayesian computation is to find ("learn") the posterior distribution of a particular variable.

*This term is the **prior probability** of A. It's the probability of A* ***before*** *any conditional data is taken into account.*

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

In a movie it's raining. What's the chance the movie is shot in Holland?

$$= \frac{P(\textit{raining in Holland})\; P(\textit{random movie shot in Holland})}{P(\textit{raining anywhere in the world})}$$

This term is the **prior probability** *of A. It's the probability of A* *before* *any conditional data is taken into account.*

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

The value of the prior is often observed from general knowledge, the actual data, or even some desired scale or distribution.

This term is the **likelihood** *function. This one swaps the conditional probabilities: it's the probability of your condition B, given A*

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

In a movie it's raining.What's the chance the movie is shot in Holland?

$$= \frac{P(\text{raining in Holland})\ P(\text{random movie shot in Holland})}{P(\text{raining anywhere in the world})}$$

This term is the **likelihood** *function. This one swaps the conditional probabilities: it's the probability of your condition B, given A*

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

The value of the likelihood function is observed from the actual data.

This term is a **normalization constant**. *It doesn't depend on A, and is generally ignored while optimizing for maximum probabilities.*

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

In a movie it's raining.What's the chance the movie is shot in Holland?

$$= \frac{P(\text{raining in Holland})\;P(\text{random movie shot in Holland})}{P(\text{raining anywhere in the world})}$$

This term is a **normalization constant**. *It doesn't depend on A, and is generally ignored while optimizing for maximum probabilities.*

For example, while running through countries to assess their weather and movie business to find the most likely one, the chance of "rain somewhere" is not relevant.

$$\text{In a movie it's raining. What's the chance the movie is shot in Holland?} = \frac{P(\text{raining in Holland})\;P(\text{random movie shot in Holland})}{P(\text{raining anywhere in the world})}$$

Many machine learning techniques use Bayesian statistics to estimate the parameters of their model

Many machine learning techniques use Bayesian statistics to estimate the parameters of their model

$$P(\beta\,|\,\text{data}) = \frac{P(\text{data}\,|\,\beta)\;P(\beta)}{P(\text{data})}$$

Many machine learning techniques use Bayesian statistics to estimate the parameters of their model

$$P(\beta \mid \text{data}) = \frac{P(\text{data} \mid \beta)\, P(\beta)}{P(\text{data})}$$

Coefficients of regression
Class labels of samples
Student proficiency and question difficulty

Many machine learning techniques use Bayesian statistics to estimate the parameters of their model

$$P(\beta\,|\,\mathrm{data}) = \frac{P(\mathrm{data}\,|\,\beta)\;P(\beta)}{P(\mathrm{data})}$$

Data points in Euclidean space
List of labeled samples
Student responses

Starting out with a prior belief of the parameters β ...

$$P(\beta\,|\,\text{data}) = \frac{P(\text{data}\,|\,\beta\,)\;P(\beta\,)}{P(\text{data})}$$

What are reasonable coefficients?
What are common class labels?
How are student proficiencies generally distributed?

… and updating the likelihood as new data comes in.

$$P(\beta \mid \text{data}) = \frac{P(\text{data} \mid \beta)\, P(\beta)}{P(\text{data})}$$

Given these parameters, are my data reasonable?
Given these proficiencies and difficulties, how likely are these seen student responses?

Now you see why the normalization constant is generally ignored.

$$P(\beta \mid \text{data}) = \frac{P(\text{data} \mid \beta)\; P(\beta)}{P(\text{data})}$$

How likely is this data anyway?

The idea of Bayesian inference, then, is to **update our beliefs** *about the distribution of A using the data ("evidence") at our disposal*

$$P(\beta\,|\,\mathrm{data}) = \frac{P(\mathrm{data}\,|\,\beta)\;P(\beta)}{P(\mathrm{data})}$$

The **maximum likelihood estimator (MLE)** *finds the parameters that make the data most likely*

$$P(\beta \mid \text{data}) = \frac{P(\text{data} \mid \beta)\, P(\beta)}{P(\text{data})}$$

The **maximum a posteriori estimate (MAP)** *finds the parameters that are most likely, given the data and the prior*

$$P(\beta \mid \mathrm{data}) = \frac{P(\mathrm{data} \mid \beta)\; P(\beta)}{P(\mathrm{data})}$$

As a final remark, Bayes' Theorem offers a "wormhole" between two different "interpretations" of probability

As a final remark, Bayes' Theorem offers a “wormhole” between two different “interpretations” of probability

The **frequentist** *interpretation regards an event’s probability as its limiting frequency across a very large number of trials*

As a final remark, Bayes' Theorem offers a "wormhole" between two different "interpretations" of probability

The **frequentist** *interpretation regards an event's probability as its limiting frequency across a very large number of trials*

The **Bayesian** *interpretation regards an event's probability as a "degree of belief," which applies even to events that haven't occurred yet*

If this sounds crazy to you, don’t worry…we won’t dwell on the theoretical details.

If this sounds crazy to you, don't worry…we won't dwell on the theoretical details.

If this sounds interesting: this a good direction to head if you're serious in becoming a rock star data scientist.

Method	***Predictions***
The **frequentist** *interpretation*	*point estimates*
The **Bayesian** *interpretation*	*distributions*

III. BAYESIAN COIN FLIPS EXAMPLE (SIT BACK & RELAX)

(A FREQUENTIST) QUIZ QUESTION

Problem:

We observe the following coin flips:

HTHH

What is $P(X = \text{Heads})$?

Problem:

We observe the following coin flips:

HTHH

What is P(X = Heads) ?   3/4, Why?

Problem:

We observe the following coin flips:

HTHHTHT

What is P(X = Heads) ?

Problem:

We observe the following coin flips:

HTHHTHT

What is P(X = Heads) ?   4/7, Why?

Problem:

We observe the following coin flips:

HTHHTHT

What is P(X = Heads) ?   4/7, Why?

With the classical method,

$$P(X = head) = \frac{\#\ heads}{\#\ tosses}$$

Which is not so reliable with little data

Problem:

We observe the following coin flips:

H

What is $P(X = \text{Heads})$?

Problem:

We observe the following coin flips:

H

What is P(X = Heads) ? Exactly 1.

Why do you care?

Many problems are binary and are estimated using counts...

Why do you care?

Many problems are binary and are estimated using counts…

Ex. 1: Sample 100 people and ask if they support a politician.
23 say Yes

Why do you care?

Many problems are binary and are estimated using counts…

Ex. 1: Sample 100 people and ask if they support a politician.
23 say Yes – Is the correct prediction 23/100?

Why do you care?

Many problems are binary and are estimated using counts…

Ex. 2: Sample 100 people and ask *which* politician they support
3 say Trump

Why do you care?

Many problems are binary and are estimated using counts…

Ex. 2: Sample 100 people and ask *which* politician they support
3 say Trump – Is the correct prediction P(Trump) = 3/100?

For the frequentist method, you need a lot of data to succeed

Let’s try the Bayesian approach

Let’s try the Bayesian approach

This is going to be a lot of high-level math to illustrate Bayes

- *If you’d like to fully understand, see the enclosed notebook*
- *If you’re fine with hand-waiving: sit back and relax*

$$P(P(H) = p | \text{tosses}) = \frac{P(\text{tosses} | H) \times P(P(H) = p)}{P(\text{tosses})}$$

$$P(P(H) = p|\text{ tosses}) = \frac{P(\text{tosses }|H) \times P(P(H) = p)}{P(\text{tosses})}$$

We don’t not estimate the probability *P(H)* directly

$$P(P(H) = p \mid \text{tosses}) = \frac{P(\text{tosses} \mid H) \times P(P(H) = p)}{P(\text{tosses})}$$

We don't not estimate the probability *P(H)* directly, but we ask:

> *Given the observed tosses, what is the chance that this probability P(H) is equal to some value p?*

$$P(P(H) = p|\text{ tosses}) = \frac{P(\text{tosses }|H) \times P(P(H) = p)}{P(\text{tosses})}$$

We don't not estimate the probability *P(H)* directly, but we ask:

> *Given the observed tosses, what is the chance that this probability P(H) is equal to some value p?*

We look for which *p* the probability of *P(H) = p* is the most likely.

$$P(P(H) = p \mid \text{tosses}) = \frac{P(\text{tosses} \mid H) \times P(P(H) = p)}{P(\text{tosses})}$$

$$P(P(H) = p | \text{ tosses}) = \frac{P(\text{tosses } | H) \times P(P(H) = p)}{P(\text{tosses})}$$

$$P(p|D) = \frac{P(D|p) \times P(p)}{P(D)}$$

let's clean up notation

$$P(P(H) = p|\text{ tosses}) = \frac{P(\text{tosses }|H) \times P(P(H) = p)}{P(\text{tosses})}$$

$$P(p|D) = \frac{P(D|p) \times P(p)}{P(D)}$$

let's clean up notation

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

look for which *p* the probability of *P(H) = p* is the **most likely**

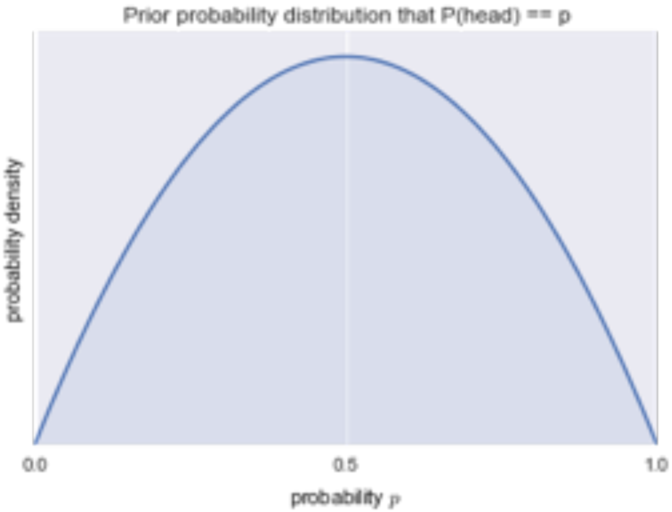
What is the **prior distribution** *of p?*

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

What is the **prior distribution** *of p?*

Let's pick a simple Beta distribution

$$P(p) = 6\,p\,(1 - p)$$



What is the **likelihood function** *of p?*

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

What is the **likelihood function** *of p?*

This is the reversed question: Given any probability p, what is the chance I'd see the observed coin tosses D?

What is the **likelihood function** *of p?*

This is the reversed question: Given any probability p, what is the chance I'd see the observed coin tosses D?

That's the binomial distribution

$$P(D|p) = \binom{N}{n} p^n (1-p)^{N-n}$$

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

$$\max_p P(p|D) = \max_p P(D|p)P(p) = \max_p \underbrace{\binom{N}{n} p^n (1-p)^{N-n}}_{\text{likelihood function}} \underbrace{6p(1-p)}_{\text{prior belief}}$$

$$\max_p P(p|D) = \max_p P(D|p)P(p) = \max_p \underbrace{\binom{N}{n} p^n (1-p)^{N-n}}_{\text{likelihood function}} \underbrace{6p(1-p)}_{\text{prior belief}}$$

$$\frac{d}{dp} P(p|D) = 0$$

derivative is zero at maximum

$$\max_p P(p|D) = \max_p P(D|p)P(p) = \max_p \underbrace{\binom{N}{n} p^n (1-p)^{N-n}}_{\text{likelihood function}} \underbrace{6p(1-p)}_{\text{prior belief}}$$

$$\frac{d}{dp} P(p|D) = 0$$

derivative is zero at maximum

$$p = \frac{n+1}{N+2}$$

solution follows algebraically

COIN TOSSING – THE BAYESIAN WAY

$$p = \frac{n+1}{N+2} \quad = \quad \begin{cases} 1/2 \;\; \text{if no coins have been tossed yet } (N = 0) \\ \end{cases}$$

$$p = \frac{n+1}{N+2} \quad = \quad \begin{cases} 1/2 & \text{if no coins have been tossed yet } (N = 0) \\ \longrightarrow n/N & \text{if many coins have been tossed (i.e., frequentist)} \end{cases}$$

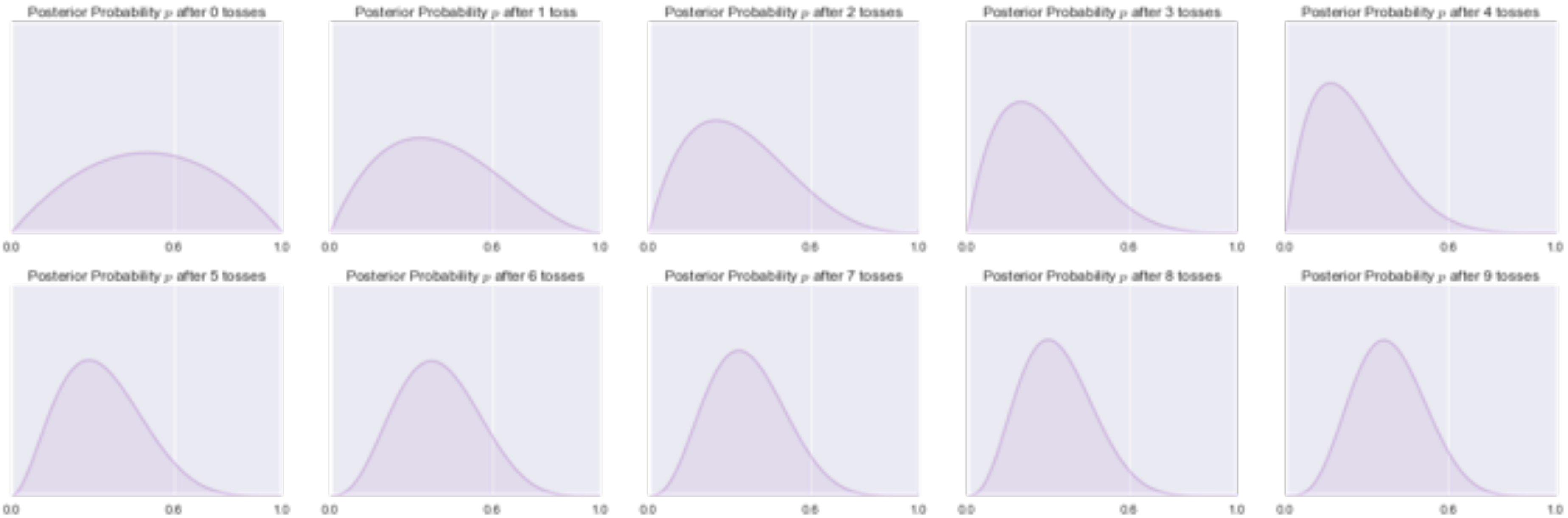
In this case, you could interpret this as the classical approach, after having added two imaginary coin flips at the beginning. We count one more head, and one more tail:

[HT]HTHHHTHT

$$p = \frac{n+1}{N+2} = \begin{cases} 1/2 & \text{if no coins have been tossed yet } (N = 0) \\ \longrightarrow n/N & \text{if many coins have been tossed (i.e., frequentist)} \end{cases}$$

$$p = \frac{n+1}{N+2} = \begin{cases} 1/2 & \text{if no coins have been tossed yet } (N = 0) \\ \longrightarrow n/N & \text{if many coins have been tossed (i.e., frequentist)} \end{cases}$$

Bayes provides you distributions, rather than point estimates



(A BAYESIAN) QUIZ QUESTION

Problem:

We observe the following coin flips:

HTHH

What is P(X = Heads) ?

Problem:

We observe the following coin flips:

HTHH

What is P(X = Heads) ?

Well, it depends on...

... on what?

Problem:

We observe the following coin flips:

HTHH

What is P(X = Heads) ?

Well, it depends on the prior

Which one shall we take?

Problem:

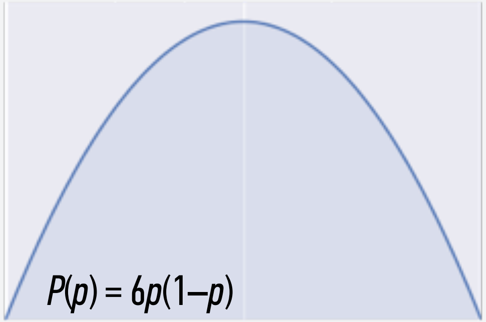
We observe the following coin flips:

HTHH

What is P(X = Heads) ?

Well, it depends on the prior

- Let's take the Beta prior



Problem:

We observe the following coin flips:

HTHH

What is P(X = Heads) ?

Well, it depends on the prior

- Let's take the Beta prior

$$P(X = \text{head}) = \frac{\#\,\textit{heads} + 1}{\#\,\textit{tosses} + 2}$$

Problem:

We observe the following coin flips:

HTHH

What is P(X = Heads) ? 2/3

Well, it depends on the prior

- Let's take the Beta prior

$$P(X = \text{head}) = \frac{\#\,heads + 1}{\#\,tosses + 2}$$

IV. NAIVE BAYES

Confused?

Confused? Relax, it gets easier!

Suppose we have a dataset with features $x_1, \ldots, x_n$ and class labels C. What can we say about classification using Bayes' theorem?

Suppose we have a dataset with features $x_1, \ldots, x_n$ and class labels c. What can we say about classification using Bayes' theorem?

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

What is the chance that these words (and n-grams) have class label C?

Suppose we have a dataset with features $x_1, \ldots, x_n$ and class labels C. What can we say about classification using Bayes' theorem?

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, given the data we observe.

However, the likelihood function can often be intractably difficult in practice to determine

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

$$P(\{x_i\}|C) = P(\{x_1, x_2, \ldots, x_n\}|C)$$

What is the chance that a random sample from a given class C has exactly all these words (and n-grams)?

So let's make a simplifying assumption. In particular, we assume that the features x_i are conditionally independent from each other:

$$P(\{x_i\}|C) = P(x_1, x_2, \ldots, x_n|C) \approx P(x_1|C) * P(x_2|C) * \ldots * P(x_n|C)$$

What is the chance that a random word (or n-gram) from a given class C is exactly word x_1?

So let's make a simplifying assumption. In particular, we assume that the features x_i are conditionally independent from each other:

$$P(\{x_i\}|C) = P(x_1, x_2, \ldots, x_n|C) \approx P(x_1|C) * P(x_2|C) * \ldots * P(x_n|C)$$

This "naive" assumption simplifies the likelihood function to make it tractable.

The Naive Bayes algorithm combines the probability of a class C overall with the probabilities of each individual feature appearing in class C

$$P(C|\{x_i\}) \sim P(C) \prod_i P(x_i|C)$$

DISCUSSION