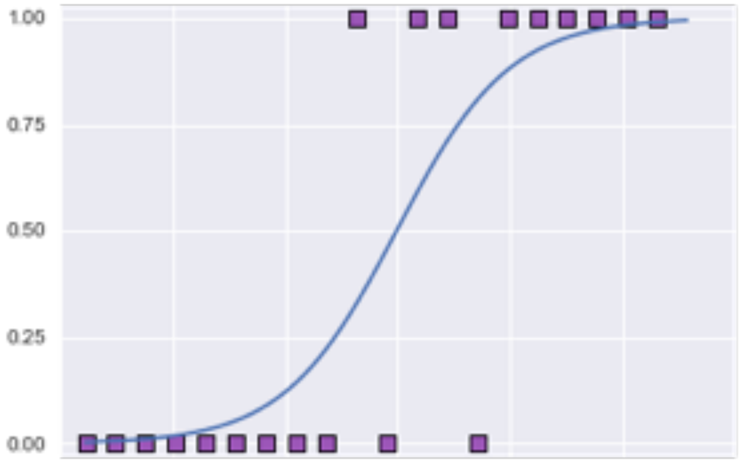
GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 10: BAYESIAN STATISTICS

I. REGRESSION RECAP
II. LOGISTIC REGRESSION
III. INTERPRETING RESULTS
IV. DECISION BOUNDARIES
V. EVALUATING CLASSIFIERS



any questions?

DATA EXPLORATION

SUPERVISED LEARNING: REGRESSION

SUPERVISED LEARNING: CLASSIFICATION

UNSUPERVISED LEARNING

VARIOUS TOPICS

- DATA EXPLORATION
- SUPERVISED LEARNING: REGRESSION
- SUPERVISED LEARNING: CLASSIFICATION
- UNSUPERVISED LEARNING
- VARIOUS TOPICS

- LOGISTIC REGRESSION
- NAIVE BAYES (TODAY)
- RANDOM FORESTS
- SUPPORT VECTOR MACHINES
- COMPETITION

0. DEMO SAMPLE PROJECTS
I. PROBABILITY
II. BAYES’ THEOREM
III. EXAMPLE: BAYSAIN COIN FLIPS (OPTIONAL)
IV. NAIVE BAYES

I. PROBABILITY

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$$P(\Omega) = 1$$
$$P(\varnothing) = 0$$

$\varnothing$ *is the empty set* { }

It makes sense to think of events of subsets in the sample space Ω

Two events A *and* B *are* **mutually exclusive** or **disjoint**
if they are not overlapping:

$$A \cap B = \varnothing$$

Their intersection is an empty set

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$P($ ⚃ and ⚅ $) = 0$ *you cannot have both 4 and 6 in one throw*

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If two events A *and* B *are* **mutually exclusive** or **disjoint**, *i.e., if they are not overlapping, then we can add their probabilities*

$$P(A \text{ or } B) = P(A) + P(B)$$

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If two events A and B are **mutually exclusive** or **disjoint**, *i.e., if they are not overlapping, then we can add their probabilities*

$$P(A \text{ or } B) = P(A) + P(B)$$

If A and B are ***not*** *mutually exclusive, then we have*

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

It makes sense to think of events of subsets in the sample space Ω

If two events A and B are **mutually exclusive**

$$P(\text{⚃} \; or \; \text{⚅}) = P(\text{⚃}) + P(\text{⚅}) = 1/6 + 1/6 = 1/3$$

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If two events A and B are **mutually exclusive**

$$P(\text{⚃ or ⚅}) = P(\text{⚃}) + P(\text{⚅}) = 1/6 + 1/6 = 1/3$$

If A and B are ***not*** *mutually exclusive*

$$P(\text{7 or ♠}) = P(\text{7}) + P(\text{♠}) - P(\text{7♠})$$

$$= 1/13 + 1/4 - 1/52 = 11/26$$

Two events A and B are called **independent** *if their joint probability is the product of their individual probabilities:*

$$P(A \, and \, B) = P(A) \, P(B)$$

We often write $A \perp A$

Two events A *and* B *are called* **independent** *if their joint probability is the product of their individual probabilities:*

two throws $P(\text{⚃}\ and\ \text{⚅}) = P(\text{⚃})\,P(\text{⚅}) = 1/6 \times 1/6 = 1/36$

Q: Suppose event B has occurred. What quantity represents the probability of A **given** *this information about B?*

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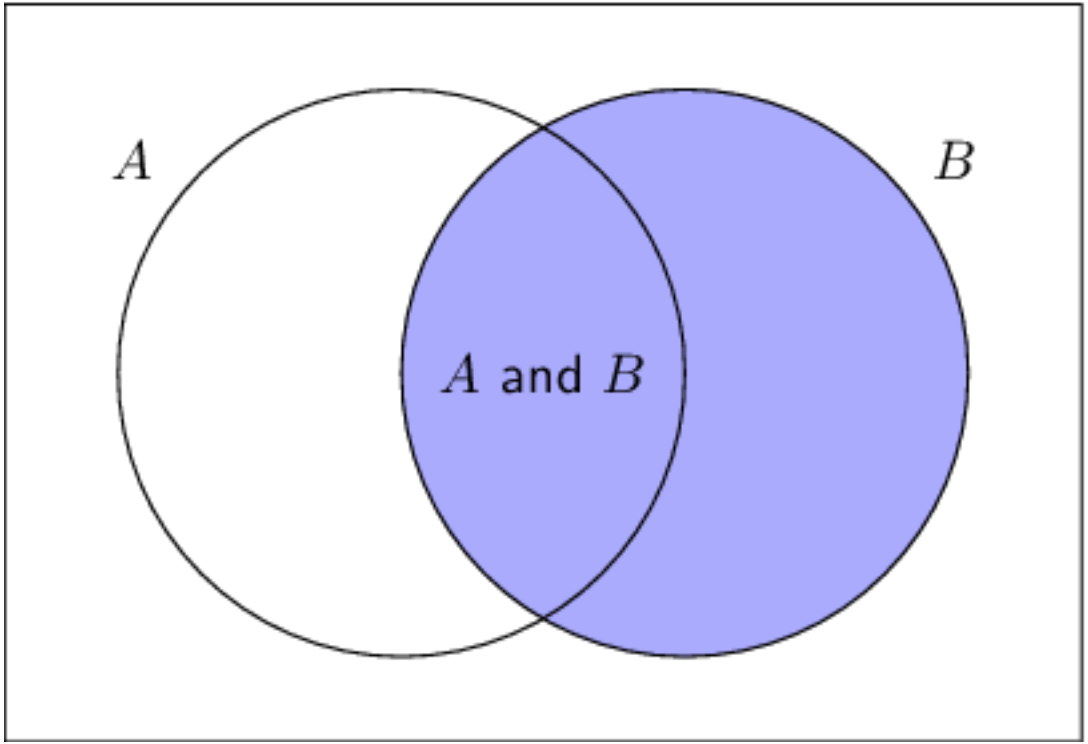
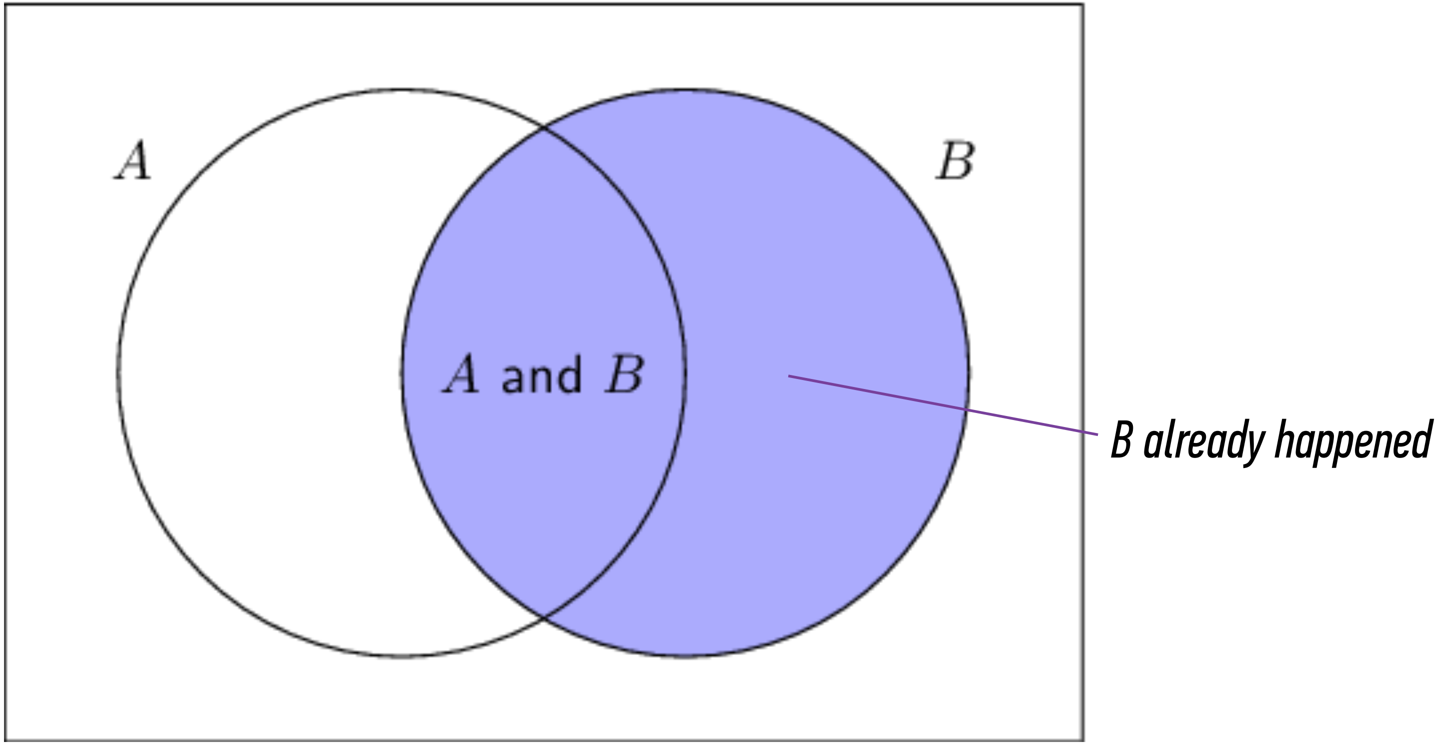
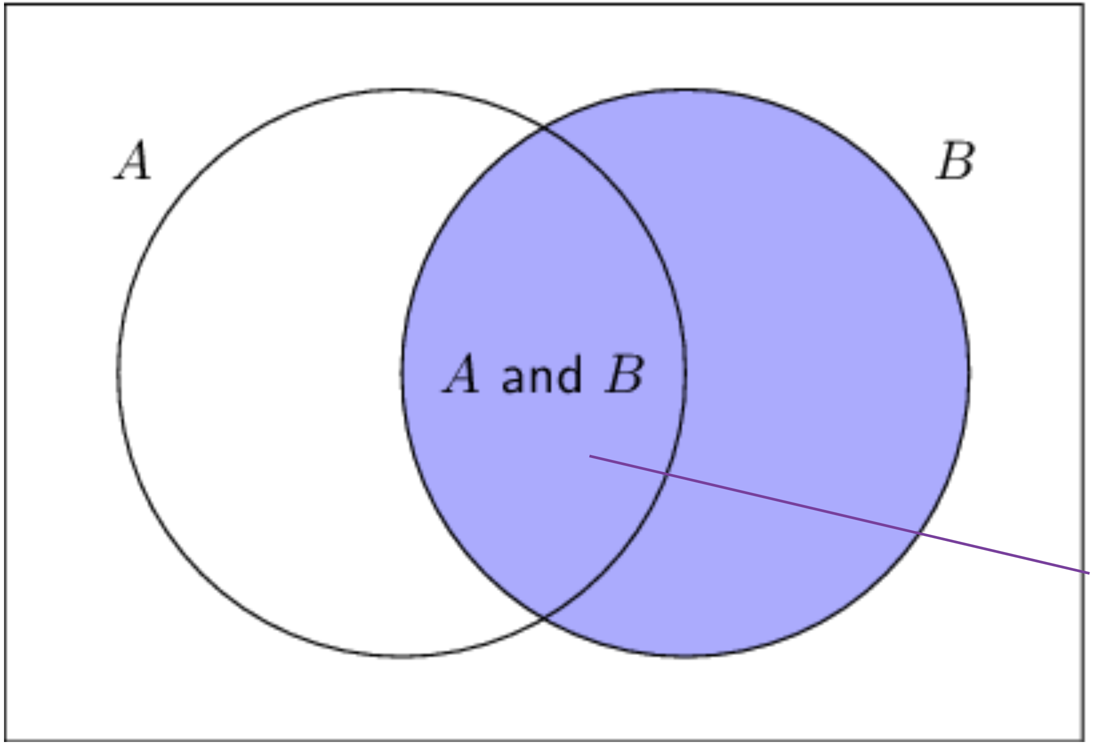
A: The intersection of A & B divided by region B.

This is called the **conditional probability** *of A given B, written $P(A|B) = P(AB) / P(B)$.*


A
B
A and B


A
B
A and B
B already happened



B already happened

The probability of A is now given by A ∩ B (or AB)

Two **conditional probability** *is the probability of some event A, given the occurrence of some other event B.*

$$P(A \mid B) = \frac{P(A\ and\ B)}{P(B)}$$

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$$P(A \mid B) = \frac{P(A\, and\, B)}{P(B)}$$

It follows that if $P(A \mid B) = P(A)$ *if and only if* $A \perp B$

QUIZ QUESTION

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1) *Linda is a bank teller.*
2) *Linda is a bank teller and active in the feminist movement.*

Q: What does it mean for two events to be **independent***?*

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A: Information about one does not affect the probability of the other.

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And we have $P(A\, and\, B) = P(A)\, P(B)$

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

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Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is larger?

1) P(bank teller)

2) P(bank teller and feminist movement)

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is larger?
1) P(bank teller)
2) P(bank teller) × P(feminist movement)

II. BAYES' THEOREM

Recall the **conditional probability**

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Recall the **conditional probability**

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

We can rewrite that as $P(AB) = P(A|B)\,P(B)$

As well as $P(AB) = P(B|A)\,P(A)$

It follows that $P(A \mid B) = \dfrac{P(B|A)\,P(A)}{P(B)}$

This result is called **Bayes' Theorem**

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In a movie it's raining.What's the chance the movie is shot in Holland?

This result is called **Bayes' Theorem**

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

It means you can swap conditional probabilities

In a movie it's raining. What's the chance the movie is shot in Holland?

$$= \frac{P(\text{raining in Holland})\;P\left(\begin{array}{l}\text{random movie}\\ \text{shot in Holland}\end{array}\right)}{P\left(\begin{array}{l}\text{raining anywhere}\\ \text{in the world}\end{array}\right)}$$

Each term in this relationship has a name, and each plays a distinct role in any probability calculation (including ours).

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

This term is the **posterior probability** *of A. It's the probability of A* *<u>after</u> the conditional data is taken into account.*

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

In a movie it's raining.What's the chance the movie is shot in Holland?

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$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

The goal of any Bayesian computation is to find ("learn") the posterior distribution of a particular variable.

This term is the **prior probability** *of A. It's the probability of A* *before* *any conditional data is taken into account.*

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This term is the **prior probability** *of A. It's the probability of A <u>before</u> any conditional data is taken into account.*

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

The value of the prior is often observed from general knowledge, the actual data, or even some desired scale or distribution.

This term is the **likelihood** *function. This one swaps the conditional probabilities: it's the probability of your condition B, given A*

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

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$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

The value of the likelihood function is observed from the actual data.

This term is a **normalization constant**. *It doesn't depend on A, and is generally ignored while optimizing for maximum probabilities.*

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This term is a **normalization constant**. *It doesn't depend on A, and is generally ignored while optimizing for maximum probabilities.*

For example, while running through countries to assess their weather and movie business to find the most likely one, the chance of "rain somewhere" is not relevant.

In a movie it's raining.What's the chance the movie is shot in Holland?

$$= \frac{P(\text{raining in Holland})\ P(\text{random movie shot in Holland})}{P(\text{raining anywhere in the world})}$$

Many machine learning techniques use Bayesian statistics to estimate the parameters of their model

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$$P(\beta \mid \text{data}) = \frac{P(\text{data} \mid \beta)\ P(\beta)}{P(\text{data})}$$

Coefficients of regression
Class labels of samples
Student proficiency and question difficulty

Many machine learning techniques use Bayesian statistics to estimate the parameters of their model

$$P(\beta\,|\,\text{data}) = \frac{P(\text{data}\,|\,\beta\,)\;P(\beta\,)}{P(\text{data})}$$

Data points in Euclidean space
List of labeled samples
Student responses

Starting out with a prior belief of the parameters β ...

$$P(\beta \mid \text{data}) = \frac{P(\text{data} \mid \beta)\, P(\beta)}{P(\text{data})}$$

What are reasonable coefficients?
What are common class labels?
How are student proficiencies generally distributed?

… and updating the likelihood as new data comes in.

$$P(\beta\,|\,\text{data}) = \frac{P(\text{data}\,|\,\beta\,)\,P(\beta\,)}{P(\text{data}\,)}$$

Given these parameters, are my data reasonable?
Given these proficiencies and difficulties, how likely are these seen student responses?

Now you see why the normalization constant is generally ignored.

$$P(\beta\,|\,\text{data}) = \frac{P(\text{data}\,|\,\beta\,)\;P(\,\beta\,)}{P(\text{data})}$$

How likely is this data anyway?

The idea of Bayesian inference, then, is to **update our beliefs** *about the distribution of A using the data ("evidence") at our disposal*

$$P(\beta \mid \mathrm{data}) = \frac{P(\mathrm{data} \mid \beta)\; P(\beta)}{P(\mathrm{data})}$$

The **maximum likelihood estimator (MLE)** *finds the parameters that make the data most likely*

$$P(\beta\,|\,\text{data}) = \frac{P(\text{data}\,|\,\beta)\,P(\beta)}{P(\text{data})}$$

The **maximum a posteriori estimate (MAP)** *finds the parameters that are most likely, given the data and the prior*

$$P(\beta\,|\,\mathrm{data}) = \frac{P(\mathrm{data}\,|\,\beta)\;P(\beta)}{P(\mathrm{data})}$$

As a final remark, Bayes' Theorem offers a “wormhole” between two different “interpretations” of probability

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The **frequentist** *interpretation regards an event's probability as its limiting frequency across a very large number of trials*

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The **frequentist** *interpretation regards an event's probability as its limiting frequency across a very large number of trials*

The **Bayesian** *interpretation regards an event's probability as a "degree of belief," which applies even to events that haven't occurred yet*

If this sounds crazy to you, don’t worry…we won’t dwell on the theoretical details.

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If this sounds interesting: this a good direction to head if you're serious in becoming a rock star data scientist.

Method	***Predictions***
The **frequentist** *interpretation*	*point estimates*
The **Bayesian** *interpretation*	*distributions*

III. BAYESIAN COIN FLIPS EXAMPLE (SIT BACK & RELAX)

(A FREQUENTIST) QUIZ QUESTION

Problem:

We observe the following coin flips:

HTHH

What is P(X = Heads) ?

Problem:

We observe the following coin flips:

HTHH

What is $P(X = \text{Heads})$?   3/4, Why?

Problem:

We observe the following coin flips:

HTHHTHT

What is P(X = Heads) ?

Problem:

We observe the following coin flips:

HTHHTHT

What is P(X = Heads) ?   4/7, Why?

Problem:

We observe the following coin flips:

HTHHTHT

What is P(X = Heads) ?   4/7, Why?

With the classical method,

$$P(X = head) = \frac{\#\,heads}{\#\,tosses}$$

Which is not so reliable with little data

Problem:

We observe the following coin flips:

H

What is P(X = Heads) ?

Problem:

We observe the following coin flips:

H

What is P(X = Heads) ? Exactly 1.

Why do you care?

Many problems are binary and are estimated using counts...

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Ex. 1: Sample 100 people and ask if they support a politician.
23 say Yes

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23 say Yes – Is the correct prediction 23/100?

Why do you care?

Many problems are binary and are estimated using counts…

Ex. 2: Sample 100 people and ask *which* politician they support
3 say Trump

Why do you care?

Many problems are binary and are estimated using counts…

Ex. 2: Sample 100 people and ask *which* politician they support
3 say Trump – Is the correct prediction P(Trump) = 3/100?

For the frequentist method, you need a lot of data to succeed

Let's try the Bayesian approach

Let’s try the Bayesian approach

This is going to be a lot of high-level math to illustrate Bayes

- *If you’d like to fully understand, see the enclosed notebook*
- *If you’re fine with hand-waiving: sit back and relax*

$$P(P(H) = p|\text{ tosses}) = \frac{P(\text{tosses }|H) \times P(P(H) = p)}{P(\text{tosses})}$$

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We don't not estimate the probability *P(H)* directly

$$P(P(H) = p \mid \text{tosses}) = \frac{P(\text{tosses} \mid H) \times P(P(H) = p)}{P(\text{tosses})}$$

We don't not estimate the probability *P(H)* directly, but we ask:

Given the observed tosses, what is the chance that this probability P(H) is equal to some value p?

$$P(P(H) = p | \text{ tosses}) = \frac{P(\text{tosses } | H) \times P(P(H) = p)}{P(\text{tosses})}$$

We don't not estimate the probability *P(H)* directly, but we ask:

> *Given the observed tosses, what is the chance that this probability P(H) is equal to some value p?*

We look for which *p* the probability of *P(H) = p* is the most likely.

$$P(P(H) = p | \text{ tosses}) = \frac{P(\text{tosses } | H) \times P(P(H) = p)}{P(\text{tosses})}$$

$$P(P(H) = p | \text{ tosses}) = \frac{P(\text{tosses } | H) \times P(P(H) = p)}{P(\text{tosses})}$$

$$P(p|D) = \frac{P(D|p) \times P(p)}{P(D)}$$

let's clean up notation

$$P(P(H) = p | \text{ tosses}) = \frac{P(\text{tosses } | H) \times P(P(H) = p)}{P(\text{tosses})}$$

$$P(p|D) = \frac{P(D|p) \times P(p)}{P(D)}$$

let’s clean up notation

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

look for which *p* the probability of *P(H) = p* is the **most likely**

What is the **prior distribution** *of p?*

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

What is the **prior distribution** *of p?*

Let's pick a simple Beta distribution

$$P(p) = 6\,p\,(1 - p)$$

What is the **likelihood function** *of p?*

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

What is the **likelihood function** *of p?*

This is the reversed question: Given any probability p, what is the chance I'd see the observed coin tosses D?

What is the **likelihood function** *of p?*

This is the reversed question: Given any probability p, what is the chance I'd see the observed coin tosses D?

That's the binomial distribution

$$P(D|p) = \binom{N}{n} p^n (1-p)^{N-n}$$

$$\max_p P(p|D) = \max_p P(D|p)P(p)$$

$$\max_p P(p|D) = \max_p P(D|p)P(p) = \max_p \underbrace{\binom{N}{n} p^n (1-p)^{N-n}}_{\text{likelihood function}} \underbrace{6p(1-p)}_{\text{prior belief}}$$

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$$\frac{d}{dp} P(p|D) = 0$$

derivative is zero at maximum

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$$\frac{d}{dp} P(p|D) = 0$$

derivative is zero at maximum

$$p = \frac{n+1}{N+2}$$

solution follows algebraically

$$p = \frac{n+1}{N+2} \quad = \quad \left\{ \begin{array}{l} 1/2 \ \text{ if no coins have been tossed yet } (N = 0) \\ \\ \end{array} \right.$$

$$p = \frac{n+1}{N+2} \quad = \quad \begin{cases} 1/2 & \text{if no coins have been tossed yet } (N = 0) \\ \longrightarrow n/N & \text{if many coins have been tossed (i.e., frequentist)} \end{cases}$$

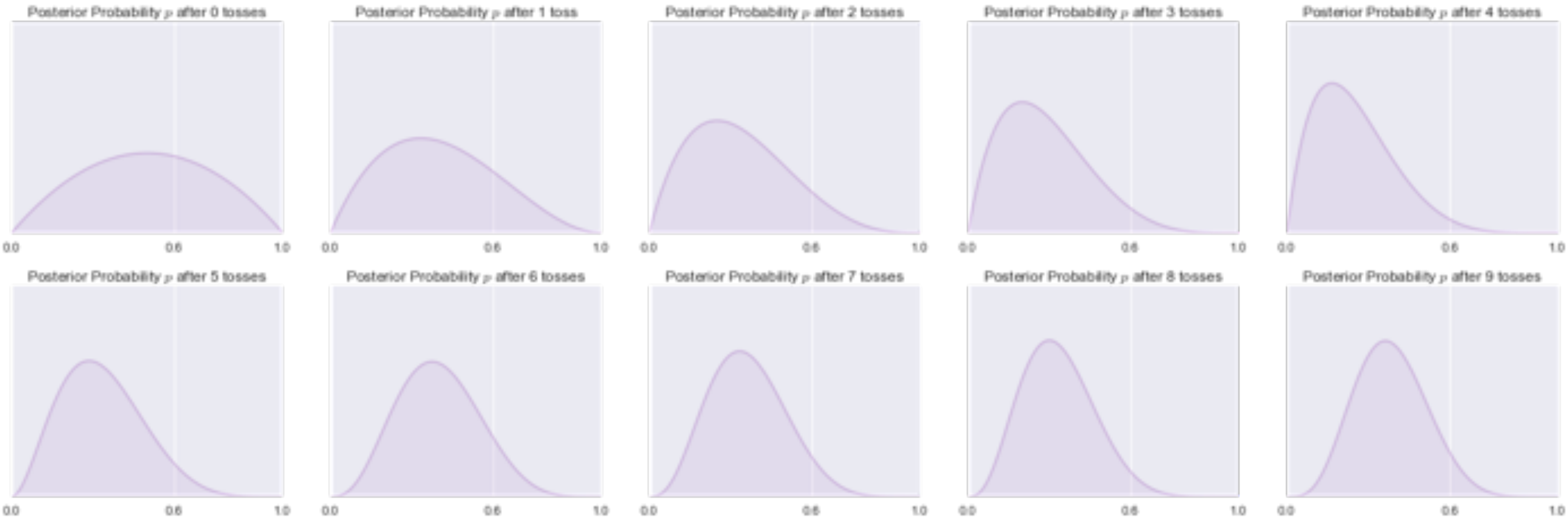
In this case, you could interpret this as the classical approach, after having added two imaginary coin flips at the beginning. We count one more head, and one more tail:

[HT]HTHHTHT

$$p = \frac{n+1}{N+2} \quad = \begin{cases} 1/2 & \text{if no coins have been tossed yet } (N = 0) \\ \longrightarrow n/N & \text{if many coins have been tossed (i.e., frequentist)} \end{cases}$$

$$p = \frac{n+1}{N+2} = \begin{cases} 1/2 & \text{if no coins have been tossed yet } (N = 0) \\ \longrightarrow n/N & \text{if many coins have been tossed (i.e., frequentist)} \end{cases}$$

Bayes provides you distributions, rather than point estimates



(A BAYESIAN)
QUIZ QUESTION

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We observe the following coin flips:

HTHH

What is P(X = Heads) ?

Problem:

We observe the following coin flips:

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What is P(X = Heads) ?

Well, it depends on...

... on what?

Problem:

We observe the following coin flips:

HTHH

What is P(X = Heads) ?

Well, it depends on the prior

Which one shall we take?

Problem:

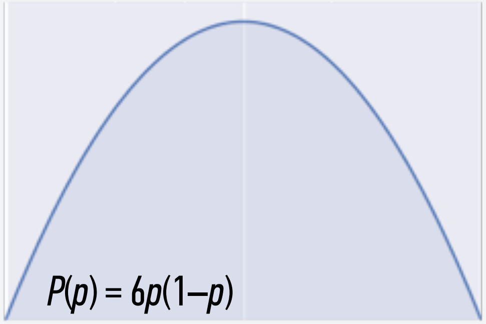
We observe the following coin flips:

HTHH

What is P(X = Heads) ?

Well, it depends on the prior

- Let's take the Beta prior



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We observe the following coin flips:

HTHH

What is P(X = Heads) ?

Well, it depends on the prior

- Let's take the Beta prior

$$P(X = \text{head}) = \frac{\#\,\textit{heads} + 1}{\#\,\textit{tosses} + 2}$$

Problem:

We observe the following coin flips:

HTHH

What is P(X = Heads) ? 2/3

Well, it depends on the prior

- Let's take the Beta prior

$$P(X = \text{head}) = \frac{\#\, heads + 1}{\#\, tosses + 2}$$

IV. NAIVE BAYES

Confused?

Confused? Relax, it gets easier!

Suppose we have a dataset with features $x_1, \ldots, x_n$ and class labels C. What can we say about classification using Bayes' theorem?

Suppose we have a dataset with features $x_1, \ldots, x_n$ and class labels C. What can we say about classification using Bayes' theorem?

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

What is the chance that these words (and n-grams) have class label C?

Suppose we have a dataset with features $x_1, \ldots, x_n$ and class labels c. What can we say about classification using Bayes' theorem?

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, given the data we observe.

However, the likelihood function can often be intractably difficult in practice to determine

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

$$P(\{x_i\}|C) = P(\{x_1, x_2, \ldots, x_n\}|C)$$

What is the chance that a random sample from a given class C has exactly all these words (and n-grams)?

So let's make a simplifying assumption. In particular, we assume that the features x_i are conditionally independent from each other:

$$P(\{x_i\}|C) = P(x_1, x_2, \ldots, x_n|C) \approx P(x_1|C) * P(x_2|C) * \ldots * P(x_n|C)$$

What is the chance that a random word (or n-gram) from a given class C is exactly word x_1?

So let's make a simplifying assumption. In particular, we assume that the features x_i are conditionally independent from each other:

$$P(\{x_i\}|C) = P(x_1, x_2, \ldots, x_n|C) \approx P(x_1|C) * P(x_2|C) * \ldots * P(x_n|C)$$

This "naive" assumption simplifies the likelihood function to make it tractable.

The Naive Bayes algorithm combines the probability of a class C overall with the probabilities of each individual feature appearing in class C

$$P(C|\{x_i\}) \sim P(C) \prod_i P(x_i|C)$$

DISCUSSION