PRACTICE PROBLEMS ON RECTIFICATION

TYPE – I

1. Find the length of the arc of the parabola $y^2 = 8x$ cut off by the latus rectum.
2. Find the arc length of $x^2 = 4y$ cut off by its latus rectum.
3. Show that the length of the parabola $y^2 = 4ax$ from the vertex to the end of the latus rectum is $a[\sqrt{2} + log(1 + \sqrt{2})]$. Find the length of arc cut off by the line $3y = 8x$
4. Find the arc length of $y^2 = 4x$ cut off by the line $y = 2x$.
5. Find the length of arc of parabola $y^2 = 4a(a - x)$ cut off by the y-axis.
6. Find the length of the parabola $x^2 = 4y$ which lies inside the circle $x^2 + y^2 = 6y$
7. Find the length of the arc of parabola $y^2 = 4x$ which lies inside the curve $x^2 + y^2 = 5$
8. Find the arc length of $ay^2 = x^3$ from (0,0) to (a, a)
9. Find the length of the arc of $y = e^x$ from $(0, 1)$ to $(1, e)$
10. Prove that the length of the arc of the curve $y = log\left(\frac{e^x-1}{e^x+1}\right)$ from $x = 1$ and $x = 2$ is $log\left(e + \frac{1}{e}\right)$
11. Find the total length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$
12. Find the length of the loop of the curve $3ay^2 = x(x-a)^2$
13. Show that the length of the loop $3ay^2 = x^2(a - x) is\ 4a/\sqrt{3}$
14. Show that the length of the loop $9ay^2 = x(x - 3a)^2\ is\ 4\sqrt{3}.a$
15. Find the total length of the loop of the curve $9y^2 = (x + 7)(x + 4)^2$
16. Find the perimeter of the loop of the curve $9ay^2 = (x - 2a)(x - 5a)^2$

ANSWERS

1. $2\pi\, a$ **2.** $2\sqrt{2} + 2log(\sqrt{2} + 1)$ **3.** $a\left[\log 2 + \frac{15}{16}\right]$ **4.** $\sqrt{2} + log(1 + \sqrt{2})$

5. $2a[\sqrt{2} + log(1 + \sqrt{2})]$ **6.** $2[\sqrt{6} + log(\sqrt{2} + \sqrt{3})]$ **7.** $2\sqrt{2} + 2log(1 + \sqrt{2})$

8. $\frac{a}{27}(13\sqrt{13} - 8)$. 9. $\sqrt{1 + e^2} - \sqrt{2} - log\left[\frac{1+\sqrt{1+e^2}}{e(1+\sqrt{2})}\right]$ **11.** $6a$

12. $\frac{4a}{\sqrt{3}}$ **15.** $4\sqrt{3}$ **16.** $4\sqrt{3}a$

TYPE - II

1. Find the length of one arc of the cycloid $x = a(\theta - sin\theta), y = a(1 + cos\theta)$.
2. Find the length of the cycloid $x = a(\theta + sin\theta), y = a(1 - cos\theta)$ from one cusp to another cusp. If s is the length of the arc from the origin to a point $P\ (x, y)$ show that $s^2 = 8ay$

3. Trace the curve $x = a\,(\theta - sin\theta)$, $y = a(1 - cos\theta)$ as θ varies from 0 to 2π. Show that the line $\theta = {}^{2\pi}/_{3}$ divides it in ratio 1 : 3.

4. Prove that the length of the arc of the curve $x = a\,sin2\theta(1 + cos2\theta), y = a\cos 2\theta(1 - cos2\theta)$ measured from the origin to (x, y) is $\frac{4}{3}a\sin 3\theta$.

5. Find the length of the loop of the curve. $x = t^2, y = t\left(1 - \frac{t^2}{3}\right)$

6. Prove that the length of the curve $x = e^{\theta}\left[\sin\frac{\theta}{2} + 2\,cos\frac{\theta}{2}\right], y = e^{\theta}\left[cos\frac{\theta}{2} - 2\,sin\frac{\theta}{2}\right]$ measured from $\theta = 0\ to\ \theta = \pi\ is\ \frac{5}{2}[e^{\pi} - 1]$

7. Find the length of the astroid $x = a\cos^3 t\,, y = a\sin^3 t$

8. Find the total length of the curve $(x/a)^{2/3} + (y/b)^{2/3} = 1$. Hence, deduce the total length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$. Also show that the line $\theta = \pi/3$ divides the length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ in the first quadrant in the ratio 1: 3

9. Find the length of the following curves:

(i) $x = a(2\,cos\theta + cos2\theta), y = a(2\,sin\theta + \sin 2\theta)\ from\ \theta = 0$ to any point θ.

(ii) $x = a(\theta - sin\theta), y = a(1 - cos\theta)$ from $\theta = 0$ to $\theta = 2\pi$

(iii) $x = a(cos\,\theta + \theta\,sin\theta), y = a(sin\,\theta - \theta cos\,\theta)$ from $\theta = 0$ to $\theta = 2\pi$

(iv) $x = ae^{\theta}\,sin\theta,\ y = ae^{\theta}\ cos\theta$ from $\theta = 0$ to $\theta = 2\pi$

(v) $x = a(3cos\theta - cos3\theta), y = a(3sin\theta - sin3\theta)$ from $\theta = \pi/2$ to any point θ

(vi) $x = a(\theta + sin\,\theta), y = a(1 + cos\theta)$ between two consecutive cusps.

(vii) $x = log(sec\theta + tan\theta) - sin\theta, y = cos\theta$ from $\theta = 0$ to any point θ

(viii) $x = a(t - \tan ht), y = a\,\mathrm{sec}\,ht$ from $t = 0$ to any point t.

(ix) $x = 1 - \cos t + (3/5)t, y = (4/5)\sin t$ from $t = 0$ to $t = \pi$

(x) $x = a\,cost + at\,sin\,t, y = a\,sin\,t - at\,cos\,t$ from $t = 0$ to $t = \pi/2$

ANSWERS

1. $8a$ **2.** $8a$ **5.** $4\sqrt{3}$

7. $6a$

9. **(i)** $8a\sin\left(\frac{\theta}{2}\right)$ **(ii)** $8a$ **(iii)** $2\pi^2 a$ **(iv)** $\sqrt{2}\left(e^{\pi/2} - 1\right)a$

(v) $6a\cos\theta$ **(vi)** $8a$ **(vii)** $logsec\theta$ **(viii)** $a\log\cos h\,t$

(ix) $\pi + (\frac{6}{5})$ **(x)** $\pi^2 a/8$

TYPE – III

1. Find the length of the cardioide $r = a(1 + \sin\theta)$

2. Find the length of the perimeter $r = a(1 + \cos\theta)$. Prove also that the upper half of cardiode is bisected by the line $\theta = \pi/3$.

3. Show that upper half of $r = 2a\, cos^2\left(\frac{\theta}{2}\right)$ is bisected by the line $\theta = \pi/3$.

4. Find the perimeter of the cardioide $r = a(1 - \cos\theta)$ and prove that the line $\theta = 2\pi/3$ bisects the upper half of the cardioide.

5. Find the length of the arc of the curve $r = a\sin^2\left(\frac{\theta}{2}\right)$ from $\theta = 0$ to any point $P(\theta)$

6. Find the length of the cardioide $r = a(1 - \cos\theta)$ lying outside the circle $r = a\cos\theta$

7. Find the length of the cardioide $r = a(1 + \cos\theta)$ which lies outside the circle $r + a\cos\theta = 0$

8. Find the length of the cardioide $r = a(1 - \cos\theta)$ lying inside the circle $r = a\cos\theta$

9. Show that the length of the arc of that part of cardioide $r = a(1 + \cos\theta)$ which lies on the side of the line $4r = 3\, asec\theta$ away from the pole is $4a$

OR Show that the perimeter of cardioid $r = a(1 + \cos\theta)$ is bisected by the line $4\,r = 3a\sec\theta$

10. Find the length of the arc of the parabola $r = \frac{6}{1+\cos\theta}$ from $\theta = 0$ to $\theta = \pi/2$

11. Find the length of the Cissoid $r = 2a\tan\theta\sin\theta$ from $\theta = 0\ to\ \theta = \pi/4$

12. Find the length of the upper arc of one loop of Lemniscate $r^2 = a^2\cos 2\theta$

13. Show that the total perimeter of $r^2 = a^2\cos 2\theta\ \ is\ \frac{a}{\sqrt{2\pi}}\left(\overline{|1/4}\right)^2$

14. Find the total length of the curve $r = a\ sin^3(\theta/3)$

ANSWERS

1. $8a$

2. $8a$

4. $8a$

5. $4asin^2\left(\frac{\theta}{4}\right)$

6. $4a\sqrt{3}$

7. $4a\sqrt{3}$

8. $8a\left(1 - \frac{\sqrt{3}}{2}\right)$

11. $2a(\sqrt{5} - 2) + a\sqrt{3}log\left(\frac{4-\sqrt{15}}{7-4\sqrt{3}}\right)$

12. $\frac{a}{4\sqrt{2\pi}}\left(\overline{|1/4}\right)^2$

14. $\frac{3}{2}\pi a$