

Course Code: 216U06C104 Name of the Course: Engineering Mechanics

Question No.		Max. Marks
Q1	Attempt any two.  i. Explain with a suitable example the resolution of a force into a force and couple.  Explanation (03 marks)  Diagram (02 marks)  Consider a force F acting on a body at point A. This is to be replaced by a force and couple at some point B as shown  Introduce two equal and opposite forces at B, each of magnitude F and acting parallel to the force at A.  Of the three equal forces, consider the two forces acting in opposite directions at points A and B. They form a couple of moment, M = F × d(5)  Thus the original force F acting at point A can be replaced.  (i) By a single force F and  (ii) A couple M = F × d  ii. A car starts from rest at t = 0 along a circular track of radius 200 m. The rate of increase in speed of the car is uniform. At the end of 90 seconds, the speed of car is 36 kmph. Find the tangential and normal components of acceleration at t = 30 seconds.  Tangential component (02 marks)	10
	Normal components of acceleration at t = 30 seconds (03 marks)	
	Solution: Given data:  Initial velocity, $u = 0$	
	Radius of curvature, $\rho = 200 \text{ m}$	
	Final velocity, $v = 36 \times \frac{5}{18} = 10 \text{ m/s}$	
	Time of travel, $t = 90 \text{ s}$	
	To find: Tangential acceleration, $a_t = ?$ at $t = 30$ s	
	Normal acceleration, $a_N = ?$ at $t = 30$ s	
	In this problem, car is moving with a uniform tangential acceleration. Hence tangential acceleration at any time remain constant.	

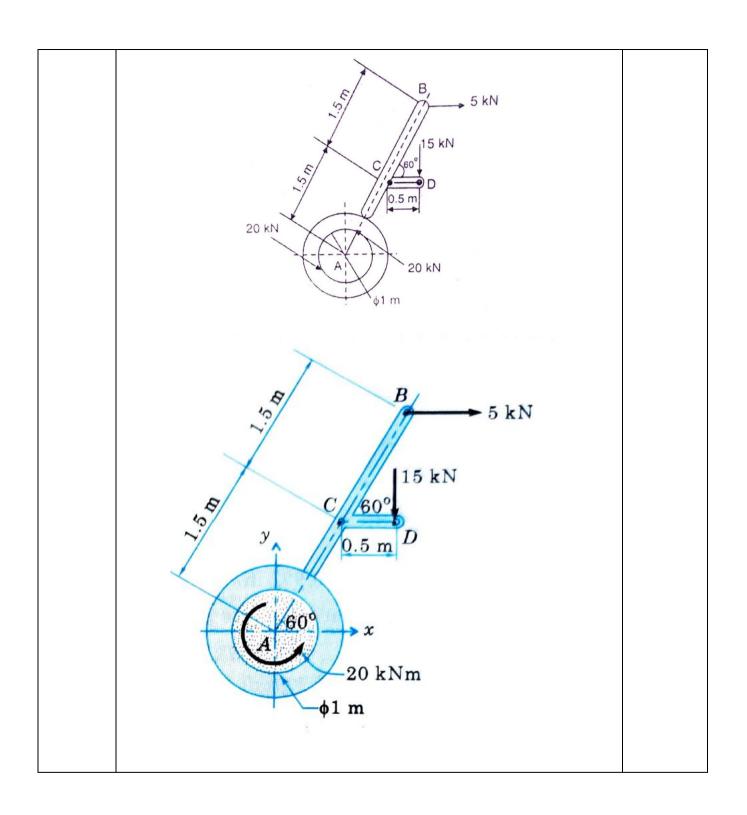
Using the equation of motion  $v = u + a_t \times t$  $10 = 0 + a_t \times 90$  $a_t = 0.1111 \text{ m/s}^2$ To find speed when t = 30 s. Use again  $v = u + a_t \times t$  $v = 0 + 0.1111 \times 30$  $\therefore v = 3.333 \text{ m/s}$ Normal acceleration,  $a_N = \frac{v^2}{\rho} = \frac{3.333^2}{200} = 0.056 \text{ m/s}^2$  $\therefore$  Tangential acceleration at  $t = 30 \text{ s is } a_t = 0.1111 \text{ m/s}^2$ Normal acceleration at  $t = 30 \text{ s is } a_N = 0.056 \text{ m/s}^2$ A force acts at the origin in the direction defined by angles  $\Theta_v = 65^0$ iii. and  $\Theta_z = 40^{\circ}$ . Knowing that the x component of force is -750 N find the value of  $\Theta_x$  and magnitude of the force. value of  $\Theta_x$  (03 marks) Magnitude of the force (02 marks) **Solution**: Given:  $\theta_y = 65^\circ$ ,  $\theta_z = 40^\circ$  and  $F_x = -750$  N. Using the equation  $\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$  $\cos^2 \theta_x + \cos^2 65^\circ + \cos^2 40^\circ = 1$  $\cos^2\theta_x = 0.2346$  $\cos \theta_x = \pm 0.4843$  $\theta_r = 61.03^{\circ} \text{ or } 180 - 61.03 = 118.97^{\circ}$  $\theta_r = 118.97^{\circ}$  since  $F_x = -750$  N has negative magnitude We know that  $F_x = |F| \cos \theta_x$  $-750 = |F| \cos 118.97^{\circ}$ |F| = 1548.46 N10

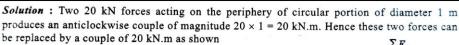
A machine part is subjected to forces as shown in the figure below. Find the resultant force in magnitude and direction. Also, locate the point where the resultant cuts the center line of bar AB.

Calculation of resultant and orientation (04 marks)

Moment, X and Y intersepts (05 marks)

**Location of resultant (01 mark)** 

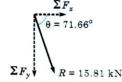




Now

ow,  

$$(+ \to) \Sigma F_x = 5 \text{ kN } (\to)$$
  
 $(+ \downarrow) \Sigma F_y = -15 \text{ kN } = 15 \text{ kN } (\downarrow)$   
 $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(5)^2 + (-15)^2} = 15.81 \text{ kN}$ 



$$\theta = \tan^{-1} \left| \frac{\sum F_y}{\sum F_x} \right| = \tan^{-1} \left| \frac{15}{5} \right| = 71.56^{\circ}$$

 $\theta$  lies in fourth quadrant as  $\Sigma F_x$  is positive and  $\Sigma F_y$  is negative. Resultant is represented as shown

To locate the position of resultant, we use Varignon's theorem as

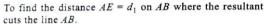
$$\left| \sum M_A \right| = \left| \sum F_x \times y \right| = \left| \sum F_y \times x \right| = \left| R \times d \right| \dots (1)$$
  
Now.

$$(+ \circlearrowleft) \Sigma M_A = 20 - 5 \times 3 \sin 60^\circ - 15[1.5 \cos 60^\circ + 0.5]$$
  
= -11.74 kN.m = 11.74 kN.m ( $\circlearrowright$ )

Now using equation (I), we get

11.74 = 
$$5 \times y = 15 \times x = 15.81 \times d$$
  
 $\therefore x = 0.783 \text{ m}, y = 2.35 \text{ m} \text{ and } d = 0.742 \text{ m}$ 

The position of the resultant is represented as shown



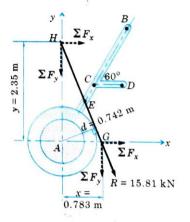
Using geometry of the figure , we have from similar triangles EFG and HAG.

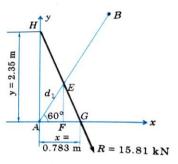
$$\frac{FG}{FE} = \frac{AG}{AH}$$

But  $FG = AG - AF = 0.783 - d_1 \cos 60^\circ = 0.783 - 0.5 d_1$ and  $FE = d_1 \sin 60^\circ = 0.866 d_1$ 

$$\therefore \frac{0.783 - 0.5 d_1}{0.866 d_1} = \frac{0.783}{2.35}$$
$$\therefore d_1 = 0.992 \text{ m}$$

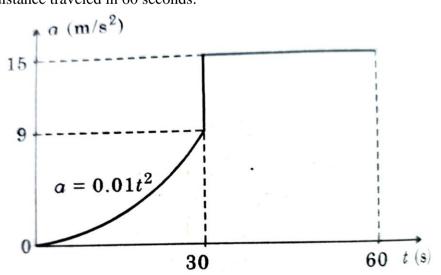
Resultant R cuts the line AB at a distance  $d_1 = 0.992$  m





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A two-stage rocket is fired vertically from rest at s = 0 with an acceleration as shown in the figure below. After 30 seconds, the first stage A burns and stage B ignites. Plot v-t and s-t graphs, which describe the motion, and find the distance traveled in 60 seconds.



V-t Diagram (02 marks)
Calculations (03 marks)
S-t Diagram (02 marks)
Calculations (03 marks)

Solution: Given initial condition.

At 
$$t = 0$$
,  $x_0 = 0$ ,  $v_0 = 0$ 

Area under a-t diagram = Change in velocity ( $\Delta v$ )

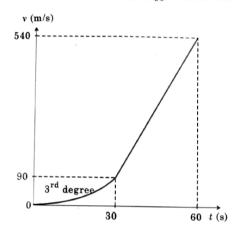
For 
$$0 \le t \le 30$$
 s, Area  $A_1 = \frac{ab}{n+1}$ . Here  $a = 30, b = 9, n = 2$  (degree)

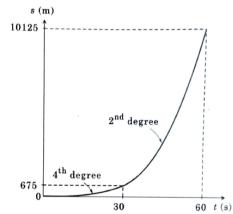
$$\therefore A_1 = \frac{30 \times 9}{2+1} = 90 = v_{30} - v_0 = v_{30} - 0$$

$$\therefore v_{30} = 90 \text{ m/s}$$

For 
$$30 \le t \le 60$$
 s, Area  $A_2 = 30 \times 15 = 450 = v_{60} - v_{30} = v_{60} - 90$ 

$$v_{60} = 540 \text{ m/s}$$





For drawing s-t diagram from v-t diagram, we use

Area under v-t diagram = Change in position ( $\Delta s$ )

For 
$$0 \le t \le 30$$
 s, Area  $A_1 = \frac{ab}{n+1}$ . Here  $n = 3$ ,  $a = 30$ ,  $b = 90$ 

$$\therefore A_1 = \frac{30 \times 90}{3+1} = 675 = s_{30} - s_0 = s_{30} - 0$$

$$s_{30} = 675 \text{ m}$$

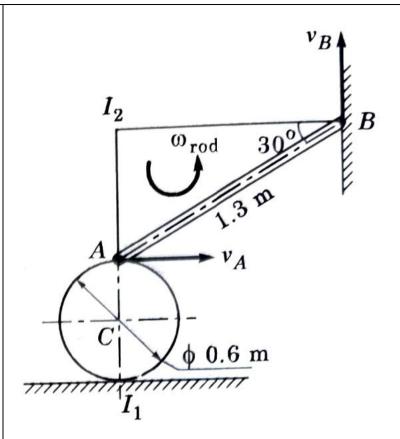
For 
$$30 \le t \le 60$$
 s, Area  $A_2 = \left(\frac{90 + 540}{2}\right) \times 30 = 9450 = s_{60} - s_{30} = s_{60} - 675$ 

$$s_{60} = 10125 \text{ m}$$

## OR

C is a uniform cylinder to which rod AB is pinned at A and the other end of the rod is moving along a vertical wall as shown in the figure below. If end B of the rod is moving upwards along the wall with the speed of 3.3 m/s find the angular velocity of the wheel and rod assuming that the cylinder is rolling without slipping.

<u>Location of ICR (04 marks)</u> <u>angular velocity of the wheel (03 marks)</u> angular velocity of the rod (03 marks)



**Solution**: Since velocity of point B of rod is upward, velocity of point A is rightwards. Point A is common to rod AB and cylinder C which rolls on horizontal surface.  $I_1$  (point of contact with horizontal surface) is the instantaneous centre of rotation for the cylinder. Intersection of lines drawn perpendicular to velocity  $v_B$  and velocity  $v_A$  [along line  $I_1A$ ] gives the point of rotation  $I_2$  for the rod.

$$v_B = I_2 B \omega_{\text{rod}}$$
 i.e.  $3.3 = I_2 B \omega_{\text{rod}}$  ..... (I)

$$v_A = I_2 A \times \omega_{\text{rod}} = I_1 A \times \omega_C$$
 ..... (II)

To find lengths  $I_2A$  and  $I_2B$ , we use geometry.

From triangle  $I_2AB$ 

From triangle 
$$I_2AB$$
  
 $\cos 30^\circ = \frac{I_2B}{AB} = \frac{I_2B}{1.3}$   $\therefore I_2B = 1.126 \text{ m}$  and  $\sin 30^\circ = \frac{I_2A}{AB} = \frac{I_2A}{1.3}$   $\therefore I_2A = 0.65 \text{ m}$   
Put these values in equation (I) and (II) to get

From equation (1) 
$$3.3 = I_2B \times \omega_{\text{rod}} = 1.126 \times \omega_{\text{rod}}$$
  $\therefore \omega_{\text{rod}} = 2.931 \text{ r/s}$ 

From equation (II) 
$$v_A = I_2 A \times \omega_{rod} = I_1 A \times \omega_C$$

$$v_A = 0.65 \times 2.931 = 0.6 \times \omega_C$$
  $\omega_{\text{cylinder}} = 3.175 \text{ r/s}$