EEEE lec 23 1/10/23

Concept of Admittance:

① Admittance (Y) is reciprocal of impedance (Z)

$\bar{Z} = R \pm jX$

② $\bar{Y} = \frac{1}{\bar{Z}} = \frac{1}{R \pm jX}$

i.e $\bar{Y} = \frac{1}{R \pm jX} \times \frac{R \mp jX}{R \mp jX}$

i.e $\bar{Y} = \frac{R \mp jX}{R^2 + X^2}$

i.e $\bar{Y} = \underbrace{\frac{R}{R^2+X^2}}_{G} \mp j \underbrace{\frac{X}{R^2+X^2}}_{B}$

$\boxed{\bar{Z} = R \pm jX}$

i.e $\boxed{\bar{Y} = G \mp jB}$

Admittance (℧) mho

Conductance (℧) (Real part of $\bar{Y}$)

Susceptance (℧) (Imaginary part of $\bar{Y}$)

where,

$G = \frac{R}{R^2 + X^2}$

$B = \frac{X}{R^2 + X^2}$

③ For RL circuits,

$Z = R + jX_L \longrightarrow Y = G - jB$

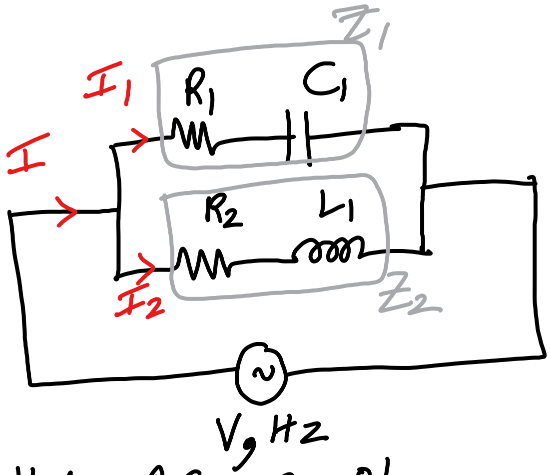
④ For RC circuits,

$Z = R - jX_c \longrightarrow Y = G + jB$

Parallel AC circuits:

① 'V' is same across each branch

② Current 'I' is different in two branches (I_1, I_2)



③ 3 methods of solving parallel AC circuits

ⓐ By phasor diagram

ⓑ By phasor algebra

ⓒ By Admittance method

I] Phasor diagram : (applicable for 2 branches)

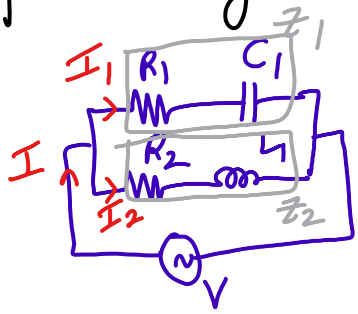
a) Find magnitude & phase of each branch current

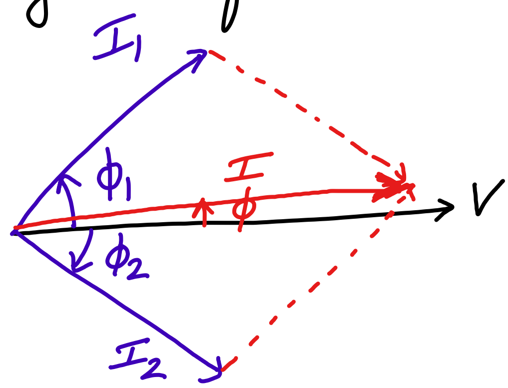
b) Draw phasor diagram taking voltage as reference

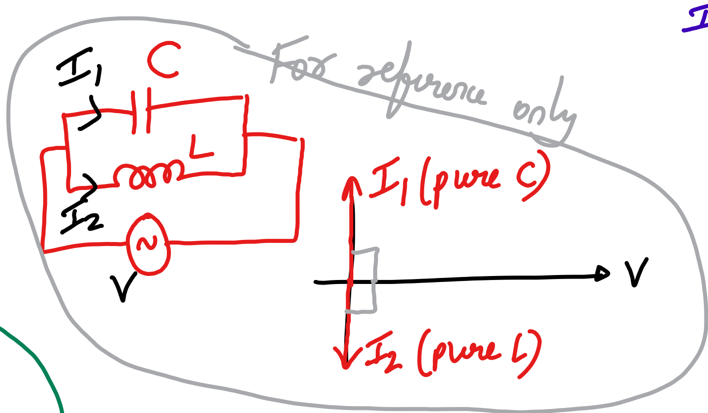
$\phi_1 < 90°$

$\phi_2 < 90°$

ϕ - phase angle







$\overline{I} = \overline{I}_1 + \overline{I}_2$

$I_1 Z_1 = V = I_2 Z_2$

i) $\overline{Z}_1 = R_1 - jX_{C_1}$

$\overline{Z}_2 = R_2 + jX_{L_1}$

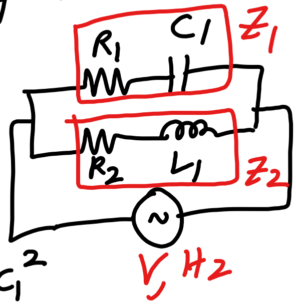
mag $\left\{ \begin{array}{l} Z_1^2 = R_1^2 + X_{C_1}^2 \\ Z_2^2 = R_2^2 + X_{L_1}^2 \end{array} \right.$

ii) $I_1 = \frac{V}{Z_1} \Rightarrow$; $\phi_1 = \tan^{-1}\left(\frac{X_{C_1}}{R_1}\right)$

$I_2 = \frac{V}{Z_2} \Rightarrow$; $\phi_2 = \tan^{-1}\left(\frac{X_{L_1}}{R_2}\right)$ $\Big\}$ phase.

II] Phasor Algebra:

→ Here voltage, current & impedances are expressed in rectangular or polar form



i) $\bar{V} = V - j0 = V\angle 0^\circ$ Volts

ii) $\bar{Z}_1 = R_1 - jX_{C_1} = Z_1\angle -\phi_1$; $Z_1^2 = R_1^2 + X_{C_1}^2$

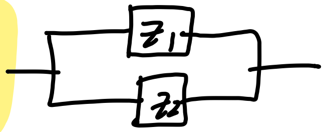
$\phi_1 = \tan^{-1}\left(\frac{X_{C_1}}{R_1}\right)$

iii) $\bar{Z}_2 = R_2 + jX_{L_1} = Z_2\angle \phi_2$;

$Z_2^2 = R_2^2 + X_{L_1}^2$; $\phi_2 = \tan^{-1}\left(\frac{X_{L_1}}{R_2}\right)$

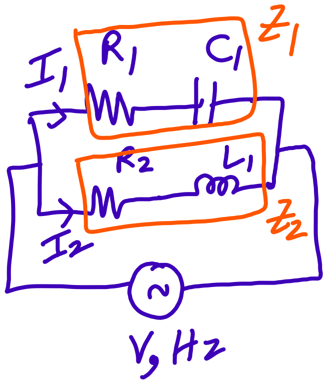
iv) $\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1}$; $\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2}$

$\bar{Z}_{eq} = \bar{Z}_1 \parallel \bar{Z}_2$



v) $\bar{I} = \bar{I}_1 + \bar{I}_2$

III] By admittance method: $\bar{Y} = \frac{1}{\bar{Z}}$ ℧



a) For given parallel AC ckt,

$$\frac{1}{\bar{Z}_{eq}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2}$$

i.e $\boxed{\bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2}$ Y_{eq} → admittance of the ckt

$\bar{V} = \bar{I}\,\bar{Z}_{eq}$

b) Line current

$$\bar{I} = \frac{\bar{V}}{\bar{Z}_{eq}} = \bar{V}\,\bar{Y}_{eq}$$

c) $\bar{Z}_1 = R_1 - jX_{C_1}$ Ω $\bar{Z}_2 = R_2 + jX_{L_1}$ Ω

$\bar{Y}_1 = (G_1 + jB_{C_1})$ ℧ $\bar{Y}_2 = (G_2 - jB_{L_1})$ ℧

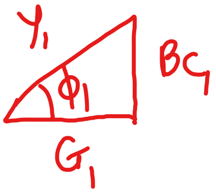
B_{C_1} - capacitive susceptance

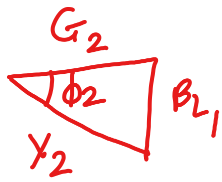
$G_1 = \frac{R_1}{R_1^2 + X_{C_1}^2}$ $G_2 = \frac{R_2}{R_2^2 + X_{L_1}^2}$

$B_{L_1} \to$ Inductive susceptance

$$B_{C_1} = \frac{X_{C_1}}{R_1^2 + X_{C_1}^2}$$

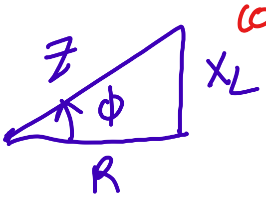
$$B_{L_1} = \frac{X_{L_1}}{R_2^2 + X_{L_1}^2}$$





Admittance triangle:

a) For an inductive ckt:



$\cos\phi = \frac{R}{Z}$

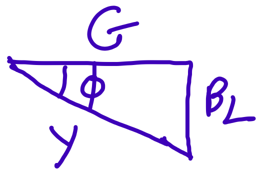
Impedance triangle

$$Z^2 = R^2 + X_L^2$$

$$B_L = Y\sin\phi = \frac{1}{Z} \times \frac{X_L}{Z}$$

$$B_L = \frac{X_L}{Z^2} = \frac{X_L}{R^2 + X_L^2}$$

$\cos\phi$



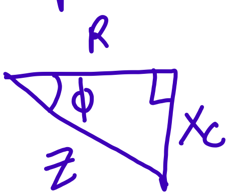
Admittance triangle

$$Y^2 = G^2 + B_L^2$$

$$G = Y\cos\phi = \frac{1}{Z}\,\frac{R}{Z}$$

$$G = \frac{R}{Z^2} = \frac{R}{R^2 + X_L^2}$$

b) For a capacitive ckt,



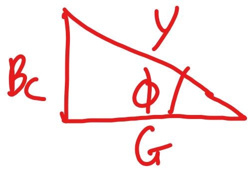
$\cos\phi = \frac{R}{Z}$

$\sin\phi = \frac{X_C}{Z}$

$$Z^2 = R^2 + X_C^2$$

Impedance triangle

$$Y^2 = B_C^2 + G^2$$

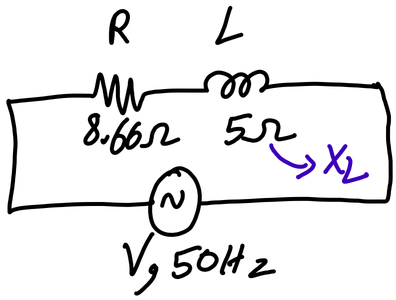


Admittance triangle

i) $G = Y\cos\phi = \frac{1}{Z}\,\frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X_C^2}$ $\mho$

ii) $B_C = Y\sin\phi = \frac{1}{Z}\,\frac{X_C}{Z} = \frac{X_C}{Z^2} = \frac{X_C}{R^2 + X_C^2}$ $\mho$

Numerical 1: Calculate the admittance & draw the admittance triangle for the ckt shown below



Solution: 1) Circuit impedance

$$\bar{Z} = R + jX_L$$

$$\bar{Z} = 8.66 + j5 \quad \text{— Rectangular form}$$

$$\bar{Z} = 10 \angle 30^\circ \ \Omega \qquad \text{Shift Pol}(8.66, 5)$$

2) Circuit admittance, $\bar{Y} = \dfrac{1}{\bar{Z}} = \dfrac{1}{10\angle 30^\circ}$

$$\bar{Y} = 0.1 \angle -30^\circ \ \mho \qquad \bar{Y} = Y \angle \phi^\circ \ \mho$$

Polar form of $\bar{Y}$

$(Y = 0.1\mho,\ \phi = -30^\circ)$

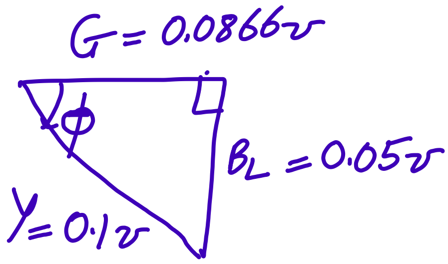
$$\bar{Y} = (0.0866 - j0.05)\ \mho \qquad \text{Shift Rec}(0.1, -30^\circ)$$

$$\bar{Y} = (G - jB_L)\ \mho$$

Rectangular form of $\bar{Y}$

$(G = 0.0866\ \mho,\ B_L = 0.05\ \mho)$

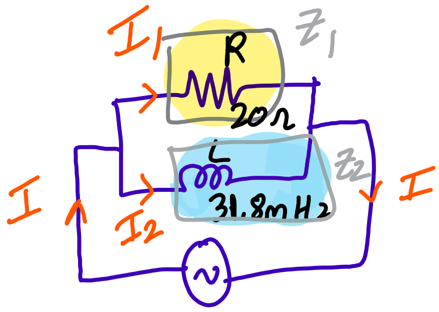
3) Admittance triangle,



Numerical 2 next

Numerical 2: A resistance of 20Ω & a pure coil of inductance 31.8 mH are connected in parallel across 230V, 50Hz supply. Calculate i) Line current ii) power factor iii) Power consumed by the ckt

Solution:



230V, 50Hz

$Z_1 = R \pm jX$

$Z_2 = R \pm jX$

$X_L = 2\pi f L$

$X_L = 100\pi \times 31.8 \times 10^{-3}$

$X_L = \underline{10\,\Omega}$

1) $\bar{V} = 230\angle 0^\circ$ V

2) $\bar{Z}_1 = 20 + j0 = 20\angle 0^\circ\,\Omega$

3) $\bar{Z}_2 = 0 + j10 = 10\angle 90^\circ\,\Omega$

4) $\bar{I}_1 = \dfrac{\bar{V}}{\bar{Z}_1} = \dfrac{230\angle 0^\circ}{20\angle 0^\circ} = 11.5\angle 0^\circ$ A

$\bar{I}_1 = (11.5 + j0)$ A

5) $\bar{I}_2 = \dfrac{\bar{V}}{\bar{Z}_2} = \dfrac{230\angle 0^\circ}{10\angle 90^\circ} = 23\angle -90^\circ = (0 - 23j)$ A

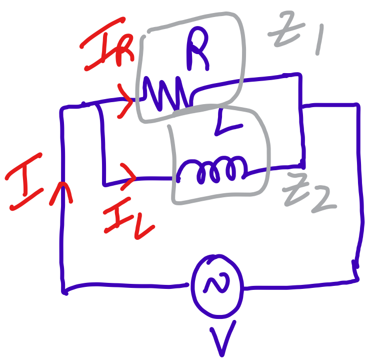
6) $\bar{I} = \bar{I}_1 + \bar{I}_2 = (11.5 + j0) + (0 - 23j)$

$\bar{I} = (11.5 - 23j)$A $= \underline{25.71\angle -63.43^\circ}$ A

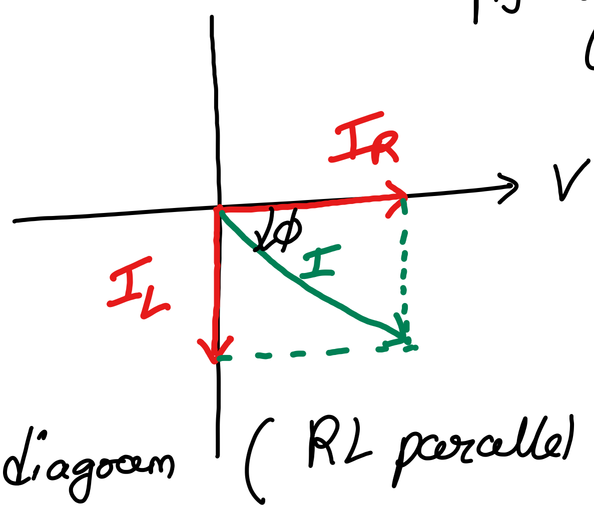
7) $p.f = \cos\phi = \cos(-63.43^\circ) = \underline{0.447}$

8) $P = VI\cos\phi = 230 \times 25.71 \times 0.447$

$P = 2643.24$W



p.f = $\cos\phi$ (lagging)



Phasor diagram (RL parallel ckt)

$$\bar{I} = \bar{I}_R + \bar{I}_L$$

$$Z_{eq} = Z_1 \parallel Z_2$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{jX_L}$$

i.e $y = y_R - j y_L$

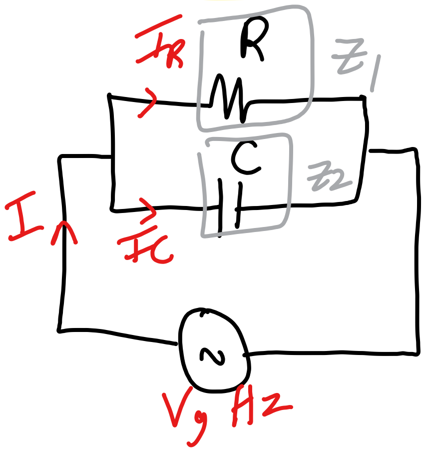
$$\boxed{\bar{y} = \frac{1}{R} - j\frac{1}{X_L}}$$

Parallel RL ac ckt

$\bar{y} = G - jB_L$

$G = \frac{1}{R}$; $B_L = \frac{1}{X_L}$

2) RC parallel ckt :



$$\bar{I} = \bar{I}_R + \bar{I}_C$$

$$\bar{Z}_1 = R + j0$$

$$\bar{Z}_2 = 0 - jX_C$$

$\rightarrow Z_{eq} = Z_1 \parallel Z_2$

$\rightarrow \frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$

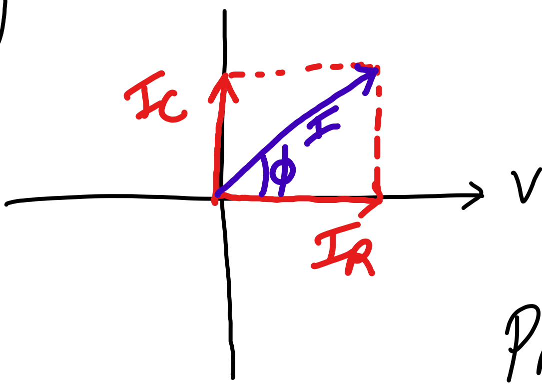
$\rightarrow Y_{eq} = \frac{1}{R} + \frac{1}{jX_C}$

$$Y_{eq} = G + jB_C$$

$$\boxed{\begin{aligned} G &= \frac{1}{R} \\ B_C &= \frac{1}{X_C} \end{aligned}}$$

$p.f = \cos\phi = $ (leading)

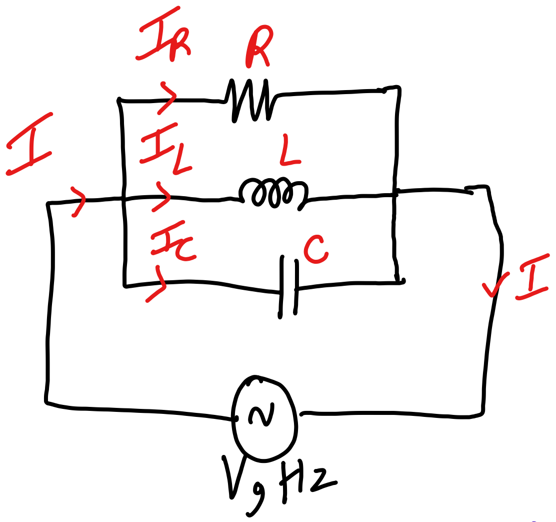
I_C, I_R, I, V



Taken 'V' as reference

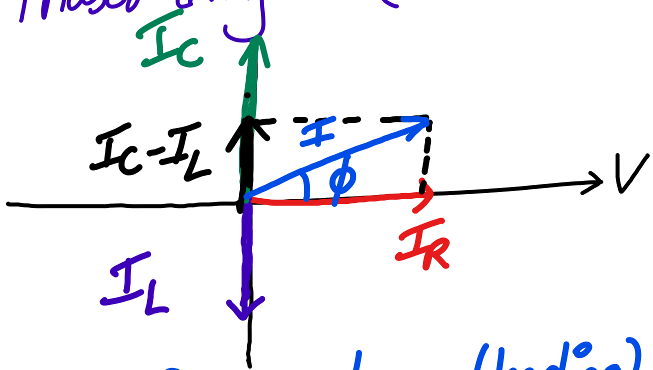
Phasor diagram (Parallel RC ckt)

RLC parallel AC ckt:



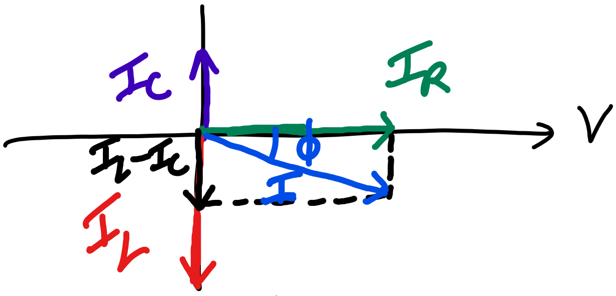
$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

Phasor diagram ($I_C > I_L$)



p.f = cos ϕ → (leading)

Phasor diagram ($I_L > I_C$)



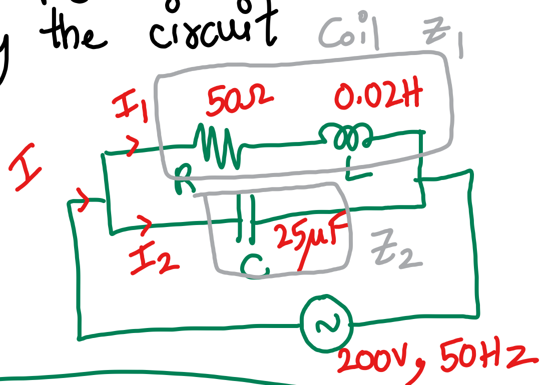
p.f = cosϕ (lagging)

—— x ——

Numerical: A coil having a resistance of 50Ω & an inductance of $0.02H$ is connected in parallel with a capacitor of $25\mu F$ across $200V$, $50Hz$ supply.

Calculate: a) the current in the coil and capacitor
b) total current drawn and p.f
c) total power consumed by the circuit

Solution: Given $R = 50\Omega$, $L = 0.02H$
$V = 200V$, $f = 50Hz$



To find: a) I_1 & I_2
b) I & p.f
c) P

Thinking
a) X_L
b) X_C
c) $\bar{Z}_1 = R + jX_L$
d) $\bar{Z}_2 = 0 - jX_C$
e) $\bar{V}$ in polar form
f) $\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1}$
$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2}$
g) $\bar{I} = \bar{I}_1 + \bar{I}_2$
h) $p.f = \cos\phi$
i) $P = VI\cos\phi$

① $X_L = 2\pi f L = 100\pi \times 0.02$
$X_L = 6.28\Omega$

② $X_C = \frac{1}{2\pi f C} = \frac{1}{100\pi \times 25 \times 10^{-6}}$
$X_C = 127.32\Omega$

③ $\bar{Z}_1 = R + jX_L = (50 + j6.28)\Omega$
$\bar{Z}_1 = 50.39\angle 7.16^\circ\ \Omega$

④ $\bar{Z}_2 = 0 - jX_C = 0 - j127.32$
$\bar{Z}_2 = 127.32\angle -90^\circ\ \Omega$

⑤ $\bar{V} = 200\angle 0^\circ\ V$

⑥ $\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{200\angle 0^\circ}{50.39\angle 7.16^\circ} = 3.97\angle -7.16^\circ$
$\bar{I}_1 = 3.94 - 0.49j$

⑦ $\bar{I}_2 = \dfrac{\bar{V}}{\bar{Z}_2} = \dfrac{200\angle 0^\circ}{127.32\angle -90^\circ} = 1.57\angle 90^\circ\ A$

$\bar{I}_2 = 0 + 1.57j$

⑧ $\bar{I} = \bar{I}_1 + \bar{I}_2 = (3.94 - 0.49j) + (0 + 1.57j)$

$\bar{I} = 3.94 + 1.08j$ $\qquad\qquad \bar{V} \,\&\, \bar{I} \rightarrow \phi = 15.33^\circ$

$\bar{I} = 4.08\angle 15.33^\circ\ A$

⑨ $p.f = \cos\phi = \cos(15.33) = 0.964$

⑩ $P = VI\cos\phi = 200 \times 4.08 \times 0.964$

$P = 786.62\,W$

Numerical: For the circuit shown below, calculate

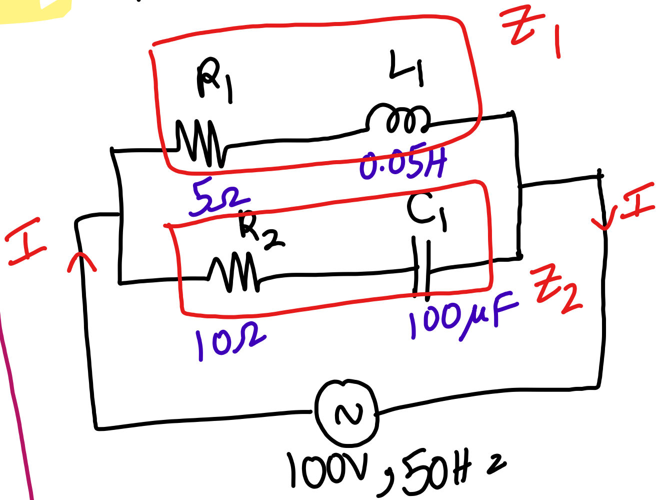
a) Impedance & admittance of each branch

b) Equivalent admittance & impedance of ckt

c) Supply current & p.f

Thinking:

1) Z_1 & Z_2
2) Y_1 & Y_2
3) Y_{eq} & Z_{eq}
4) $\bar{I} = \dfrac{V}{Z_{eq}}$
5) $p.f = \cos\phi$



Solution: 1) $X_{L_1} = 2\pi f L_1 = 100\pi \times 0.05 = 15.71\,\Omega$

2) $X_{C_1} = \dfrac{1}{2\pi f C_1} = \dfrac{1}{100\pi \times 100\times 10^{-6}} = 31.83\,\Omega$

3) $\bar{Z}_1 = R_1 + jX_{L_1} = (5 + j15.71)\,\Omega = 16.49\angle 72.3^\circ\,\Omega$

4) $\bar{Z}_2 = R_2 - jX_{C_1} = (10 - j31.83)\,\Omega = 33.36\angle -72.5^\circ\,\Omega$

5) $\bar{Y}_1 = \dfrac{1}{\bar{Z}_1} = \dfrac{1}{16.49\angle 72.3^\circ} = 0.06\angle -72.3^\circ \ \mho$

$\bar{Y}_1 = (0.018 - 0.057j)\ \mho$

6) $\bar{Y}_2 = \dfrac{1}{\bar{Z}_2} = \dfrac{1}{33.36\angle -72.5^\circ} = 0.03\angle 72.5^\circ \ \mho$

$\bar{Y}_2 = (8.99\times10^{-3} + 0.0286j)\ \mho$

7) $\bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 = (0.018 - 0.057j) + (8.99\times10^{-3} + 0.0286j)$

$\bar{Y}_{eq} = (0.027 - 0.0284j)\ \mho$

$\bar{Y}_{eq} = 0.039\angle -46.45^\circ \ \mho$

8) $\bar{Z}_{eq} = \dfrac{1}{\bar{Y}_{eq}} = \dfrac{1}{0.039\angle -46.45^\circ} = 25.64\angle 46.45^\circ$

9) $\bar{V} = 100\angle 0^\circ$

10) $\bar{I} = \dfrac{\bar{V}}{\bar{Z}_{eq}} = \dfrac{100\angle 0^\circ}{25.64\angle 46.45^\circ} = 3.9\angle -46.45^\circ$

$\bar{I} = I\angle\phi$

$\phi = -46.45^\circ$

11) $p.f = \cos\phi = \cos(-46.45)$

$p.f = \underline{0.689}$

— x —

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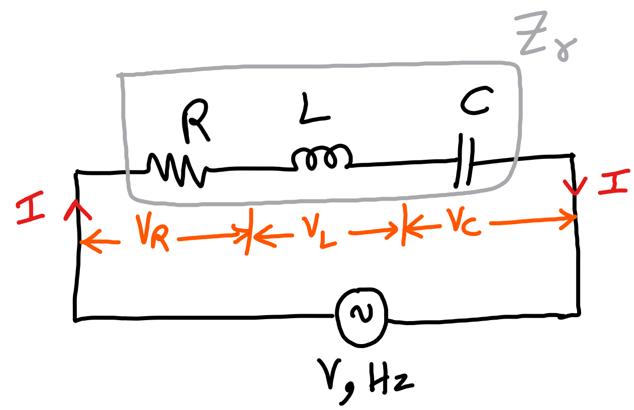
* RLC Series Resonance

① The ability of a radio-receiver to select a certain frequency transmitted by the station & to eliminate frequencies from other station's is based on the 'Principle of Resonance'

② In series RLC ckt,

→ X_L causes total current 'I' lags behind the applied voltage 'V'

→ X_C causes total current 'I' leads the applied voltage 'V'



RLC Series resonant circuit

Z_r - Circuit impedance at resonance frequency 'f_r'

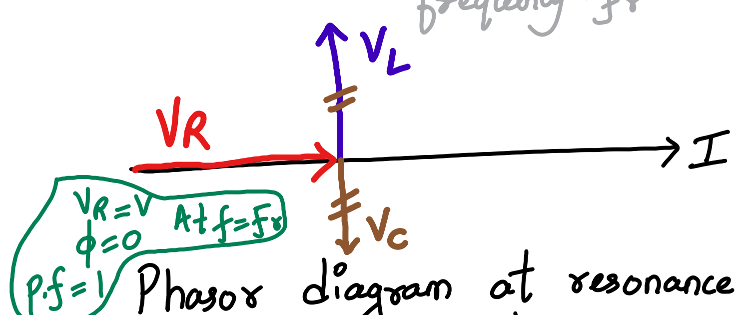
③ When $X_L > X_C$ → ckt nature is inductive

When $X_C > X_L$ → ckt nature is capacitive

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

④ $\bar{Z} = R + jX$

⑤ Net reactance → $X = |X_L - X_C|\ \Omega$



Phasor diagram at resonance 'f_r'

⑥ $X_L = 2\pi f L\ \Omega$
$= \omega L\ \Omega$

↓ inductive reactance $(X_L \propto f)$ ↓ supply frequency

; $X_C = \dfrac{1}{2\pi f C}\ \Omega$
$= \dfrac{1}{\omega C}\ \Omega$

↓ capacitive reactance $\left(X_C \propto \dfrac{1}{f}\right)$

At resonance,
$X_L = X_C$; $V_R = I_r R$
$I_r X_L = I_r X_C$
→ $(V_L = V_C) > V$
→ Voltage magnification ckt

⑦ At resonance frequency 'f_r' → the inductive reactance becomes equal to capacitive reactance i.e $X_L = X_C$

$Z_r = R + jX$
$= R\ \Omega$

∴ Net reactance i.e $X = X_L - X_C = 0$ becomes zero

8. ∴ Impedance 'Z' of the circuit becomes purely resistive (i.e $Z_r = R$)

At resonance, (p.f = 1) $\cos\phi = 1$
$X_L = X_C$, $Z = R + j(X_L - X_C)$ $\phi = 0$
$\rightarrow Z_r = R + j0 = R$ (minimum)
$\rightarrow I_r = \frac{V}{Z} = \frac{V}{R}$ (max)

9. i.e Whole ckt behaves as a purely resistive ckt
i.e Current 'I' remains in phase with applied voltage V
i.e Power factor p.f is unity $p.f = \cos 0 = 1$

10. These condition's results in 'Electrical series resonance'

11. Effect's of Series resonance

a) $X = 0$ i.e $X_L - X_C = 0$ i.e $X_L = X_C$

b) Impedance at resonance 'Z_r' is minimum & is equal to the resistance of the circuit

i.e $\boxed{Z_r = R}$ Ω

c) The current 'I_r' in the circuit is maximum

i.e $\boxed{I_r = \frac{V}{Z_r} = \frac{V}{R}}$ A

d) As I_r is maximum ⟶ the power absorbed by the ckt will also be at its maximum value $P = I_r^2 R = P_{max}$

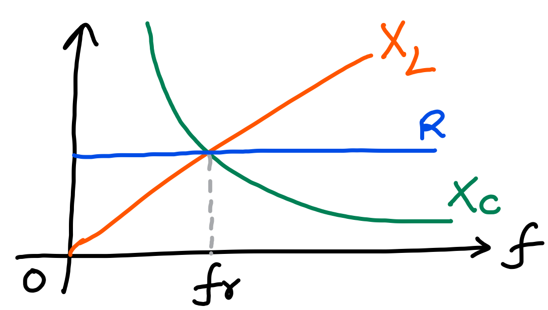
12. Resonant frequency (f_r) :

$X_L = \omega L$ $X_C = \frac{1}{\omega C}$

a) X_L & X_C are frequency dependent

b) Higher 'f' ⟶ X_L higher & lower X_C

c) The frequency at which $X_L = X_C$ in a series RLC ckt is called the 'resonant frequency' (f_r)



(13) At series resonance, $X_L = X_C$

i.e $2\pi f_r L = \dfrac{1}{2\pi f_r C}$

i.e $f_r^2 = \dfrac{1}{4\pi^2 LC}$

$\omega_r = 2\pi f_r$

i.e $\boxed{f_r = \dfrac{1}{2\pi\sqrt{LC}}}$ in Hz series RLC resonance

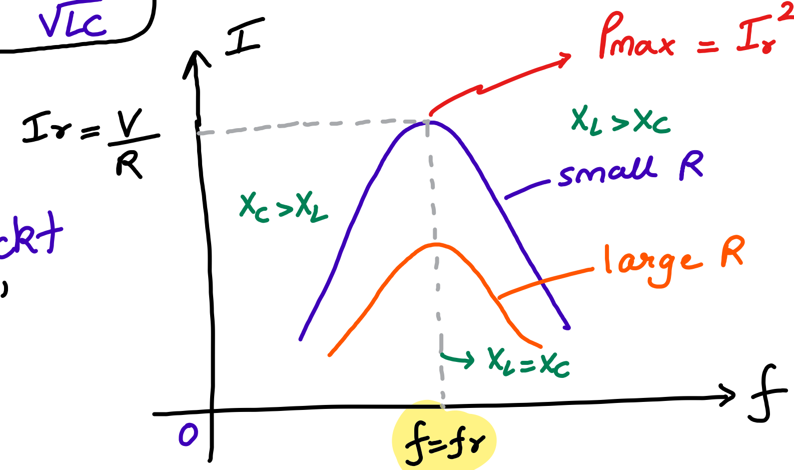
Resonant frequency

$\boxed{\omega_r = \dfrac{1}{\sqrt{LC}}}$ rad/sec

(14) Series Resonance curve:

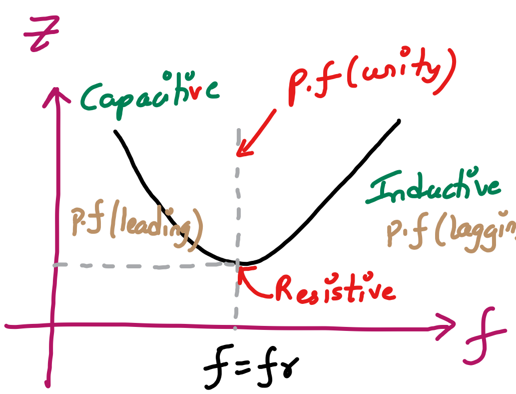
a) It is a curve between ckt current I and frequency 'f'

b) Current 'I' reaches it's maximum value at the resonant frequency 'f_r'



Variation of circuit current 'I' with frequency 'f'

c) Current 'I' fall off rapidly on either side since



i) If 'f' is below f_r ($f < f_r$)

$X_C > X_L \rightarrow$ p.f is leading

ii) If 'f' is above f_r ($f > f_r$)

$X_L > X_C \rightarrow$ pf is lagging

→ $f < f_r$ → Impedance → capacitive, pf leading

→ $f = f_r$ → Impedance → resistive, pf unity

→ $f > f_r$ → Impedance → inductive, pf lagging

15. Magnitude of current falls rapidly as the frequency 'f' deviates from 'f_r'

16. Quality factor (Q-factor):

a) It is a measure of selectivity/sharpness of series resonant circuit

b) $$\text{Q-factor} = \frac{\text{Voltage across L \& C at resonance}}{\text{Voltage across R at resonance}} = \frac{V_C \text{ or } V_L}{V_R}$$

i.e $$\boxed{\text{Q-factor} = \frac{V_L \text{ or } V_C}{V_R}}$$

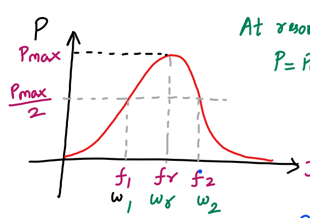
(At resonance, $V_R = V$)

c) $\because f_r = \dfrac{1}{2\pi\sqrt{LC}} \rightarrow \text{Q-factor} = \dfrac{I_r X_L}{I_r R} = \dfrac{X_L}{R}$

i.e $$\text{Q-factor} = \frac{2\pi f_r L}{R} = \frac{\cancel{2\pi} \times \frac{1}{\cancel{2\pi}\sqrt{LC}} \times L}{R} = \frac{\sqrt{L}\sqrt{L}}{\sqrt{LC}\, R}$$

i.e $$\boxed{\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Series resonant circuit



At resonance, $I = I_r(max)$

$P = P_{max} = I_r^2 R$

$P_{max} = I_r R$

$P_{half} = \dfrac{P_{max}}{2}$

$P_{half} = \dfrac{I_r^2 R}{2}$

Let at $P = P_{half} \rightarrow I_r = I_b$

$I_r^2 \cancel{R} = \dfrac{I_b^2 \cancel{R}}{2}$

$\rightarrow I_b = \dfrac{I_r}{\sqrt{2}} = 0.707 I_r$

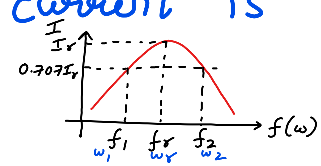
→ Half power frequencies $(\omega_1 \& \omega_2)$ or $(f_1 \& f_2)$

→ Bandwidth BW)

$BW = (f_2 - f_1)$ or $(\omega_2 - \omega_1)$

17. Bandwidth of Series RLC resonant circuit:

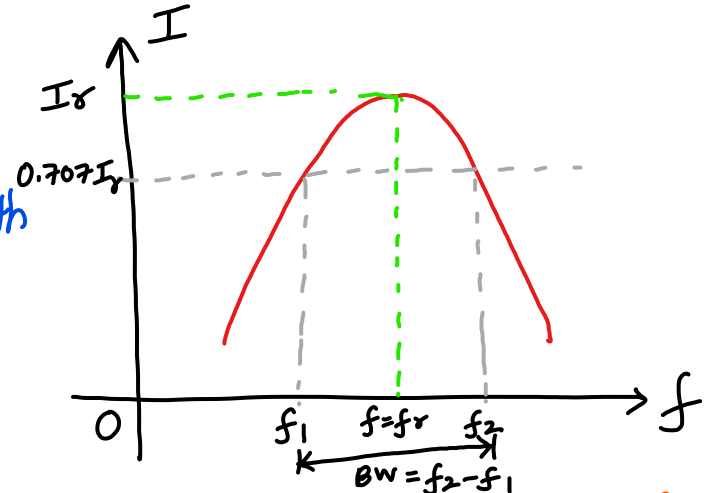
a) Bandwidth (BW) of a series resonance ckt is defined as the range of frequency over which circuit current is equal to 70.7% of it's maximum current



b) I_r – Circuit current at resonance

c) $(f_2 - f_1)$ is the bandwidth of the circuit

i.e $\boxed{BW = f_2 - f_1}$ Hz



d) $\boxed{BW = \frac{R}{2\pi L}}$ Hz

This can be proven

e) f_1 & f_2 are the limiting frequencies at which current is exactly equal to 70.7% of the maximum value

f) f_1 – Lower cut off frequency

f_2 – higher cut-off frequency

g) The resonant frequency is sufficiently centered w.r.t two cut-off frequencies (f_1 & f_2)

Proof: (Only for reading)

① Bandwidth $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}}$ — ①

At any frequency

i.e $I = \frac{V}{\sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}}$

② At half power point, $I = \frac{I_r}{\sqrt{2}} = \frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{2} R}$ — ②

③ Put ② in eqn ①,

$\frac{\cancel{V}}{\sqrt{2} R} = \frac{\cancel{V}}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}} \rightarrow \sqrt{2} R = \sqrt{R^2+\left(\omega L - \frac{1}{\omega C}\right)^2}$

④ Squaring both sides, $2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \rightarrow \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$

⑤ $\left(\omega L - \frac{1}{\omega C}\right) = \pm R \rightarrow \frac{\omega^2 LC - 1}{\omega C} = \pm R \rightarrow \omega^2 LC - 1 = \pm \omega CR$

⑥ $\omega^2 LC - 1 \pm \omega CR = 0 \rightarrow LC\left[\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC}\right] = 0$

⑦ $\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0 \rightarrow$ Solving ax^2+bx+c Roots $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

⑧ Finding roots,

$\omega = \frac{\mp\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2}+\frac{4}{LC}}}{2} = \pm\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2}+\frac{1}{LC}}$

⑨ For low values of R, $\frac{R^2}{4L^2} << \frac{1}{LC}$

⑩ $\boxed{\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{1}{LC}}} \rightarrow \omega = \pm\frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$ — ③

⑪ But, $\omega_r = \frac{1}{\sqrt{LC}} \rightarrow \boxed{\omega = \pm\frac{R}{2L} \pm \omega_r}$ — ④

⑫ Considering ω_r as positive,

$\boxed{\omega_1 = \omega_r - \frac{R}{2L}}$, $\boxed{\omega_2 = \omega_r + \frac{R}{2L}}$

⑬ Bandwidth (BW) → $BW = \omega_2 - \omega_1 = \cancel{\omega_r} + \frac{R}{2L} - \cancel{\omega_r} + \frac{R}{2L}$

$BW = \frac{2R}{2L} = \frac{R}{L}$ rad/sec

BW in Hz, $BW = f_2 - f_1 = \frac{\omega_2}{2\pi} - \frac{\omega_1}{2\pi} = \frac{1}{2\pi}(\omega_2-\omega_1)$

$\boxed{BW = \frac{R}{2\pi L} \text{ Hz}}$ — ⑤

⑭ $f_1 = f_r - \frac{R}{4\pi L}$, $f_2 = f_r + \frac{R}{4\pi L}$

$f_1 = f_r - \frac{BW}{2}$; $f_2 = f_r + \frac{BW}{2}$

$$\boxed{\begin{aligned} f_1 &= f_r - \frac{BW}{2} \\ f_2 &= f_r + \frac{BW}{2} \end{aligned}}$$

$$\omega_1 = \omega_r - \frac{R}{2L} = \omega_r - \frac{BW}{2} \quad BW = \frac{R}{L} \text{ rad/s}$$

$$\omega_2 = \omega_r + \frac{R}{2L} = \omega_r + \frac{BW}{2}$$

(18) Relation between Q-factor & bandwidth

a) $Q \text{ factor} = 2\pi f_r \frac{L}{R} = \frac{2\pi L}{R} f_r = \frac{f_r}{BW}$

i.e $\boxed{f_r = Q\text{-factor} \times BW}$

(19) Voltage drop at resonance:

a) At resonance $\rightarrow$ V_L & V_C are equal & antiphase with each other

b) The drop across the resistor (R) V_R is equal to the applied voltage (V)

c) At resonance, $V_R = I_r R = \frac{V}{R} R = V$ i.e $\boxed{V_R = V}$

$$V_L = I_r X_L = \frac{V}{R} 2\pi f_r L = \frac{V}{R} \cancel{2\pi} \frac{1}{\cancel{2\pi}\sqrt{LC}} L = \frac{V}{R}\sqrt{\frac{L}{C}} \, V$$

$$\boxed{V_L = \frac{V}{R}\sqrt{\frac{L}{C}}} \, V$$

$$V_C = I_r X_C = \frac{V}{R} \frac{1}{2\pi f_r C} = \frac{V}{R} \frac{1}{\cancel{2\pi}} \frac{\cancel{2\pi}\sqrt{LC}}{C} = \frac{V}{R}\sqrt{\frac{L}{C}} \, V$$

$$\boxed{V_C = \frac{V}{R}\sqrt{\frac{L}{C}}}$$

Numerical 1: A series RLC circuit has the following parameter values: $R = 10\Omega$, $L = 0.014\,H$, $C = 100\mu F$

Calculate: a) Resonance (ω_r) frequency in rad/sec

b) Q-factor (Q) of the circuit

c) Bandwidth BW in Hz

d) Lower & higher frequency points of the bandwidth

e) Maximum value of the voltage 'Vc' appearing across the capacitor if voltage $v = \sin 1000t$ is applied to series RLC circuit

Given: $R = 10\Omega$, $L = 0.01\,H$, $C = 100\mu F$, series RLC resonance ckt

To find: a) ω_r b) Q-factor c) BW d) f_1 and f_2

e) V_c

Thinking

a) $\omega_r = \dfrac{1}{\sqrt{LC}}$

b) $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

c) $BW = \dfrac{R}{2\pi L}$

d) $f_1 = f_r - \dfrac{BW}{2}$

e) $f_2 = f_r + \dfrac{BW}{2}$

f) $V_c = \dfrac{V}{R}\sqrt{\dfrac{L}{C}}$

Solution:

1) $\omega_r = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.014 \times 100 \times 10^{-6}}}$

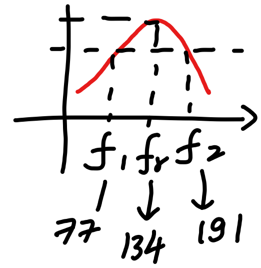
$\omega_r = \underline{845.15 \text{ rad/sec}}$

$f_r = \dfrac{\omega_r}{2\pi} = \underline{134.51}$ Hz

2) $Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.014}{100\times10^{-6}}}$

i.e $Q\text{-factor} = \underline{1.18}$

3) $BW = \frac{R}{2\pi L} = \frac{10}{2\pi\times 0.014} = \underline{113.68}\ Hz$



4) $f_1 = f_r - \frac{BW}{2} = 134.51 - \frac{113.68}{2}$

$f_2 = f_r + \frac{BW}{2} = 134.51 -$

5) $v = \sin 1000t \rightarrow V = \frac{V_m}{\sqrt{2}} = \frac{1}{\sqrt{2}} =$

6) At resonance → voltage that appears across the capacitor is max

$V_C = \frac{V}{R}\sqrt{\frac{L}{C}} = \frac{0.707}{10}\sqrt{\frac{0.014}{100\times10^{-6}}} =$

$V_C = \underline{\qquad}$

↓

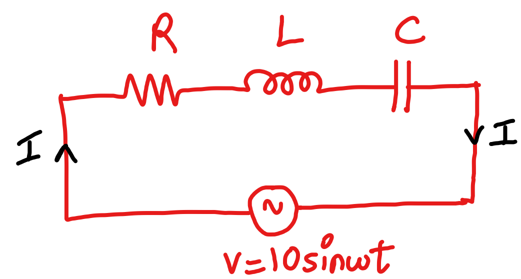
Numerical ahead

HW: A voltage of $v = 10\sin\omega t$ is applied to RLC series circuit. At the resonant frequency of the circuit, the maximum value across the capacitor is found to be 500V, BW = 400 rad/sec and impedance at resonance is 100Ω

Calculate: i) Resonant frequency

ii) Find the upper & lower limits of BW

iii) L and C



Hint: 1) $v = 10\sin\omega t$, $V_{rms} = \frac{V_m}{\sqrt{2}}$

$V_C = 500V$

$Z_r = 100\Omega$

2) $BW \text{ in Hz} = \frac{BW \text{ in rad/s}}{2\pi}$

3) $Z_r = R$ at resonance

4) $Q\text{-factor} = \frac{V_C}{V}$

5) $f_r = BW \text{ in Hz} \times Q\text{-factor}$

6) $f_1 = f_r - \frac{BW}{2}$

$f_2 = f_r + \frac{BW}{2}$

7) $BW = \frac{R}{2\pi L} \rightarrow L = \frac{R}{2\pi \times BW}$

8) $f_r = \frac{1}{2\pi\sqrt{LC}} \rightarrow C = ?$

Answers: $f_r = 4502\text{Hz}$

$f_1 = 4470\text{Hz}$, $f_2 = 4533\text{Hz}$

$L = 0.25\text{H}$

$C = 5\times10^{-9}\text{F}$

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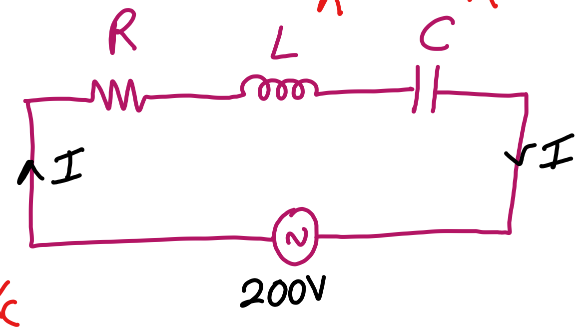
Numerical 2 : Determine R, L, C of an RLC series circuit that will resonate at 10 KHz, has a bandwidth of 1000Hz & draw 15.3W from 200V supply, operating at the resonant frequency of the circuit

$P = \frac{V_R^2}{R} \Rightarrow R?$ $\quad Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R}$



Given: $f_r = 10000\,Hz$, $BW = 1000\,Hz$,

$P = 15.3\,W$, $V_{rms} = 200\,V$

$X_L = X_C$

To find : R, L and C

Solution:

1) At series resonance, $V_R = V = 200$ volts

2) $P = \frac{V_R^2}{R} \rightarrow R = \frac{V_R^2}{P} = \frac{200^2}{15.3} = \underline{2614.38\,\Omega}$

3) $Q\text{factor} = \frac{f_r}{BW} = \frac{10{,}000}{1000} = \underline{10}$

4) $Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R} \rightarrow L = \frac{QR}{2\pi f_r} = \frac{10 \times 2614.38}{2\pi \times 10 \times 10^3}$

$L = \underline{0.416\,H}$

5) At resonance, $X_L = X_C$

$2\pi f_r L = \frac{1}{2\pi f_r C} \rightarrow C = \frac{1}{4\pi^2 f_r^2 L}$

i.e $C = \frac{1}{4\pi^2 \times (10 \times 10^3)^2 \times 0.416}$

i.e $C = 0.609 \times 10^{-9}\,F$

$C = 0.609\,nF$

$\times 10^{-3} \rightarrow m$

$\times 10^{-6} \rightarrow \mu$

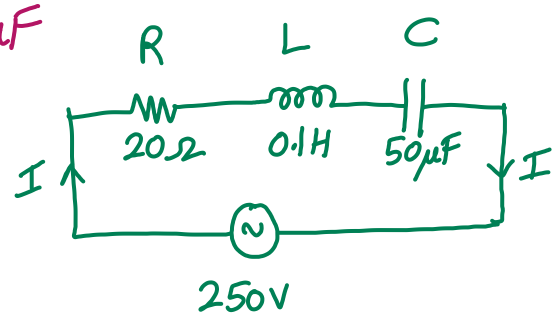
$\times 10^{-9} \rightarrow n$

$\times 10^{-12} \rightarrow p$

Numerical 3 : A coil having $R = 20\Omega$ and $L = 0.1H$ is connected in series with $C = 50\mu F$. An AC voltage of 250V is applied to the circuit. At what value of frequency will the current in the circuit be maximum? What is the value of this current? Also, find V_L and Q-factor

Given: $R = 20\Omega$, $V = 250V$, $L = 0.1H$, $C = 50\mu F$

To find: f_r, V_L, Q-factor, I_r



Solution:

1) $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.1 \times 50 \times 10^{-6}}}$

resonant frequency

i.e $f_r = \underline{71.18\ Hz}$

2) At series resonance, value of maximum current is,

$$I_r = \frac{V}{R} = \frac{250}{20} = \underline{12.5A}$$

3) $V_L = I_r X_L$

i.e $V_L = I_r \times 2\pi f_r L = 12.5 \times 2\pi \times 71.18 \times 0.1$

i.e $V_L = \underline{559.04V}$

4) $\text{Q-factor} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{20}\sqrt{\dfrac{0.1}{50 \times 10^{-6}}}$

$\text{Q-factor} = \underline{2.236}$

Thinking

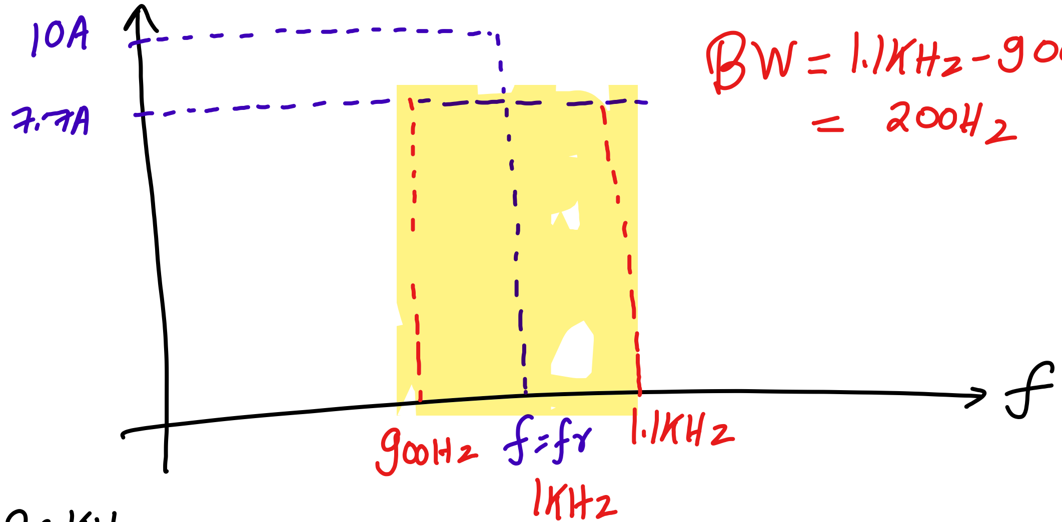
1) $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

2) $I_r = \dfrac{V}{R}$

3) $V_L = I_r X_L$

4) $\text{Q-factor} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

— x —



$BW = 1.1KHz - 900Hz$
$= 200Hz$

20Hz – 20KHz

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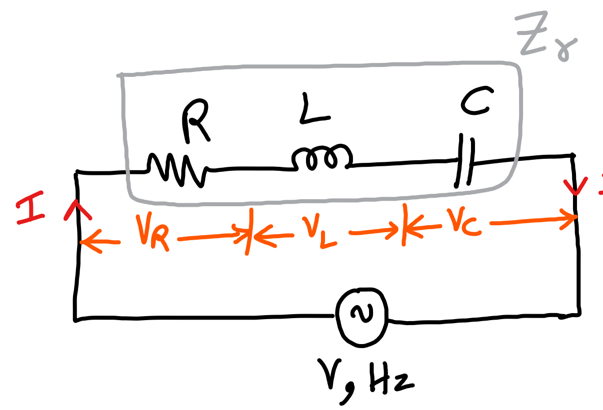
* RLC Series Resonance

① The ability of a radio-receiver to select a certain frequency transmitted by the station & to eliminate frequencies from other station's is based on the 'Principle of Resonance'

② In series RLC ckt,

→ X_L causes total current 'I' lags behind the applied voltage 'V'

→ X_C causes total current 'I' leads the applied voltage 'V'



RLC Series resonant circuit

Z_r - Circuit impedance at resonance frequency 'f_r'

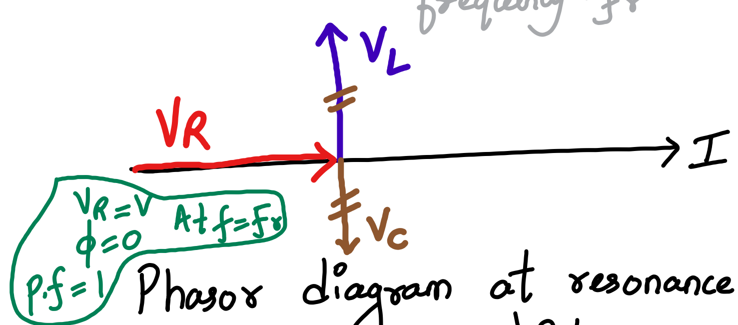
③ When $X_L > X_C$ → ckt nature is inductive

When $X_C > X_L$ → ckt nature is capacitive

$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$

④ $\bar{Z} = R + jX$

⑤ Net reactance → $X = |X_L - X_C|\ \Omega$



Phasor diagram at resonance 'f_r'

⑥ $X_L = 2\pi f L\ \Omega$
$= \omega L\ \Omega$

inductive reactance $(X_L \propto f)$

supply frequency

; $X_C = \dfrac{1}{2\pi f C}\ \Omega$
$= \dfrac{1}{\omega C}\ \Omega$

capacitive reactance $\left(X_C \propto \dfrac{1}{f}\right)$

At resonance,
$X_L = X_C$; $V_R = I_r R$
$I_r X_L = I_r X_C$
→ $(V_L = V_C) > V$
→ Voltage magnification ckt

⑦ At resonance frequency 'f_r' → the inductive reactance becomes equal to capacitive reactance i.e $X_L = X_C$

$Z_r = R + jX$
$= R\ \Omega$

∴ Net reactance i.e $X = X_L - X_C = 0$ becomes zero

(8) ∴ Impedance 'Z' of the circuit becomes purely resistive (i.e $Z_r = R$)

At resonance, (p.f = 1) $\cos\phi = 1$
$X_L = X_C$, $Z = R + j(X_L - X_C)$ $\phi = 0$
$\rightarrow Z_r = R + j0 = R$ (minimum)
$\rightarrow I_r = \frac{V}{Z} = \frac{V}{R}$ (max)

(9) i.e Whole ckt behaves as a purely resistive ckt
i.e Current 'I' remains in phase with applied voltage V
i.e Power factor p.f is unity $p.f = \cos 0 = 1$

(10) These condition's results in 'Electrical series resonance'

(11) Effect's of Series resonance

a) $X = 0$ i.e $X_L - X_C = 0$ i.e $X_L = X_C$

b) Impedance at resonance 'Z_r' is minimum & is equal to the resistance of the circuit

i.e $\boxed{Z_r = R}$ Ω

c) The current 'I_r' in the circuit is maximum

i.e $\boxed{I_r = \frac{V}{Z_r} = \frac{V}{R}}$ A

d) As I_r is maximum → the power absorbed by the ckt will also be at its maximum value $P = I_r^2 R = P_{max}$

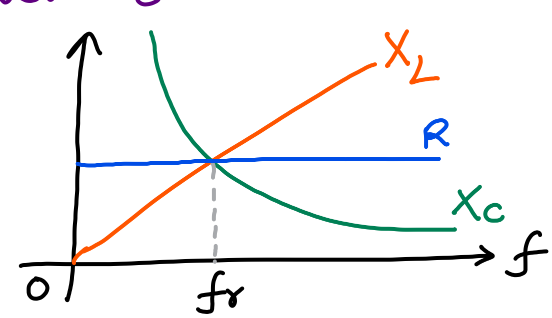
(12) Resonant frequency (f_r) :

$X_L = \omega L$ $X_C = \frac{1}{\omega C}$

a) X_L & X_C are frequency dependent

b) Higher 'f' → X_L higher & lower X_C

c) The frequency at which $X_L = X_C$ in a series RLC ckt is called the 'resonant frequency' (f_r)



(13) At series resonance, $X_L = X_C$

i.e $2\pi f_r L = \dfrac{1}{2\pi f_r C}$

i.e $f_r^2 = \dfrac{1}{4\pi^2 LC}$

$\omega_r = 2\pi f_r$

i.e $\boxed{f_r = \dfrac{1}{2\pi\sqrt{LC}}}$ in Hz series RLC resonance

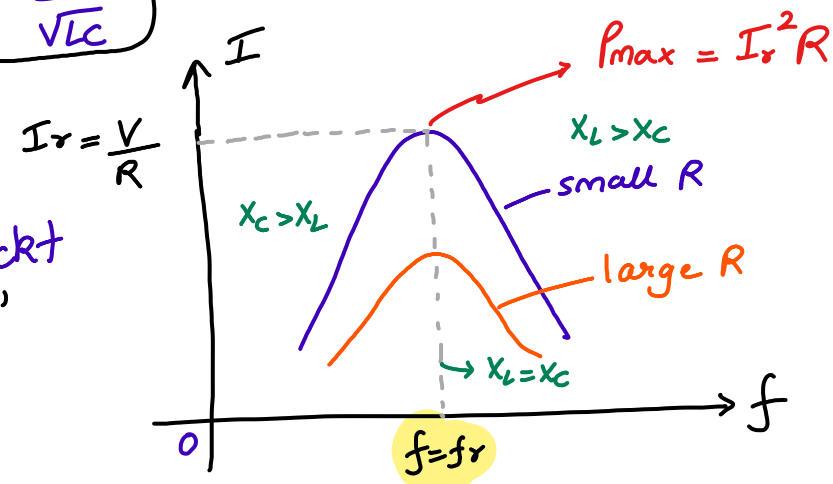
Resonant frequency

$\boxed{\omega_r = \dfrac{1}{\sqrt{LC}}}$ rad/sec

(14) Series Resonance curve :

a) It is a curve between ckt current I and frequency 'f'

b) Current 'I' reaches it's maximum value at the resonant frequency 'f_r'



Variation of circuit current 'I' with frequency 'f'

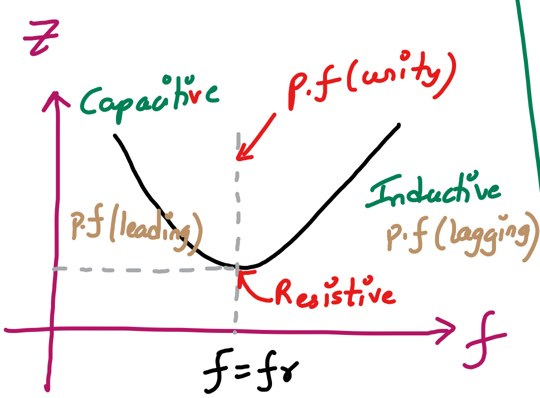
c) Current 'I' fall off rapidly on either side since

i) If 'f' is below f_r ($f < f_r$)

$X_C > X_L \rightarrow$ p.f is leading

ii) If 'f' is above f_r ($f > f_r$)

$X_L > X_C \rightarrow$ pf is lagging



$\rightarrow f < f_r \rightarrow$ Impedance $\rightarrow$ capacitive, pf leading

$\rightarrow f = f_r \rightarrow$ Impedance $\rightarrow$ resistive, pf unity

$\rightarrow f > f_r \rightarrow$ Impedance $\rightarrow$ inductive, pf lagging

(15) Magnitude of current falls rapidly as the frequency 'f' deviates from 'f_r'

(16) Quality factor (Q-factor) :

a) It is a measure of selectivity/sharpness of series resonant circuit

b) $$\text{Q-factor} = \frac{\text{Voltage across L \& C at resonance}}{\text{Voltage across R at resonance}} = \frac{V_C \text{ or } V_L}{V_R}$$

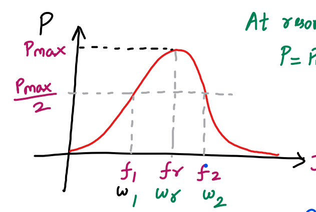
i.e $$\boxed{\text{Q-factor} = \frac{V_L \text{ or } V_C}{V_R}}$$

(At resonance, $V_R = V$)

c) $\because f_r = \dfrac{1}{2\pi\sqrt{LC}} \rightarrow \text{Q-factor} = \dfrac{I_r X_L}{I_r R} = \dfrac{X_L}{R}$

i.e $$\text{Q-factor} = \frac{2\pi f_r L}{R} = \frac{\cancel{2\pi} \times \frac{1}{\cancel{2\pi}\sqrt{LC}} \times L}{R} = \frac{\sqrt{L}\sqrt{L}}{\sqrt{LC}\; R}$$

i.e $$\boxed{\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$ Series resonant circuit



At resonance, $I = I_r(max)$

$P = P_{max} = I_r^2 R$

→ Half power frequencies (ω_1 & ω_2) or (f_1 & f_2)

→ Bandwidth

$BW = (f_2 - f_1)$ or $(\omega_2 - \omega_1)$

$P_{max} = I_r R$

$P_{half} = \dfrac{P_{max}}{2}$

$P_{half} = \dfrac{I_r^2 R}{2}$

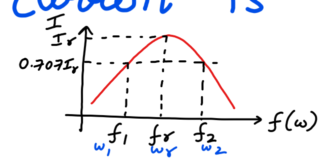
Let at $P = P_{half}$ → $I_r = I_b$

$I_r^2 \cancel{R} = \dfrac{I_b^2 \cancel{R}}{2}$

→ $I_b = \dfrac{I_r}{\sqrt{2}} = 0.707 I_r$

(17) Bandwidth of Series RLC resonant circuit :

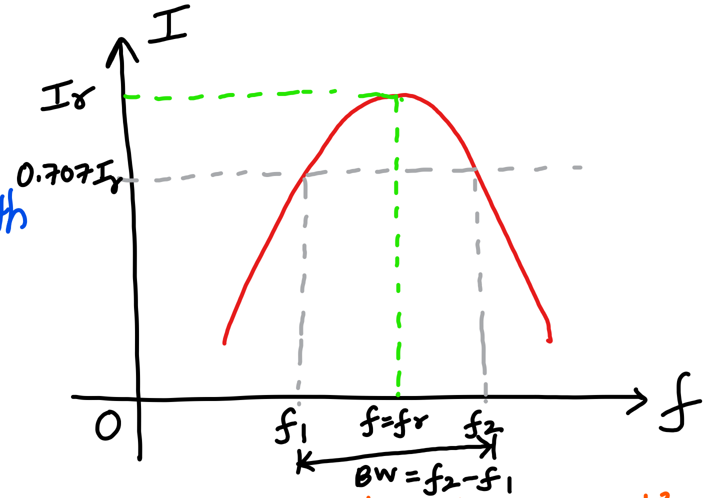
a) Bandwidth (BW) of a series resonance ckt is defined as the range of frequency over which circuit current is equal to 70.7% of it's maximum current



b) I_r – Circuit current at resonance

c) $(f_2 - f_1)$ is the bandwidth of the circuit

i.e $\boxed{BW = f_2 - f_1}$ Hz



d) $\boxed{BW = \frac{R}{2\pi L}}$ Hz

→ This can be proven

e) f_1 & f_2 are the limiting frequencies at which current is exactly equal to 70.7% of the maximum value

f) f_1 – Lower cut off frequency

f_2 – higher cut-off frequency

g) The resonant frequency is sufficiently centered w.r.t two cut-off frequencies (f_1 & f_2)

Proof: (Only for reading)

① Bandwidth $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}}$ — ①

At any frequency

i.e $I = \frac{V}{\sqrt{R^2+(\omega L - \frac{1}{\omega C})^2}}$

② At half power point, $I = \frac{I_r}{\sqrt{2}} = \frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{2} R}$ — ②

③ Put ② in eqn ①,

$\frac{\cancel{V}}{\sqrt{2} R} = \frac{\cancel{V}}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}} \rightarrow \sqrt{2} R = \sqrt{R^2+(\omega L - \frac{1}{\omega C})^2}$

④ Squaring both sides, $2R^2 = R^2 + (\omega L - \frac{1}{\omega C})^2 \rightarrow (\omega L - \frac{1}{\omega C})^2 = R^2$

⑤ $(\omega L - \frac{1}{\omega C}) = \pm R \rightarrow \frac{\omega^2 LC - 1}{\omega C} = \pm R \rightarrow \omega^2 LC - 1 = \pm \omega CR$

⑥ $\omega^2 LC - 1 \pm \omega CR = 0 \rightarrow LC\left[\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC}\right] = 0$

⑦ $\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0 \rightarrow$ Solving ax^2+bx+c Roots $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

⑧ Finding roots,

$\omega = \frac{\mp\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2}+\frac{4}{LC}}}{2} = \pm\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2}+\frac{1}{LC}}$

⑨ For low values of R, $\frac{R^2}{4L^2} << \frac{1}{LC}$ $f_r = \frac{1}{2\pi\sqrt{LC}}$

⑩ $\boxed{\omega = \pm\frac{R}{2L} \pm \sqrt{\frac{1}{LC}}} \rightarrow \omega = \pm\frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$ — ③

⑪ But, $\omega_r = \frac{1}{\sqrt{LC}} \rightarrow \boxed{\omega = \pm\frac{R}{2L} \pm \omega_r}$ — ④

⑫ Considering ω_r as positive,

$\boxed{\omega_1 = \omega_r - \frac{R}{2L}}$, $\boxed{\omega_2 = \omega_r + \frac{R}{2L}}$

⑬ Bandwidth (BW) → $BW = \omega_2 - \omega_1 = \cancel{\omega_r} + \frac{R}{2L} - \cancel{\omega_r} + \frac{R}{2L}$

$BW = \frac{2R}{2L} = \frac{R}{L}$ rad/sec

BW in Hz, $BW = f_2 - f_1 = \frac{\omega_2}{2\pi} - \frac{\omega_1}{2\pi} = \frac{1}{2\pi}(\omega_2 - \omega_1)$

$\boxed{BW = \frac{R}{2\pi L} \text{ Hz}}$ — ⑤ $\omega = 2\pi f$

⑭ $f_1 = f_r - \frac{R}{4\pi L}$, $f_2 = f_r + \frac{R}{4\pi L}$

$f_1 = f_r - \frac{BW}{2}$; $f_2 = f_r + \frac{BW}{2}$

$$\boxed{\begin{aligned} f_1 &= f_r - \frac{BW}{2} \\ f_2 &= f_r + \frac{BW}{2} \end{aligned}}$$

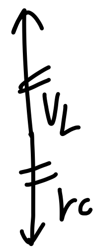
$\omega_1 = \omega_r - \frac{R}{2L} = \omega_r - \frac{BW}{2}$ $\quad BW = \frac{R}{L}$ rad/s

$\omega_2 = \omega_r + \frac{R}{2L} = \omega_r + \frac{BW}{2}$

(18) Relation between Q-factor & bandwidth

a) $Q \text{ factor} = 2\pi f_r \frac{L}{R} = \frac{2\pi L}{R} f_r = \frac{f_r}{BW}$

i.e $\boxed{f_r = Q\text{-factor} \times BW}$ $\qquad \boxed{Q = \frac{f_r}{BW}}$



(19) Voltage drop at resonance:

a) At resonance $\rightarrow V_L$ & V_C are equal & antiphase with each other

b) The drop across the resistor (R) V_R is equal to the applied voltage (V) $\quad V = V_R + \underbrace{V_L + V_C}$

c) At resonance, $V_R = I_r R = \frac{V}{R} R = V$ i.e $\boxed{V_R = V}$

$V_L = I_r X_L = \frac{V}{R} 2\pi f_r L = \frac{V}{R} \cancel{2\pi} \frac{1}{\cancel{2\pi}\sqrt{LC}} L = \frac{V}{R}\sqrt{\frac{L}{C}}\ V$

$\boxed{V_L = \frac{V}{R}\sqrt{\frac{L}{C}}}\ V$ $\qquad \boxed{V_L = V_C = \frac{V}{R}\sqrt{\frac{L}{C}}}$

$V_C = I_r X_C = \frac{V}{R} \frac{1}{2\pi f_r C} = \frac{V}{R} \frac{1}{\cancel{2\pi}} \frac{\cancel{2\pi}\sqrt{LC}}{C} = \frac{V}{R}\sqrt{\frac{L}{C}}\ V$

$\boxed{V_C = \frac{V}{R}\sqrt{\frac{L}{C}}}$

Numerical 1: A series RLC circuit has the following parameter values: $R = 10\Omega$, $L = 0.014\,H$, $C = 100\mu F$

Calculate: a) Resonance (ω_r) frequency in rad/sec

b) Q-factor (Q) of the circuit

c) Bandwidth BW in Hz

d) Lower & higher frequency points of the bandwidth

e) Maximum value of the voltage 'Vc' appearing across the capacitor if voltage $v = \sin 1000t$ is applied to series RLC circuit

Given: $R = 10\Omega$, $L = 0.01\,H$, $C = 100\mu F$, series RLC resonance ckt

To find: a) ω_r b) Q-factor c) BW d) f_1 and f_2

e) V_c

Thinking

a) $\omega_r = \dfrac{1}{\sqrt{LC}}$

b) $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

c) $BW = \dfrac{R}{2\pi L}$

d) $f_1 = f_r - \dfrac{BW}{2}$

e) $f_2 = f_r + \dfrac{BW}{2}$

f) $V_c = \dfrac{V}{R}\sqrt{\dfrac{L}{C}}$

Solution:

1) $\omega_r = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.014 \times 100 \times 10^{-6}}}$

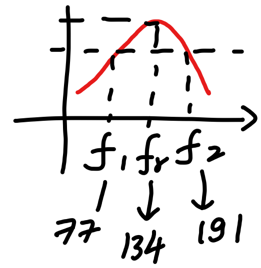
$\omega_r = \underline{845.15 \text{ rad/sec}}$

$f_r = \dfrac{\omega_r}{2\pi} = \underline{134.51}$ Hz

2) Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.014}{100\times10^{-6}}}$

ie Q-factor $= \underline{1.18}$

3) $BW = \frac{R}{2\pi L} = \frac{10}{2\pi\times0.014} = \underline{113.68}$ Hz



4) $f_1 = f_r - \frac{BW}{2} = 134.51 - \frac{113.68}{2}$

$f_2 = f_r + \frac{BW}{2} = 134.51 -$

5) $v = \sin 1000t \rightarrow V = \frac{V_m}{\sqrt{2}} = \frac{1}{\sqrt{2}} =$

$v = V_m \sin \omega t$
$V = V_m$

6) At resonance → voltage that appears across the capacitor is max

$$V_C = \frac{V}{R}\sqrt{\frac{L}{C}} = \frac{0.707}{10}\sqrt{\frac{0.014}{100\times10^{-6}}} =$$

An

$V_C = \underline{\qquad}$

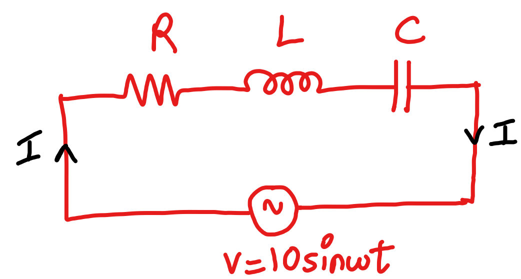
Numerical ahead

HW: A voltage of $v = 10\sin\omega t$ is applied to RLC series circuit. At the resonant frequency of the circuit, the maximum value across the capacitor is found to be 500V, BW = 400 rad/sec and impedance at resonance is 100Ω

Calculate: i) Resonant frequency

ii) Find the upper & lower limits of BW

iii) L and C



Hint: 1) $v = 10\sin\omega t$, $V_{rms} = \frac{V_m}{\sqrt{2}}$

$V_C = 500V$

$Z_r = 100\Omega$

2) $\text{BW in Hz} = \frac{\text{BW in rad/s}}{2\pi}$

3) $Z_r = R$ at resonance

4) $Q\text{-factor} = \frac{V_C}{V}$

5) $f_r = \text{BW in Hz} \times Q\text{-factor}$

6) $f_1 = f_r - \frac{BW}{2}$

$f_2 = f_r + \frac{BW}{2}$

7) $BW = \frac{R}{2\pi L} \rightarrow L = \frac{R}{2\pi \times BW}$

8) $f_r = \frac{1}{2\pi\sqrt{LC}} \rightarrow C = ?$

Answers: $f_r = 4502\,Hz$

$f_1 = 4470\,Hz$, $f_2 = 4533\,Hz$

$L = 0.25\,H$

$C = 5 \times 10^{-9}\,F$

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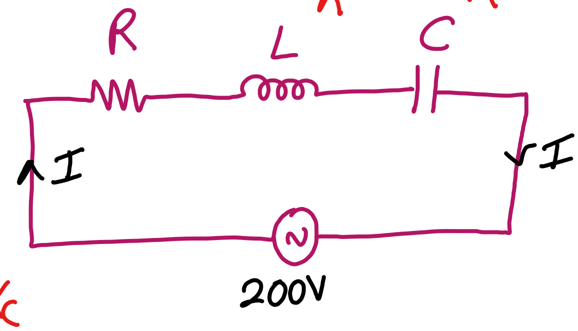
Numerical 2 : Determine R, L, C of an RLC series circuit that will resonate at 10 KHz, has a bandwidth of 1000Hz & draw 15.3W from 200V supply, operating at the resonant frequency of the circuit

$P = \frac{V_R^2}{R} \Rightarrow R?$ $\quad Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R}$



Given: $f_r = 10000Hz$, $BW = 1000Hz$,

$P = 15.3W$, $V_{rms} = 200V$

$X_L = X_C$

To find : R, L and C

Solution:

1) At series resonance, $V_R = V = 200$ volts

2) $P = \frac{V_R^2}{R} \rightarrow R = \frac{V_R^2}{P} = \frac{200^2}{15.3} = \underline{2614.38\Omega}$

3) $Q\text{factor} = \frac{f_r}{BW} = \frac{10{,}000}{1000} = \underline{10}$

4) $Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R} \rightarrow L = \frac{QR}{2\pi f_r} = \frac{10 \times 2614.38}{2\pi \times 10 \times 10^3}$

$L = \underline{0.416\ H}$

5) At resonance, $X_L = X_C$

$2\pi f_r L = \frac{1}{2\pi f_r C} \rightarrow C = \frac{1}{4\pi^2 f_r^2 L}$

i.e $C = \frac{1}{4\pi^2 \times (10 \times 10^3)^2 \times 0.416}$

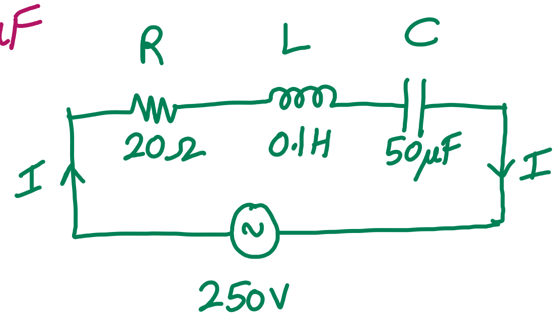
i.e $C = 0.609 \times 10^{-9} F$

$C = 0.609nF$

$\times 10^{-3} \rightarrow m$
$\times 10^{-6} \rightarrow \mu$
$\times 10^{-9} \rightarrow n$
$\times 10^{-12} \rightarrow p$

Numerical 3: A coil having $R = 20\Omega$ and $L = 0.1H$ is connected in series with $C = 50\mu F$. An AC voltage of 250V is applied to the circuit. At what value of frequency will the current in the circuit be maximum? What is the value of this current? Also, find V_L and Q-factor

Given: $R = 20\Omega$, $V = 250V$, $L = 0.1H$, $C = 50\mu F$



To find: f_r, V_L, Q-factor, I_r

Solution:

1) $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.1 \times 50 \times 10^{-6}}}$

resonant frequency

i.e $f_r = \underline{71.18\,Hz}$

2) At series resonance, value of maximum current is,

$I_r = \dfrac{V}{R} = \dfrac{250}{20} = \underline{12.5\,A}$

3) $V_L = I_r X_L$

i.e $V_L = I_r \times 2\pi f_r L = 12.5 \times 2\pi \times 71.18 \times 0.1$

i.e $V_L = \underline{559.04\,V}$

4) $Q\text{-factor} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{20}\sqrt{\dfrac{0.1}{50 \times 10^{-6}}}$

$Q\text{-factor} = \underline{2.236}$

Thinking

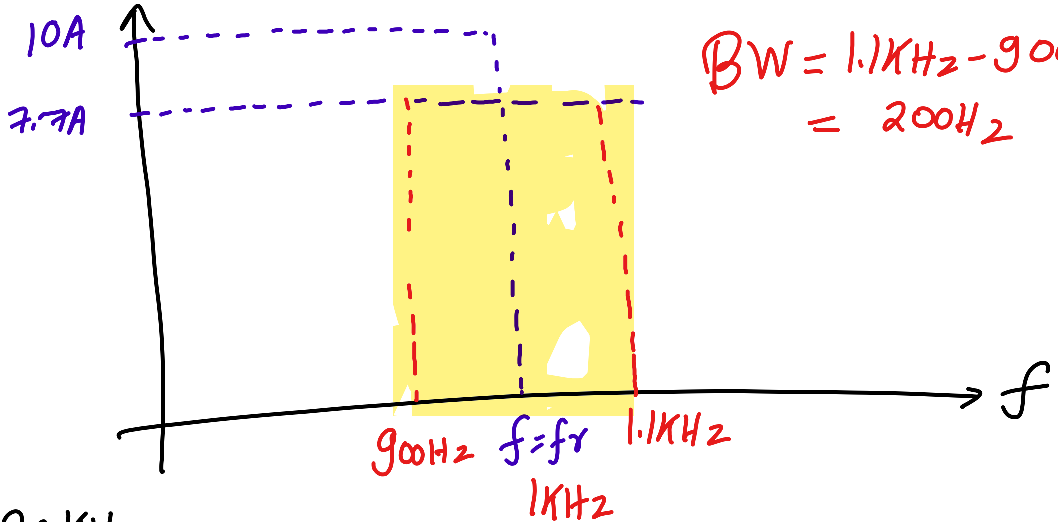
1) $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

2) $I_r = \dfrac{V}{R}$

3) $V_L = I_r X_L$

4) $Q\text{-factor} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

— x —



$BW = 1.1KHz - 900Hz$
$= 200Hz$

20Hz – 20KHz