EEEE lec 40a 1/6/2023

Module 3.5 : Three-phase balanced circuits, voltage & current relations in star & Delta connections

* 3 phase system:

① Single phase (1ϕ) system → domestic application's eg mixer, coolers, fans, ACs, refrigerators, etc

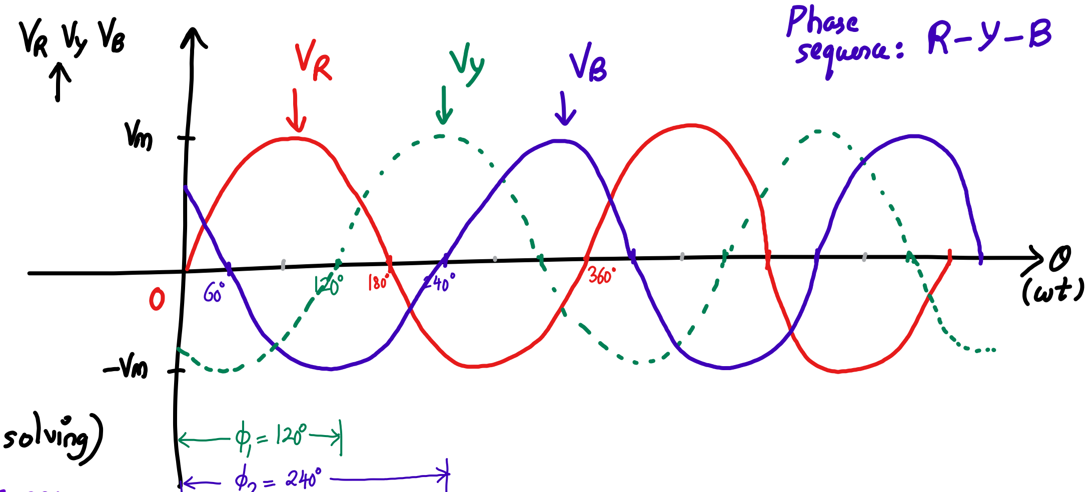
② For generation, transmission and distribution of electric power, three phase (3ϕ) system is universally accepted as standard system

③ Three-phase voltages w.r.t time:

$V_R = V_m \sin(\omega t - 0°)$

$V_y = V_m \sin(\omega t - 120°)$

$V_B = V_m \sin(\omega t - 240°)$

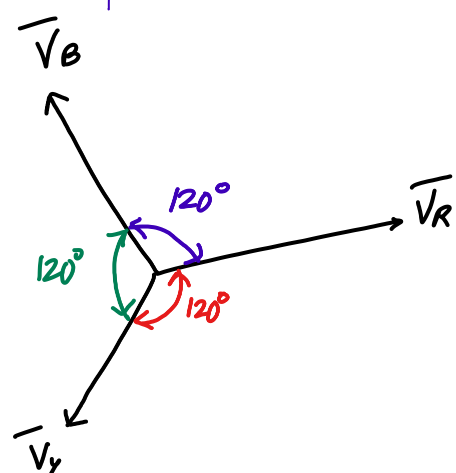


$V_R + V_y + V_B = 0$ (Try solving)

↳ sum of 3 voltages at any instant is zero

④ 3ϕ supply system have three independent alternating voltages (V_R, V_y, V_B)

⑤ These voltage's have same magnitude & frequency but they have a phase difference of 120° between them.



Phasor diagram

* Advantages of 3ϕ system over 1ϕ system:

① In a 1ϕ system, the instant eous power is fluctuating → hence causes considerable vibrations ir 1ϕ motor

But, in 3ϕ sys-em → the instantaneous power is constant at all times

② Output of a 3ϕ system (415V, 50Hz) is greater than that of 1ϕ system(230V, 50Hz)

③ 3ϕ otors (employing 3ϕ supply) are more efficient and have higher p.f than 1ϕ motors running al same frequency

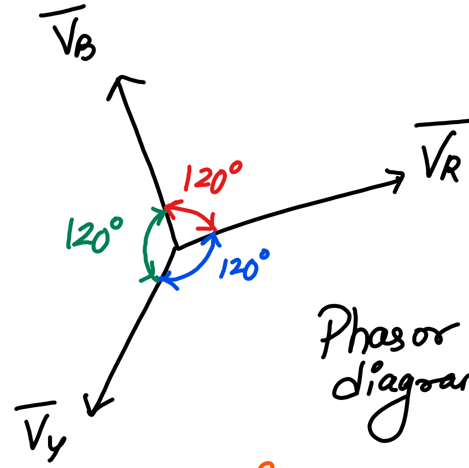
④ Γr transmission & dist ·bution, employing 3ϕ system, is cheaper than 1ϕ system → since a 3ϕ syste needs less copper than a 1ϕ system.

⑤ 3ϕ motors self-starting, where 1ϕ system are not self starting

⑥ 1ϕ supply can e obtained from a 3 | system but not vice-versa

↓
Terminology ahead

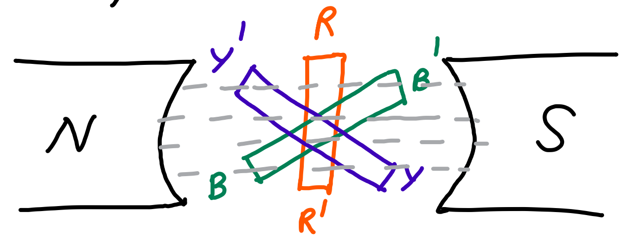
* Terminology related to 3ϕ systems:



Phasor diagram

① **Phase sequence:** The order in which the voltages in the 3 phases reach their maximum positive value is called the 'Phase sequence' or 'phase order'

eg R-Y-B in 3ϕ supply



3 coils: RR', YY', BB'

② **Phase voltage: (V_{ph})** The voltage induced in each winding is called 'phase voltage'

③ **Phase current: (I_{ph})** The current flowing through each winding is called 'phase current'

④ **Line voltage: (V_L)** The voltage available between any pair of terminals / Lines is called 'Line voltage'

⑤ **Line current: (I_L)** The current flowing through each Line is called the 'Line current'

⑥ **Symmetrical / Balanced 3ϕ system:**

A 3ϕ system is said to be balanced if

① Voltages in 3 phases are equal in magnitude & differ in phase from each other by 120°

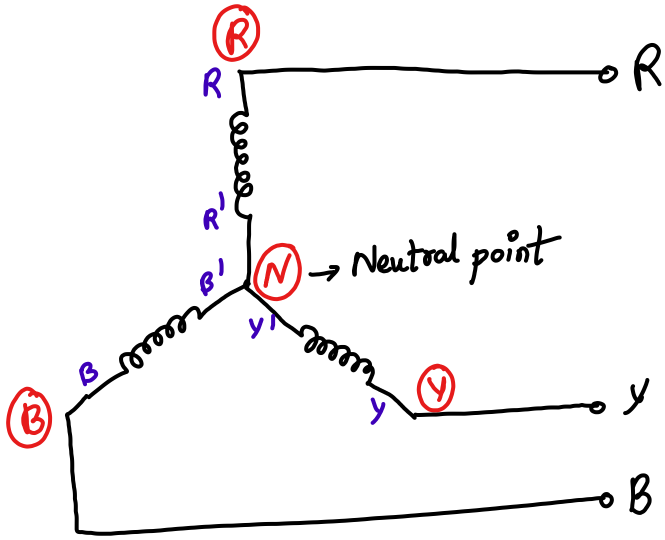
② Currents in 3 phases are equal in magnitude & differ in phase from each other by 120°

③ The loads connected across the 3 phases are identical i.e all the loads have same magnitude & power factor

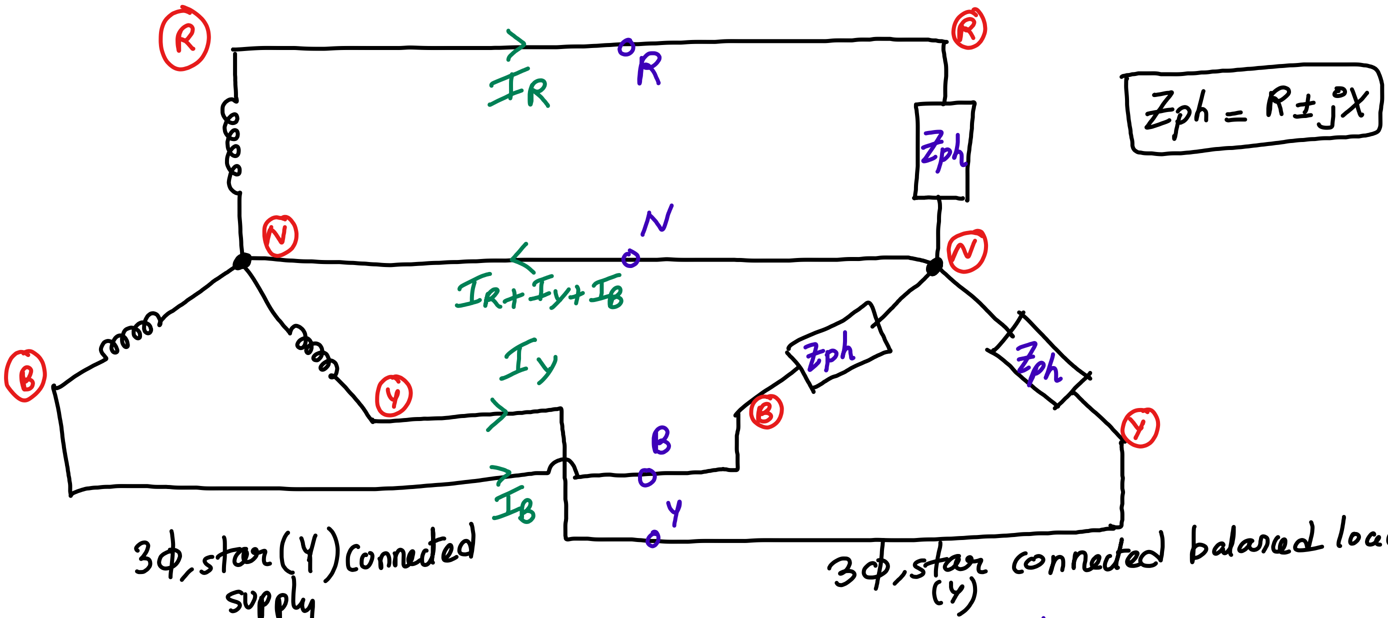
* Star connection (3 phase) :

→ Similar terminals (start or finish) of the 3 windings are joined together as shown

→ The common point is called "Neutral point")



3ϕ star connected supply



3ϕ, 4 wire, star connected system

① 3 Line current's are I_R, I_y, I_B

$$I_R = I_m \sin(\omega t - 0°)$$

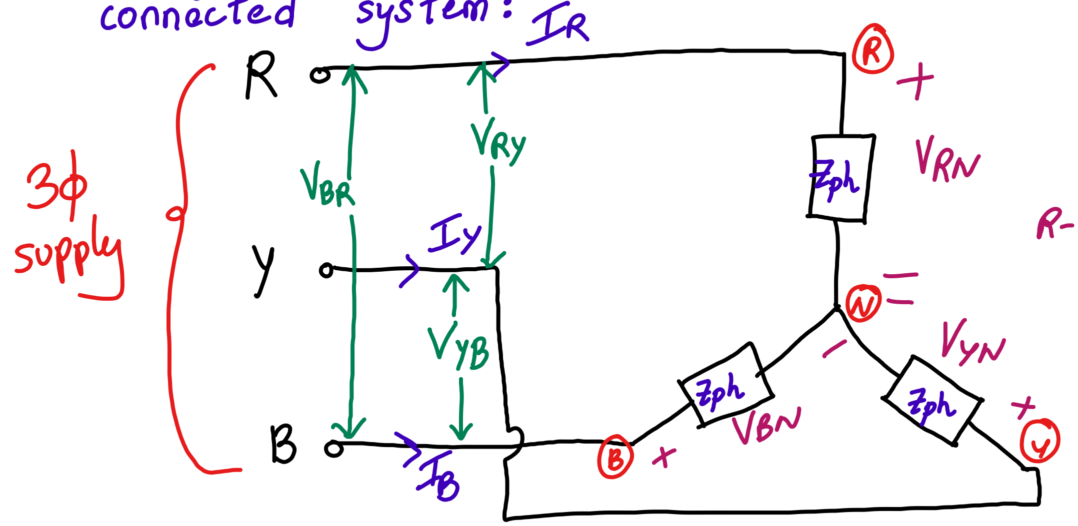
$$I_y = I_m \sin(\omega t - 120°)$$

$$I_B = I_m \sin(\omega t - 240°)$$

② Current flowing through neutral (N) wire is $I_R + I_y + I_B = 0$ (Check this)

∴ If the load is balanced → the neutral wire carries zero current

* Voltage, current & power relation's in a balanced 3ϕ star connected system:



R-Y-B

$\underbrace{V_{RN}, V_{YN}, V_{BN}}_{\text{Phase voltages}}$

$\underbrace{V_{RY}, V_{BR}, V_{YB}}_{\text{Line voltages}}$

$\underbrace{I_R = I_Y = I_B = I_L = I_{ph}}_{\text{Line and Phase current's}}$

3ϕ, star(Y) connected balanced load

① As the load is balanced, (let magnitude of each phase is V_{ph})

$$\therefore \boxed{V_{ph} = V_{RN} = V_{YN} = V_{BN}}$$

↳ only magnitude's are same

$V_{RN} = V_{ph} \angle 0^\circ$

$V_{YN} = V_{ph} \angle -120^\circ$

$V_{BN} = V_{ph} \angle -240^\circ$

② The 3 Lines voltages are V_{RY}, V_{YB}, V_{BR} (For balanced system $V_L = V_{RY} = V_{YB} = V_{BR}$)

③ Writing Line voltages w.r.t phase voltages:

$$\overline{V_{RY}} = \overline{V_{RN}} + \overline{V_{NY}} = \overline{V_{RN}} - \overline{V_{YN}}$$

$\overline{V_{RY}} \rightarrow \overline{V_{RN}} - \overline{V_{YN}}$, $\overline{V_{RN}} + \overline{V_{NY}}$

i.e $\overline{V_{RY}} = V_{ph}\angle 0^\circ - V_{ph}\angle -120^\circ$

i.e $\overline{V_{RY}} = (V_{ph} + j0) - (-0.5V_{ph} - j0.866V_{ph})$

i.e $\overline{V_{RY}} = 1.5V_{ph} + j0.866V_{ph}$

i.e $\boxed{\overline{V_{RY}} = \sqrt{3}\, V_{ph} \angle 30^\circ}$

Similarly, $\overline{V_{YB}} = \overline{V_{YN}} + \overline{V_{NB}} = \sqrt{3}\, V_{ph} \angle 30^\circ$

$\overline{V_{BR}} = \overline{V_{BN}} + \overline{V_{NR}} = \sqrt{3}\, V_{ph} \angle 30^\circ$

④ $\therefore$ In a star connected 3ϕ system $\rightarrow$ line voltages lead the respective phase voltages by 30°

$$\boxed{V_L = \sqrt{3}\,V_{ph}} \quad 3\phi,\text{star}$$

⑤ Line current is equal to the phase current

$$\boxed{I_L = I_{ph}} \quad 3\phi,\text{star system}$$

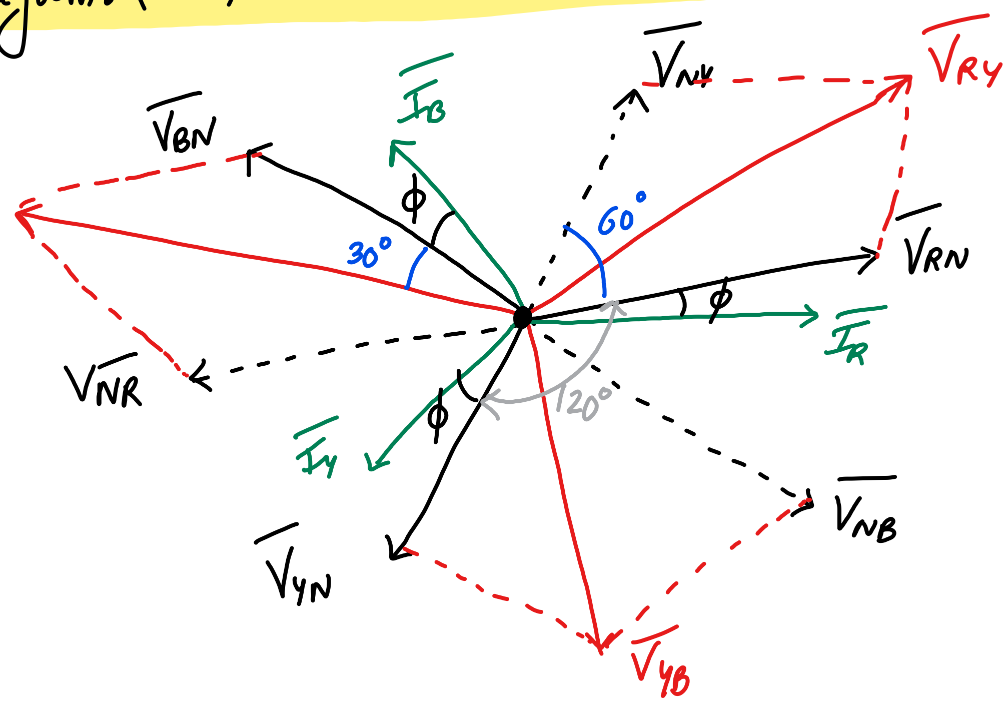
* Phasor diagram: (3ϕ star connected balanced system)

$\overline{V_{RY}} = \overline{V_{RN}} + \overline{V_{NY}}$

$\overline{V_{YB}} = \overline{V_{YN}} + \overline{V_{NB}}$

$\overline{V_{BR}} = \overline{V_{BN}} + \overline{V_{NR}}$

Assuming inductive load
↓
draw 3 phase current
$\overline{I_R}, \overline{I_Y}, \overline{I_B}$



* Power (3ϕ star connected balanced system):

① In a 3ϕ star connected balanced load, power consumed in each load phase is the same $\longrightarrow$ so all 3 phases are identical
$\therefore$ Total power is the sum of powers in the 3phases i.e

② Power in each phase $P' = V_{ph} I_{ph} \cos\phi$ (W)

Total power, $P = 3 \times$ power in each phase

$$\boxed{P = 3\,V_{ph} I_{ph} \cos\phi} \quad (\text{W or kW})$$

③ For a star connected, $V_L = \sqrt{3}\, V_{ph}$, $I_L = I_{ph}$

$$P = 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

$$P = \frac{\sqrt{3} \times \cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} V_L I_L \cos\phi$$

$$\boxed{P = \sqrt{3}\, V_L I_L \cos\phi}$$ 3ϕ star system

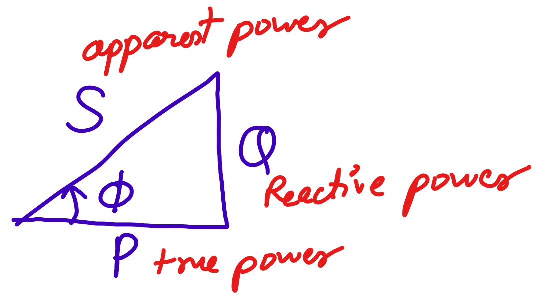
where, ϕ is the phase difference between the phase voltage & the corresponding phase current.

④ Total reactive power,

$$\boxed{Q = 3 V_{ph} I_{ph} \sin\phi = \sqrt{3} V_L I_L \sin\phi}$$

VAR or KVAR

Power triangle



$S^2 = P^2 + Q^2$

⑤ Total apparent power,

$$\boxed{S = 3 V_{ph} I_{ph} = \sqrt{3} V_L I_L}$$ VA or KVA

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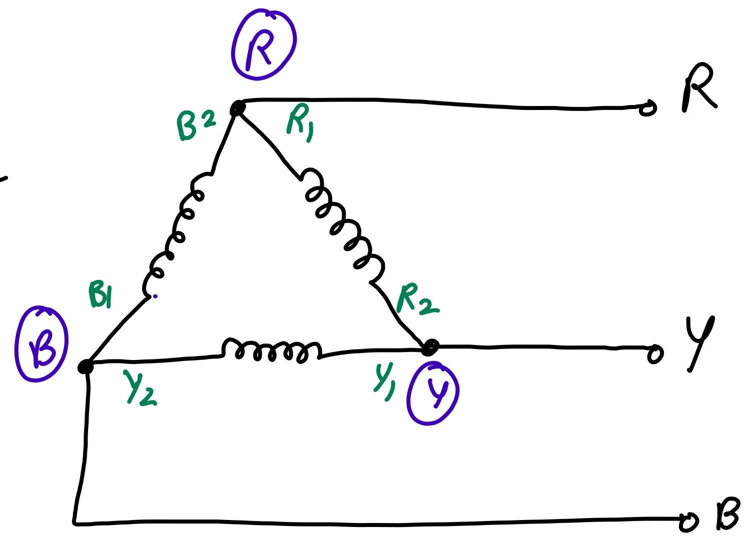
* 3ϕ Delta connection:

① Here, dissimilar terminals of the 3 windings are joined together.



3ϕ, Delta connected supply

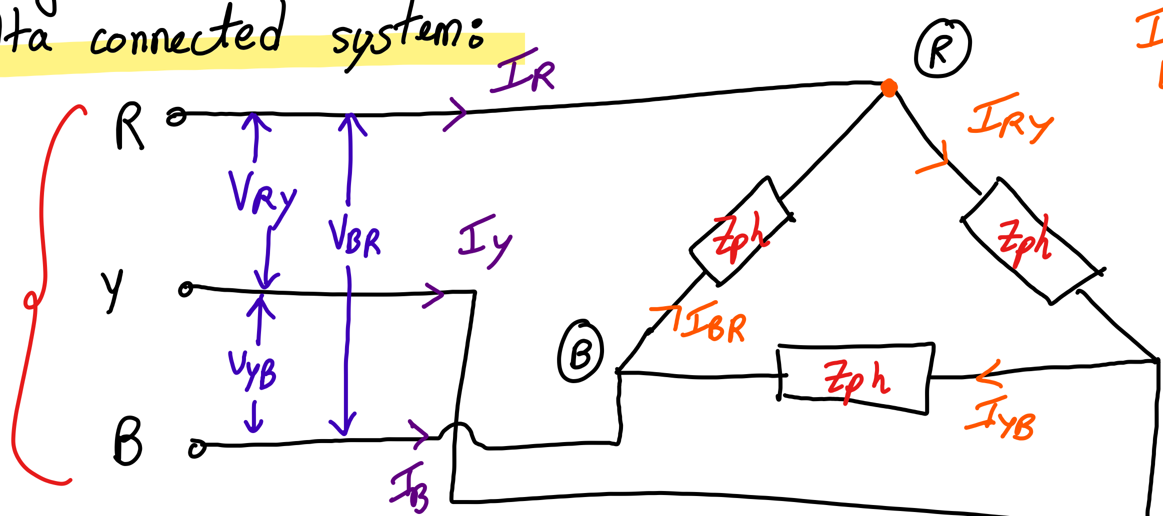
② Here, the 3 phase voltages are equal in magnitude & differ in phase from each other by 120°

$V_R = V_m \sin(\omega t - 0°)$, $V_Y = V_m \sin(\omega t - 120°)$

$V_B = V_m \sin(\omega t - 240°)$

$V_R + V_Y + V_B = 0$ (can be proven!)

* Voltage, current and power relations in a balanced 3ϕ Delta connected system:



I_{RY}, I_{BR}, I_{YB} — phase currents

I_R, I_Y, I_B — Line currents

3ϕ, delta connected balanced load

RY
YB
BR

$\underbrace{V_{RY}, V_{YB}, V_{BR},}_{\text{Line voltages}}$

① $V_L = V_{RY} = V_{YB} = V_{BR}$ ---- load is balanced

② In a 3ϕ delta connected system, Line voltage is same as phase voltage

$$\boxed{V_L = V_{ph}}$$

③ As the load is balanced, all the 3 phase currents are equal in magnitude but differ in phase from each other by 120°

i.e $I_{RY} = I_{YB} = I_{BR} = \underbrace{I_{ph}}_{\text{magnitude of phase current}}$

i.e $\overline{I_{RY}} = I_{ph} \angle 0°$

$\overline{I_{YB}} = I_{ph} \angle -120°$

$\overline{I_{BR}} = I_{ph} \angle -240°$

④ 3 Line currents are I_R, I_Y, I_B

$I_R = I_Y = I_B = \underbrace{I_L}_{\text{magnitude of line current}}$

⑤ KCL at node Ⓡ, $\overline{I_R} + \overline{I_{BR}} = \overline{I_{RY}}$

i.e $\overline{I_R} = \overline{I_{RY}} - \overline{I_{BR}} = I_{ph} \angle 0° - I_{ph} \angle -240°$

i.e $\overline{I_R} = (I_{ph} + j0) - (-0.5 I_{ph} + j0.866 I_{ph})$

i.e $\overline{I_R} = 1.5 I_{ph} - j\,0.866 I_{ph}$

$$\boxed{\overline{I_R} = \sqrt{3}\, I_{ph} \angle -30°}$$

⑥ Similarly, $\overline{I_Y} + \overline{I_{RY}} = \overline{I_{YB}}$; $\overline{I_Y} = \overline{I_{YB}} - \overline{I_{RY}} = \sqrt{3}\, I_{ph} \angle -30°$

$\bar{I}_B + \bar{I}_{YB} = \bar{I}_{BR}$; $\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} = \sqrt{3}\, I_{ph} \angle -30^\circ$

7) ∴ In a 3φ delta connected system → line currents (I_R, I_B, I_Y) are 30° behind the respective phase currents (I_{RY}, I_{YB}, I_{BR}) $\boxed{I_L = \sqrt{3}\, I_{ph}}$

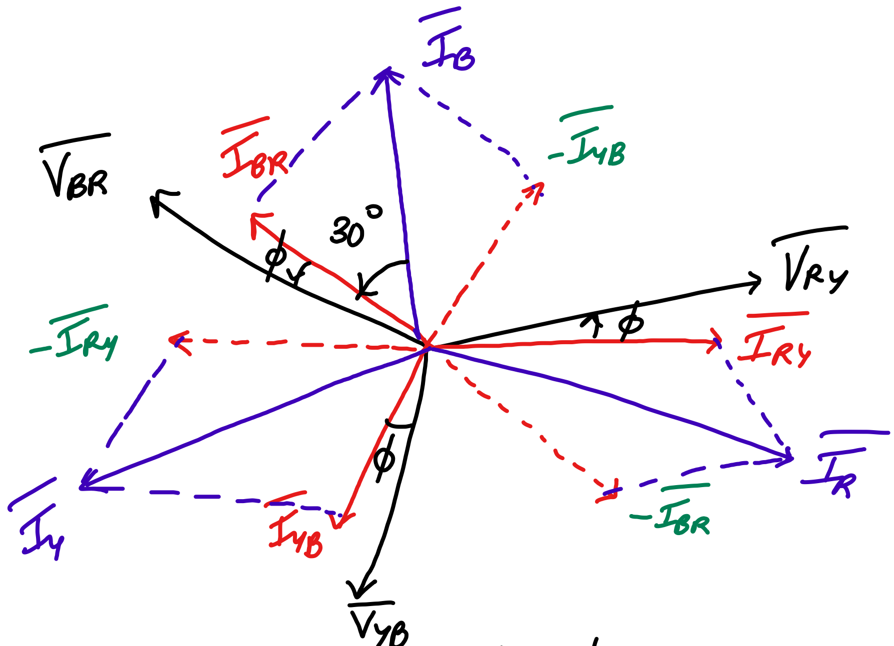
8) Phasor diagram (3 φ Delta connected balanced system)

Assuming inductive load

$\bar{I}_R = \bar{I}_{RY} + (-\bar{I}_{BR})$

$\bar{I}_Y = \bar{I}_{YB} + (-\bar{I}_{RY})$

$\bar{I}_B = \bar{I}_{BR} + (-\bar{I}_{YB})$



9) Power: (3φ Delta connected balanced load)

a) Here, as all the 3 phases are identical, power consumed in each load phase is the same

b) ∴ Total power is the sum of power in each phases

c) Power in a single phase ⇒ $P' = V_{ph} I_{ph} \cos\phi$

d) Total power ⇒ $P = 3 \times$ power in each phase

ie $\boxed{P = 3 V_{ph} I_{ph} \cos\phi}$

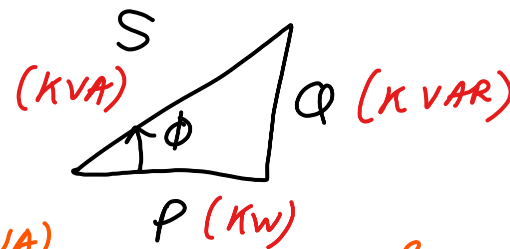
e) For a 3ϕ delta connection, $V_L = V_{ph}$, $I_L = \sqrt{3}\, I_{ph}$

$$\therefore \quad P = 3 V_L \frac{I_L}{\sqrt{3}} \cos\phi$$

i.e $\boxed{P = \sqrt{3}\, V_L I_L \cos\phi}$ 3ϕ delta connected system

ϕ - phase difference betn phase voltage & phase current

f) Power triangle



→ Apparent power (S):

$$\boxed{S = 3V_{ph} I_{ph} = \sqrt{3}\, V_L I_L} \quad (KVA)$$

$$S^2 = P^2 + Q^2$$

→ Reactive power (Q):

$$\boxed{Q = 3V_{ph} I_{ph} \sin\phi = \sqrt{3}\, V_L I_L \sin\phi} \quad (KVAR)$$

—x—

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* 3ϕ balanced Star to delta & Delta to star conversion:

① For a balanced star connected load,

V_L, I_L — Line voltage, line current

V_{ph}, I_{ph} — Phase voltage, Phase current

a) $V_L = \sqrt{3}\, V_{ph}$ b) $I_{ph} = I_L$

b) $Z_{ph} = \dfrac{V_{ph}}{I_{ph}} = \dfrac{V_L/\sqrt{3}}{I_L} = \dfrac{V_L}{\sqrt{3} I_L}$ — ①

$$\boxed{Z_{ph} = \frac{V_L}{\sqrt{3} I_L}}$$

→ 3ϕ star

$$\boxed{Z_Y = \frac{V_L}{\sqrt{3} I_L}}$$

c) $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{V_L}{\sqrt{3} Z_{ph}} = I_L$

d) $P = \sqrt{3} V_L I_L \cos\phi$

$$P = \cancel{\sqrt{3}} V_L \frac{V_L}{\cancel{\sqrt{3}} Z_{ph}} \cos\phi$$

$$\boxed{P_Y = \frac{V_L^2}{Z_{ph}} \cos\phi}$$ star(Y) connected load (P_Y)

$P_\Delta = \sqrt{3} V_L I_L \cos\phi$

$P_\Delta = \sqrt{3} V_L \dfrac{\sqrt{3} V_L}{Z_{ph}} \cos\phi$

$P_\Delta = \dfrac{3 V_L^2}{Z_{ph}} \cos\phi$

$$\boxed{P_\Delta = 3 \times P_Y \qquad P_Y = \frac{1}{3} P_\Delta}$$

→ Power consumed by a balanced 3ϕ star load is $\frac{1}{3}$rd of that in case of 3ϕ Delta load

② For a balanced delta connected load,

V_L, I_L, I_{ph}, V_{ph}

a) $V_{ph} = V_L$ b) $I_{ph} = \dfrac{I_L}{\sqrt{3}}$

c) $Z_{ph} = \dfrac{V_{ph}}{I_{ph}} = \dfrac{V_L \sqrt{3}}{I_L}$

d) $\boxed{Z_{ph} = \dfrac{V_L \sqrt{3}}{I_L}}$ 3ϕ delta connected load → e) $\boxed{Z_\Delta = 3 Z_Y}$

$Z_\Delta = 3 \dfrac{V_L}{\sqrt{3} I_L} = \dfrac{\sqrt{3} V_L}{I_L} = Z_Y$

delta impedance (Z_Δ) $= \dfrac{V_L \sqrt{3}}{I_L} =$ i.e $Z_Y = \dfrac{1}{3} Z_\Delta$

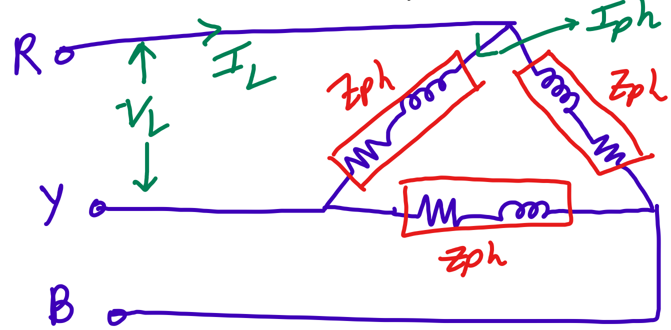
star impedance

e) $Z_Y = \dfrac{V_L}{\sqrt{3} I_L}$

* 3ϕ Star connection Vs 3ϕ Delta connection

	3ϕ star connection	3ϕ Delta connection
① Circuit diagram	Draw circuit	Draw circuit
② Line voltage	$V_L = \sqrt{3}\, V_{ph}$	$V_L = V_{ph}$
③ Line current	$I_L = I_{ph}$	$I_L = \sqrt{3}\, I_{ph}$
④ Phase betn angle line & phase quantity	Line voltage leads the respective phase voltage by 30°	Line current lags behind the respective phase current by 30°
⑤ Power	$P_Y = \frac{1}{3} P_\Delta$	$P_\Delta = 3 P_Y$
⑥ Possible connection	3phase, 3wire & 3phase, 4wire systems are possible	only 3phase, 3wire system is possible
⑦ Neutral point	There is a neutral point	No neutral point in delta connection
⑧ Motor speed	The speeds of star connected motors are slow as they receive $1/\sqrt{3}$ voltage	The speeds of Delta connected motors are high because each Phase gets the total of Line voltage
⑨ Cost saving	In star connection, the phase voltage is low as $\frac{1}{\sqrt{3}}$ of the Line voltage ∴ It needs a low number of turns, hence, saving in copper and low insulation cost	In delta connection, $V_{ph} = V_L$ hence, it needs more number of turns which increase the total cost & require high insulation cost
⑩ Application	It is common & general system used in Power transmission	They are used in distribution systems and industries

Numerical 1: Three similar coils, each of resistance 20Ω & inductance 0.5H are connected in Delta to a 3ϕ, 400V, 50Hz supply. Calculate the line current & total power absorbed



Solution:

① 3ϕ Delta balanced load

② As the same 3 coils are connected in Delta, Zph is same

$$\overline{Z}_{ph} = R + jX_L, \quad \overline{Z}_{ph} = Z_{ph}\angle\phi^\circ$$

③ Resistance of each coil $R = 20\Omega$

④ Inductance of each coil $L = 0.5H$

⑤ Reactance of each coil $X_L = 2\pi f L = 100\pi \times 0.5 = 157\Omega$

⑥ $\overline{Z}_{ph} = R + jX_L = (20 + j157)\Omega = \underbrace{158.34}_{Z_{ph}} \angle \underbrace{82.74^\circ}_{\phi}\ \Omega$

$\overline{Z}_{ph} = Z_{ph}\angle\phi$

⑦ $\phi = 82.74^\circ$

⑧ In a delta connection, $V_L = V_{ph} = 400V$

⑨ Phase current, $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{400}{158.34} = 2.53\,A$

⑩ Line current, $I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 2.53 = 4.38A$

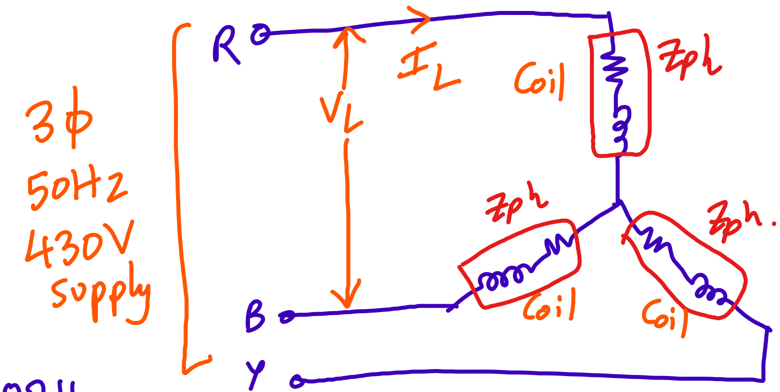
⑪ Total power absorbed, $P = \sqrt{3}\, V_L I_L \cos\phi$

$\therefore P = \sqrt{3} \times 400 \times 4.38 \cos(82.74)$

$P = 383.48W$

Numerical 2 : 3 similar coils, each of resistance 8Ω, & inductance 0.02H are connected in star across a 3ϕ, 430V, 50Hz supply. Calculate the line current (I_L), total power absorbed (P), reactive volt amperes ($Q \rightarrow VAR$), and total volt-amperes ($S \rightarrow VA$)

Solution:



a) Impedance of each coil: Zph

b) $Z_{ph} = R + jX_L$

c) $R = 8\Omega$, $L = 0.02H$

d) $X_L = 2\pi f L = 100\pi \times 0.02 = 6.28\Omega$

e) $Z_{ph} = (8 + j6.28)\Omega = \underbrace{10.17}_{Z_{ph}} \angle \underbrace{38.13^\circ}_{\phi}\ \Omega$

f) In a star connected load, $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{430}{\sqrt{3}} = 248.26V$

g) Phase current, $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{248.26}{10.17} = \underline{24.4A}$

h) Line current, $I_L = I_{ph} = \underline{24.4A}$

i) Total power absorbed, $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 430 \times 24.4 \times \cos(38.13)$

$P = 14.3 \times 10^3 W = \underline{14.3 kW}$

j) Reactive volt ampere, $Q = \sqrt{3} V_L I_L \sin\phi = \sqrt{3} \times 430 \times 24.4 \times \sin(38.13)$

$Q = 11.22 \times 10^3$ VAR $= \underline{11.22\ K\ VAR}$

h) Volt ampere, $S = \sqrt{3} V_L I_L = \sqrt{3} \times 430 \times 24.4$

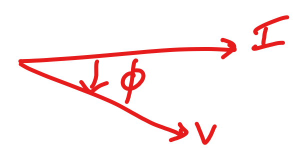
$S = 18.18 \times 10^3$ VA $= \underline{18.18\ KVA}$

Numerical 3 : A balanced 3ϕ Load connected in delta, draws a power of 10KW at 440V at a p.f of 0.6 leading. Find the values of circuit elements & reactive power drawn

Solution: Given: $P = 10KW$, $V_L = 440V$, $p.f = 0.6$ leading

(load is capacitive)

(I leading the voltage)



R–C series

① For delta connection, $V_L = V_{ph} = 440V$

② $P = \sqrt{3}\, V_L I_L \cos\phi$; $p.f = \cos\phi = 0.6$

$$I_L = \frac{P}{\sqrt{3}\, V_L (p.f)} = \frac{10\times10^3}{\sqrt{3}\times440\times0.6}$$

$$I_L = \underline{21.87A}$$

③ Phase current, $I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{21.87}{\sqrt{3}} = \underline{12.62A}$

④ $Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{440}{12.62} = \underline{34.86\Omega}$

$\overline{Z_{ph}} = Z_{ph}\angle\phi$

$\overline{Z_{ph}} = R - jX_C$

⑤ Phase angle of Z_{ph} (ϕ):-

$0.6 = p.f = \cos\phi$ leading

$\phi = \cos^{-1}(0.6) = -53.13^\circ$ (as p.f is leading, phase angle is -ve)

⑥ $\overline{Z_{ph}} = 34.86\angle{-53.13^\circ}\ \Omega$ --- $\overline{Z_{ph}} = Z_{ph}\angle\phi$

⑦ $\overline{Z_{ph}} = (\underbrace{20.91}_{R} - j\underbrace{27.89}_{X_C})\ \Omega$ --- $\overline{Z_{ph}} = R - jX_C$

⑧ $R = 20.91\Omega$, $X_C = 27.89\Omega$

X_C ↓ Capacitive reactance

$$X_C = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_C} = 114.18\mu F$$

($f = 50Hz$ assume)

⑨ Reactive power

$Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3}\times440\times21.87\sin(-53.13^\circ)$

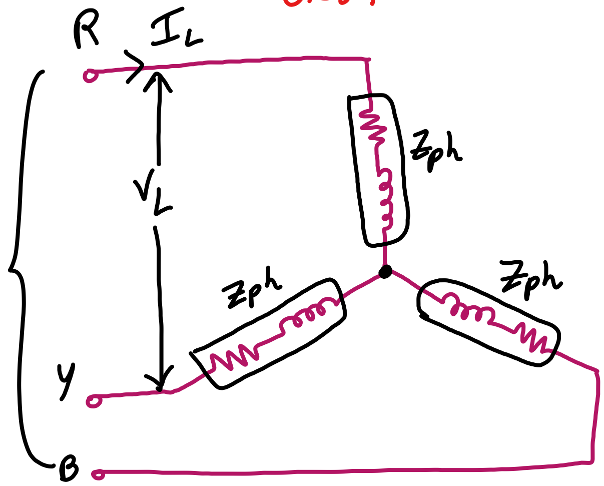
$Q = -13.33$ KVAR

H.W: 3 similar coils connected in star, take a power of 1.5 KW at a p.f of 0.2 lagging from a 3ϕ, 440V, 50Hz supply. Calculate resistance & inductance of each coil

5.165Ω

0.08H

Solution:



Given: $P = 1.5\,KW$, $p.f = 0.2$ (lag)

$V_L = 440V$, $f = 50Hz$

① For 3ϕ star connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} =$$

② $P = \sqrt{3}\,V_L I_L \cos\phi$; $p.f = \cos\phi$

i.e $$I_L = \frac{P}{\sqrt{3}\,V_L\,p.f} =$$

③ In a star connection, $I_{ph} = I_L =$

④ $$Z_{ph} = \frac{V_{ph}}{I_{ph}} =$$

⑤ Phase angle of Z_{ph}, ($p.f = \cos\phi$

⑥ $\overline{Z_{ph}} =$ ---- polar form

⑦ $\overline{Z_{ph}} =$ ---- rectangular form ($Z_{ph} = R + jX_L$)

i.e $R =$, $X_L =$

⑧ ∴ Resistance of each coil is and inductive reactance of each coil is

⑨ $X_L = 2\pi f L \rightarrow L = \dfrac{X_L}{2\pi f} =$

∴ Inductance of each coil is