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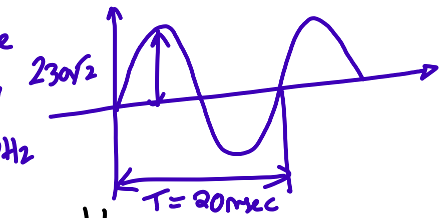
Module 2 : AC Circuits

Single phase supply 230V, 50Hz



Unit 2.1 : Generation of alternating voltage, average value, RMS value, form factor, phasor representation in rectangular and polar form, crest factor

① **Alternating current** : When the current in the circuit varies in magnitude as well as direction periodically → it is called "alternating current"

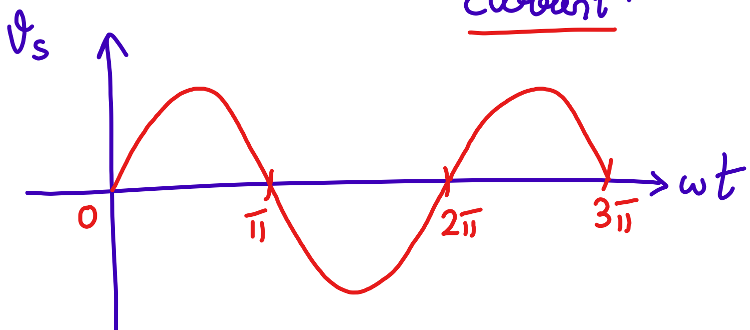


fig1: Sinusoidal alternating voltage

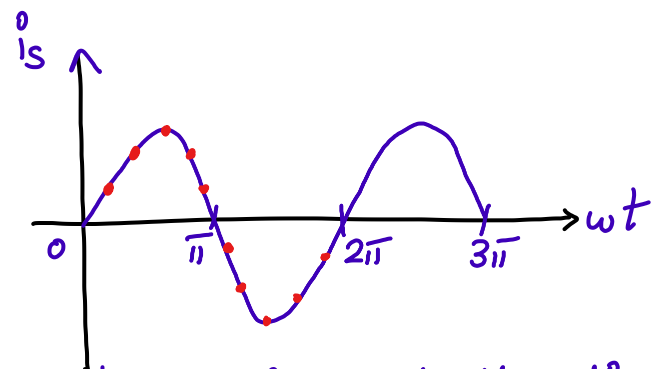
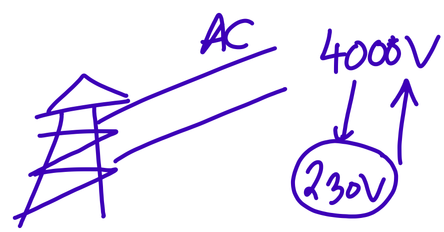


fig 2: Sinusoidal alternating current

② The circuits in which alternating currents flows are called "AC circuits"



③ **Advantages of AC** :

a) Voltages in AC systems can be raised or lowered with the help of transformer.

b) Used in AC motor — simpler — lesser maintanance

c) AC can be easily converted to DC

Single phase

Thus, AC is extensively used in practise (230-250V, 50Hz, 1ϕ)

in residencial areas.

* Generation of Alternating (AC) voltages:

① An alternating voltage can be generated either by

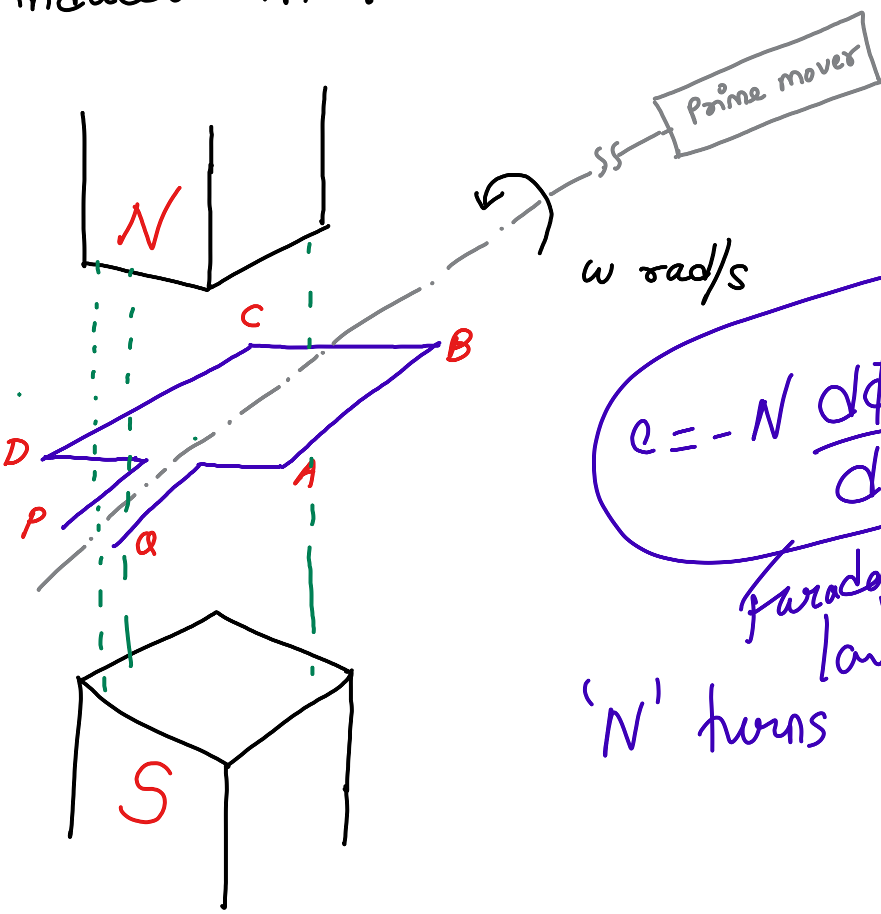
a) by rotating a coil in a stationary magnetic field OR

b) by rotating a magnetic field within a stationary coil

② In both cases, the magnetic field is cut by the coils & an emf is induced in the coil.

Faraday's law of electromagnetic induction

③ Scheme for generation of AC voltage is shown in fig3.



$$e = -N\frac{d\phi}{dt}$$

Faraday's law

'N' turns

fig 3: Generation of alternating voltage

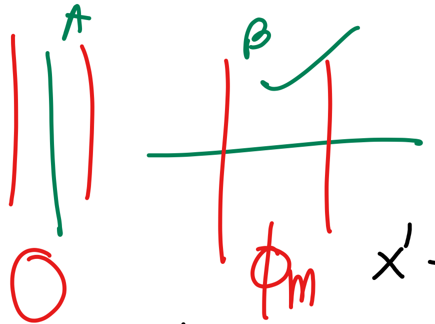
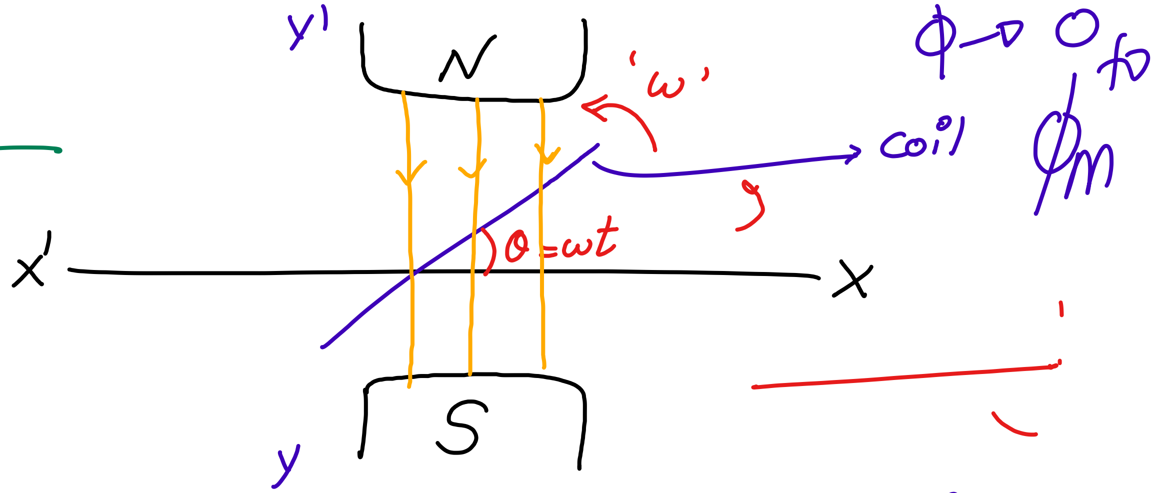


fig 4



(4) Here, it consists of permanent magnet with two poles

(5) A rectangular coil (ABCDPQ) of 'N' turns is rotating in anti-clockwise with angular velocity 'ω' radians per seconds in a uniform magnetic field

(6) When the coil cut's the magnetic line of flux → an emf gets induced in it.

(7) Let 'ϕ_m' be the max flux cutting the coil when it's axis coincides with XX' axis

(8) When the coil is along yy' i.e parallel to lines of flux → the flux linking with it is zero

(9) Coil rotates through an angle $\boxed{\theta = \omega t}$ at any instant 't'

a) θ - angle between motion of the coil & direction of the magnetic field

(10) The flux Linkage of the coil is given by,

$\omega t = \theta$

$\omega = 2\pi f = \frac{2\pi}{1}$

$$\phi = \phi_m \cos\omega t = \phi_m \cos\theta \quad \text{— (1)}$$

(11) As per Faraday's law of electromagnetic induction,

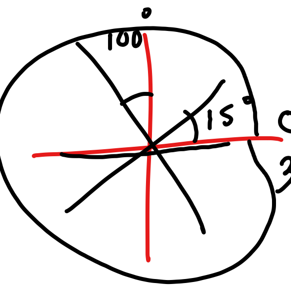
$$e = -N \frac{d\phi}{dt}$$

$$\boxed{v = V_m \sin\omega t}$$

$$\text{i.e } e = -N \frac{d}{dt}(\phi_m \cos\omega t)$$

$$\text{i.e } e = \boxed{N \phi_m \omega} \sin\omega t$$

"$\omega t = \theta$"



$$\text{i.e } \boxed{e = E_m \sin\omega t} \quad \text{— (2)}$$

$e(\omega t)$ $e = E_m \sin\omega t$
$= E_m \sin\theta$

where, $E_m = N \phi_m \omega$
↳ maximum value of induced emf

(12) 'e' is the instantaneous value of the induced emf (voltage) in a coil

$$\boxed{e = E_m \sin\omega t}$$
$\sin\theta$

a) e depends on θ ($\theta = \omega t$)

b) As the coil continously rotates with constant angular velocity 'ω' (i.e $\theta = \omega t$ varies from $0°$ to $360°$) the magnitude of induced emf in a coil varies continously.

$$\boxed{e = E_m \sin\theta = E_m \sin\omega t}$$

(13) $\theta = 0°$ → no emf → $e = 0$

$\theta = 0$ to $360°$ → coil complete one rotation

(14) Taking different instants ie position of the coil → the induced emf in the coil is calculated as follows:-

$$\boxed{e = E_m \sin\theta = E_m \sin\omega t}$$

$90 \times \frac{\pi}{180^\circ}$

i) $\theta = 0^\circ \rightarrow \sin 0^\circ \rightarrow e = 0V$

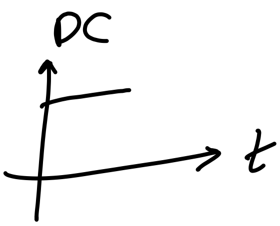
ii) $\theta = 90^\circ \rightarrow (\omega t = \frac{\pi}{2}) \rightarrow \sin 90^\circ \rightarrow e = E_m$ Volts

iii) $\theta = 180^\circ \rightarrow \sin(180^\circ) \rightarrow e = 0V$
$(\omega t = \pi)$

iv) $\theta = 270^\circ \rightarrow \sin(270^\circ) \rightarrow e = -E_m$ volts
$(\omega t = \frac{3\pi}{2})$

v) $\theta = 360^\circ \rightarrow \sin(360^\circ) \rightarrow e = 0V$
$(\omega t = 2\pi)$

$$\boxed{e = E_m \sin\omega t} \; \boxed{E_m \sin\theta}$$



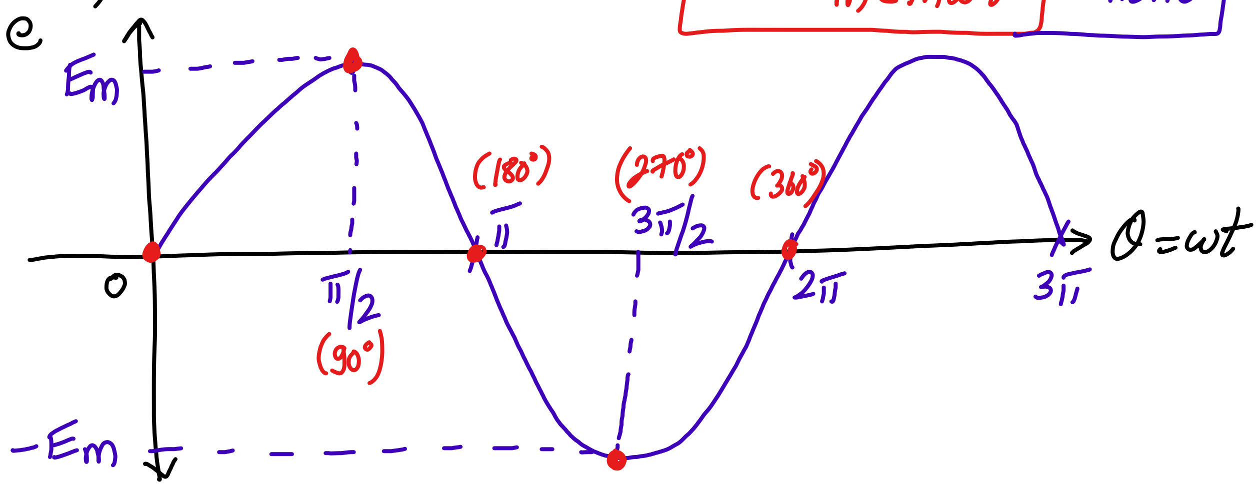


fig4: Generated sinusoidal alternating voltage

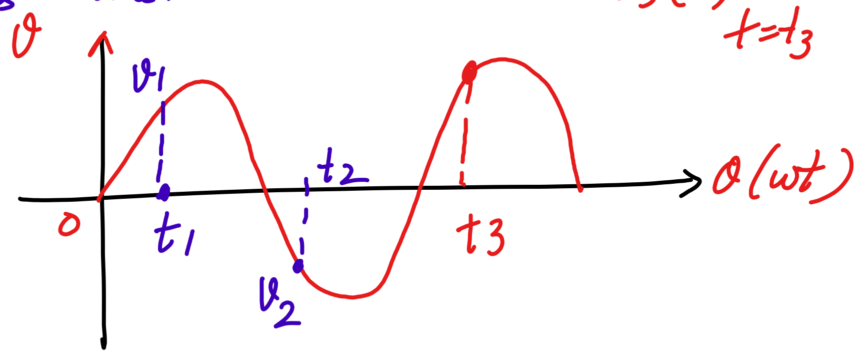
(14) By plotting induced emf 'e' against the position of the coil (i.e $\theta = \omega t$), we get sinusoidal alternating waveforms as shown in fig 4

(15) Thus, the alternating voltage can be generated by rotating a coil with constant velocity 'ω' in a uniform magnetic field

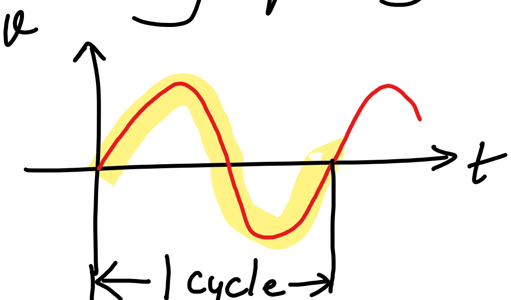
* AC Terminology:

① **Instantaneous value:** The value of the alternating quantity at a particular instant of time is known as its instantaneous value.

eg $v_1(t)$ & $v_2(t)$ $\qquad$ $v_3(t)\big|_{t=t_3}$



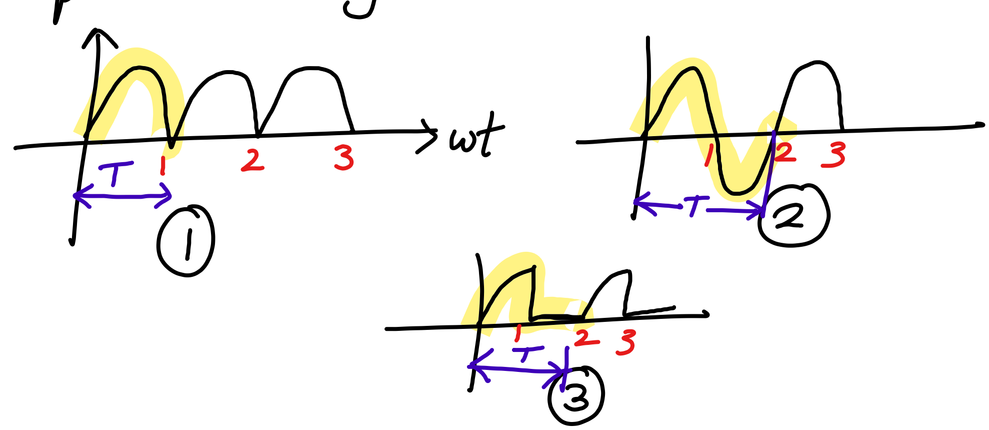
② **Cycle:** One complete set of positive and negative values of an alternating quantity is known as cycle



eg 1 cycle of sinusoidal AC voltage

③ **Time period: (T):** The time taken by an alternating quantity to complete one cycle is known as its time period.

time for 1 cycle (sec/cycle)



④ **Frequency (f):** The number of cycles completed by an alternating quantity per second is known as it's frequency.
It is denoted by 'f'

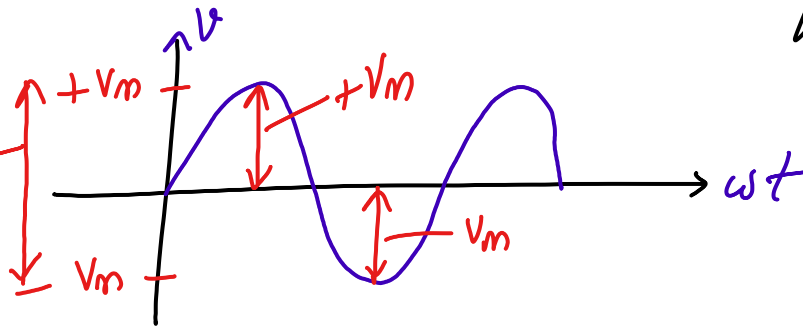
It is measured in cycles/seconds or hertz (Hz)

Frequency is reciprocal of the time period "50Hz"

$$\boxed{f = \frac{1}{T}} \text{ Hz} \qquad T = 20\text{msec}$$

⑤ **Amplitude:** The maximum +ve or -ve value of an alternating quantity

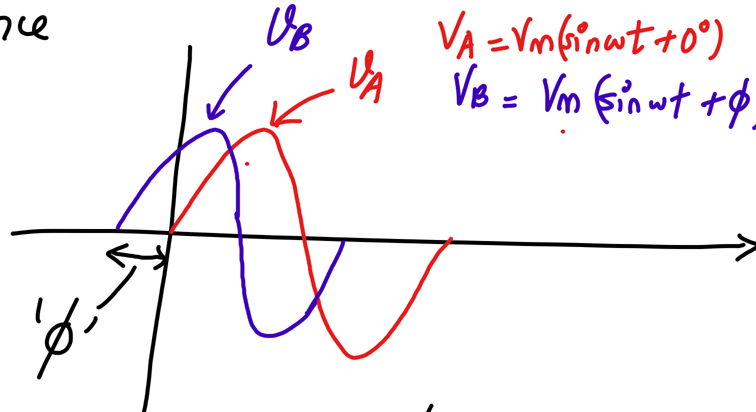
Peak to peak value of AC s/g → $2V_m$



⑥ **Phase:** The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference

⑦ **Phase difference (ϕ):**
↳ Two alternating quantities are said to be in phase when they reach their maximum & zero values at the same time

$V_A = V_m(\sin\omega t + 0°)$
$V_B = V_m(\sin\omega t + \phi)$



ϕ - phase angle

$$\left.\begin{aligned} v_A &= V_m \sin\omega t \\ v_B &= V_m \sin(\omega t + \phi) \end{aligned}\right\} \rightarrow v_B \text{ leads } v_A \text{ by phase angle } \phi$$

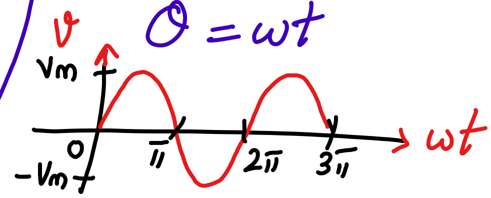
* Standard forms of Alternating quantities:

$$v = V_m \sin\theta$$
$$v = V_m \sin\omega t \quad (\theta = \omega t)$$
$$v = V_m \sin 2\pi f t \quad (\omega = 2\pi f)$$
$$v = V_m \sin\left(\frac{2\pi}{T}\right) t \quad \left(f = \frac{1}{T}\right)$$

$\rightarrow$ AC voltage

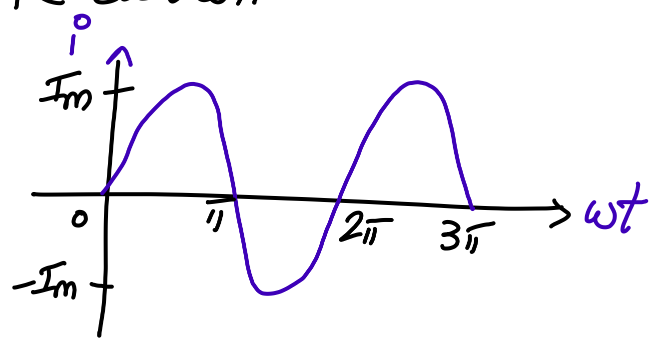
$$\omega = \frac{2\pi}{T}$$

$$\theta = \omega t$$



$$i = I_m \sin\theta$$
$$i = I_m \sin\omega t$$
$$i = I_m \sin(2\pi f) t$$
$$i = I_m \sin\left(\frac{2\pi}{T}\right) t$$

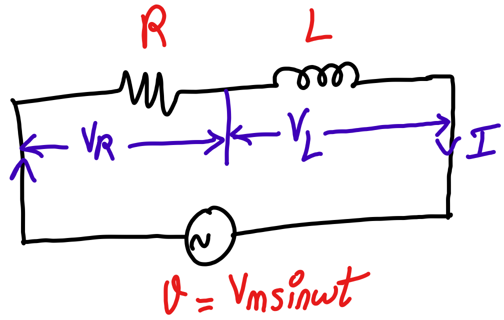
$\rightarrow$ AC current



—— x ——

$V_R = IR$
$V_L = IX_L$
$V = IZ$

RL series circuit :



RL series circuit

① Consider a resistor 'R' Ω & an inductor 'L' H connected in series across an AC supply

② Let V = rms value of applied voltage
I = rms value of circuit current
Z = impedance of circuit

③ By ohm's law,

$$\boxed{\begin{aligned} V_R &= IR \\ V_L &= IX_L \\ V &= IZ \end{aligned}}$$

$V_R = IR$ → V_R is in phase with I

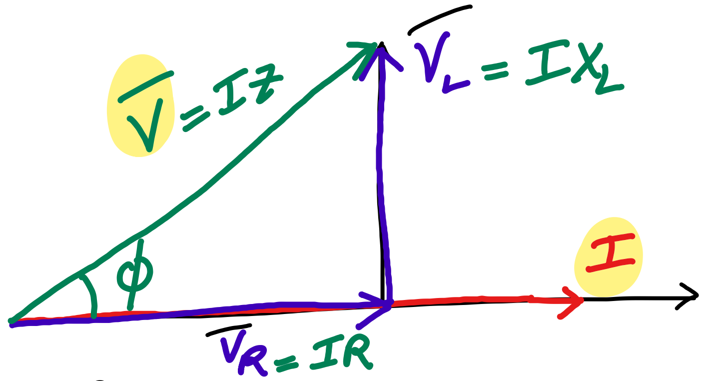
$V_L = IX_L$ → V_L leads I by 90°

④ Phasor diagram

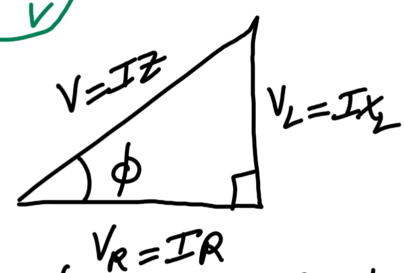
$\bar{V} = \bar{V}_L + \bar{V}_R$

$\tan\phi = \dfrac{V_L}{V_R}$

$\cos\phi = \dfrac{V_R}{V}$



RL series ckt



Voltage triangle

a) ϕ is the phase angle of the circuit

b) From phasor diagram → Current 'I' lags behind the applied voltage 'V' by ϕ $(\phi < 90°)$

c) Nature of the circuit is Inductive

(5) $\tan\phi = \frac{V_L}{V_R} \rightarrow \phi = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{IX_L}{IR}\right)$

i.e $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

(6) Power factor (p.f) of the circuit

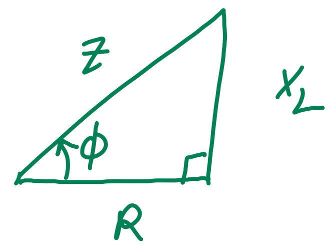
$p.f = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$

$\boxed{p.f = \frac{R}{Z}}$ lagging

(7) As the circuit current 'I' lags behind the applied voltage 'V' $\rightarrow$ the nature of p.f is lagging

(8) Impedance (Z):

$Z^2 = R^2 + X_L^2$



Impedance triangle

a) Total opposition to the flow of AC current is Z

b) In voltage triangle →
→ Divide each voltage phasor by I
↓
we get Impedance triangle

c) From impedance triangle,

$\boxed{Z = (R^2 + X_L^2)^{1/2}}\ \Omega$

d) Impedance Z can be expressed in rectangular & polar form

$\bar{Z} = (R + jX_L)\ \Omega$ .... rectangular form

$$\bar{Z} = Z \angle \phi \ \Omega \quad \text{---- Polar form}$$

where,

$$Z = \sqrt{R^2 + X_L^2} \ ; \ \phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

g) Power (P) :

a) Power that is actually consumed in the circuit is called "Active or True" Power (in case of R)

b) Circulating power is called "Reactive power" (in case of L or C)

c) In case of resistance → current 'I' is in phase with voltage 'V'

d) In case of inductance & capacitive
↳ Current 'I' is 90° out of phase with voltage

e) Active power 'P' = V × I (in phase with voltage)

Reactive power 'Q' = V × I (90° out of phase with voltage)

f) Circulating power → A pure inductor & capacitor consume no power because all power received from AC source in a half cycle is returned to the source in the next half cycle → This is circulating power

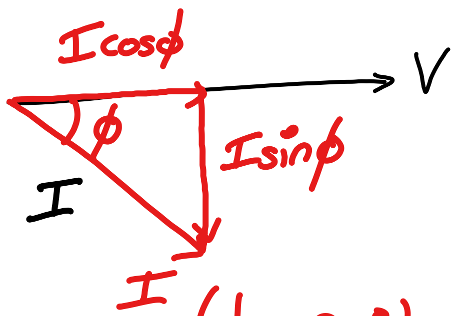
9) Apparent power (S): It is the product of voltage & current in AC circuit

(10) Taking V as reference phasor

→ Circuit current 'I' can be resolved into two components

1) $I\cos\phi$ is in phase with voltage

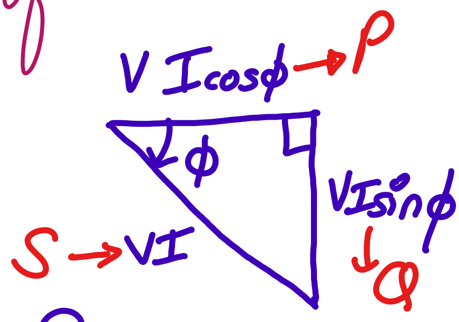
2) $I\sin\phi$ is $90°$ out of phase with voltage



Phasor diagram figure '1'

(11) In figure 1, if we multiply each of current phasor by V, we get the power triangle

$$\boxed{S^2 = P^2 + Q^2}$$



Power triangle

(12) Active power, $P = VI\cos\phi$ unit is W or KW

Reactive power, $Q = VI\sin\phi$ unit is VAR or KVAR

Apparent power, $S = VI$ unit is VA or KVA

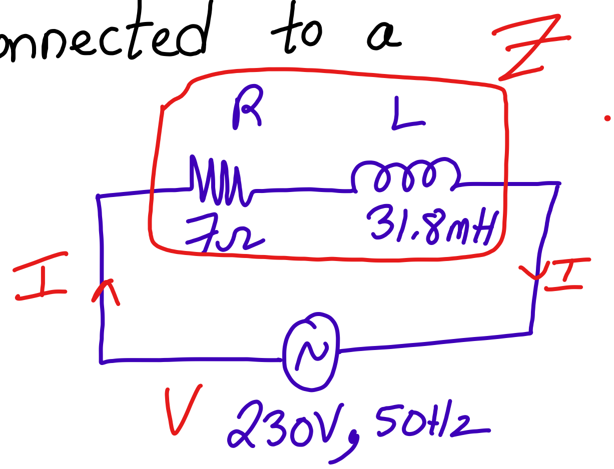
(volt-ampere-reactive)

(volt-ampere)

$$\boxed{\cos\phi = \frac{P}{S}}$$

$$\boxed{\begin{aligned} P &= I^2 R \\ Q &= I^2 X_L \\ S &= I^2 Z \end{aligned}}$$

Numerical 1: A coil has a resistance of 7Ω & an inductance of 31.8mH is connected to a 230V, 50Hz supply



Calculate : i) Circuit current (I)

ii) Phase angle (ϕ)

iii) Power factor (p.f)

iv) Power consumed (P)

Solution: (1) X_L (2) Z (3) $I = \frac{V}{Z}$ (4) ϕ

(5) $p.f = \cos\phi$ (6) $P = VI\cos\phi = I^2 R$

(1) $X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\Omega$

(2) $Z^2 = R^2 + X_L^2 = 7^2 + 10^2 = 149 \rightarrow Z = \sqrt{149}$

$Z = 12.21\Omega$

③ $I = \frac{V}{Z} = \frac{230}{12.21} = 18.84A$

④ Phase angle $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{10}{7}\right)$

$\phi = 55°$

⑤ $p.f = \cos\phi = \cos 55° = 0.574$ lagging

⑥ $P = VI\cos\phi = 230 \times 18.84 \times 0.574 = 2487\ W$

$P = I^2R = I^2R = 18.84^2 \times 7 = 2487\ W$

OR (Different method):

① $\bar{V} = 230\angle 0°\ V$

② $\bar{Z} = R + jX_L = 7 + 10j\ \Omega$

Shif Pol(7,10)

③ $\bar{Z} = 12.20\angle 55°\ \Omega$ $(\bar{Z} = Z\angle\phi)$

④ $\phi = 55°$

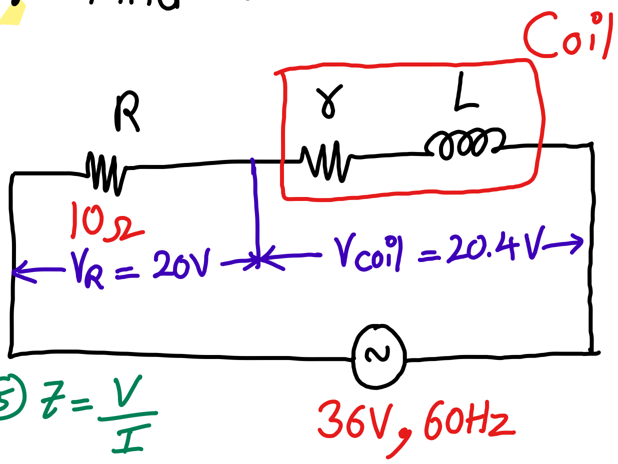
⑤ $\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{230\angle 0°}{12.20\angle 55°} = 18.85\angle -55°A$

⑥ $p.f = \cos\phi = \cos(55°) = 0.574$ lagging

⑦ $P = VI\cos\phi = \underline{2487W}$

Numerical below

Numerical 2: Find r and L



$V_{coil} = I Z_{coil}$

$V_R = I R$

① $I = \frac{V_R}{R}$

② $Z_{coil} = \frac{V_{coil}}{I}$

③ $Z_{coil}^2 = r^2 + X_L^2$

④ $Z^2 = (R+r)^2 + X_L^2$

③ & ④ ; ⑤ $Z = \frac{V}{I}$

$r \rightarrow L$

Solution:

① $V_R = IR \rightarrow I = \frac{V_R}{R} = \frac{20}{10} = 2A$

② $V_{coil} = I Z_{coil} \rightarrow Z_{coil} = \frac{V_{coil}}{I} = \frac{20.4}{2} = 10.2\Omega$

③ $Z_{coil}^2 = r^2 + X_L^2 = 10.2^2$

i.e $r^2 + X_L^2 = 10.2^2$ — ①

④ $V = IZ \rightarrow Z = \frac{V}{I} = \frac{36}{2} = 18\Omega$

⑤ $Z^2 = (R+r)^2 + X_L^2$

$18^2 = (10+r)^2 + X_L^2$ — ②

⑥ Eqn ② − Eqn ① $\rightarrow (10+r)^2 + \cancel{X_L^2} - r^2 - \cancel{X_L^2} = 18^2 - 10.2^2$

i.e $100 + 20r + \cancel{r^2} - \cancel{r^2} = 219.96$

$20r = 119.96$

$\boxed{r \approx 6\Omega}$

⑦ From eqn ①, $r^2 + X_L^2 = 10.2^2$

i.e $6^2 + X_L^2 = 10.2^2$

$X_L^2 = 10.2^2 - 6^2 = 68.04$

$X_L = 8.25\Omega$

⑧ $X_L = 2\pi f L \rightarrow L = \frac{X_L}{2\pi f} = \frac{8.25}{2\pi \times 50}$

$\boxed{L = 0.022\,H}$

——x——

Numerical 3 : A current of 5A flows through pure resistance and a coil supplied at 250V, 50Hz
"H.W"
If the voltage across the resistance is 125V and across the coil is 200V

Calculate : a) Impedance, reactance & resistance of coil

b) Power absorbed by the coil

c) Power factor of the coil & circuit

d) Draw phasor diagram

Solution :

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25/9/23

Numerical 1 : The voltage applied to a circuit is $v = 100\sin(\omega t + 30^\circ)$ and the current flowing in the circuit is $i = 15\sin(\omega t + 60^\circ)$. Find impedance, resistance, reactance, power factor and power consumed.

Given: $v = 100\sin(\omega t + 30^\circ)$

$i = 15\sin(\omega t + 60^\circ)$

To find: Z, R, X, $p.f$ and P

Thinking

$\downarrow$

$v = V_m\sin(\omega t + \phi)$

$V_{rms} = \frac{V_m}{\sqrt{2}}$

Polar form

$\bar{V} = V_{rms}\angle\phi^\circ$

$\bar{I} = I_{rms}\angle\phi^\circ$

$\bar{Z} = \frac{\bar{V}}{\bar{I}}$

$Z = R \pm jX$

$R =$

$X =$

$p.f = \cos\phi$

$P = VI\cos\phi$

Solution:

① $v = V_m\sin(\omega t + \phi)$

$v = 100\sin(\omega t + 30^\circ)$

$V_m = 100V$, $\phi = 30^\circ \rightarrow V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}}\,V$

② Writing 'V' in Polar form

$\bar{V} = V_{rms}\angle\phi$

$\bar{V} = \frac{100}{\sqrt{2}}\angle 30^\circ\ V$

$\bar{V} = \underbrace{V}_{rms}\angle\phi$

③ $i = I_m\sin(\omega t + \phi) = 15\sin(\omega t + 60^\circ)$

$I_m = 15$; $I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{15}{\sqrt{2}}\ A$

④ Writing I in polar form, $\bar{I} = \underbrace{I}_{rms}\angle\phi$

$\bar{I} = \frac{15}{\sqrt{2}}\angle 60^\circ\ A$

60°)

$\bar{V} = \bar{I}\bar{Z}$

⑤ $\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{\frac{100}{\sqrt{2}}\angle 30^\circ}{\frac{15}{\sqrt{2}}\angle 60^\circ} = 6.67\angle -30^\circ\ \Omega$

⑥ $\bar{Z} = (5.776 - j3.33)\,\Omega$ Rec $(6.67, -30)$

$\bar{Z} = R \pm jX$

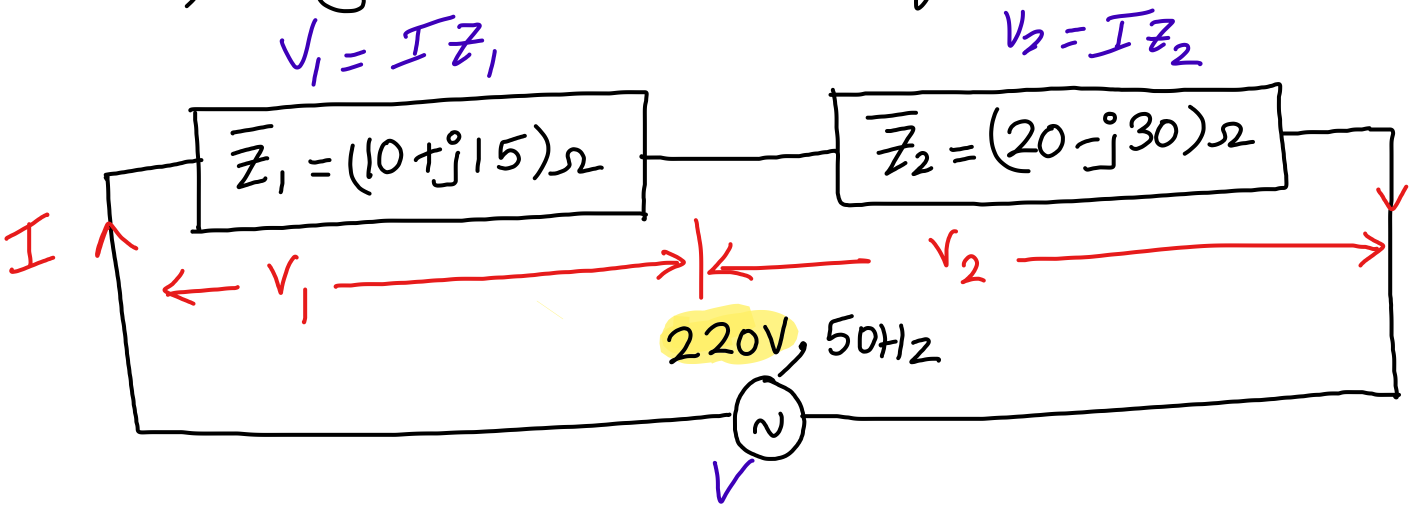
⑦ $R = 5.77\,\Omega$
$X_C = 3.33\,\Omega$

{ i leading V by $30° \rightarrow$ ckt is capacitive
$\bar{Z} = R - jX_C$ }

⑧ $p.f = \cos\phi = \cos(-30°) = 0.868$ (leading)

⑨ $P = VI\cos\phi = \frac{100}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \times 0.868 = \underline{649.5\,W}$

Numerical 2 : For the circuit shown in figure below, find current I, voltages V_1 & V_2 and power factor



Given: $\bar{Z}_1 = (10 + j15)\,\Omega$, $V_{rms} = V = 220V$
$\bar{Z}_2 = (20 - j30)\,\Omega$, $f = 50Hz$

To find : I, V_1, V_2 and p.f

Solution : $\bar{Z}_1 = 18.03\angle 56.31°$ $\bar{Z}_2 = 36.05\angle -56.31°$

① $\bar{Z}_1 = 10 + j15\,\Omega$; $\bar{Z}_2 = 20 - j30\,\Omega$

② $\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = (30 - 15j)\,\Omega$

③ $\bar{Z} = 33.54\angle -26.56\,\Omega$ → Pol$(30, -15)$

④ $\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{220\angle 0°}{33.54\angle -26.56}$

Thinking
$\bar{Z}_1, \bar{Z}_2 \rightarrow$ given
$\bar{Z} = \bar{Z}_1 + \bar{Z}_2$
$\bar{Z}$ in polar form
$\bar{Z} = Z\angle\phi°\,\Omega$
$\bar{I} = \frac{\bar{V}}{\bar{Z}}$
$V_1 = I\bar{Z}_1$, $V_2 = I\bar{Z}_2$
$p.f = \cos\phi$

$$\bar{I} = 6.56\angle 26.56^\circ \text{ A}$$

(5) $\bar{V}_1 = \bar{I}\,\bar{Z}_1 = (6.56\angle 26.56^\circ) \times (18.03\angle 56.31^\circ)$

polar polar

$$\bar{V}_1 = 118.27\angle 82.87^\circ \text{ V}$$

(6) $\bar{V}_2 = \bar{I}\,\bar{Z}_2 = (6.56\angle 26.56^\circ) \times (36.05\angle -56.31^\circ)$

$$\bar{V}_2 = 236.48\angle -29.75^\circ \text{ V}$$

(7) $p.f = \cos\phi = \cos(26.56^\circ) = 0.894$ leading

Numerical 3 : An inductive coil having inductance of 0.04H and resistance of 25Ω has been connected in series with another inductive coil of inductance 0.2H and resistance 15Ω. The whole circuit is powered with 230V, 50Hz supply. Calculate the power dissipation in each coil and the total power factor

Given: $r_1 = 25\Omega$, $L_1 = 0.04H$

H.W $r_2 = 15\Omega$, $L_2 = 0.2H$

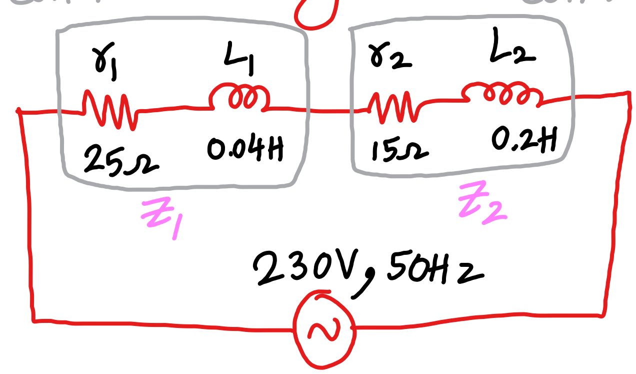
$V = 230V$, $f = 50Hz$

rms value

To find : P_1, P_2 and $p.f_{total}$

Thinking:

Coil 1 Coil 2



$\bar{Z}_1 = r_1 + jX_{L_1}$; $\bar{Z}_2 = r_2 + jX_{L_2}$

$\bar{Z} = \bar{Z}_1 + \bar{Z}_2$; $I = \frac{\bar{V}}{\bar{Z}}$ (Polar)

$P_1 = I^2 r_1$, $P_2 = I^2 r_2$

$p.f = \cos\phi$

Numerical 4: A capacitor of 35μF is connected in series with a variable resistor. The circuit is connected across 50Hz, 230V mains. Find the value of a resistor for a condition when the voltage across the capacitor (V_C) is half the supply voltage (V)

$$\boxed{V_C = \frac{1}{2}V}$$

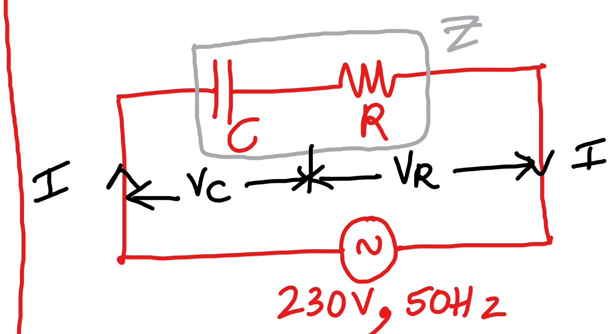
Given: $C = 35\times10^{-6}F$, $V = 230V$ ← rms value

$f = 50Hz$

To find: R

Solution:

Thinking



$$V_C = \frac{1}{2}V$$

$$IX_C = \frac{1}{2}IZ$$

$$X_C = \frac{1}{2\pi fC}$$

$$Z = \sqrt{R^2+X_C^2}$$

$$R^2 = Z^2 - X_C^2$$

ⓐ $X_C = \dfrac{1}{2\pi fC}$

$$X_C = \frac{1}{2\pi\times50\times35\times10^{-6}}$$

$$X_C = 90.947\Omega$$

ⓑ $V_C = \dfrac{1}{2}V$

$V_C = IX_C$

$V = IZ$

$$\cancel{I}X_C = \frac{1}{2}\cancel{I}Z$$

$$Z = 2X_C = 2\times90.947\Omega$$

$$Z = 181.89\Omega$$

③ $Z^2 = R^2 + X_C^2$

$$R^2 = Z^2 - X_C^2 = 181.89^2 - 90.947^2$$

$$R^2 = 24812.6$$

$$\boxed{R = 157.52\Omega}$$

—x—

Numerical 5 : A resistance and a capacitance in series is connected across 250V supply draws 5.8A current at a frequency of 60Hz. Find the value of capacitor and also power drawn and power factor. Given resistor value is 20Ω

Given: $R = 20\Omega$, $V = 250V$, $I = 5.8A$, $f = 60Hz$

Solution:

$$V = IZ$$

① $V = 250V$

$I = 5.8A$

$$Z = \frac{V}{I} = \frac{250}{5.8} = 43.103\Omega$$

$$X_c = \frac{1}{2\pi fC}$$

② $Z^2 = R^2 + X_c^2$

$$X_c^2 = Z^2 - R^2 = 43.103^2 - 20^2$$

$$X_c^2 = 1457.86$$

$$X_c = 38.18\Omega$$

③ $X_c = \frac{1}{2\pi fC} \rightarrow C = \frac{1}{2\pi f X_c}$

$$C = \frac{1}{2\pi \times 60 \times 38.18} = 69.4\,\mu F$$

④ $\phi = \tan^{-1}\left(\frac{X_c}{R}\right) = \tan^{-1}\left(\frac{38.18}{20}\right)$

$$\phi = 62.35^\circ$$

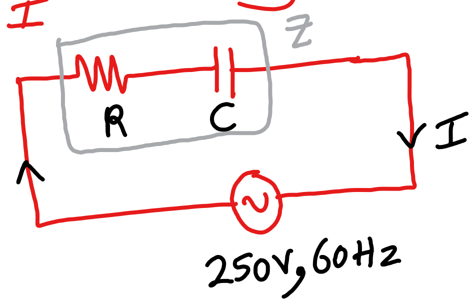
⑤ $p.f = \cos\phi = \cos(62.35) = 0.4640$ Leading

⑥ $P = VI\cos\phi = 250 \times 5.8 \times 0.464$

$$\boxed{P = 672.8\ W}$$

Thinking

$Z = \frac{V}{I}$



Z, $Z^2 = R^2 + X_C^2$

$X_C \rightarrow C$,

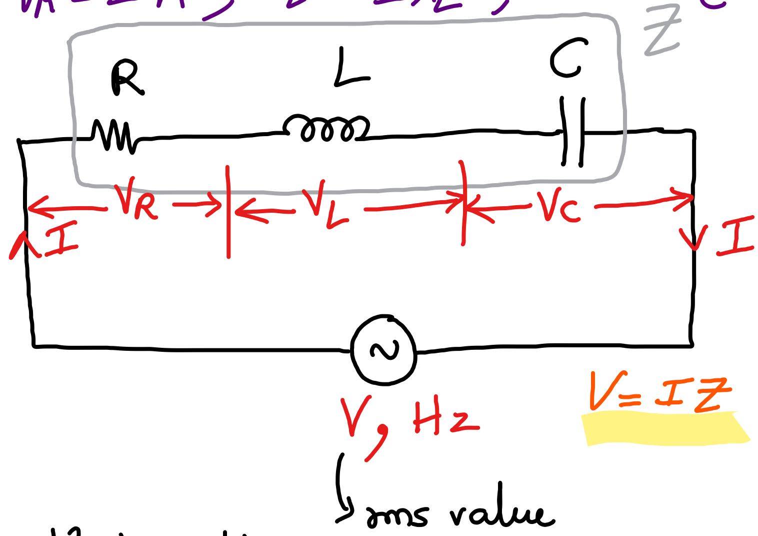
$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

$p.f = \cos\phi$

$P = VI\cos\phi$

* RLC series circuit :

$V_R = IR$; $V_L = IX_L$; $V_C = IX_C$



$V = IZ$

① Consider an AC circuit containing R, L, C are connected in series across an AC supply

② Let V = rms value of applied voltage

I = rms value of circuit current

Z = impedance of the circuit

③ By ohm's law,

$V_R = IR$; where V_R is in phase with I ✓

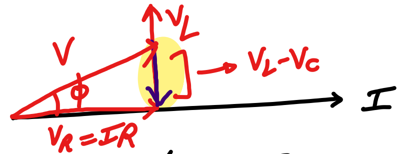
$V_L = IX_L$; V_L leads I by $90°$ ✓

$V_C = IX_C$; V_C lags behind I by $90°$ ✓

$V = IZ$

④ RLC series circuit can be effectively inductive, capacitive or resistive depending upon the values of X_L and X_C

⑤ Case 1 : $X_L > X_C$



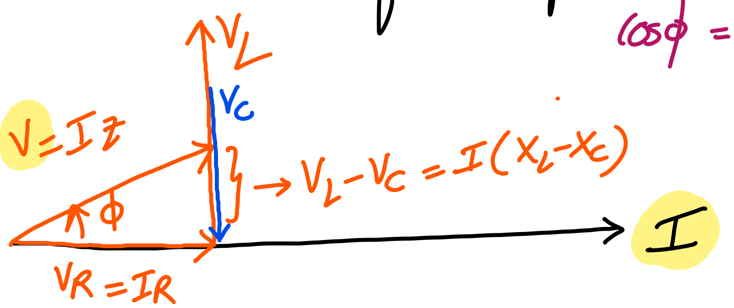
i) If $X_L > X_C$ $\longrightarrow$ then $V_C = IX_C$, $V_L = IX_L$ i.e $V_L > V_C$
i.e circuit behaves like an RL series circuit

ii) Phasor diagram $(X_L > X_C)$

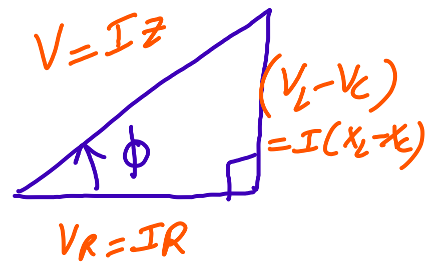
a) Taking current as reference point

$\tan\phi = \left(\frac{V_L - V_C}{V_R}\right)$

$\cos\phi = \frac{V_R}{V}$



Phasor diagram



Voltage triangle

b) It is clear from phasor diagram that the current I lags behind the applied voltage V by ϕ $(\phi < 90^\circ)$

c) $\therefore$ Nature of circuit is $\boxed{\text{Inductive}}$

d) Phase angle of the circuit,

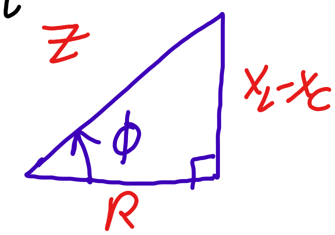
$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{\cancel{I}(X_L - X_C)}{\cancel{I}R}\right)$$ Refer voltage triangle

$$\boxed{\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)}$$

e) Power factor,

$$p.f = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \text{ lagging}$$

iii) Impedance (Z): Dividing each of voltage phasor by 'I' in a voltage triangle → we get impedance triangle $Z^2 = R^2 + (X_L - X_C)^2$



Impedance triangle (when $X_L > X_C$)

a) From impedance triangle,

$$Z^2 = R^2 + (X_L - X_C)^2$$

$$\boxed{Z = \sqrt{R^2 + (X_L\ X_C)^2}\quad \Omega}$$

b) The impedance can be expressed in rectangular and polar form as,

$$\bar{Z} = [R + j(X_L - X_C)]\,\Omega \rightarrow \text{Rectangular form}$$

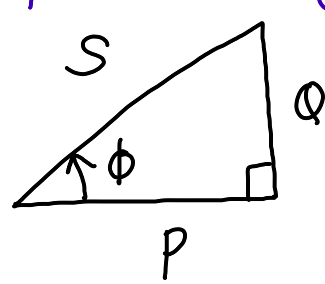
$$\bar{Z} = Z\angle\phi \rightarrow \text{Polar form}$$

where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

iv) Power triangle:

"Power triangle"



$(X_L > X_C)$

$$P = VI\cos\phi = I^2R$$

$$Q = VI\sin\phi = I^2X_L$$

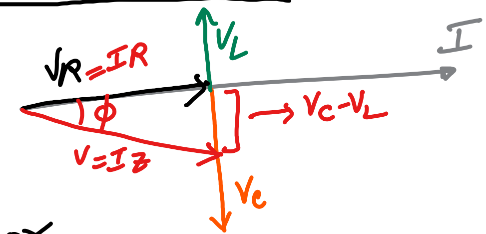
$$S = VI = I^2Z$$

$$S^2 = P^2 + Q^2$$

6) Case 2: $X_C > X_L$

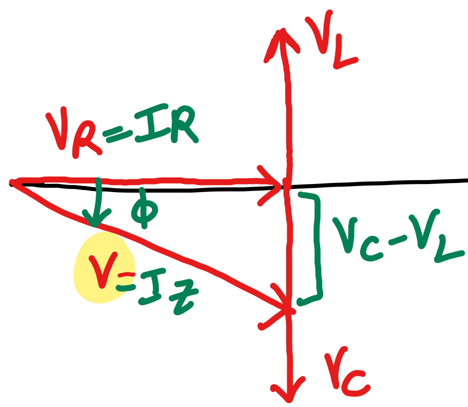
i) If $X_C > X_L \rightarrow$ then $V_C = IX_C$, $V_L = IX_L \rightarrow V_C > V_L$

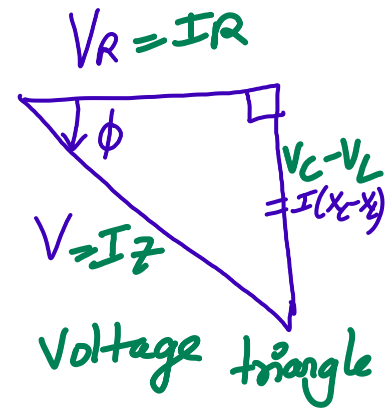
i.e circuit behaves like an RC series circuit



ii) Phasor diagram ($X_C > X_L$)

a) Taking current as a reference phasor





b) It is clear from phasor diagram $\rightarrow$ current I leads the applied voltage V by ϕ ($\phi < 90°$)

c) $\therefore$ Nature of the circuit is $\boxed{\text{Capacitive}}$

d) Phase angle of the circuit

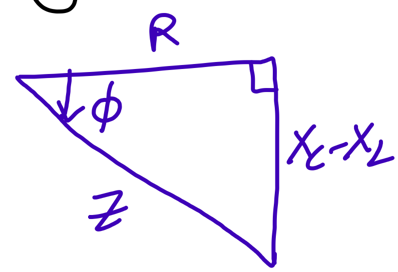
$$\phi = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right) = \tan^{-1}\left(\frac{I(X_C - X_L)}{IR}\right)$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)}$$

e) Power factor

$$p.f = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \text{ leading}$$

iii) Impedance (Z): Dividing each of the voltage phasor by I, we get impedance triangle



Impedance triangle (when $X_C > X_L$)

ⓐ From impedance triangle,

$$Z^2 = R^2 + (X_C - X_L)^2$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

ⓑ Circuit impedance (Z) can be expressed in rectangular and polar form as

$$\bar{Z} = R - j(X_C - X_L) \longrightarrow \text{Rectangular form}$$

$$\bar{Z} = Z \angle -\phi \ \Omega \longrightarrow \text{Polar form}$$

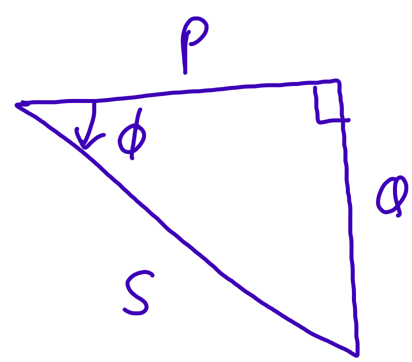
ⓒ Power triangle:

$$S^2 = P^2 + Q^2$$

$$P = VI\cos\phi = I^2 R$$

$$Q = VI\sin\phi = I^2 X_C$$

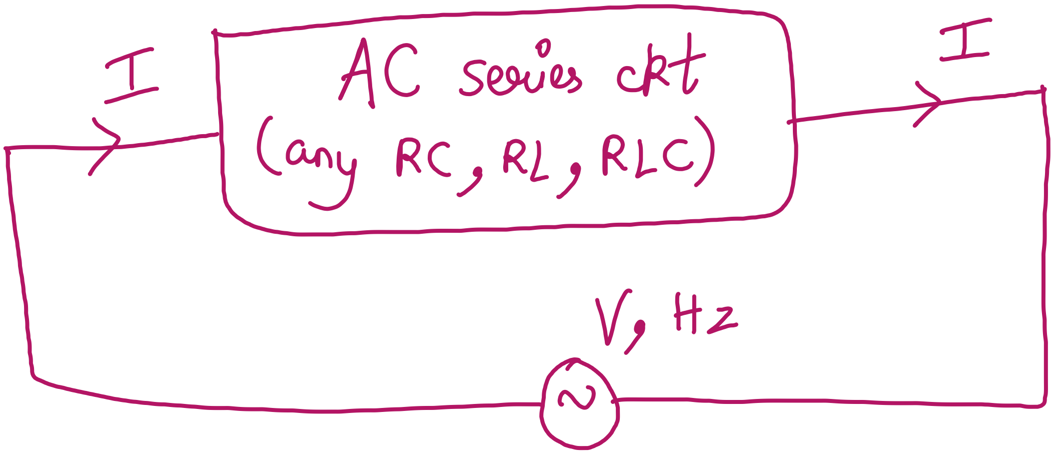
$$S = VI = I^2 Z$$



Power triangle ($X_C > X_L$)

— x —

* Analysis of AC Series circuit:



① Express circuit impedance ($\bar{Z}$) into it's rectangular & polar forms

$$\bar{Z} = (R \pm jX)\ \Omega$$

R → total resistance of the circuit

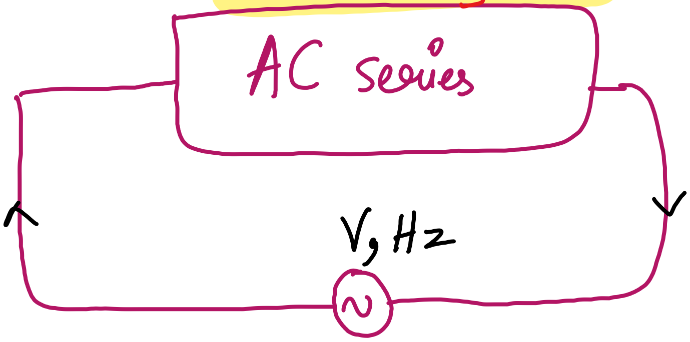
jX → imaginary part of impedance

X → Net reactance of the circuit

ⓐ If circuit is Inductive → $X_L > X_C$ → Imaginary part impedance will be positive

ⓑ If circuit is Capacitive → $X_C > X_L$ → Imaginary part impedance will be negative

$Z = (20 - j30)\,\Omega$



② For any unknown AC series circuit

↓

if rectangular form of (Z) impedance is given,

↓

then we get the following information

↓

a) Effective component of circuit

→ Real part of $\bar{Z}$ → is the total resistive of ckt

→ Imaginary part of $\bar{Z}$ → is the net reactance of ckt

b) Nature of the circuit

→ if imaginary part is +ve → then ckt is inductive

→ if imaginary part is -ve → then ckt is capacitive

$\bar{Z} = R + j(X_L - X_C)$

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

c) Phase angle of the circuit

$$\phi = \tan^{-1}\left(\frac{\text{Imaginary part of } \bar{Z}}{\text{Real part of } \bar{Z}}\right)$$

eg $\bar{Z} = (20 - 30j)\,\Omega$

a) $R = 20\,\Omega$, $X = 30\,\Omega$

b) Nature of ckt is capacitive

c) $\phi = \tan^{-1}\left(\frac{30}{20}\right) = \underline{56.31^\circ}$

$\bar{Z} = Z\angle\phi$

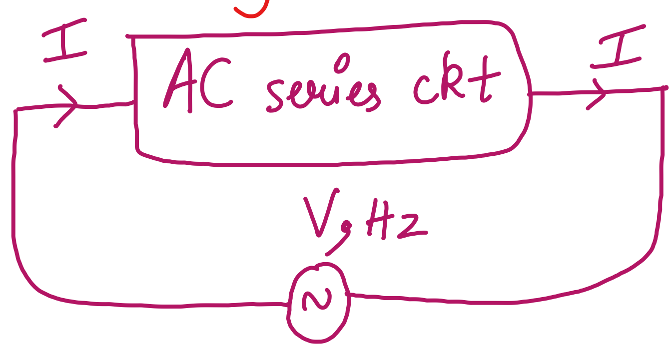
eg $Z = 10\angle 45^\circ\,\Omega$

③ For any unknown AC series circuit → if polar form of Z is given

↓

then we get the following information.

↓



a) Magnitude of impedance Z

b) Phase angle of the circuit

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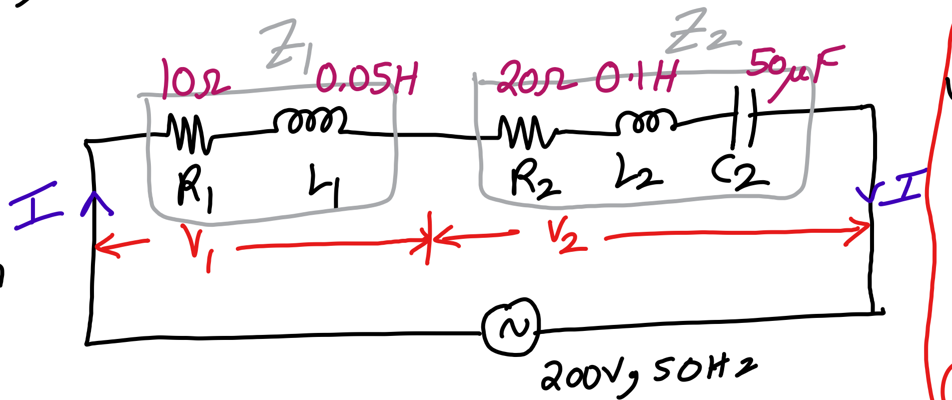
Numerical 2: For the circuit shown below, calculate
1) Circuit current 2) V_1 & V_2



$I = ?$
$V_1 = ?$, $V_2 = ?$

Thinking
① $\bar{I} = \frac{\bar{V}}{\bar{Z}}$
② $\bar{Z}_1$ ③ $\bar{Z}_2$
④ $\bar{V}_1 = \bar{I}\,\bar{Z}_1$
⑤ $\bar{V}_2 = \bar{I}\,\bar{Z}_2$

Solution:-

① $\bar{V} = 200\angle 0^\circ$ V , $R = R_1 + R_2 = 10 + 20 = 30\Omega$

② $X_{L1} = 2\pi f L_1 = 100\pi \times 0.05 = 15.71\Omega$

③ $X_{L2} = 2\pi f L_2 = 100\pi \times 0.1 = 31.42\Omega$

$X_L = X_{L1} + X_{L2}$
$X_L = 47.13\Omega$

④ $X_C = \frac{1}{2\pi f C_2} = \frac{1}{100\pi \times 50\times10^{-6}} = 63.66\Omega$

$X_C = 63.66\Omega$

⑤ As $X_C > X_L \rightarrow$ ckt is capacitive in nature

⑥ $X = X_C - X_L = 16.53\Omega$

⑦ Total impedance : $\bar{Z} = R - jX = (30 - j\,16.53)\Omega$

⑧ $\bar{Z} = 34.25\angle -28.85^\circ\Omega$ Shift Pol (30, -16.53)

$Z = 34.25\Omega$ $\phi = -28.85^\circ$

⑨ $\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{200\angle 0^\circ}{34.25\angle -28.85} = 5.84\angle 28.85^\circ$ A

⑩ $\bar{Z}_1 = R_1 + jX_{L1} = 10 + j15.71\Omega = 18.62\angle 57.52^\circ\Omega$

(11) $\bar{Z}_2 = R_2 - j(X_C - X_{L_2}) = 20 - j(63.66 - 31.42)$

$\bar{Z}_2 = (20 - j\,32.24)\,\Omega$

$\bar{Z}_2 = 37.94 \angle -58.18^\circ$

(12) $\bar{V}_1 = \bar{I}\bar{Z}_1 = (5.84 \angle 28.85^\circ)(18.62 \angle 57.52^\circ)$

$\bar{V}_1 = 108.74 \angle 86.37^\circ \text{ V}$

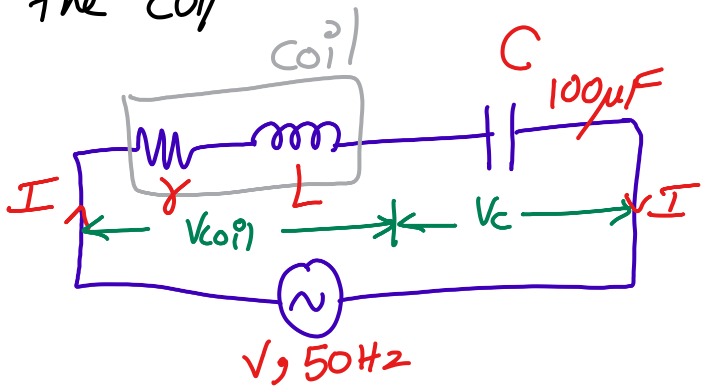
(13) $\bar{V}_2 = \bar{I}\bar{Z}_2 = (5.84 \angle 28.85^\circ)(37.94 \angle -58.18^\circ)$

$\bar{V}_2 = 221.57 \angle -29.34^\circ \text{ V}$

Numerical 3: A coil of 0.6 power factor in series with a 100μF connected to 50Hz supply. The voltage across the coil is equal to the voltage across the capacitor. Find inductance & resistance of the coil

Given: $C = 100\mu F$, $f = 50Hz$

$p.f_{coil} = 0.6$, $V_{coil} = V_C$



To find: L & r ?

Solution:

(1) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{100\pi \times 100\times 10^{-6}}$

$X_C = 31.83\,\Omega$

(2) $V_{coil} = V_C$

$\cancel{I} Z_{coil} = \cancel{I} X_C$

$Z_{coil} = X_C = 31.83\,\Omega$

Thinking

(1) $L = ?$ $\quad X_L = 2\pi f L$

(2) $Z_{coil}^2 = r^2 + X_L^2$

(3) $p.f_{coil} = \dfrac{r}{Z_{coil}}$

(4) $V_{coil} = V_C$

$I Z_{coil} = I X_C$

(5) $X_C = \dfrac{1}{2\pi f C} \longrightarrow$

③ $p.f_{coil} = 0.6 = \dfrac{r}{Z_{coil}} \rightarrow r = 0.6 \times 31.83$

$r = \underline{19.1\,\Omega}$

④ $Z_{coil}^2 = r^2 + X_L^2$

$\rightarrow X_L^2 = Z_{coil}^2 - r^2 = 31.83^2 - 19.1^2$

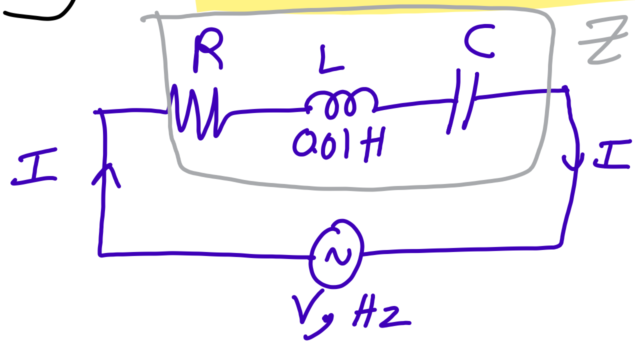
$X_L^2 = 648.41$

$X_L = \underline{25.46\,\Omega}$

⑤ $X_L = 2\pi f L \rightarrow L = \dfrac{X_L}{2\pi f} = \dfrac{25.46}{100\pi}$

$\boxed{L = 0.081\ H}$

Numerical 4: A resistance R & inductance 0.01 H & a capacitor are connected in series. When a voltage $v = 400\sin(3000t - 10°)$ V is applied to the series combination → the current flowing is $10\sqrt{2}\cos(3000t - 55°)$ A. Find R and C



Solution:

$V_m \sin(\omega t + \phi)$

$v = 400\sin(3000t - 10°)$ V

$i = 10\sqrt{2}\cos(3000t - 55°)$ A

$i = 10\sqrt{2}\sin(3000t + 35°)$ A

$I_m \sin(\omega t + \phi)$

$v = V\angle\phi$ → V_{rms}

$i = I\angle\phi$ → I_{rms}

① $V_{rms} = V_m/\sqrt{2} = \dfrac{400}{\sqrt{2}} = 282.84$ V

(2) $I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10A$

(3) $\bar{V} = 282.84 \angle -10^\circ$ V

$\bar{V} = \bar{I}\bar{Z}$

(4) $\bar{I} = 10 \angle 35^\circ$ A

(5) $\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{282.84 \angle -10^\circ}{10 \angle 35^\circ}$

$\bar{Z} = 28.284 \angle -45^\circ \ \Omega$

Shift Rec $(28.284, -45)$

(6) $\bar{Z} = (20 - j20)\,\Omega$

$\bar{Z} = R - jX$

$X = X_C - X_L$

Capacitive.

(7) $R = 20\Omega$

$X = X_C - X_L = 20\Omega$

(8) $X_L = \omega L = 3000 \times 0.01 = 30\Omega$

(9) $X_C = X + X_L = 20 + 30\Omega = 50\Omega$

(10) $X_C = \frac{1}{\omega C} \rightarrow C = \frac{1}{\omega X_C} = \frac{1}{3000 \times 50}$

$$\boxed{C = 6.67\,\mu F}$$

—x—

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Numerical 1: For the circuit shown below,

Find a) Circuit impedance (Z)

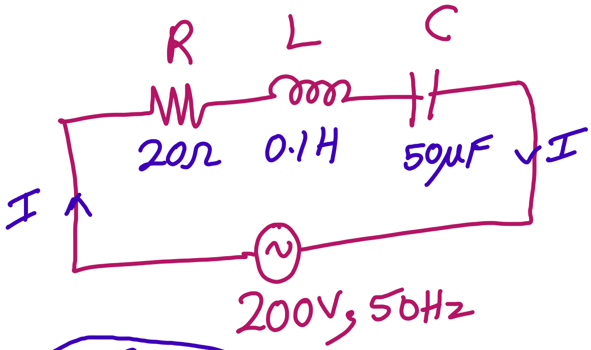
b) Circuit current (I)

c) Power factor (p.f)

d) Active power (P)

e) Reactive power (Q)

f) Apparent power (S)



Thinking

1) Find ckt nature

a) $X_L = 2\pi f L$ b) $X_C = \frac{1}{2\pi f C}$

If $X_L > X_C \rightarrow$ inductive

$X_C > X_L \rightarrow$ capacitive

2) Z in polar form

3) $\bar{I} = \frac{\bar{V}}{\bar{Z}}$

4) $p.f = \cos\phi$

5) $P = VI\cos\phi$

6) $Q = VI\sin\phi$

7) $S = VI$

Solution:

1) $X_L = 2\pi f L = 2\pi \times 50 \times 0.1$

$X_L = 31.42\,\Omega$

2) $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50\times10^{-6}}$

$X_C = 63.66\,\Omega$

3) $X_C > X_L \rightarrow X = X_C - X_L$

$X = 32.24\,\Omega$

Net reactance

Nature of ckt is capacitive

$\bar{Z} = R - jX$

$\bar{Z} = Z\angle\phi$

4) $\bar{V} = 200\angle 0^\circ\ V$

5) $\bar{Z} = 20 - j32.24$

6) $\bar{Z} = 37.94\angle{-58.19^\circ}\,\Omega$ Pol(20, -32.24)

$Z = 37.94\,\Omega$, $\phi = -58.19^\circ$

7) $\bar{V} = \bar{I}\bar{Z} \longrightarrow \bar{I} = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{200\angle 0^\circ}{37.94\angle 58.19}$

$\boxed{\bar{I} = 5.27\angle 58.19^\circ}$ A

8) $p.f = \cos\phi = \cos(-58.19) = 0.527$ leading

9) $P = VI\cos\phi = 200 \times 5.27 \times 0.527 = 555.45W$

10) $Q = VI\sin\phi = 200 \times 5.27 \times \sin(-58.19) = -895.69$ VAR

11) $S = VI = 200 \times 5.27 = 1054$ VA

—x—