Alternating Current Short Notes - RS

1. **RMS:** The RMS value of A.C. equals the steady value of direct current producing the same work or heat in the same time

$$v = V_m \sin\theta \qquad 0<\theta<2\pi$$

$$\begin{aligned}
V_{\text{rms}} &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\,d\theta} \\
&= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta\,d\theta} \\
&= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta\,d\theta} \\
&= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta} \\
&= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{2\pi}} \\
&= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{2\pi}{2}-0-0+0\right]} \\
&= \sqrt{\frac{V_m^2}{2}} \\
&= \frac{V_m}{\sqrt{2}} \\
&= 0.707\,V_m
\end{aligned}$$

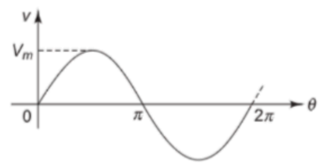


Fig. 3.6 *Sinusoidal waveform*

2. Derivation of Vrms
3. Peak factor/crest/amplitude factor = max value/rms value

$$v = V_m \sin\theta \qquad 0<\theta<2\pi$$

Since this is a symmetrical waveform, the average value is calculated over half the cycle.

$$\begin{aligned}
V_{\text{avg}} &= \frac{1}{\pi}\int_0^{\pi} v(\theta)\,d\theta \\
&= \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\,d\theta \\
&= \frac{V_m}{\pi}\int_0^{\pi} \sin\theta\,d\theta \\
&= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi} \\
&= \frac{V_m}{\pi}[1+1] \\
&= \frac{2V_m}{\pi} \\
&= 0.637\,V_m
\end{aligned}$$

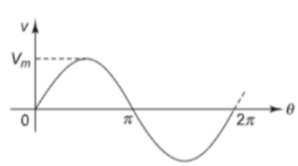


Fig. 3.7 *Sinusoidal waveform*

4. half cycle since the waveform is symmetrical
5. Form factor = rms / avg

Example 24

A full-wave rectified wave is clipped at 70.7% of its maximum value as shown in Fig. 3.21. Find its average and rms values.

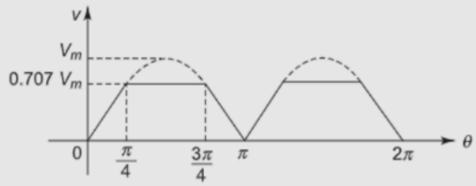


Fig. 3.21

Solution

$$\begin{aligned} v &= V_m \sin\theta && 0 < \theta < \pi/4 \\ &= 0.707\, V_m && \pi/4 < \theta < 3\pi/4 \\ &= V_m \sin\theta && 3\pi/4 < \theta < \pi \end{aligned}$$

(i) Average value of the waveform

$$\begin{aligned} V_{\text{avg}} &= \frac{1}{\pi}\int_0^{\pi} v(\theta)\,d\theta \\ &= \frac{1}{\pi}\left[\int_0^{\pi/4} V_m \sin\theta\, d\theta + \int_{\pi/4}^{3\pi/4} 0.707 V_m\, d\theta + \int_{3\pi/4}^{\pi} V_m \sin\theta\, d\theta\right] \\ &= \frac{V_m}{\pi}\left\{\left[-\cos\theta\right]_0^{\pi/4} + 0.707[\theta]_{\pi/4}^{3\pi/4} + \left[-\cos\theta\right]_{3\pi/4}^{\pi}\right\} \\ &= \frac{V_m}{\pi}(0.293 + 1.11 + 0.293) \\ &= 0.54\, V_m \end{aligned}$$

//mistake made: in the middle interval of root2vm, i did not integrate 1 to theta.

A phasor is a rotating line:

- - Length = Maximum value of alternating quantity.
- - Angular velocity = Alternating quantity's angular velocity.
- - Y-axis projection = Instantaneous value.

Phasor diagrams use RMS values for convenience :

- - Instruments show RMS values, not maximum values.
- - RMS phasors simplify diagrams but need √2 scaling for sine waves.

Phasor representation forms:

1. **Rectangular:** V=X±jYV = X \pm jY, Magnitude: X2+Y2\sqrt{X^2 + Y^2}, Phase: tan−1(YX)\tan^{-1}(\frac{Y}{X}).
2. **Trigonometric:** V=V(cosθ+jsinθ)V = V (\cos\theta + j\sin\theta).
3. **Exponential:** V=VejθV = Ve^{j\theta}.
4. **Polar:** V=V∠θV = V\angle\theta.

Significance of operator jj:

- **Definition:** j=−1j = \sqrt{-1}.

- **Function:** Represents 90° anticlockwise rotation of a phasor.
- **Power of jj:** Indicates multiple 90° anticlockwise rotations.

Example 11

Two currents, $\overline{I}_1 = 10\angle 50°$ A and $\overline{I}_2 = 5\angle -100°$ A, flow in a single-phase ac circuit. Estimate (i) $\overline{I}_1 + \overline{I}_2$ (ii) $\overline{I}_1 . \overline{I}_2$ (ii) $\dfrac{\overline{I}_1}{\overline{I}_2}$.

Solution

$$\overline{I}_1 = 10\angle 50° \text{ A}$$

$$\overline{I}_2 = 5\angle -100° \text{ A}$$

(i) $$\overline{I}_1 + \overline{I}_2 = 10\angle 50° + 5\angle -100° = 6.2\angle 26.21° \text{ A}$$

(ii) $$\overline{I}_1 . \overline{I}_2 = (10\angle 50°)(5\angle -100°) = 50\angle -50° \text{ A}$$

(ii) $$\frac{\overline{I}_1}{\overline{I}_2} = \frac{10\angle 50°}{5\angle -100°} = 2\angle 150° \text{ A}$$

Example 12

*revisit 343

Single phase AC Circuits

Parameter	Pure Resistor	Pure Inductor	Pure Capacitor
Impedance (Z)	$Z = R$	$Z = j\omega L$	$Z = \frac{1}{j\omega C}$
Current (i)	$i = \frac{V_m}{R}\sin\omega t$	$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$	$i = \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right)$
Voltage (v)	$v = V_m \sin\omega t$	$v = V_m \sin\omega t$	$v = V_m \sin\omega t$
Phase Difference	$\phi = 0^\circ$	$\phi = +90^\circ$	$\phi = -90^\circ$
Power (p)	$p = V_m I_m \sin^2\omega t$	$p = 0$	$p = 0$
Average Power (P)	$P = V_{\text{rms}} I_{\text{rms}}$	$P = 0$	$P = 0$
Power Factor ($\cos\phi$)	$\cos\phi = 1$	$\cos\phi = 0$	$\cos\phi = 0$
Current-Voltage Relation	i in phase with v	i lags v by 90°	i leads v by 90°
Reactance (X)	$X = R$	$X_L = \omega L$	$X_C = \frac{1}{\omega C}$

***revisit 366 for numericals**

A coil connected across a 250 V, 50 Hz supply takes a current of 10 A at 0.8 lagging power factor. What will be the power taken by the choke coil when connected across a 200 V, 25 Hz supply? Also calculate resistance and inductance of the coil.

*power nahi aata hai

Parameter	Series Pure R-L Circuit	Series Pure R-C Circuit	Series Pure L-C Circuit
Impedance (Z)	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + X_C^2}$	$Z = \sqrt{X_L^2 + X_C^2}$
Current (i)	$i = \frac{V_m}{Z}\sin(\omega t - \phi)$	$i = \frac{V_m}{Z}\sin(\omega t - \phi)$	$i = \frac{V_m}{Z}\sin(\omega t - \phi)$
Voltage (v)	$v_R = RI, v_L = X_L I$	$v_R = RI, v_C = X_C I$	$v_L = X_L I, v_C = X_C I$
Phase Difference (ϕ)	$\phi = \tan^{-1}(\frac{X_L}{R})$	$\phi = \tan^{-1}(\frac{X_C}{R})$	$\phi = \pm 90^\circ$ (depending on X_L and X_C)
Power Factor	$\cos\phi = \frac{R}{Z}$	$\cos\phi = \frac{R}{Z}$	$\cos\phi = 0$ (at resonance)
Reactance (X)	$X_L = \omega L$	$X_C = \frac{1}{\omega C}$	$X_L = X_C$ (at resonance)
Current-Voltage Relation	Current lags voltage by 90°	Current leads voltage by 90°	Voltage and current are in opposite directions (at resonance)
Resonance Condition	No resonance (inductive reactance dominates)	No resonance (capacitive reactance dominates)	Resonance occurs when $X_L = X_C$
Power	$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$	$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$	$P = 0$ (at resonance)

Parallel AC Circuits Summary:

- **Components:** Resistor, inductor, and capacitor (or combinations) connected in parallel.
- **Voltage:** Same across each branch of the circuit.
- **Total Current:** Equal to the phasor sum of the currents in each branch.

$$\frac{1}{z} = \frac{1}{z1} + \frac{1}{z2}$$

If $\overline{Z}_1 = R + jX_L$, and $\overline{Z}_2 = -jX_C$

then,
$$\begin{aligned}\frac{1}{\overline{Z}} &= \frac{1}{R + jX_L} + \frac{1}{-jX_C} \\ &= \frac{R - jX_L}{R^2 + X_L^2} + j\frac{1}{X_C} \\ &= \frac{R}{R^2 + X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}\right) \\ &= G + jB\end{aligned}$$

1. Real part is g and is called conductance
2. Imaginary part is B and is called susceptance
3. Both have unit mho or siemen

*numericals 413 onwards

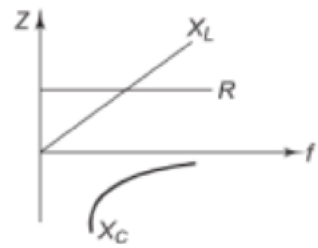
Series resonance occurs in an R-L-C circuit when the inductive reactance (XLX_L) and capacitive reactance (XCX_C) are equal, resulting in minimum impedance. At resonance, the circuit's impedance is purely resistive, and current is in phase with voltage.

$$f0=\frac{1}{2\pi\sqrt{LC}}$$

Power factor is given by R/Z. But here the only impedance is R, so power factor is one.

An RLC circuit under resonance is called an acceptor circuit because it allows maximum current to pass through, accepting energy.

At resonance in an RLC circuit, the voltage across the inductor and capacitor increases significantly due to voltage magnification. This occurs because their reactances cancel each other, leading to a minimum impedance and maximum current flow and maximum voltage. This effect is called "Voltage Magnification."



- XL is directly proportional to frequency
- XC is inversely proportional to frequency
- R is constant and independent of frequency.

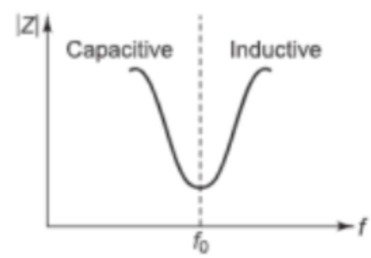


Fig. 4.96 Impedance

- **At Resonance ($f = f_0$):** Impedance is minimum, current is maximum, and power factor is 1 (unity).
- **When $f > f_0$:** Inductive reactance dominates, increasing impedance and decreasing current, with a lagging power factor.
- **When $f < f_0$:** Capacitive reactance dominates, increasing impedance and decreasing current, with a leading power factor.

Bandwidth in a series R-L-C circuit is the range of frequencies where the power (given to R) is at least half of the maximum power at resonance. It is the difference between the upper and lower frequencies where current drops to 0.707 of its maximum value.

*check derivation pg 440 and numericals

Q factor = voltage across i or c / voltage at resonance

1. **When f<f0**
   Capacitive reactance dominates, impedance increases, current decreases, **current leads voltage**, and **power factor** is **leading**.

2. **At f=f0:**
   Inductive and capacitive reactances cancel, impedance is minimum, current is

maximum, **current is in phase with voltage**, and **power factor** is **1** (unity).

3. **When f>f0:**
 Inductive reactance dominates, impedance increases, current decreases, **current lags voltage**, and **power factor** is **lagging**.

Bandwidth is the same as in series resonance

$$\text{Quality factor} = \frac{current\ through\ the\ inductor\ or\ capacitor}{current\ at\ resonance}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

*460 onwards do numerical

1. P=VIcosϕ , *unit is kW*
2. S=VI , *unit is kVA*
3. Q=VIsinϕ *unit is kVAR*

Asdakjdjasdkasd,nasdnasndkadsghankmjhgfdftyuijhbgv cvbghjuiujhgvcvfgtytfdcftyygfvghuijn

Example 7

An R-L-C series circuit has a current which lags the applied voltage by 45°. The voltage across the inductance has a maximum value equal to twice the maximum value of voltage across the capacitor. Voltage across the inductance is 300 sin (1000t) and R = 20 Ω. Find the value of inductance and capacitance.

Solution

$$\phi = 45°$$

$$v_L = 300 \sin(1000t)$$

$$R = 20\ \Omega$$

(i) Value of inductance

$$V_{L(\max)} = 2V_{C(\max)}$$

$$V_L = 2V_C$$

$$IX_L = 2IX_C$$

$$X_L = 2X_C$$

$$\cos\phi = \frac{R}{Z}$$

$$\cos(45°) = \frac{20}{Z}$$

$$Z = 28.28\ \Omega$$

For a series *R-L-C* circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$28.28 = \sqrt{(20)^2 + (2X_C - X_C)^2}$$

$$= \sqrt{400 + X_C^2}$$

$$X_C = 20\ \Omega$$

$$X_L = 2X_C = 40\ \Omega$$

$$X_L = \omega L$$

$$40 = 1000 \times L$$

$$L = 0.04\ \text{H}$$

(ii) Value of capacitance

$$X_C = \frac{1}{\omega C}$$

$$20 = \frac{1}{1000 \times C}$$

$$C = 50\ \mu\text{F}$$