Tutorial 3

QNo.	Question	Marks
Q1.	(A)Let, A = {a, b, c, d, e} R = {(a, a), (a,b), (b,c), (c,e), (c,d), (d,e)} Obtain: i) R^2 ii) R^3 iii) R^4 iv) R^∞ Sol: **Set A = {a,b,c,d,e}** **Relation R = {(a,a), (a,b), (b,c), (c,d), (c,e), (d,e)}** **i) R^2 = R ◦ R = {(a,a), (a,b), (a,c), (b,d), (b,e), (c,e)}** **ii) R^3 = R^2 ◦ R = {(a,a), (a,b), (a,c), (a,d), (a,e), (b,e)}** **iii) R^4 = R^3 ◦ R = {(a,a), (a,b), (a,c), (a,d), (a,e)}** **iv) R^∞ (transitive closure) = {(a,a), (a,b), (a,c), (a,d), (a,e), (b,b), (b,c), (b,d), (b,e), (c,c), (c,d), (c,e), (d,d), (d,e), (e,e)}** **Explanation: Composition R^n lists pairs (x,z) where there is a path of length n from x to z in the digraph. R^∞ includes all reachable pairs (including reflexive pairs).** (B)Let, A = {H, S, R, G, W, E}, B = {7, 6, 5, 4, 3, 2 }, R = {(H,7), (S, 5), (R, 3), (W, 4), (E, 6)} Obtain: i. Domain of R ii. Range of R iii. Diagraph of R iv. M_R **Sol:** **i) Domain(R) = {E, H, R, S, W}** **ii) Range(R) = {3, 4, 5, 6, 7}**	4+4=08
Q2.	Let A={1,2,3,4,5} and R be the relation defined by aRb if and only if $a \leq b$ compute R, R^2 and Connectivity Relation. (04)	07

	Sol: Relation R = {(1,1), (1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5), (3,3), (3,4), (3,5), (4,4), (4,5), (5,5)} R^2 = R ◦ R = {(1,1), (1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5), (3,3), (3,4), (3,5), (4,4), (4,5), (5,5)} Connectivity relation (reachability / transitive closure) for $\leq$ equals R itself because $\leq$ is transitive. So connectivity = R. Draw Digraph of R, R^2 and Connectivity Relation.(03)	
Q.3	(A)Suppose R is a relation defined on the set $A = \{-2, -1, 0, 1, 2\}$, as $x, y \in A, x\, R\, y \Leftrightarrow \lvert x - y \rvert < 2$ (i)What are the ordered pairs in the relation R ? (1M) (ii)Draw the directed graph (diagraph) that represents R (1M) (iii)Determine whether the relation R is reflexive, symmetric, antisymmetric, and/or transitive. (3M) Sol: (i)Let A = {-2,-1,0,1,2} and define xRy iff |x - y| < 2. Ordered pairs in R: {(-1,-1), (-1,-2), (-1,0), (-2,-1), (-2,-2), (0,-1), (0,0), (0,1), (1,0), (1,1), (1,2), (2,1), (2,2)} (ii)Directed graph: (iii)Reflexive? Yes, because |x-x| = 0 < 2 for all x. Symmetric? Yes, because |x-y| = |y-x|, so if (x,y) in R then (y,x) in R. Antisymmetric? No: antisymmetry would require x=y whenever (x,y) and (y,x) hold; here distinct neighbors show antisymmetry fails. Transitive? Not necessarily: e.g., -2 R -1 and -1 R 0 but -2 R 0? | -2 - 0| = 2 which is not <2, so transitivity fails. (B)Let R be a relation defined on the set $A = \{-2, -1, 0, 1, 2\}$ $a, b \in A\ ,\ a\, R\, b \Leftrightarrow a.\, b \geq 2$	05+05

	Determine whether the relation R is Equivalence Relation Sol: Let A = {-2,-1,0,1,2} and define aRb iff $a \cdot b \geq 2$. Pairs in R: {(-1,-2), (-2,-1), (-2,-2), (1,2), (2,1), (2,2)} Reflexive? No Symmetric? Yes Transitive? No Conclusion: For equivalence relation all three must hold. Since reflexivity fails (0,0) not in R because $0 \cdot 0 = 0 < 2$, R is not an equivalence relation.	