Relations

→ if $R = \phi$, R is called void/empty relation

→ $R = A1 \times A2 \ldots \times An$ universal relation

→ $n = 1, 2, 3$ unary, binary, ternary relation

Domain → 1st element

Range → 2nd element.

Q) $A = \{1, 2, 3, 4, 5\}$ $B = A$

aRb iff $a < b$

$R = \{(1,2), (1,3), (1,4), (1,5) \ldots$ ~~(4,5)}~~
$(2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$

$Dom(R) = \{1, 2, 3, 4\}$

$Range(R) = \{2, 3, 4, 5\}$

Representation of Relation

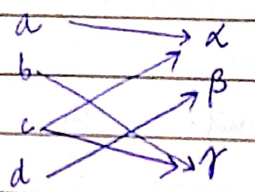
Graphical, Tabular, Matrix

$A = \{a, b, c, d\}$ $B = \{\alpha, \beta, \gamma\}$

$R \to (A \to B)$

$R = \{(a,\alpha), (b,\gamma), (c,\alpha), (c,\gamma), (d,\beta)\}$

	α	β	γ
a	✓		
b			✓
c	✓		✓
d		✓	

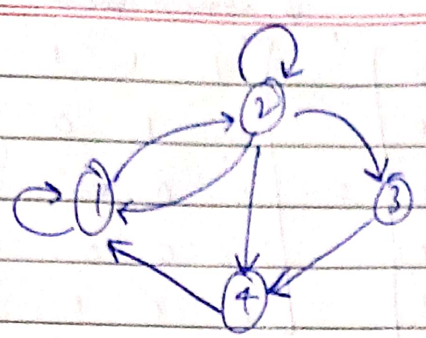


$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Diagraph

If A is a finite set and R is a relation on A, we can represent R as follows:

1) Draw a circle with elements in A. like
2) Draw edges between related elements.
3) Result → directed graph.



$R = \{(1,2), (1,1), (2,1), (2,2), (2,4) (2,3), (3,4), (4,1)\}$

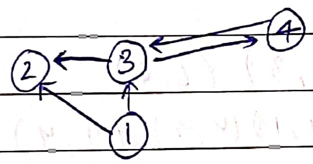
Degree of vertex in a directed graph.

→ in a diagraph, each vertex has an in-degree & an out-degree.

→ in-degree : edges coming INTO vertex V

→ out-degree : edges going out of vertex V.

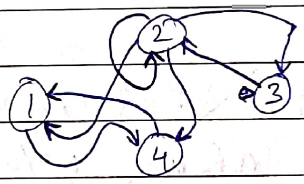
Q)



V	1	2	3	4
in	0	2	2	1
out	2	0	2	1

Q)

V	1	2	3	4
in	2	2	1	2
out	1	4	1	1



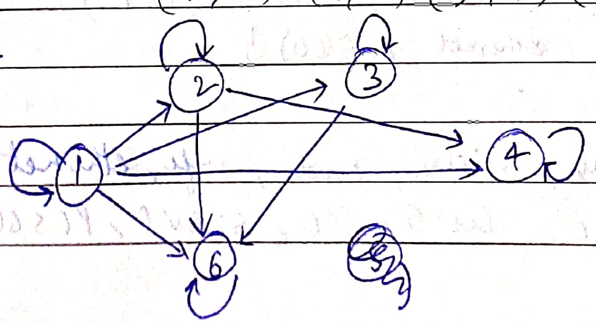
Q) $A = \{1, 2, 3, 4, 6\}$

$R \to A$ such that $x \% y = 0$

→ draw diagraph, find R, matrix R, inverse R.

$R = \{(1,1), (1,2), (1,3)(1,4) (1,6) (2,4) (2,6) (3,6) (2,2) (3,3) (4,4)(6,6)\}$

dig :



	1	2	3	4	6
1	✓	✗	✗	✗	✗
2	✓	✓	✗	✗	✗
3	✓	✗	✓	✗	✗
4	✓	✓	✗	✓	✗
6	✓	✓	✓	✗	✓

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$$M_{R^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}^{\prime} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

↓ domain

$R^{-1} = \{(1,1), (2,1), (3,1) \ldots\}$

* just ~~or~~ exchange range & domain.

Q) $A = \{1, 2, 3, 4, 6\} = B$
a R b if a is a multiple of B. R^{-1}
Find R, diagraph, matrix

$R(3) = (3,1), (3,3)$
$R(6) = (6,1), (6,2), (6,3), (6,6)$
$R(\{2,4,6\}) = \{(2,1)\ (2,2)\ (4,1)\ (4,2)\ (4,4)$
$(6,1)(6,2)\ (6,3)(6,6)\}$

~~Q) $A = \{1,2,3,4,6\}$~~
~~aRb $y \% x = 0$~~
~~Find R, draw diagraph, find M_R & $M_{R^{-1}}$~~

~~$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)$~~

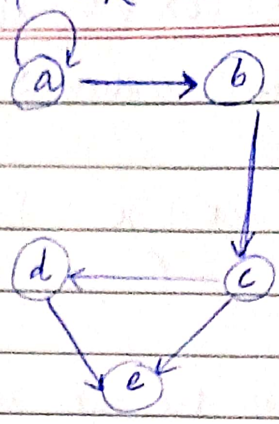
Q) A = {Hub, Switch, Router, Gateway, WiFi, Ethernet}
B = {750C, PCS60, Cis5, 523V6, 66L, 805IMC}

R: {(Hub, 750C); (Switch, Cis5), (Router, 66L), (WiFi, 523V6), (Ethernet, PCS60)}

Domain of R → {Hub, Switch, Router, wifi, ethernet}
Range → {750C, Cis 5, 66C, 523V6, PCS60}

R^n

i) find R^2



5)

$a R^2 a$	aRa	aRa
$a R^2 b$	aRa	aRb
$a R^2 c$	aRb	bRc
$b R^2 e$	bRc	cRe
$b R^2 d$	bRc	$bcRd$
$bc R^2 e$	cRd	dRe

logic : R^2 means in $x R^2 y$ x can reach y in '2' ~~iterations / pathways~~ steps!

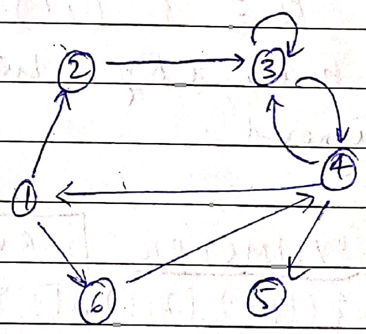
→ aR^2a is valid as we have closed loop → stands for all R^n $(n \in R)$

→ aR^2e is invalid as we pass 3 nodes to reach e. This falls under: aR^3e.

ii) find R^∞ → everything can reach everything.

[(a,a), (a,b), (b,c), (a,c), (a,e), (a,d)
(b,e), (b,d), (c,d), (c,e) (d,e)]

Note domain & range! only arrow tail becomes domain & head becomes tail.



Q)
i) list all paths of length = 1
ii) list all path $l = 2$ starting ②
list " " " . ~~4~~ ③
iii) " " " " 3 start ③
" " " " " .

iv) Find a cycle starting ② & ⑥ each.

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Types of Rel

① REFLEXIVE RELATION

i) The relation $\leq$ on the set of real no.

let $x = \{0, 1, 2, 3, 4 \ldots\}$

$R = \{(0,0), (1,1), (2,2) \ldots\}$ is reflexive

ii) If the set is $\{1, 2, 3\}$ and rel is
→ $\{(1,1), (2,2), (3,3), (3,2)\}$ is also reflexive.
→ As long as all $x \times (x,x) \in R$, we dont care about extras.
→ for parent $\{1,2,3\}$, rel $= \{(1,1), (2,2)\}$ is NOT reflexive (miss (3,3))

② ANTISYMMETRIC RELATION

→ If xRy and yRx and $x = y$ for all $\forall$ $x, y \in R$

$\{(1,1), (2,2), (3,3)\}$ for $R = \{1, 2, 3\}$

→ do ~~not~~ allow heterogenous pairs.

$\{(1,2), (3,2)\}$ is not antisym.

$\{(1,1), (2,1)\}$ is " "

→ All ~~reflexive~~ relations are not antisym ~~but~~ all antisym are reflexive.

→ $R = \{(1,2), (1,1)\} \longrightarrow$ Antisymmetric

Here, xRy but $!yRx$. so $x = y$ con. is ignored.

③ SYMMETRIC $\boxed{xRy \quad yRx}$ ✓ $\forall x, y \in R$

$\{(1,2), (2,1), (1,1)\}$

$\{(1,2), (1,5)\}$ ✗

④ ASYMMETRIC $\boxed{xRy \quad !yRx}$

$\{(1,2), (2,5)\}$ ✓ is asym

$\{(1,2), (1,1)\}$ ✗ is NOT asym because (1,1) is present and nullifies condⁿ: (1,1) xRy and yRx.

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(5) Transitive · Relation

Cond^n : xRy , yRz and xRz

if set A = {1, 2, 5, 7}

R_1 = {(1,2), (2,5), (1,5)} Transitive ✓

$N^?$ R_2 = {(1,2), (1,5), (1,1)} Transitive.

xRy $x = 1$ $y = 2$. (2, some) is not present.

if con^d 2 fails, trans. hai.

R_3 = {(1,2), (2,5), (1,1)} Not transitive as (1,5) is absent.

either satisfy all 3 cond or ONLY one.

(6) IRREFLEXIVE $(x,x) \notin R$ $\forall x \in R$

all not reflexive sets are irreflexive.

eg: {(1,2), (2,1)} is irreflexive.

Q) Identify equivalence rel if it satisfies reflexive symm. and transitive.

A ∈ R

rel = {(a,b) | a − b is divisible by 5}

~~(6,1)~~

→ Reflexive (a,a)

(1,1), (2,2) . . . $0\%5 = 0$

is reflexive

→ Sym. xRy yRx

{(1,6), (6,1)} $-5\%5 = 0$, $5\%5 = 0$ } yes.

→ Transitive

(5,10), (10,15), (5,15) $5-10\%5=0$, $10-15\%5=0$, $5-15\%5=0$ } yes

is transitive

∴ R is EQUIVALENCE relation

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Q2) $A = \{a, b\}$
$S = \{\phi, \{a\}, \{b\}, \{a,b\}\}$
$R = [S, \subseteq]$
A should have all subsets of S
∴ Set A is reflexive.
∴ Set A is not transitive OR sym.
as $\{b, a\}$ is not a subset.

→ POSETS (Partially Ordered Sets)

1) A relation R on set A is called 'partial order' if R is reflexive, anti sym., and transitive

2) Partially ordered set is A along with R.
Denoted by (A, R)

Q) $A = \{1, 2, 3, 4, \ldots\}$
$R \Rightarrow (a, b) \in R$ iff $a \% b = 0$.
→ as all int. divide themselves, R is reflexive
→ as a divides b
but b not divide a unless $a = b$
R is anti sym.
$(a,b) \in R$, $(b,a) \notin R$ when $a \neq b$
→ as if a divides b and b divides c, a divides c, R is transitive.
R is partial ordered. relation.

→ DUAL OF POSET

If R is a p.o. on A
R^{-1} is inverse of R, and also a p.o.
→ The poset (A, R^{-1}) is the dual of poset (A, R)
→ The p.o R^{-1} is the dual of p.o R

PRODUCT PARTIAL ORDER

If (A, R) and (B, R) are posets
$(A \times B, R)$ is a poset with p.o R
where R is defined $(a, b) \, R \, (a', b')$
iff $a R a'$ in A and $b R b'$ in B
R is called product p.o. as defined on Cartesian product $A \times B$

$A = \{1, 2, 3\}$

$B = \{0, 4\}$ $\quad R ; \leq$

$R = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\} \therefore (A, R)$

$R = \{(0,0), (0,4), (4,4)\} \therefore (B, R)$

$A \times B = \cancel{(0,1)\ (0,2)\ (0,3)}$
$\{(1,0), (2,0), (3,0), (1,4), (2,4), (3,4)\}$

$R = \cancel{\{((1,0),(1,4))\}}$
$\leq$

$\{(\underbrace{(1,0), (1,4)}_{\leq}), ((2,0), (2,4)), ((3,0), (3,4))\}$
and so on

$(A \times B, R)$ poset
↑ product partial order.

HASSE DIAGRAM

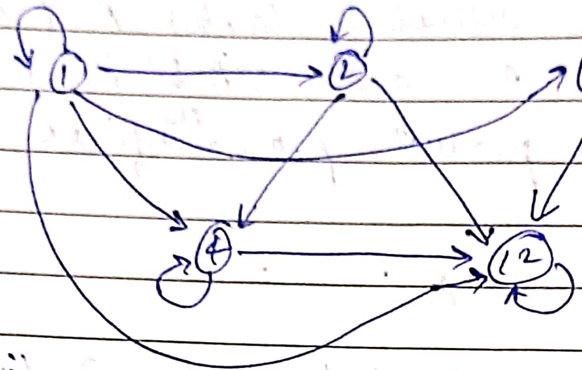
HOW?

draw diagraph → delete all cycles → eliminate all transitive edges.
↓
draw diagraph of partial order with all edges pointing up
←
replace circle with dots and remove arrow

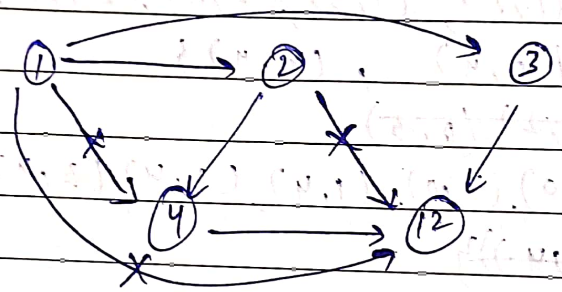
Q = $A = \{1, 2, 3, 4, 12\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (12,12), (1,2), (4,12)\ (1,3)\ (1,4)\ (1,12)\ (2,4)\ (2,12)\ (3,12)\}$

Step 1:



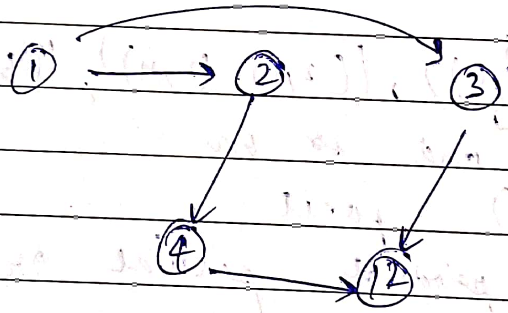
Step 2:



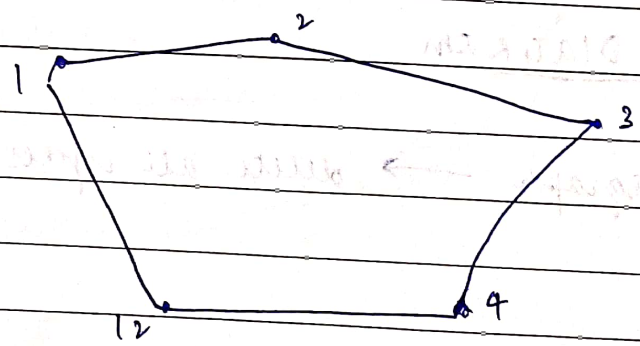
(2,4) (4,12)

remove (2,12)

Step 3:



Step 4 →



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Q) $A = \{a, b, c, d\}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

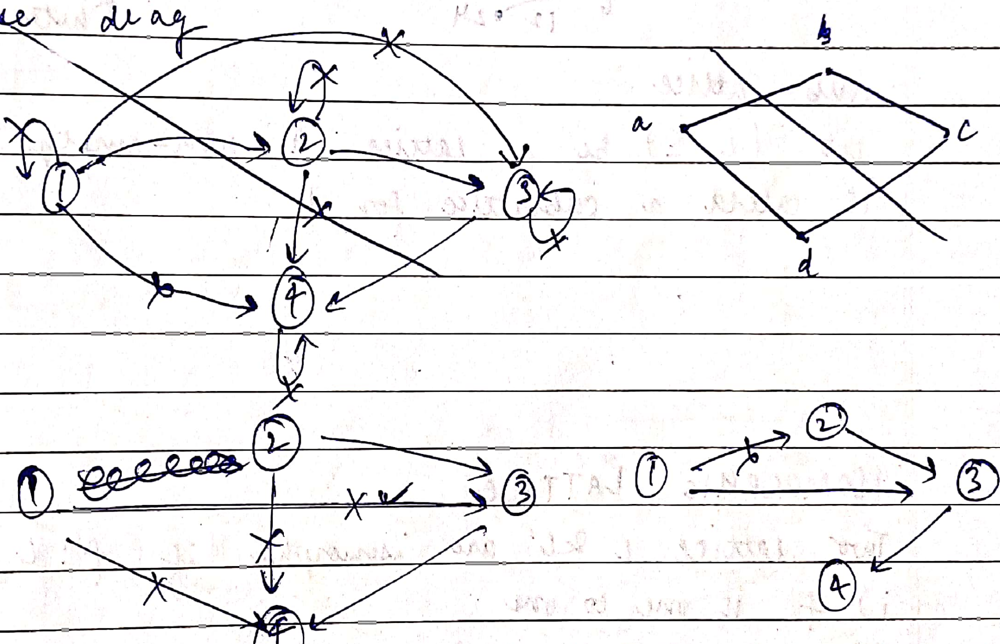
i) Prove R is p.o.

$R = \{(1,1), (2,2), (3,3), (4,4), (1,3), (1,4), (2,3), (2,4) (3,4)\}$

is reflexive & transitive & anti as

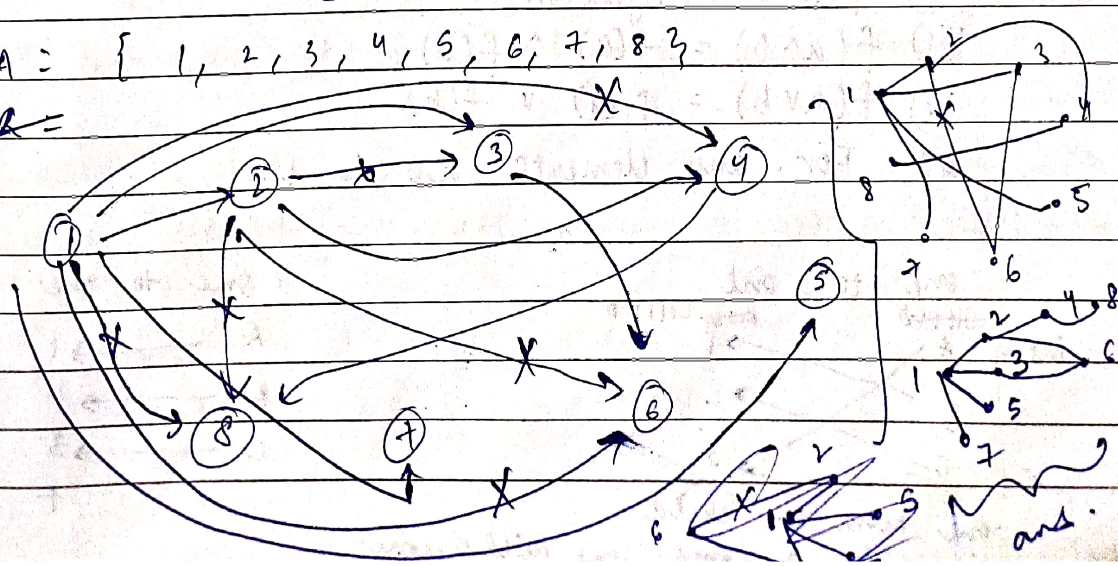
1,3 $\in$ R but 3,1 $\notin$ R unless a = b

ii) hasse diag



Q) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

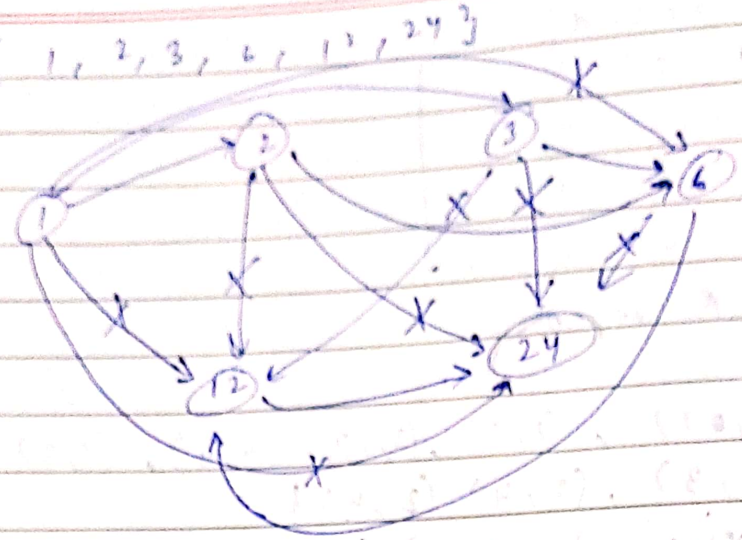
~~R~~ =

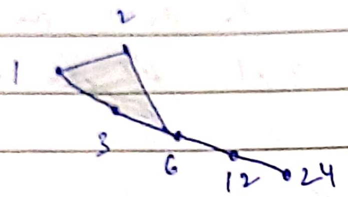


ans.

8) $A = \{1, 2, 3, 6, 12, 24\}$

$1 \rightarrow$





↑ notes missing

Sub-lattice

Let $(L, \leq)$ be a lattice. A non-empty subset S of L is called a sublattice for L

ISOMORPHIC LATTICE

Two lattice L & L' are isomorphic if $f : L \rightarrow L'$ such that

i) f is one to one

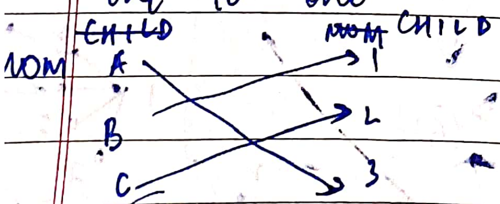
ii) f is onto (bijection)

iii) $f(a \wedge b) = f(a) \wedge f(b)$

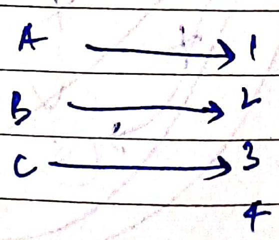
iv) $f(a \vee b) = f(a) \vee f(b)$

~~v)~~ For any elements a,b in L :

one to one



one to one.



one mom one child

No child 2 moms. Some child 0 moms.

onto;

each element in range ~~mom~~ child belongs in set ~~child~~ mom

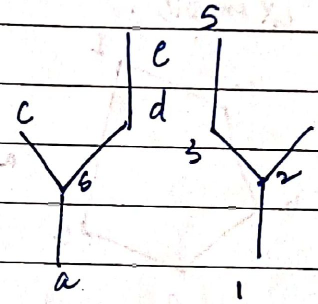
- A → 1
- B → 2
- C → 3
- D → 3

ONTO. (all 1, 2, 3 are in range)

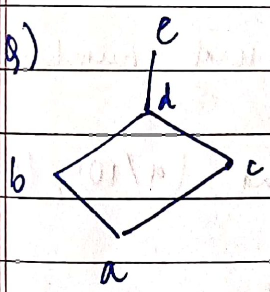
∴ each child has a mom.
2+ children can share a mom.

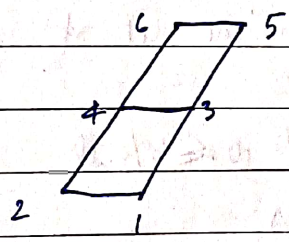
Example:
Are the 2 lattices isomorphic?

- a → 1
- b → 2
- c → 4
- d → 3
- e → 5



Q)





- 1 ← a
- 2 ← b
- 3 ← b
- 4 ← c
- 5 ← d
- 6 ← d
- e

Not isomorphic, as f is not one to one ~~& not onto~~

BOUNDED LATICE
If greatest element I & least element 0. condⁿ for bonded lattice:

$a \vee 0 = a$ $\quad$ $a \wedge 0 = 0$

$a \vee I = I$ $\quad$ $a \wedge I = a$

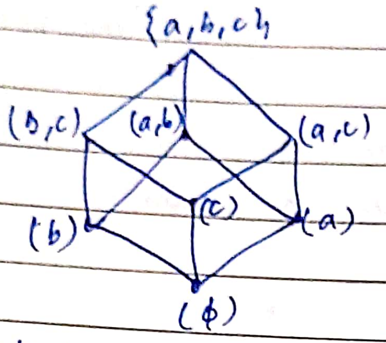
DISTRIBUTIVE LATTICE

Condition

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

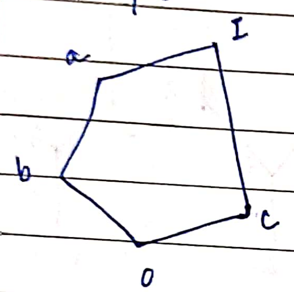
$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

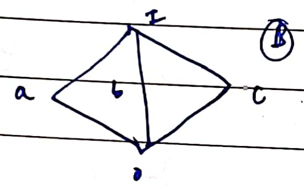
If L is not distributive, L is non-distributive.



example :

Ⓐ





Ⓑ

in Ⓐ , $a \wedge (b \vee c) = a \wedge I = a$

Q1) A relation is defined on the set of natural numbers.

i) $(0,0) \in R$

ii) $(a,b) \in R$ iff $a \% 10 \leq b \% 10$ and $(a/10), (b/10) \in R$

Check if it's partial order

Rel ; (0,0), (10,10) (10,20) (10,30) (10,40)
(20,30)

∴ reflexive & trancitive

∴ not antisymmetric

∴ R is ~~antisymmetric~~ not partial order

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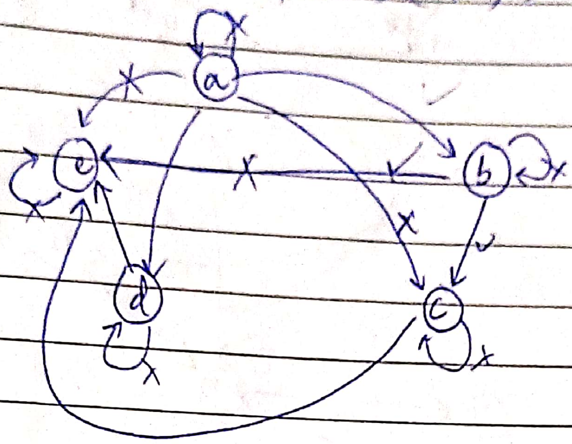
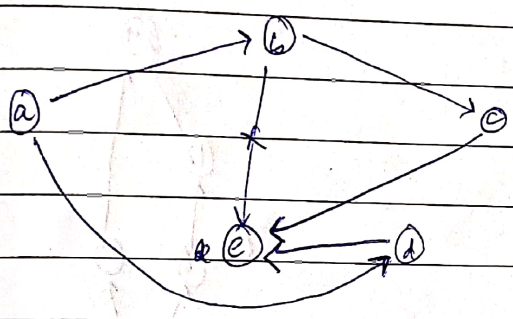
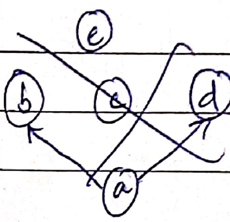
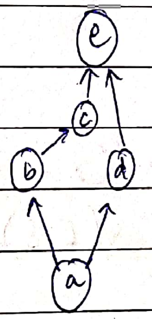
Q2) $x = \{a, b, c, d, e\}$

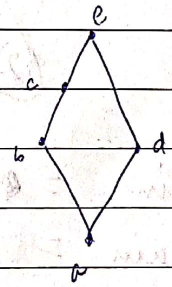
$R = \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,b), (b,c), (b,e)$
$(c,c), (c,e), (d,d), (d,e), (e,e)\}$









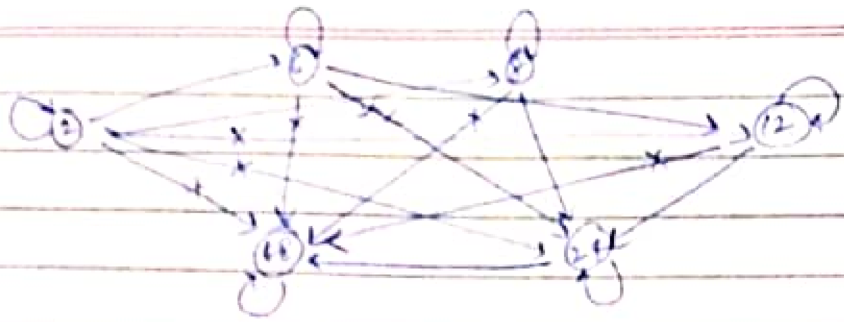


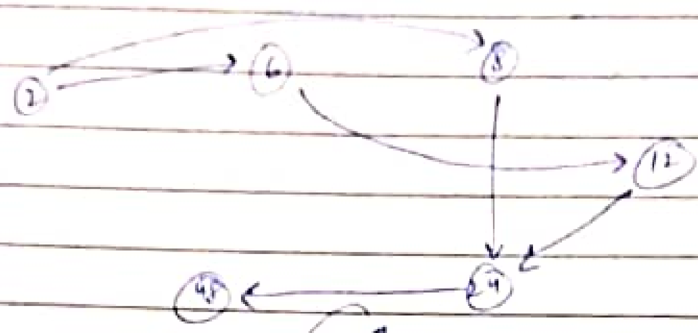
Q3) $A = \{2, 6, 8, 12, 24, 48\}$

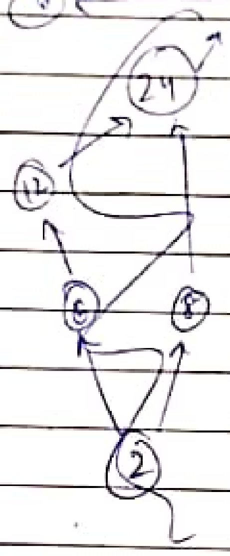
$a \leq b$ iff $a \% b = 0$.

$R = \{(2,2), (2,6), (2,8), (2,12), (2,24), (2,48),$
$(6,6), (6,12), (6,24), (6,48),$
$(8,8), (8,24), (8,48),$
$(12,12), (12,24), (12,48),$
$(24,24), (24,48),$
$(48,48)\}.$

Scanned with OKEN Scanner







longest chain = 5 → (2 → 6 → 12 → 24 → 48)

" antichain = 2 → (6, 8) or (8, 12)

minimal element = ~~48~~ 2

maximal element = 48

greatest " = 48

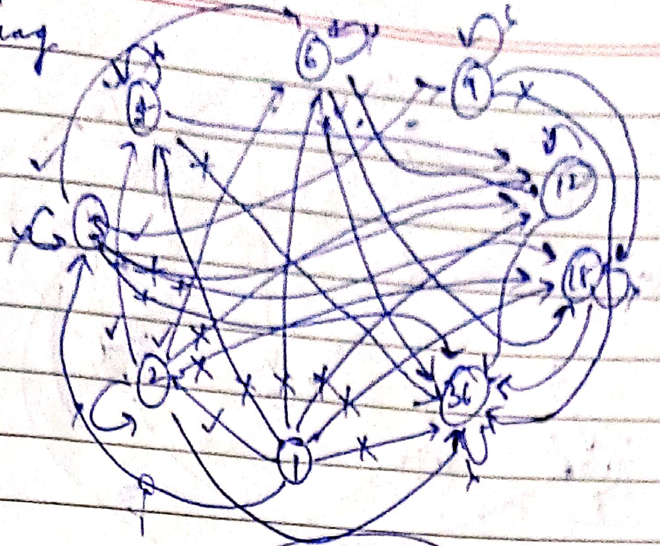
least " = 2

8) D_{56} = [1, 2, 4, 7, 8, 14, 28, 56]

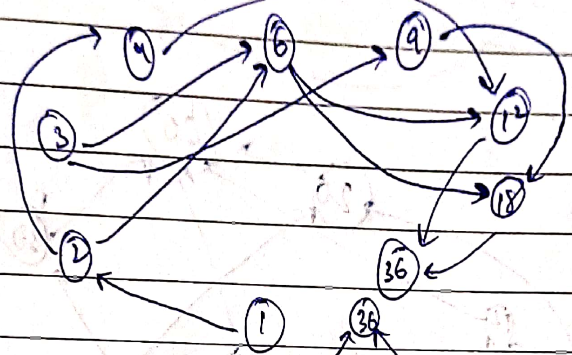
D(36) = [1, 2, 3, 4, 6, 6, 9, 12, 18, 36]

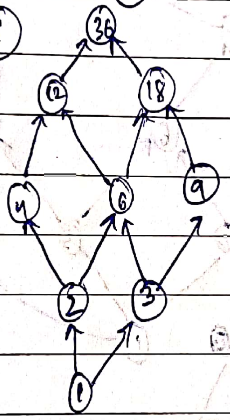
(1,1) (1,2) (1,3)

hasse diag



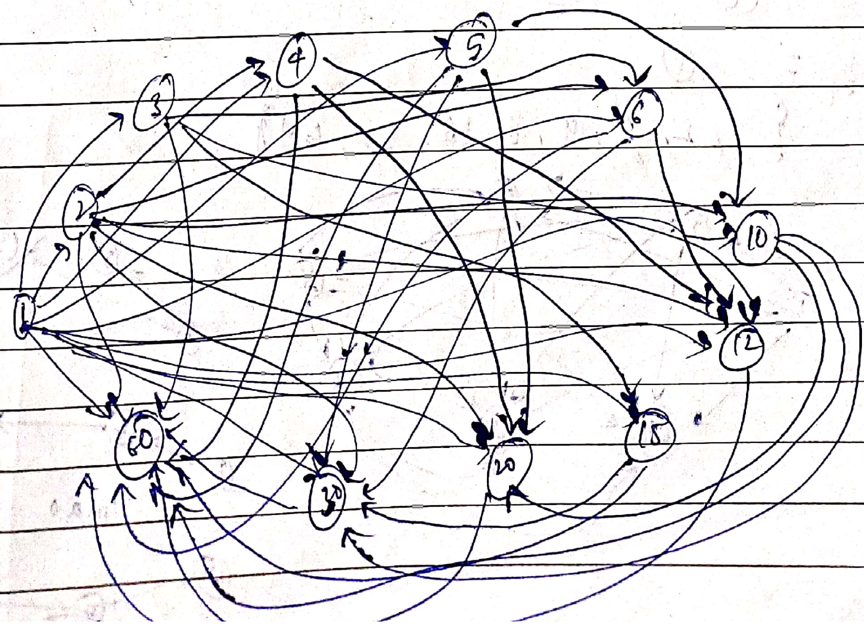
(a, b)
a divides b

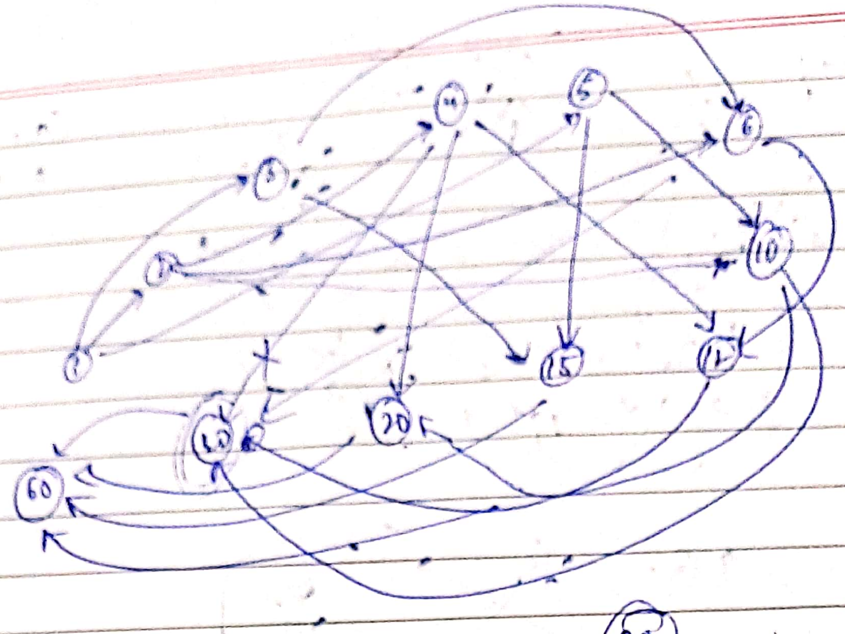




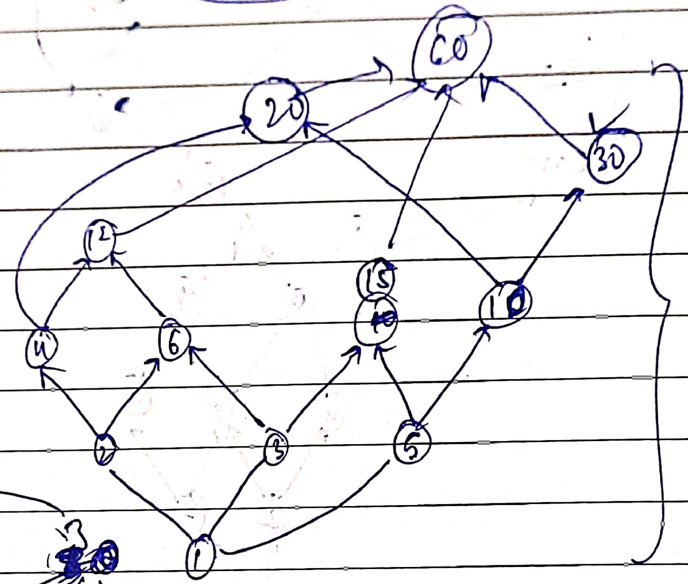
Q) D 60   D 64

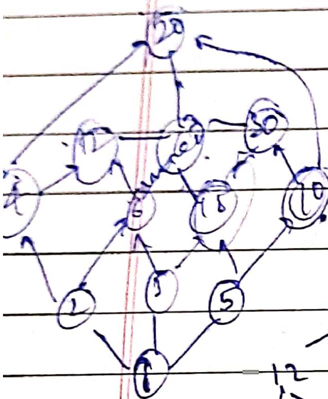
D 60 → {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}

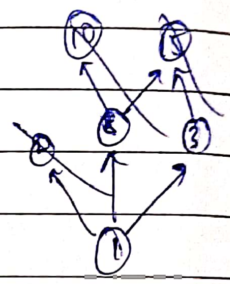


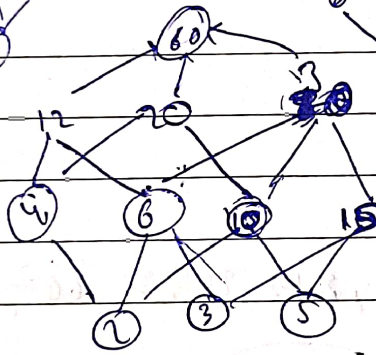


60





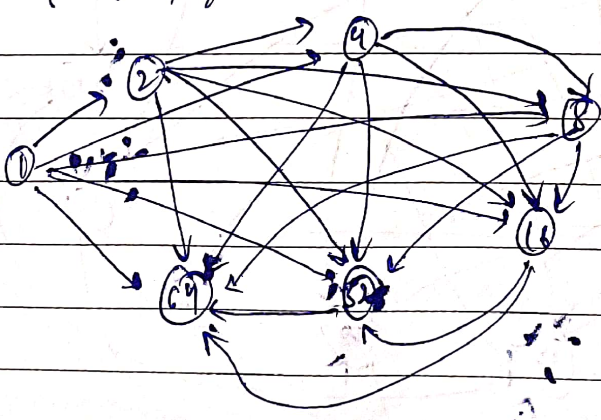


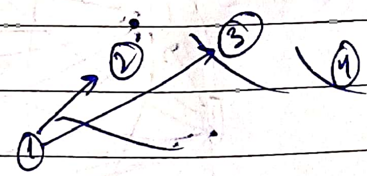


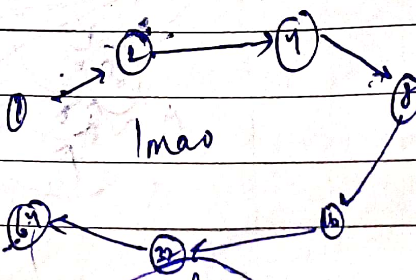
h → horse

D_{64} = 1

{1, 2, ~~3~~, 4, 8, 16, 32, 64}





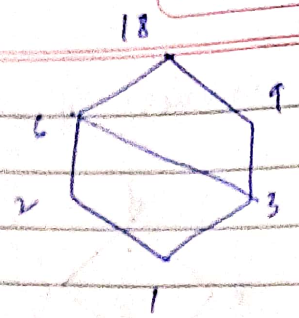


Imao

HSHSAC

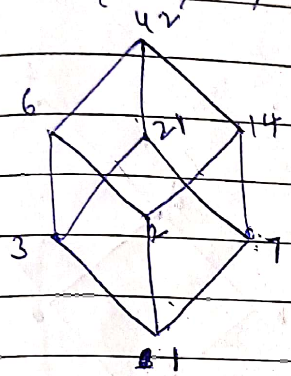
COMPLEMENTED LATTICE

Complement of an element:



2 & 9 are complements
as glb(2, 9) = 1
lub (2, 9) = 18
and 1 = lower bond of lattice
18 = upper bond of lattice ; (1 compl. of 18)
There is NO other complement elements.

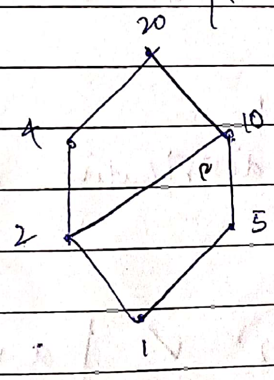
L = { 1, 2, 3, 6, 7, 14, 21, 42 } = D_{42}



glab (1, 42) = 1 } complements.
lub (1, 42) = 42 }
(3, 14), ~~(6, 2), (1, 14)~~ } other complements
(6, 7)

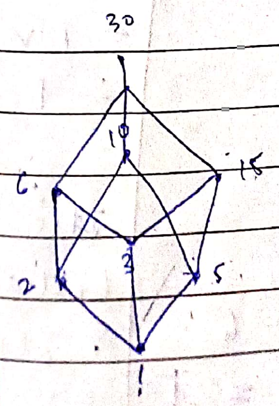
In short: all co-prime numbers are complements of each other.

→ D_{20} = L = {1, 2, 4, 5, 10, 20}



complement pairs →
(5, 4), (1, 20)
(4, 5) (20, 1)
since all elements do not have a complement, D_{20} is not a complemented lattice

→ D_{30} = L = {1, 2, 3, 5, 6, 10, 15, 30}

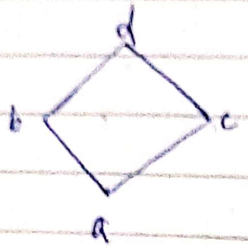


comp = (1, 30)
(6, 5)
(2, 15)
(3, 10)

poset →
digraph →
hasse →
comp table.

as all elements have a complement D_{30} is a complemented lattice

① Identity if distributive



$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ cond.

$a \vee (a) = a$

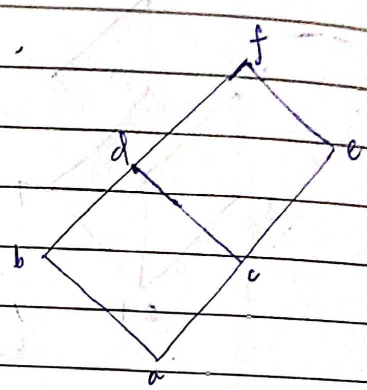
$a \vee b = a \qquad a \vee c = a \qquad a \wedge a = a$

$a \wedge (d) = \cancel{a} \; \cancel{b \vee c} \qquad a \vee a = a$

$a \wedge d = a\cancel{d} = \cancel{d}\, a$

Boolean algebra → ① every element has max 1 complement
② distributive

② Check if lattice is boolean algebra.



complement :
b & c
d & c → no complement
a & f
∴ not boolean

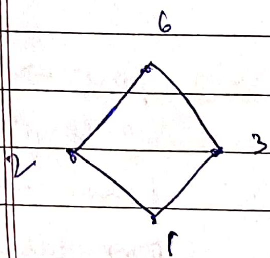
→ CONDITIONS FOR BOOLEAN ALGEBRA.

① contains 2^n elements
② A greatest & a least element
③ is complement lattice and distributive lattice

Check for boolean algebra (10 mark)

~~1, 2, 4~~ (1, 2, 3, 6) } 2^2 ✓

* Also write poset



greatest = 6 } ✓
least = 1

complement
1 & 6 } ✓
2 & 3

dis. $a \wedge (b \vee c) = a \wedge 6$
$= 1 \wedge 6 = 1$

$a \vee (b \wedge c)$
$= a \vee (1)$
$= 1 \vee 1$
$= 1$

It is boolean ~~alg~~ algebra

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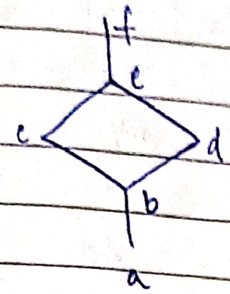
Determine if poset represented by each Hasse diag is a lattice

(1)



GLB	a	b	c	d	e	f
a	a	a	a	a	a	a
b	a	b	b	b	b	b
c	a	b	c	b	c	c
d	a	b	b	d	d	d
e	a	b	c	d	e	e
f	a	b	c	d	e	f

all have a GLB → all have an LUB (prove it)
∴ it is a lattice

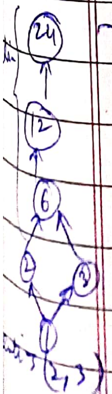
---

8) $P = \{1, 2, 3, 4, 6, 8, 12, 24\}$

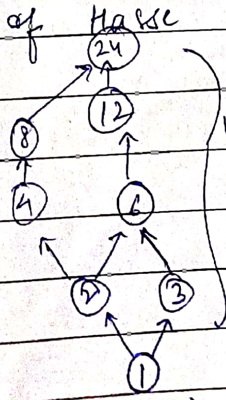
Identify minimal & maximal element
Draw Hasse diag
Draw greatest / least element if they exist
Check chain & antichain if they exist. Justify.

$R = \{ (1,1), (1,2), (1,3), (1,6), (1,12), (1,24),$
$(2,2), (2,6), (2,12), (2,24),$
$(3,3), (3,6), (3,12), (3,24), (4,4) (4,8) (4,12) (4,24)$
$(6,6), (6,12), (6,24), (8,8) (8,\cancel{16}24)$
$(12,12), (12,24),$
$(24,24) \}$

→ all steps of Hasse diagram:



without



with 4 & 8

chain = 1 → 2 → 4 → 8 → 24
1 → 3 → 6 → 12 → 24

antichain → ~~8 → 12 →~~
8, 12
4, 6
2, 3

minimal (and least) = 1
maximal (and greatest) = 24

Lcm Poset

LCM	1	2	3	6	12	24
1	1	2	3	6	12	24
2	2	2	6	6	12	24
3	3	6	3	6	12	24
6	6	6	6	6	12	24
12	12	12	12	12	12	24
24	24	24	24	24	24	24

gcd Poset

GCD	1	2	3	6	12	24
1	1	1	1	1	1	1
2	1	2	1	2	2	2
3	1	1	3	3	3	3
6	1	2	3	6	6	6
12	1	2	3	6	12	12
24	1	2	3	6	12	24

x

a) {1, 3, 9, 18}

(1,1) (1,3) (1,9) (1,18)

(3,9) (3,18) (3,3)

(9,9) (9,18)

(18,18)

$1 \rightarrow 3 \rightarrow 9 \rightarrow 18$

chain {1 — 3 — 9 — 18}

{{1,3}, {1,3,7} . . . }