

Batch -2 :

Q.1 Find the Laplace Transform of the following functions

(i) $t^5 \sin 4t$

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In [1]: var ('t s')
f = t^5 * sin(4*t)
show(f)
L = laplace(f,t,s)
show(L)
```

$$t^5 \sin(4 t)$$

$$\frac{15360 s^5}{(s^2 + 16)^6} - \frac{15360 s^3}{(s^2 + 16)^5} + \frac{2880 s}{(s^2 + 16)^4}$$

(ii) $\frac{\cos 3t - \cos 4t}{t^2}$

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In [2]: var ('t s')
f = (cos(3*t) - cos(4*t))/t^2
show(f)
L = laplace(f,t,s)
show(L)
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$$-\frac{\cos(4 t) - \cos(3 t)}{t^2}$$

$$\frac{1}{2} \pi - \frac{1}{2} s \log(s^2 + 16) + \frac{1}{2} s \log(s^2 + 9) + 3 \arctan\left(\frac{1}{3} s\right) - 4 \arctan\left(\frac{1}{4} s\right)$$

(iii) $e^{-5t} \cos 3t + t^{10}$

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In [4]: var ('t s')
f = exp(-5*t)* cos(3*t) + t^10
show(f)
L = laplace(f,t,s)
show(L)
```

$$t^{10} + \cos(3t)e^{(-5t)}$$

$$\frac{s+5}{s^2+10s+34} + \frac{3628800}{s^{11}}$$

Q.2 Find the Inverse Laplace Transform of the following Functions

(i) $\frac{s^2+16s-24}{s^4+20s^2+64}$

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In [7]: var ('t s')
f(s) = (s^2 + 16*s - 24 )/ (s^4 + 20*s^2 + 64)
show(f(s))
L = inverse_laplace(f(s),s,t)
show(L)
```

$$\frac{s^2 + 16s - 24}{s^4 + 20s^2 + 64}$$

$$-\frac{4}{3} \cos(4t) + \frac{4}{3} \cos(2t) + \frac{5}{6} \sin(4t) - \frac{7}{6} \sin(2t)$$

(ii) $\frac{s+2}{(s+3)(s+1)^3}$

$$\frac{3}{(s+3)^3} - \frac{3}{(s+3)^2} + \frac{6}{(s+3)} - 6$$

```
In [10]: var ('t s')
f(s) = (s+2)/((s+3)*((s+1)^3))
show(f(s))
L = inverse_laplace(f(s),s,t)
show(L)
```

$$\frac{s+2}{(s+3)(s+1)^3}$$

$$\frac{1}{4} t^2 e^{(-t)} + \frac{1}{4} t e^{(-t)} - \frac{1}{8} e^{(-t)} + \frac{1}{8} e^{(-3 t)}$$

Q.3 Solve the following differential equation using Laplace Transform

$$x''(t) + 3x'(t) + 2x(t) = 2(t^2 + t + 1) \quad \text{with } x(0) = 2, x'(0) = 0$$

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In [14]: s,t = var ('s,t')
x = function('x')(t)
de = diff(x,t,t)+ 3*diff(x,t)+ 2*x == 2*(t^2 + t+ 1)
show(de)
show(desolve_laplace(de, x,ics=[0,2,0]))
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$$2x(t) + 3 \frac{\partial}{\partial t} x(t) + \frac{\partial^2}{(\partial t)^2} x(t) = 2t^2 + 2t + 2$$

$$t^2 - 2t - e^{(-2t)} + 3$$
