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INVERSE

Q) $f: Z \to Z$

$f(x) = x^2 - 1$

$1 \longrightarrow 0$

$2 \longrightarrow 3$

$3 \longrightarrow 8$

$4 \longrightarrow 15$

$-1 \longrightarrow 0$

$-2 \longrightarrow 3$

as $f(1) = 0$ and $f(-1) = 0$

$f^{-1}(0) = \{1, -1\}$ $f^{-1}(3) = \{-2$

$\therefore$ f^{-1} is NOT a function

$\therefore$ f is NOT invertible

ASSIGNMENT

$1/x^2$

$x+1/x$

x^3+2

$5x-7$

$8/9-3x$

$4x+3/5x-2$

$7+4x/6-5x$

Q) $f(x) = \dfrac{4x+3}{5x-2}$

$0 \longrightarrow -3/2$

$1 \longrightarrow 7/3$

$-1 \longrightarrow -7/3$

as $f(x) \neq f(-x)$ is unique

and $f^{-1}(7/3)$

f is invertible

$$y = \frac{4x+3}{5x-2}$$

$$y(5x-2) = 4x+3$$

$$5xy - 2y - 4x - 3 = 0$$

$$x(5y-4) - (2y+3) = 0$$

$$f^{-1}(x) = \frac{2y+3}{5y-4} = \frac{2x+3}{5x-4}$$

Show that if any 30 people are selected, we may choose ~~6~~ 5 $\because$ all 5 $\rightarrow$ same day of week.

7 days in a week

~~5 people born on diff~~

$\{1, 2, 3, 4, 5\} \longrightarrow$ 6 such sets of 5 people

$\{S\ S\ M\ T\ W\}$

$\{T\ F\ S\}$

as $30 > 5$ and $30 > 7$

we are bound to have at least same. assume all have diff days.

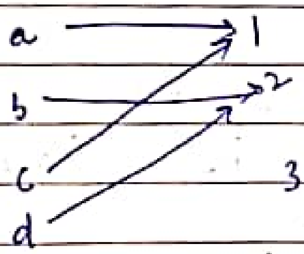
$\left\lceil \frac{30}{7} \right\rceil = 5$ people will have same.

Tutorial PQs.

$A = \{a, b, c, d\}$

$B = \{1, 2, 3\}$

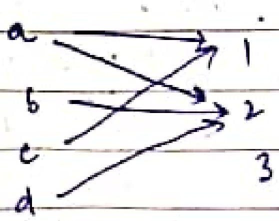
a) $\{(a,1), (b,2), (c,1), (d,2)\}$



Yes, it is a function.

Range $= \{1, 2\}$

b) $\{(a,1), (b,2), (a,2), (c,1), (d,2)\}$



It is not a func as $f(a) = \{1, 2\}$

2) $g : \{1, 2, 3\} \to \{a, b, c\}$ defined by $\begin{matrix} 1 & 2 & 3 \\ c & a & a \end{matrix}$

$h : (1, 2, 3) \to (1, 2, 3)$ defined $\begin{matrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{matrix}$

f_1 a
$1 \longrightarrow c$
$2 \longrightarrow a$
$3 \longrightarrow b$

func is neither one to one nor onto. It is everywhere defined.

f_2
$1 \to 2$
$2 \to 1$
$3 \longrightarrow 3$

func is with one to one & ~~not~~ onto.
& Yes, it is everywhere defined as each val in domain has an output

3) $A = B = C = R$.

$f : A \to B \qquad f(x) = x + 9$

$g : B \to C \qquad g(y) = y^2 + 3$.

Find :

$f \circ f(a)$
$g \circ g(a)$
$g \circ f(3)$
$f \circ g(-3)$

$f : A \to B$.

x	$x + 9$
1	10
2	11
3	12

$f \circ f$

10	19
11	20
12	21

$g : B \to C$
$y \qquad y^2 + 3$.

$f \circ f$

1	19
2	20
3	21

$f(f(x))$
$f(a + 9)$
$(a+9) + 9 = a + 18$

$g(a^2 + 3) = a^4 + 9 + 18a^2 + 3$
$= a^4 + 18a^2 + 12$

$g(a + 9) = a^2 + 81 + 18a + 3$
$= a^2 + 18a + 84$

at $a = 3$
$9 + 54 + 84 = 147$

$f(\cancel{y^2 + 3}) = \cancel{a^2 + 12} = \boxed{147}$
at -3

$a^2 + 12 =$
$9 + 12 = 21$

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i) among 100 people at least 9 are born on same month

let all 12 be born on diff months

$\left\lceil \frac{100}{12} \right\rceil = 9$

~~Still~~, at least one month will have at least 9 people born.

) at least 6 receive same grade.

possible grades = 5

$\left(\frac{N-1}{k}\right)+1 = 6$

$\cancel{\left(\frac{6-1}{5}\right)+1 =}$

$\left(\frac{N-1}{k}\right)+1 = 6$

$\frac{N-1}{5} = 5$

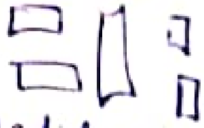
$N = 26$.

fx

$\left(\frac{N-1}{k}\right)+1 = M$

small

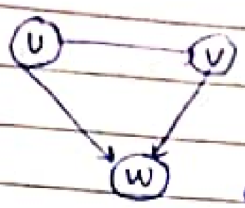
big



Module 6 – GRAPH THEORY ~~FUNCTIONS~~

Terminologies – directed graph

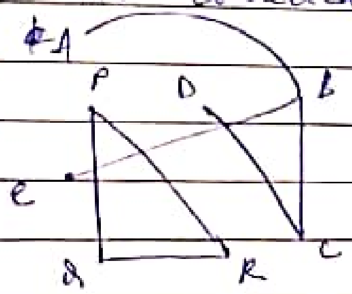
indeg – no of edges coming into a node
outdeg – " " " going away from a node



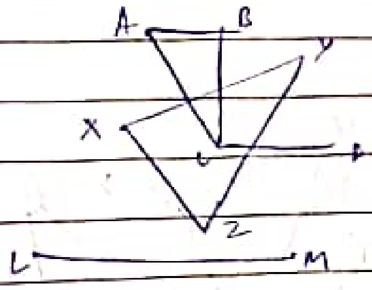
	u	v	w
indeg	0	1	2
outdeg	2	1	0

– connected graph – all nodes are connected
– disconnected graph – all " " not "
– each connected piece of a disconnected graph is called its components.

Q) Find whether the graph is connected, if not, find its connected component.



2 connected components
PQR
and ABCDE.



ABCD
XYZ
LM
} 3 components

COMPLETE GRAPH

- every vertex is connected to every vertex
- LABELLED GRAPH – if edges and vertices are assigned some data.
- WEIGHTED – each edge is assigned a +ve number called the weight of edge

SUBGRAPH - contains ~~all~~ some edges of main graph

SPANNING SUBGRAPH
→ contains all vertices of G.

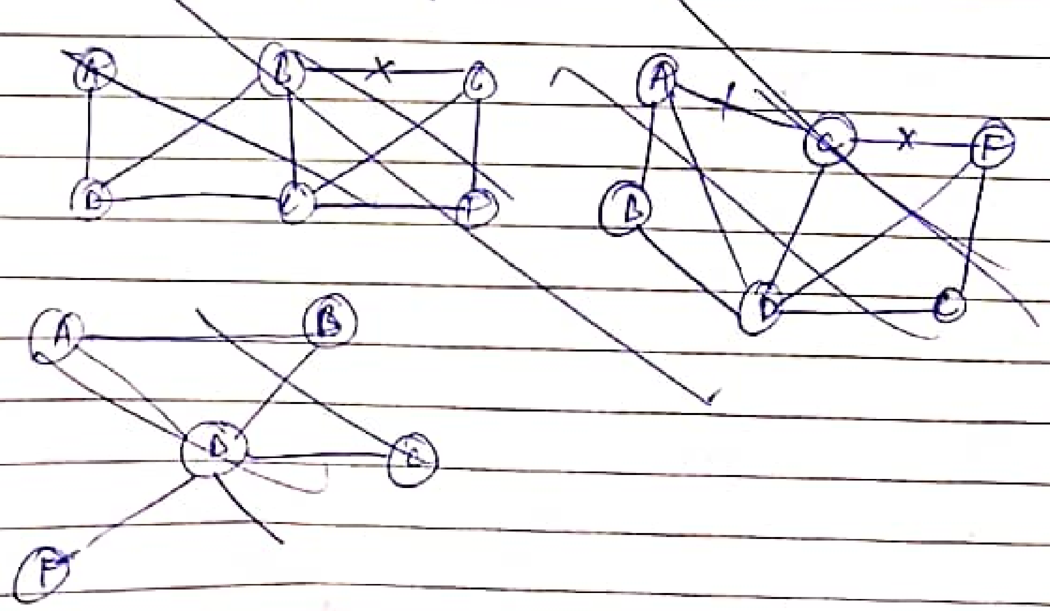
COMPLEMENT OF SUBGRAPH
→ G ka subgraph S'
Then G − S' = complement of subgraph

SUBGRAPH ISOMORPHISM
if main graph ka subgraph is isomorphic to 2nd graph.

HANDSHAKING LEMMA
→ The sum of all degrees of all vertices in G is twice the no. of edges in G

$$\Sigma\, d(v_i) = 2e$$

PROBLEM
→ 6 nodes 2 of degree 4 } no of edges
4 of degree 2



Path and circuit

- Path - series of nodes where one node is traversed once

Euler path

- visit one edge exactly once

eg.  not a euler path

 a euler path

Euler circuit

closed euler path with end point & starting point same.

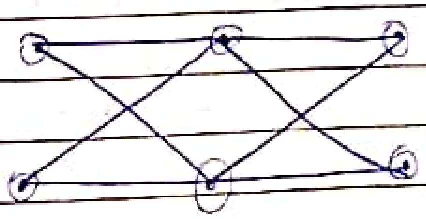
Q) how many nodes are necessary to construct a graph with exactly 6 edges ∵ each node is degree 2.

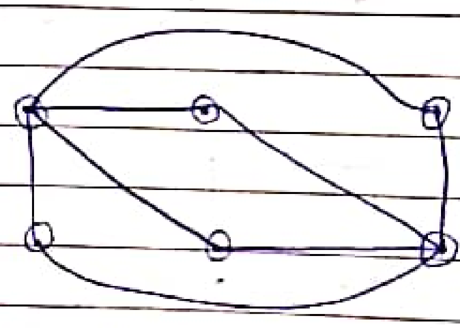
$$\sum d(v_i) = 2e$$
$$2x = 12$$
$$x = 6$$

} Based on Handshaking lemma.

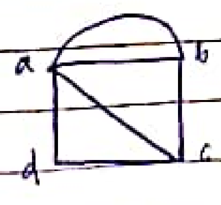
Q) Determine the no of edges with 6 nodes, 2 of degree 4 and 4 of degree 2. Draw 2 graphs.

$$\sum d(v_i) = 2e$$
$$(2 \times 4) + (4 \times 2) = 2e$$
$$16 = 2e$$
$$e = 8$$



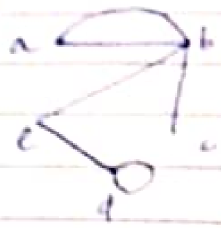


a + b + c + d + e

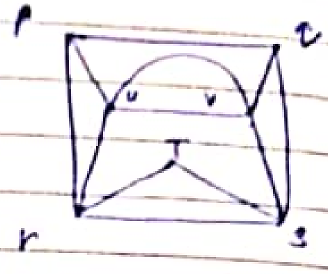
eg : 

Hamiltonian - a b c d a

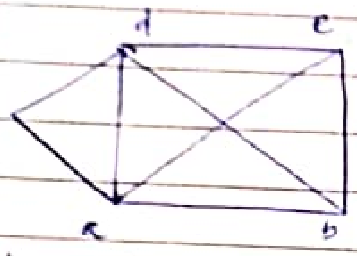
euler - c a b c d a b



hamiltonian - ~~a +~~ NO
euler - c b a b c d d

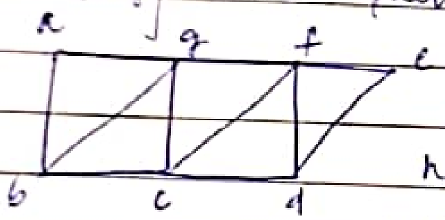


Hamiltonian -
p q s r t s v u p
euler - p r s q v u r t s v u p



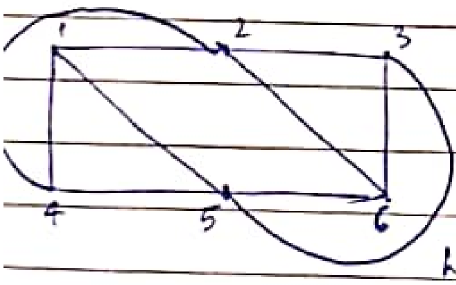
Hamiltonian a b e d c a
euler - a c d e b a e b d a (NO)

Identify euler path, circuit, hamiltonian P&C

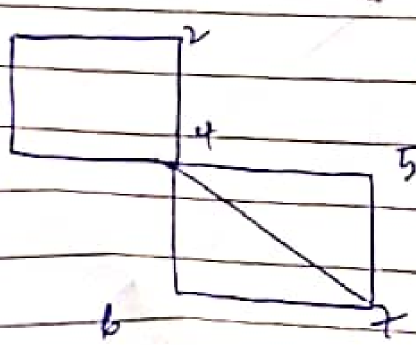


euler path - b a g b c g f c d f e d
hamiltonian - a g f e d c b a
euler circuit

Identify euler path, circuit, Hamiltonian path, circuit



euler path :
euler circuit :
hamil path :
hamil circuit : 1 4 5 6 3 2 1

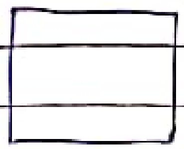


hamiltonian ~~circuit~~ path -
1 2 4 5 7 6 3 1
no circuit

ISOMORPHIC GRAPH

Graphs $G = (V, E)$ and $H = (U, F)$ are isomorphic if $f : V \to U$ ∵ x & y are adjacent.

- Same no of vertices
- same no of edges
- equal no of vertices of a given degree
- adjacency of vertices



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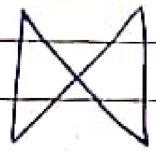
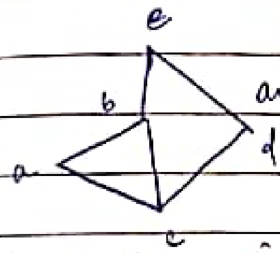


same no of edges / vertices and each vertex has 2 degree



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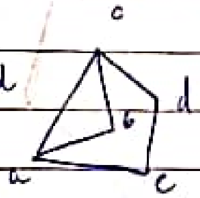
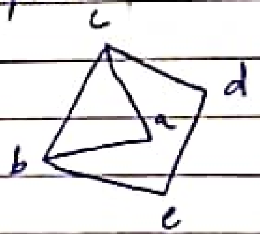


deg 2 = e, d, a
= b, d, c

not isomorphic.

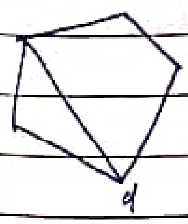


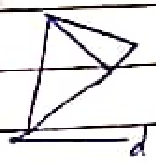
∴ 5 vertices & 6 edges.

∴ interchange b & a

∴ interchange e & c

$f(a) = b$
$f(a) = c$ as LHS mein a has degree 2
$f(a) = d$ and RHS me b, c, d have degree 2





Here, d has one edge in RHS but nothing on LHS to map with.

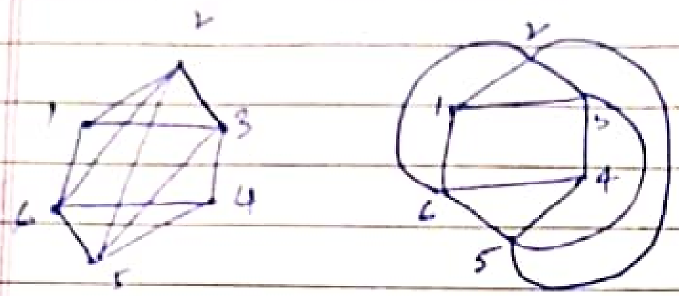
PLANAR GRAPH

→ Euler's connecting graph theorem.

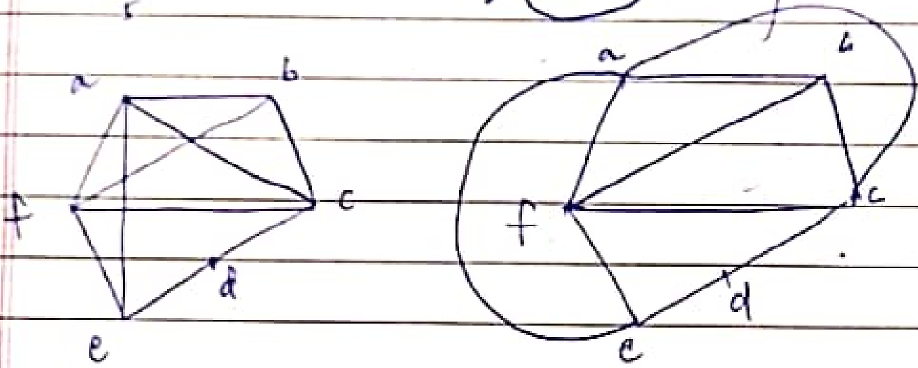
$$v - e + r = 2$$

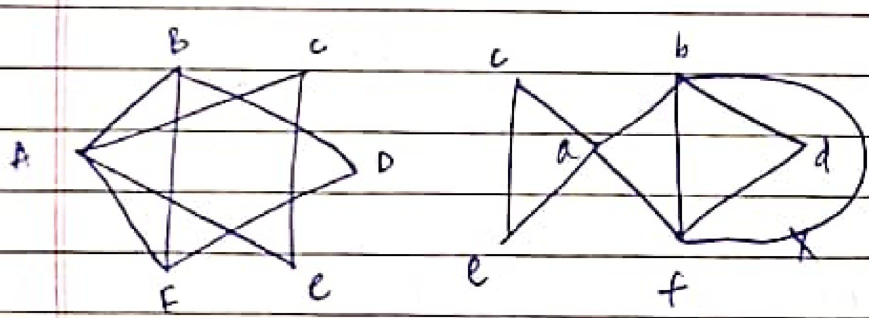
no of vertices, edges, regions.

Q1)



convert it planar.





Q2) How many edges must a planar graph have if it has $r = 7$, $n = 5$.

$$v - e + r = 2$$

$$5 - e + 7 = 2$$

$$10 = e$$