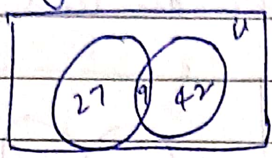
Date: 17/07 DSM

Q1) In a group of 60 people, 27 like cricket, 42 like football and each person likes at least one. Solve using venn diagram



set A = cricket
set B = football

Q2) There are 35 students in art class and 57 students in dance. Find the no of students who are either in art or at dance.

i → when classes meet at diff hours and 12 are enrolled in both.

2 → when two classes meet at the same hour.

1) $n(A) = 35$

$n(B) = 57$

$n(A \cap B) = 12$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 35 + 57 - 12$

$= 80$

2) $n(A) + n(B) \cancel{+ n(A \cap B)} = 92$

Q3) group = 100.

english = ~~70~~ 72

french = 43

Both = union − n(A) − n(B)

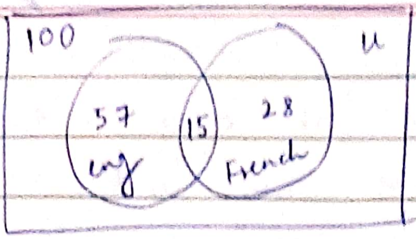
only = n(B) − both

english only = ? french only = ? both = ?

both = −100(+72+43) = ~~28~~ 15

eng only = 72 − 15 = 57

french only = 43 − 15 = 28

→ 

Q4) class = 40 → each plays at least 1 indoor game

(A) Chess = 18

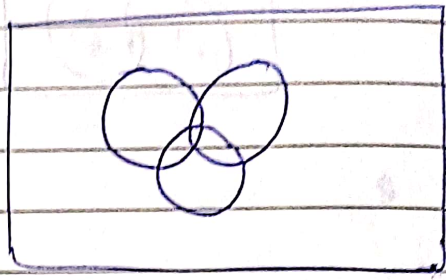
(B) Carrom = ~~20~~ 27

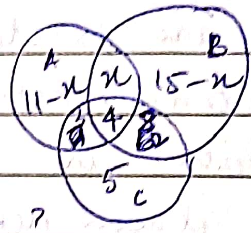
(C) Scrabble = 20

chess & scrabble = 7

Scrabble & carrom = 12

~~chess~~ all = 4





i) chess & carrom = ?

ii) " , carrom, not scrabble = ?

add; $11 - x + 15 - x + x + 11 + 4 + 5 = 40$.

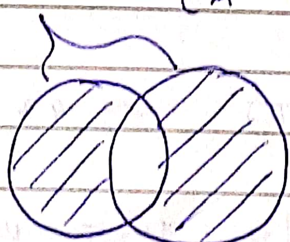
$-x = 40 - 20 - 26$.

$x = 6$.

i) = 10 (ii) = 6.

→ SYMMETRIC DIFFERENCE

$A \oplus B = (A - B) \cup (B - A)$



$A = \{1 \ 2 \ 3 \ 4 \ 5\}$

$B = \{4 \ 5 \ 6 \ 7 \ 8\}$

$A \oplus B = \{1 \ 2 \ 3 \ 6 \ 7 \ 8\}$

* remove common element.

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CARTESIAN PRODUCT

$A \times B$

$A = \{a, b\}$

$B = \{1, 2, 3\}$

$A \times B = \{(a,1), (a,2), (a,3), (b,1), \dots\}$

Q) example :

$U = \{0, 1, \dots 10\}$

$A = \{1, 2, 3, 4, 5\}$

$B = \{4, 5, 6, 7, 8\}$

i) $A \cup B = \{1, 2, 3, 4, 5, \cancel{6}\, 6, 7, 8\} \cancel{9, 10}$.

ii) $A \cap B = 4, 5$

iii) $\bar{A} = \{0, 6, 7, 8, 9, 10\}$

iv) $B' = \{0, 1, 2, 3, 9, 10\}$

v) $A - B = (1, 2, 3)$

vi) $B - A = (6, 7, 8)$

→ LAWS

1 $A \cup B = B \cup A$ } commutative
2 $A \cap B = B \cap A$

3 $A \cup (B \cup C) = (A \cup B) \cup C$ same for $\cap$ ~~ass~~ as

4 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and vice dis.

5 $A \cup \phi = A$ $A \cap U = A$ identity

6 $A \cup U = U$ $A \cap U = A$ universal set prop

7 $A \cup \{\} = A$ $A \cap \{\} = \{\}$ null prop domination law

8 $A \cup A = A$ $A \cap A = A$ idempotent

9 $(A')' = A$ double comp. law

10 $\overline{A \cup B} = \bar{A} \cap \bar{B}$ $B' \cap C$ & vv de morgan

11 $A \cup (A \cap B) = A$ $A \cup B' \cap$ vv absorption

12 $A \cup A' = U$ $A \cap A' = \phi$ complement law

$? \cup (A \cap C)$

intersection → multiply → AND
union → add → OR

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Q example

A	B	C	B∩C	A∪(B∩C)	A∪B	A∪C	exp
1	1	1	1	(1,1)	1,1	1,1	(1,1)
1	1	0	ϕ	(1,ϕ)	1,1	1,0	(1)
1	0	1	ϕ	(1,ϕ)	1,0	1,1	(1)
1	0	0	0	(1,0)	1,0	1,0	(1,0)
0	1	1	1	(0,1)	0,1	0,1	(0,1)
0	1	0	ϕ	(0,ϕ)	0,1	0,0	(0)
0	0	1	ϕ	(0,ϕ)	0,0	0,1	(0)
0	0	0	0	(0,0)	0,0	0,0	(0,0)

hence proved

→ POWER SET - P(x) is all subsets of x in a set

A = { a, b }
P(A) = { ϕ, a, b, (a,b) }
cardinality :
if n(A) = x   n(P(A)) = 2^x

16

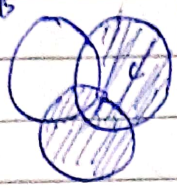
C = { 1, 2, 3, 4)   n = 4
P(C) = { ϕ 1 2 3 4 (12) (13) (14) (23) (24) (34) (123) (124) (134) (234) (1234) }   n = 16

→ PARTITION OF SETS :
A_1, A_2, etc are partitions of a set A
∵ ~~all elements of A are in~~
$A_1 \cup A_2 = A$
$A_1 \cap A_2 = \phi$
then A_1 & A_2 are partitions of A
A = { a, b, c }
partition = { {a}, { b, c } }

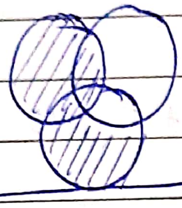
5
10
20
16
30
2
3
2
14 × 10 × 6

$B' \cap C$

B'



$A \cup B'$



Addition Principle : [10]

$|A \cup B| = |A| + |B|$

where A & B are mutually disjoint

$|A - B| = |A| - |A \cap B|$

Principle of mutual inclusion exclusion [10]

$|A \cup B| = |A| + |B| - |A \cap B|$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B|$
$- |B \cap C| - |C \cap A| + (A \cap B \cap C)$

example 1 : how many elements are in $A_1 \cup A_2$ if $n(A_1)$ $= 12$ and $n(A_2) = 18$

$A_1 \cap A_2 = \phi$

i) $|A_1 \cap A_2| = 0$

ii) $|A_1 \cup A_2| = 30$

if $(A_1 \cap A_2) = 6$, $(A \cup B) = 24$

if $A_1 \subseteq A_2 = 30 - 12 = 18$ (A_1 is a subset of A_2)

#

$A = 25$

$B = 40$ } 5 marks

$A \cap B = 10$

total needed $=$ ~~$A \cup B$~~ $= 65 - 10 = 55$

10/8 marks

#

$U = 260$

$M = 64$

$C = 94$

$B = 58$

$M \cap B = 28$

$M \cap C = 26$

$B \cap C = 22$

$B \cap C \cap M = 14$

ii) only CS $= 94 - 26 + 14 - 22$
$= 60$

* formula and venn for [8]

none $\Rightarrow U - (M + C + B - 28 - 26 - 22 + 14)$
$= 106$

Q = $U = 170$

$T = 115$

$R = 110$

$M = 130$

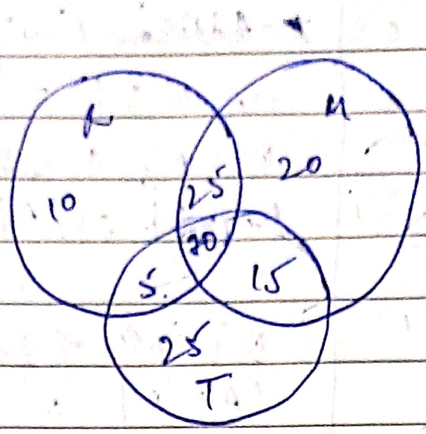
$T \cap M = 85$

$T \cap R = 75$

$M \cap R = 95$

all = 70

only R =



iii) 60

iv) exactly 2 = 25 + 5 + 15 = 45

Q) $U = 60$ = 52 union means :

$N = 25$

$T = 26$

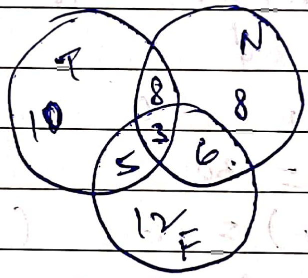
$F = 26$

$N \cap F = 9$

$N \cap T = 11$

$T \cap F = 8$

none = 8



<3

60 - none ~~52 = all~~   ~~all~~ = 25 + 26 + 26 − 9 − 11 − 8 + x

x = 3

---

p → q (implication)   p ↔ q (biconditional)

→ statement = if integer is a multipee of 2, it is even

converse → if an int is even, then it is a multiple of 2

inverse → if int is not a multiple of 2 then it is not even

contrapositive → if an int is not even then it is not a multiple of 2.

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p = Raju is rich
q = Raju is happy

→ Raju is poor ~~try~~ but happy $\sim p \wedge q$
→ " " neither rich nor happy $\sim p \wedge \sim q$
→ " " either rich OR unhappy $p \vee \sim q$
→ Raju is poor or else he's rich and unhappy $(\sim p \vee (p \wedge \sim q))$

Q either my prg runs & contains no bugs OR my prg ~~do~~ contains bug

My prg runs = p
it contains bugs = q

$((p \wedge \sim q) \vee q)$

Q) p → ~~I~~ i will study
q → i'll go to a movie
r → i'm in a good mood

→ if i'm not in a good mood, ill go movie
$\sim r \longrightarrow q$
→ i'll not go to movie & study only
$\sim q \wedge p$
→ i'll go movie iff i'll not study
~~$q \Leftrightarrow$~~ ~~$q \rightarrow \sim p$~~ $\sim p \leftrightarrow q$
→ ~~If~~ if ~~ill~~ ill not study i'm in a bad mood

$\sim p \longrightarrow \sim r$

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Q if the utility cost goes up or the req for additional funding is desired, then a new company computer wil be purchased iff we can show the current fac. are not adequate

$$\{(p \vee q) \to (r \leftrightarrow \sim s)$$

TRUTH TABLE

$((p \wedge q) \vee \sim q)$

p	q	$p \wedge q$	$\sim q$	$(p \wedge q) \vee \sim q$	
0	0	0	1	1	T
0	1	0	0	0	F
1	0	0	1	1	T
1	1	1	0	1	T

$\sim[p \wedge (p \vee \sim q)]$

p	q	$\sim q$	$p \vee \sim q$ (M)	$p \wedge (M)$ (Q)	$\sim Q$
0	0	1	1	0	1
0	1	0	0	0	1
1	0	1	1	1	0
1	1	0	1	0	1

1) Tautology → output is always true

p	q	$p \wedge q$ (x)	$x \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

∴ whenever $x \to p$ mean x is false, output is false!

2) contradiction → always false

p	$\sim p$	$\sim q$	q	$p' \vee q$ (x)	$p \wedge x$ (y)	$\sim q \wedge y$
T	F	F	T	T	T	F
T	F	T	F	F	F	F
F	T	F	T	T	F	F
F	T	T	F	T	F	F

Laws

1) idempotent law

if $p \vee p = p$

$p \wedge p = p$

2) commutative law

$p \vee q = q \vee p$

$p \wedge q = q \wedge p$

3) Associative

$(p \vee q) \vee r = p \vee (q \vee r)$

4) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

5) Identity

$p \vee 0 = p$

$p \wedge 1 = p$

$p \vee 1 = 1$

$p \wedge 0 = 0$

6) Absorption

$p \wedge (p \vee q) = p \wedge p \vee p \wedge q$

$= p \vee (p \wedge q)$

$= p \vee p \wedge p \vee q$

$= p \wedge p \vee q$

$= p$

7) implication

$p \rightarrow q = \sim p \vee q$

8) de morgan

$(p \vee q)' = p' \wedge q'$

same for $\wedge$

9) inverse law

$p \wedge p' = 0$

10) ~~$p \vee p' = 0$~~ comple. law

$p \vee p' = 1$

~~$p \wedge p' =$~~

9) Show that $a \vee (a' \wedge b) \equiv a \vee b$

i) $(a \vee a') \wedge (a \vee b)$ dis law

$1 \wedge (a \vee b) = a \vee b$ ~~identity~~ inverse law

hence proved. ↓ identity

ii) $a \wedge (a' \vee b) \equiv a \wedge b$

$a \wedge a' \vee a \wedge b$ dis law

$0 \vee a \wedge b$ complement

$a \wedge b$. identity

iii) $[(p \to q) \wedge \sim q] \to p'$ is a tautology

$p \to q = p' \vee q$

$(p' \vee q) \wedge q'$

$p' \wedge q' \vee \underbrace{q \wedge q'}$ dis law

$p' \wedge q' \vee 0$ inverse

$p' \wedge q'$ identity

$(p' \wedge q') \to p'$ implication

$(p' \wedge q')' \vee p'$ ~~complem~~

$(p \vee q) \vee p'$ de-morgan

$\cancel{p \vee q}$ $p \vee p' \vee q$ ~~complement~~ inverse

$1 \vee q = 1$ identity

iv) $[p \wedge (p \to q)] \to q$

$p \to q = p' \vee q$ implication law

$p \wedge (p' \vee q)$

$(p \wedge p') \vee (p \wedge q)$ dis

$0 \vee (p \wedge q)$ complement

$p \wedge q$ identity

$(p \wedge q) \to q$

$(p \wedge q)' \vee q$ de morg

$(p' \vee q') \vee q$

$p' \vee (q' \vee q)$ associative

$p' \vee \text{True}$ identity

$\cancel{p}$ True

v) $[(\sim p \vee \sim q) \longrightarrow (p \wedge q \wedge r)] \leftrightarrow p \wedge q$

$(p' \vee q')' \vee (p \wedge q \wedge r)$ impl.

$(p \wedge q) \vee (p \wedge q \wedge r)$ de morgan

let $p \wedge q = x$

$x \vee (x \wedge r)$ absorp.

$= x = p \wedge q \equiv \text{RHS}.$

vi) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$(p \vee q) \wedge (p \vee r)$

p	q	r	x: $q \wedge r$	$p \vee (x)$	m: $p \vee q$	n: $p \vee r$	$m \wedge n$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

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Q) Truth table

$$\overset{a}{(p \wedge \neg q)} \rightarrow \overset{b}{(p \vee q)} = x = a' + b = f(x)$$

p	q	p'	q'	$\overset{x}{p \wedge q'}$	$p \vee q$	x'	$f(x)$
0	0	1	1	0	0	1	1
0	1	1	0	0	1	1	1
1	0	0	1	1	1	0	1
1	1	0	0	0	1	1	1

∴ it is a tautology.

Q $((p \wedge q) \vee \neg q) = f(x)$

p	q	$p \wedge q$	q'	$f(x)$	$p \vee q$	$f(y)$
0	0	0	1	1	0	0
0	1	0	0	0	1	0
1	0	0	1	1	1	1
1	1	1	0	1	1	0

Q $((p \vee q) \wedge \neg q) = f(y)$

Q $\sim[p \wedge (p \vee \sim q)]$

p	q	q'	$p \vee q'$	$\overset{x}{p \wedge (p \vee q')}$	$\sim x$
0	0	1	1	0	1
0	1	0	0	0	1
1	0	1	1	1	0
1	1	0	1	1	0

9) a) $p \vee \sim q$

p	q'	$p \vee q'$
0	1	1
0	0	0
1	1	1
1	0	1

b) $\sim p \wedge \sim q$

p'	q'	$p' \wedge q'$
1	1	1
1	0	0
0	1	0
0	0	0

r) verify cont.

$(p \wedge q) \wedge \sim(p \vee q) = f(x)$

p	q	$p \wedge q$	$p \vee q$ (x)	$\sim x$	$f(x)$
0	0	0	0	1	0
0	1	0	1	0	0
1	0	0	1	0	0
1	1	1	1	0	0

$\therefore$ proved

$(p \wedge q) \wedge (p' \wedge q')$ de morgan.

$p \wedge p' \quad \wedge \quad q \wedge q'$ ass. & identity

$0 \wedge 0 = 0$. hence proved.

→

Quantifier → not propositions
do not have a truth value

eg → $x + 3 = 5$
He is lonely.

DUALITY LAW

x	x^*
$\wedge$ to $\wedge$	$\wedge$ to $\vee$
T to F	F to T
$(p \wedge q) \vee t$	$(p \vee q) \wedge T$

ELEMENTARY PRODUCT & SUM

→ elementary pd = conjunction ($\wedge$)
" sum = disjunction ($\vee$)

1. Take -ve of eq = y
2. Make TT of y
3. dnf ⇒ value = T
4. write all True as SOP
5. Now negate dnf for cnf

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Bending

i) $P(x) \Rightarrow x + 3 = 5$

Bending x by putting $x = -1 \rightarrow$ false

" " " " " $x = 2 \rightarrow$ true

ii) if $m(x)$ is " ~~He~~ x is male "

$$\exists x (m(x) \rightarrow (x))$$

There exists a man who is ____.

$$\forall x (m(x) \wedge (x))$$

For all men x is ____ & ____.

NORMAL FORMS

DNF → sum of products

$\vee$ of all $\wedge$

CNF → Prod of sums

$\wedge$ of all $\vee$.

DNF to CNF

To write a logical exp E in CNF

i) find $\neg E$

ii) write $\neg E$ in DNF

iii) Negate DNF. Apply ~~DNF~~ De morgan to get CNF

Q) $[(p \Rightarrow q) \Rightarrow r]$ in CNF

$\neg[(p \rightarrow q) \rightarrow r]$ in dnf

$\neg[(p \wedge q \wedge \neg r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r)]$

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MATHEMATICAL INDUCTION

Q)1) Prove by induction :

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

for all natural values of n.

i) Basis of induction

$$P(1) = \frac{1(1+1)}{2} = 1$$

$P(1) =$ True

ii) Assume $p(k)$ is true

$$1+2+\dots+k = \frac{k(k+1)}{2}$$

This step gives induction hypothesis. — ①

$$p(k+1) = 1+2+3+\dots k+1 = \frac{(k+1)(k+2)}{2} \quad — ②$$

iii) Using eq ①

$$\therefore \frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}$$

$$k(k+1) + 2(k+1) = (k+1)(k+2)$$

$$k^2 + k + 2k + 2 = k^2 + k + 2k + 2$$

$$LHS = RHS$$

By assuming $p(k)$ is true, $p(k+1)$ is also true.

Q2) $1^3 + 2^3 + 3^3 + \dots + n^3 = (1+2+\dots n)^2$

i) Basis of induction

$$p(1) = 1^2 = 1 = 1^3$$

$$p(2) = 1^3 + 2^3 = (1+2)^2$$

$$9 = 3^2 = 9.$$

ii) Assume $p(k)$ is true

$$1^3 + 2^3 + \dots + k^3 = (1+2+\dots k)^2$$

This gives induction hypothesis

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$p(k+1) = 1^3 + 2^3 + \dots k^3 + (k+1)^3 = (1+2+\dots+k+k+1)^2$

using eq ①:

$(1+2\dots+k)^2 + (k+1)^3 = [1+2+\dots k+k+1]^2$

$a^2 + x^3 = (a+b)^2$

$\cancel{a^2} + x^3 = \cancel{a^2} + b^2 + 2ab.$

$b^3 = (k+1)^2 + 2(k+1)(1+2\dots k)$

$\cancel{2}(k+1)(k(k+1))/\cancel{2}$

$(k+1)^3 = (k+1)^3$

Hence proved.

★ Tut 2 Practice

1) show that ;

$1^2 + 3^2 + 5^2 \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Basis of ind^n.

$p(1) = 1^2 = \frac{1(2-1)(2+1)}{3} = 1$

$p(2) = 1^2 + 3^2 = 10 = \frac{2(\cancel{3})(5)}{\cancel{3}} = 10.$

$p(1)$ & $p(2)$ is true

Induction hypothesis :

Assume $p(k)$ is true

$p(k) = 1^2 + 3^2 + \dots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$

Assume $p(k+1)$ is true

$1^2 + 3^2 + \dots + (2(k+1)-1)^2 = \frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$

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$$(2k-1)^2 +$$

$$= 1^2 + 3^2 + \cdots \cancel{} (2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}.$$

By eq ①

$$\frac{k(2k-1)(k+1)}{\cancel{3}} + 3(2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{\cancel{3}}$$

$$(2k^2-k)(2k+1) + 3(4k^2+1+4k) = (2k^2+3k+1)(2k+3)$$

LHS = RHS.

Q2) All students have taken a course in comm. skills

x has taken C.S.

$$\forall x \,(S(x) \land C(x))$$

x is a student.

There is a girl student in the class who is also a sports person.

$$\exists x \,[G(x) \land S(x) \land P(x)]$$

Some students are intelligent but not hardworking

$$\exists x \,[S(x) \land I(x) \land \neg H(x)]$$

Q3) $A = \{1, 2, 3, 4, 5\}$ determine truth value of :

a) $(\exists x \in A)(x + 3 = 10)$ False

b) $(\forall x \in A)(x + 3 < 10)$ True

c) $(\exists x \in A)(x + 3 < 5)$ True

d) $(\forall x \in A)(x + 3 \le 7)$ False