

# *User's Manual*

## e/m Experiment

Model EMX-01  
(Rev : 08/2016)

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## INTRODUCTION

Our arrangement for measuring  $e/m$ , the charge to mass ratio of the electron, is a very simple set-up. It is based on Thomson's method. The  $e/m$ -tube is bulb-like and contains a filament, a cathode, a grid, a pair of deflection plates and an anode. The filament heats the cathode which emits electrons. The electrons are accelerated through a known potential difference between the cathode and the anode. The grid and the anode have a hole through which electrons can pass. The tube is filled with helium at a very low pressure. Some of the electrons emitted by the cathode collide with helium atoms which get excited and radiate visible light. The electron beam thus leaves a visible track in the tube and all manipulations on it can be seen. The tube is placed between a pair of fixed Helmholtz coils which produce a uniform and known magnetic field. The socket of the tube can be rotated so that the electron beam is at right angles to the magnetic field. The beam is deflected in a circular path of radius  $r$  depending on the acceleration potential  $V$ , the magnetic field  $B$  and the charge to mass ratio  $e/m$ . This circular path is visible and the diameter  $d$  can be measured and  $e/m$  obtained from the relation

$$e/m = 8V/B^2 d^2.$$

The deflecting plates play no role in the  $e/m$  experiment. They are interesting for a visual observation of how the electron beam gets deflected when a potential difference is applied between the deflecting plates.

This set-up can also be used to study various features of Lorentz force  $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$  by observing the electron beam deflection for different directions of the magnetic field and different orientation of the  $e/m$ -tube.

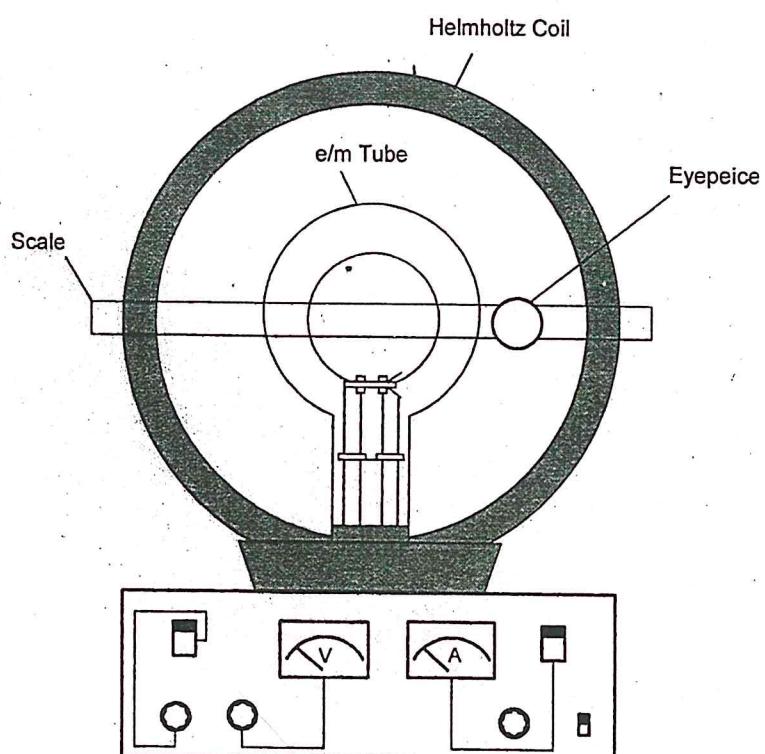


Fig. 1:  $e/m$  Experiment, EMX-01

## THEORY

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**Relation connecting  $e/m$  to accelerating potential  $V$ , magnetic field  $B$  and radius  $r$  of the circular path**

When the electrons are accelerated through the potential  $V$ , they gain kinetic energy equal to their charge times the accelerating potential. Therefore  $eV = mv^2/2$ . The final (non-relativistic) velocity of the electrons is therefore

$$v = (2eV/m)^{1/2} \quad (1)$$

When these electrons pass through a region having a magnetic field  $B$ , they are acted upon by a force, called the Lorentz force, given by  $e\vec{v} \times \vec{B}$ . If the electrons are initially moving along  $x$ -axis and the magnetic field is along  $z$ -axis, the electrons describe a circular path in the  $xy$ -plane with the centripetal force balancing the Lorentz force,

$$\begin{aligned} evB &= mv^2/r \\ \text{or } v &= eBr/m \end{aligned} \quad (2)$$

Eliminating  $v$  between Eqs.(1) and (2), we get

$$\begin{aligned} eBr/m &= (2eV/m)^{1/2} \\ \text{or } e/m &= 2V/B^2r^2 = 8V/B^2d^2 \end{aligned} \quad (3)$$

where  $d$  is diameter of the circular path. This result assumes that the magnetic field  $B$  is uniform. This in the apparatus is produced by a pair of Helmholtz coils (separated by a distance equal to their radius). If  $n$  is number of turns in a coil and  $a$  its radius, then the magnetic field  $B$ , midway between the coils is given by

$$B = 2 \times \frac{\mu_0 In}{2(5/4)^{3/2} a} = 2 \times \left( \frac{2\pi In}{(5/4)^{3/2} a} \times 10^{-7} \right) \text{ tesla}$$

when a current of  $I$  amp is flowing in the coils.  $\mu_0$  is permeability of free space and is given by  $\mu_0 = 4\pi \times 10^{-7}$  N/A<sup>2</sup>. This field is uniform in the region where the electrons move. Putting the value of  $B$  in Eq.(3), we get

$$\frac{e}{m} = \left( \frac{125a^2}{128\pi^2 n^2} \times 10^{14} \right) \frac{V}{I^2 d^2} \quad (4)$$

The coils in this apparatus have 160 turns each and their radii are 0.14 m. using these values

$$1.81 = \frac{e}{m} = \frac{(7.576 \times 10^6) \times \frac{V(\text{volt})}{I^2 (\text{amp}^2) d^2 (\text{m}^2)}}{\text{coul/kg}} \quad (5)$$

$$d^2 = \left( \frac{7.576 \times 10^6}{e/m} \right)$$

## DESCRIPTION OF THE APPARATUS

The central part of the set-up is the e/m-tube. This is energized by

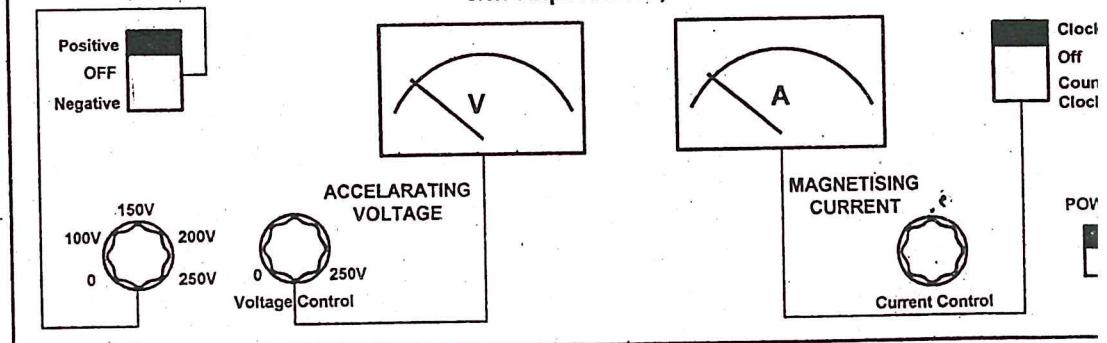
- (i) Filament current supply,
- (ii) Deflection plates voltage supply,
- (iii) Continuously variable accelerating voltage supply to the anode.

The tube is mounted on a rotatable socket and is placed between a pair of Helmholtz coils. The tube can be rotated about a vertical axis, varying the orientation of the electron beam with respect to the Helmholtz coils. This allows magnetic deflection of the beam to be demonstrated. Circular, helical or undeflected paths can be seen. The direction of the current to the Helmholtz coils can be changed. The magnetizing current  $I$  and the accelerating voltage  $V$  are respectively measured by an ammeter and a voltmeter mounted on the front of the panel. For the measurement of  $e/m$ , the socket of the tube is rotated so that the electron beam path at right angles to the magnetic field. The beam is deflected in a circular path. The diameter of the electron beam path is measured by a detachable scale mounted in front of the bulb of the tube. This scale has a slider with a hollow tube (fitted with cross wires at its both ends) to fix the line of sight while making the measurements of the beam path diameter. Base of the unit contains the power supply that provides all the required potentials and the current to the Helmholtz coils. The entire apparatus is contained in a wooden case for convenient storage.

## SPECIFICATIONS

Helmholtz coils of radii	14 cm
Number of turns	160 on each coil
Accelerating Voltage	0 – 250V
Deflection plates voltage	50V – 250V
Operating Voltage	220V AC/ 50Hz

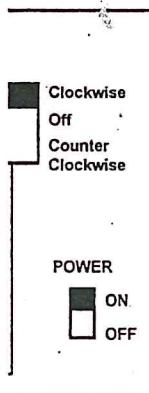
### e/m Experiment, EMX-01

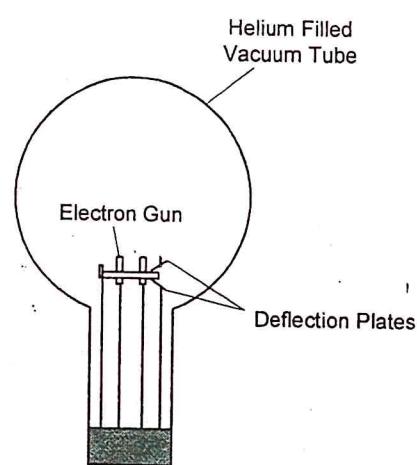


Control Panel

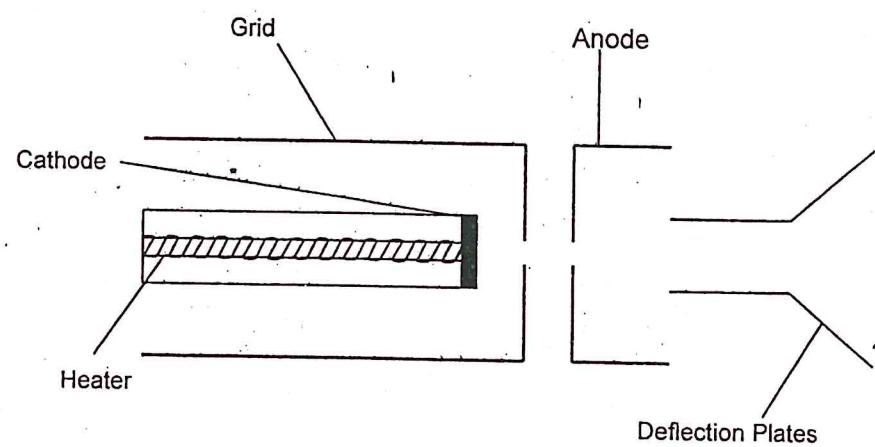
## OPERATING INSTRUCTIONS FOR e/m EXPERIMENT

1. Before the power is switched to 'ON', make sure all the control knobs are at their minimum position.
2. Turn the power switch to 'ON'. The indicator lamp will glow.
3. Wait a little for the cathode to heat up.
4. Turn the accelerator voltage adjust knob clockwise to increase the voltage. Rectilinear electron beam emerging from the cathode will be visible.. Adjust the accelerator voltage at about 200 volt.
5. It should be clear that the electrons themselves in the beam are not visible. What is observed is the glow of the helium gas in the tube when the electrons collide with the atoms of the gas. We actually see the glow of gas atoms which have been excited by collisions with electrons.
6. Rotate the e/m - tube so that the electron beam is parallel to the plane of the Helmholtz coils. *Do not take it out of its socket.*
7. Earth's magnetic field interferes with the measurements. However this magnetic field is weak compared to the field generated by the Helmholtz coils and we could ignore its effect as a first approximation.
8. Slowly turn the current adjust knob clockwise to increase the current for the Helmholtz coils. The electron beam will get curved. Increasing the current will increase the curvature of the beam.
9. In case the electron beam does not make a complete (closed) circle and the circular path is skewed, rotate the socket of the tube until the path is a closed circle. This happens when the tube pointer is set at about  $90^{\circ}$ .
10. Measure the diameter of the electron beam. This measurement has been facilitated by fixing a hollow tube (fitted with cross wires at its both ends) on the slider of the scale. This tube fixes the line of sight during measurements.
11. Note the ammeter reading for the current to the Helmholtz coils and the voltmeter reading for the accelerating voltage.
12. Decrease the accelerating voltage by a small amount (20 volt, say) and measure the diameter of the electron beam path.
13. Carry on the observations. The voltmeter reading should not be increased beyond 250 volt. A value lower than 80 volt is also not advisable. Similarly the current to the Helmholtz coils should not be more than 2 amp.
14. *Do not leave the beam ON for long periods of time.*



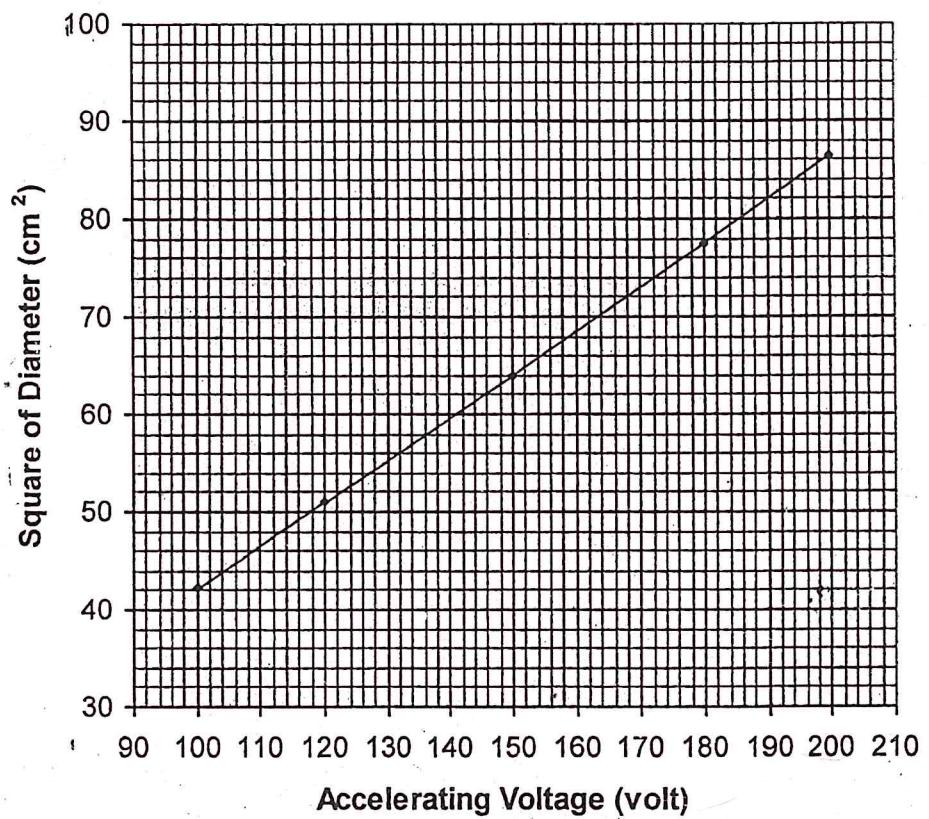


**Fig 2: e/m Tube**



**Fig. 3: Electron Gun**

$(\text{Diameter})^2$  vs. Accelerating Voltage



(c)



# **MANUAL**

**RESISTIVITY OF SEMICONDUCTOR BY FOUR PROBE METHOD AT  
DIFFERENT TEMPERATURES & DETERMINATION OF ENERGY BAND GAP**

A Product of:

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## INTRODUCTION

The Properties of the bulk material used for the fabrication of Semiconductors devices like transistors, diodes IC's etc. are essential in determining the characteristics of the completed devices. Resistivity and lifetime [of minority carries] measurements are generally made on germanium crystals to determine their suitability. The resistivity , in particular , must be measured accurately since its value is critical in many devices. The value of some transistor parameters, like the equivalent base resistance, are at least linearly related to the resistivity.

### ELECTRONIC CONDUCTION IN SOLIDS:

The electrical properties of semiconductors involve the motion of charged particles with in them. Therefore, we must have an understanding of the forces which control the motion of these particles. It is of course, the physical structure of the solid which exerts their control. This topic is very large, so only the highlights will be covered .The reader is referred to many excellent sources which are listed at the end for more details on specific aspects.

Atoms , of which a solid is composed, consist of positively charged nuclei with electron orbiting around them. The positive charge is compensated by negatively charged electrons, so that a complete atom is electrically neutral. Electrons are arranged in shells, and the closer they are to the nucleus the more strongly they are bound. If we take the particular case of silicon, a well known semiconductor, we find that it has 14 electrons which are accommodated in the shells as  $1S^2$ ,  $2S^2$ ,  $2P^6$ ,  $3S^2$ ,  $2P^2$ . Since the third shell is not even half filled, the 4 electrons are available for chemical bonding giving silicon a valency of four.(Germanium also has a valency of 4, but from the fourth shell). Fig.1 shows an energy diagram of an individual atom.

Let us now concentrate our attention on solids. If we bring many atoms close to one another, inter-atomic forces become quite strong as electronic orbits begin to overlap. The outer-shell electrons play an important role, because their orbits are the most disturbed. These electrons are no longer associated with a particular atom; the outer shell electron may make an orbit around one atom and continue about another. In this fashion, the outer shell or valency electrons are continually traded among atoms and wander all over the solid. The continuous interchange of valence electrons between atom holds the solid together. This is the predominant type of bonding in silicon and germanium, and is called the valence bonding.

In solids, atoms are usually arranged in a regular way to achieve dense packing and thereby form a crystal. The arrangement has very desirable characteristics; i.e. the transport of holes and free electrons is very smooth in these structures. When the arrangement is not crystalline complication arise. Here we will be concerned only with the properties of perfect crystals. Silicon and germanium [and carbon] crystallize with an identical crystal structure, the so-called Diamond structure. Such a structure is shown in Fig.2. The arrangement of atoms in the illustration forms a unit cell, and the crystal is made up of adjacent unit cells.

Fig.3 shows a potential diagram of an array of atoms. An actual crystal is of course three-dimensional. The most important difference between the potential plot of an isolated atom and one-dimensional array is the splitting of energy levels. In fact, bringing N atoms together we find

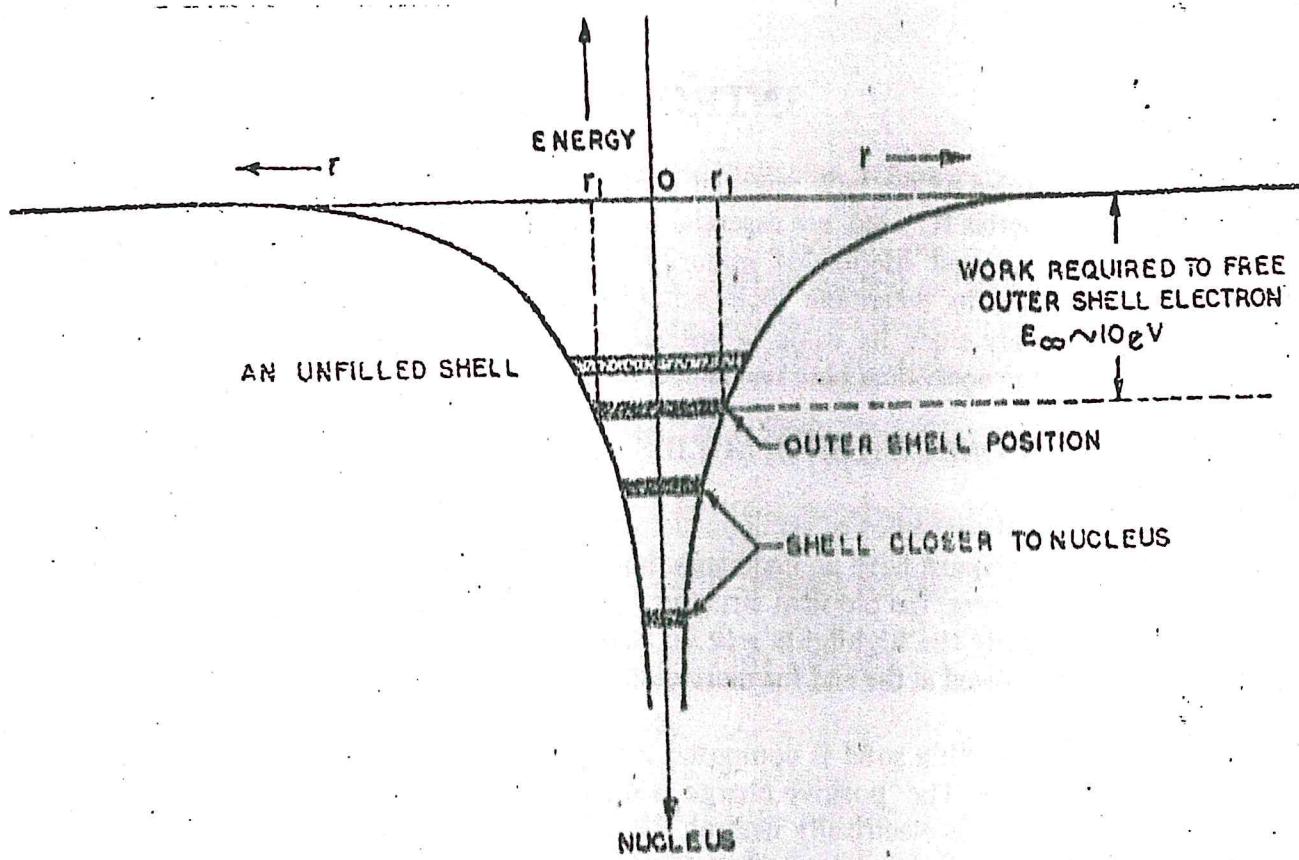


FIG. 1 POTENTIAL DISTRIBUTION AROUND AN INDIVIDUAL ATOM

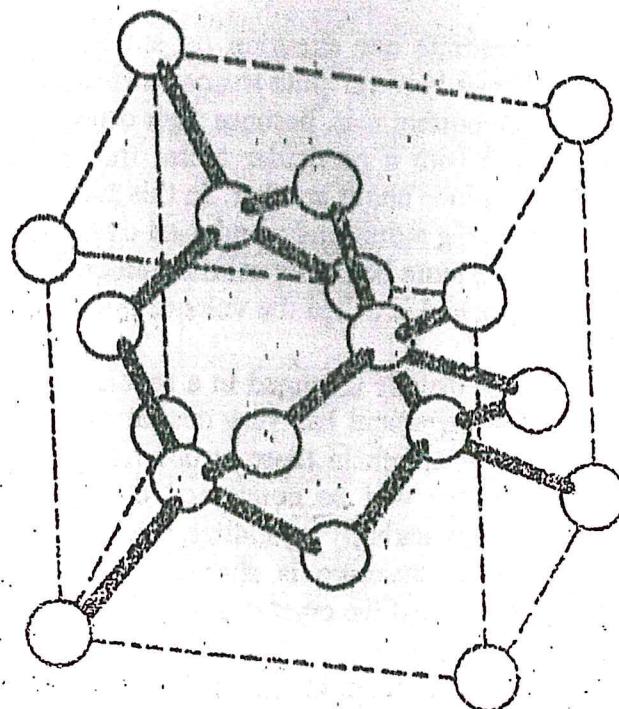


FIG. 2 STRUCTURE OF THE DIAMOND LATTICE

'N' times as many levels throughout the crystal. The spreading of energy levels depends on the degree of interaction, therefore, the inner orbits split into levels combined in a narrow energy than the outer ones.

As a result of the interaction between the tremendous number of atoms in crystal ( $10^{22}$  per  $\text{cm}^3$ ) the energy level found in isolated atoms would be split and form bands of allowed energies which contain almost continuum of levels. Accordingly, electrons are located in energy bands in crystalline solid. The band, which contains the valence electrons, is called the **valence band**. The unoccupied energy levels also split up and form another band called the **conduction band**. The interaction between the unused shells is very large and they spread widely. Therefore, while there is a band gap  $E_g$  [or forbidden region] between the valence and conduction bands, splitting of higher orbit is so wide that they usually overlap.

The bands below the energy gap  $E_g$  are completely filled at absolute zero temperature and the conduction band is empty. This is a very important point and has direct consequences on the conduction properties, as we shall see soon. The fundamental theory is that **current conduction is not possible in complete empty and complete filled bands**. The reasons about the empty band is obvious since current is not possible without carrier. The reason about the filled band is as follows: though the valence electrons move about the crystal but they can not be accelerated because the acceleration means gain of energy and there are no higher energy levels available to which they could rise.

We can now readily see that the crystal band structure shown in Fig.3 does not allow current conduction at  $T=0$ . If we increase the temperature, however thermal agitation increase and some valence electron will gain energy greater than  $E_g$  and jump into the conduction band. The electron in the conduction band is called a **free Electron**, and its former place in the valence band is called a **Hole**. Electron in conduction band can gain energy when a field is applied, because there are many higher energy states available. The fact that electrons left the valence bands leaves some empty energy levels, this allows conduction in the valence band as well. Electrons can now gain energy in the valence band also, and we observe a motion of **Holes** in the direction of the field. Because of this we begin to speak of a **Hole** as a current carrying particles.

According to the preceding theory, an insulator must have a large band gap, so that at room temperature practically the conduction band is empty and the valence band is filled and a semi-conductors must have a narrower band gaps so that appreciable number of carriers are present in the valence and conduction bands at room temperature.

In metals, however, the valence and conduction bands overlap and application of an electric field can, therefore, accelerate a great sea of electrons. The non-existence of a band gap makes conduction in metal almost independent of temperature, as compared to semiconductors. This summarizes the above points.

The fact that electrons can be found in energy bands with in a crystal and other conclusions mentioned so far can be shown by quantum mechanical calculations also which we shall not carry out here, but will use the results for quantitative analysis.

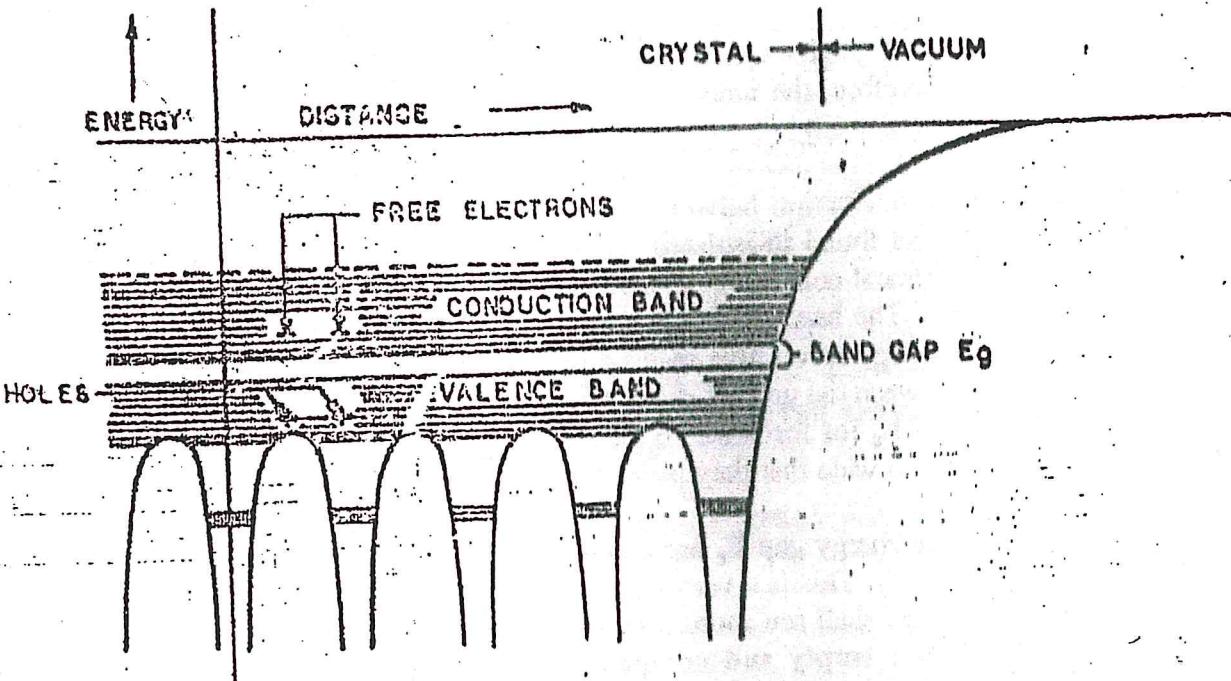


FIG. 3: UNUSED SHELLS FORM THE CONDUCTION BAND

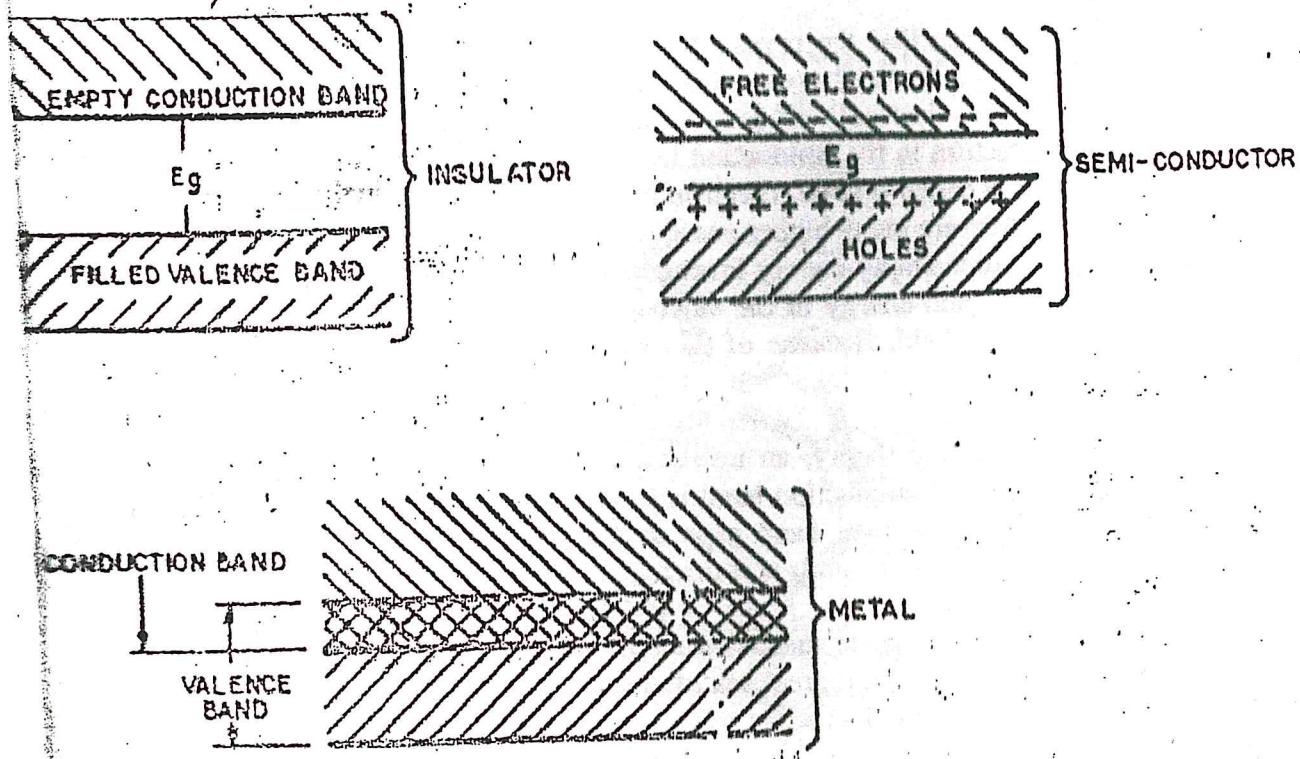


FIG. 4: BAND STRUCTURES FOR INSULATORS, SEMICONDUCTOR AND METALS

## CONCENTRATION OF INTRINSIC CARRIERS:

The concentration of intrinsic carriers i.e. the number of electrons in conduction band per unit volume is given by the expression:

$$n = 2 \left( \frac{m_e k T}{2 \pi h^2} \right)^{3/2} e^{(\mu - E_g)/kT} \quad (1)$$

And the concentration of holes in valence band is given by the expression

$$p = 2 \left( \frac{m_h \cdot k \cdot T}{2 \pi h^2} \right)^{3/2} e^{(-\mu/kT)} \quad (2)$$

If we multiply together the expression for n and p to obtain equilibrium relation:

$$np = 4 \left( \frac{kT}{2\pi h^2} \right)^3 (m_e m_h)^{3/2} e^{(-E_g/kT)} \quad (3)$$

This does not involve the Fermi level  $\mu$  and is known as the expression of ' law of mass action ' Where

$m_e$  = effective mass of Electron

$m_h$  = effective mass of Holes

$K$  = Boltzmann's constant,

$\mu$  = fermi level

$E_g$  = Bandgap,

$T$  = Temperature in °K

In case of intrinsic (highly purified) crystals, the number of electrons is equal to the number of holes, because the thermal excitation of an electron leaves behind a hole in the valance band.

Thus, from equation (3) we have, let the subscript i denote intrinsic.

$$n_i = p_i = 2 \left( \frac{kT}{2\pi h^2} \right)^{3/2} (m_e m_g)^{3/4} e^{(-E_g/kT)^2} \quad \dots \dots (4)$$

Thus, we see that the concentration of intrinsic carrier depend exponentially on  $E_g/2kT$ .

## CONDUCTIVITY OF INTRINSICS SEMICONDUCTOR:

The electrical conductivity will be the sum of the contribution of both electrons and holes.

$$\sigma = (n_i e \mu_e + p_i e \mu_h) \quad (5)$$

Where  $e$  is the electron charge.  $\mu_e$  and  $\mu_h$  are the average velocities acquired by the electrons and holes in a unit electric field and are known as mobilities. OR

$$\sigma = e n_i (\mu_e + \mu_h) \quad \text{since } n_i = p_i$$

$$\sigma = (K) T^{3/2} (\mu_e + \mu_h) e^{-E_g/2kT} \quad \dots \dots (6)$$

Using equation (4) where  $k$  is a constant.

The factor  $T^{3/2}$  and the mobilities change relatively slow with temperature compared with the exponential term, and hence the logarithm of resistivity  $\rho$  ( $= 1/\sigma$ ) varies linearly with  $1/T$ . The width of the energy gap may be determined from the slope of the curve. Thus we have,

$$\boxed{\log_e \rho = \frac{E_g}{2kT} - \log_e K} \quad \dots\dots(7)$$

### Experimental Considerations.

1. High resistance or rectification appears fairly often in electrical contacts to Semi-conductors and in fact is one of the major problems.
2. In single crystal material the resistivity may vary smoothly from point to point. In fact this is generally the case. The question is the amount of this variation rather than any question of its presence. Often however, it is conventionally stated that is constant within some percentage and when the variation does in fact fall within this tolerance, it is ignored.
3. Soldered probe contacts may disturb the current flow [ shorting out part of the sample ] and add to the ambiguity in the measurement of the probe spacing. Soldering directly to the body of the sample can affect the sample properties by heating effect and by contamination unless care is taken. These problems can be avoided by using pressure contacts. The principal drawbacks of this kind of contacts are that they may be noisy.
4. The current through the sample should not be large enough to cause heating. A further precaution is necessary to prevent 'injecting effect' from affecting the measured value of  $\rho$ . Even good contacts, to germanium, for example, may inject. This is minimized by keeping the voltage drop at the contacts low. If the surface near the current contacts is rough [lappe surface] and the electric flow in the crystal is low, these injected carriers will recombine before reaching the measuring probes.

Since  $\rho$  is independent of current, it is possible to determine whether or not any of these effects are interfering with the measuring of  $\rho$  at several values of  $I$ . It should be kept in mind that these points of experimental technique affect essentially all the measurements and not the resistivity measurement only.

### FOUR PROBE METHOD:

Many conventional methods (Direct Method, Two Point Probe Method) for measuring resistivity are unsatisfactory for semiconductors because metal-semiconductor contacts are usually rectifying in nature. Also there is generally minority carrier injection by one of the current carrying contacts. An excess concentration of minority carriers will affect the potential of other contacts and modulate the resistance of the material.

The method described here overcomes the difficulties mentioned above and also offers several other advantages. It permits measurements of resistivity in samples having a wide variety of shape, including the resistivity of small volumes within bigger pieces of semiconductor. In this manner the resistivity on both sides of a p-n junction can be determined with good accuracy.

before the material is cut into bars for making devices. This method of measurement is also applicable to silicon and other semiconductor materials.

The basic model for all these measurements is indicated in Fig.5. Four sharp probes are placed on a flat surface of the material to be measured, current is passed through the two outer electrodes, and the floating potential is measured across the inner pair. If the flat surface on which the probes rest is adequately large and the crystal is big the semiconductor may be considered to be a semi-infinite volume. To prevent minority carrier injection and make good contact, the surface on which the probes rest may be mechanically lapped.

The experimental circuit used for measurement is illustrated systematically in Fig.6. A nominal value of probe spacing, which has been found satisfactory, is an equal distance of 1.25mm between adjacent probes. This permit measurement with reasonable currents of n-type or p-type semiconductor from 0.001 to 50 ohm-cm.

In order to use this Four probe method in semiconductor crystals or slices it is necessary to assume that:

- 1 The resistivity of the material is uniform in the area of measurement.
- 2 If there is minority carrier injection into the semiconductor by the current carrying electrodes most of the carriers recombine near the electrodes so that their effect on the conductivity is negligible. [This means that the measurement should be made on surface which have a high recombination rate, such as mechanically lapped surfaces]
- 3 The surface on which the probes rest is flat with no surface leakage.
- 4 The four probes used for resistivity measurement contact the surface at points that lie in a straight line.
- 5 The diameter of the contact between the metallic probes and the semiconductor should be small compared to the distance between probes.
- 6 The boundary between the current-carrying electrodes and the bulk material is hemispherical and small in diameter.
- 7 The surfaces of the semiconductor crystal may be either conducting or non-conducting.
  - [a] A conducting boundary is one on which materials of much lower resistivity than semiconductor [such as copper] has been plated.
  - [b] A non conducting boundary is produced when the surface of the crystal is in contact with an insulator.

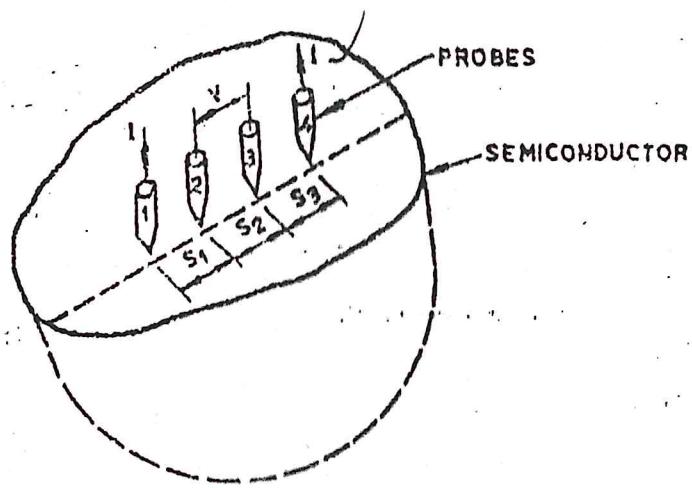


FIG.5 MODEL FOR THE FOUR PROBE RESISTIVITY MEASUREMENTS

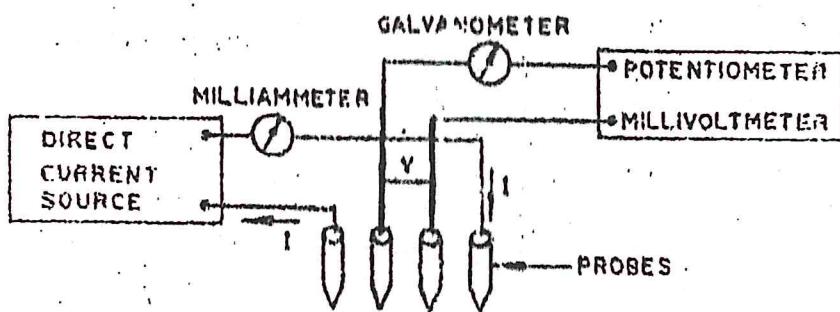


FIG.6. CIRCUIT USED FOR RESISTIVITY MEASUREMENTS

By their very natures, resistivity measurements are geometry dependent and quite sensitive to Boundary conditions, because of this sensitivity many correction factors have been calculated.

### Case 1. ✓ RESISTIVITY MEASUREMENT ON LARGE SAMPLE:

One added boundary condition is required to treat this case; namely, that the probes are far from any of the other surfaces of the sample and the sample can thus be considered a semi-infinite volume of uniform resistivity material. Fig.5 shows the geometry of this case. Four probes are spaced  $S_1, S_2$  and  $S_3$  apart. Current  $I$  is passed through the outer probes [1 and 4] and the floating potential  $V$  is measured across the inner pair of probes 2 and 3.

The floating potential  $V_f$ , a distance  $r$  from an electrode carrying a current  $I$  in a material of resistivity  $\rho_0$  is given by.

$$V_f = \frac{\rho_0 I}{2\pi r} \quad (8)$$

In the model shown in Fig.5 there are two current-carrying electrodes, numbered 1 and 4, and the floating potential  $V_f$  at any point in the semiconductor is the difference between the potential induced by each of the electrodes, since they carry currents of equal magnitude but in opposite directions. Thus:

$$V_f = \frac{\rho_0 I}{2\pi} \left( \frac{1}{r_1} - \frac{1}{r_4} \right) \quad (9)$$

Where

$r_1$  = distance from probe number 1

$r_4$  = distance from probe number 4

The floating potential at probe [2],  $V_{f_2}$  and at probe [3],  $V_{f_3}$  can be calculated from [9] by substituting the proper distances as follows:

$$V_{f_2} = \frac{\rho_0 I}{2\pi} \left( \frac{1}{S_1} - \frac{1}{S_2 + S_3} \right)$$

$$V_{f_3} = \frac{\rho_0 I}{2\pi} \left( \frac{1}{S_1 + S_2} - \frac{1}{S_3} \right)$$

The potential difference  $V$  between the probes is then

$$V = V_{f_2} - V_{f_3} = \frac{\rho_0 I}{2\pi} \left( \frac{1}{S_1} + \frac{1}{S_3} - \frac{1}{S_2 + S_3} - \frac{1}{S_1 + S_2} \right)$$

and the resistivity  $\rho$  is computable as

$$\rho_0 = \frac{V}{I} \frac{2\pi}{\left( \frac{1}{S_1} + \frac{1}{S_3} - \frac{1}{S_2 + S_3} - \frac{1}{S_1 + S_2} \right)} \quad \dots(10)$$

When the probe spacing is equal that is  $S_1 = S_2 = S_3 = S$  the above equation simplified to,

$$\boxed{\rho_0 = \frac{V}{I} 2 \pi s} \quad \dots(11)$$

### Case 2. RESISTIVITY MEASUREMENT ON THIN SLICE-CONDUCTING BOTTOM SURFACE:

Two boundary conditions must be met on this case; the top surface of the slice must be reflecting [non-conducting] surface and the bottom surface must be an absorbing [conducting] surface. Since the two boundaries are parallel a solution by the method of images requires for each current sources an infinite series of images along a line normal to the planes and passing through the current sources.

The model for this case is shown in Fig.7. The side surface of the die is assumed to be far from the area of measurement and, therefore, only the effect of the bottom surface needs to be considered. In this analysis equal probe spacing  $S$  shall be assumed. The width of the slice is  $W$ . The array of images needed is indicated in Fig.7, where the polarity and spacing of the first few images are as shown.

The floating potential  $V_{f_2}$  at electrode 2 is

$$V_{f_2} = \frac{\rho I}{2\pi} \left[ \sum_{n=-\infty}^{\infty} (-1)^n \frac{1}{\sqrt{S^2 + (2nw)^2}} - \sum_{n=-\infty}^{\infty} (-1)^n \frac{1}{\sqrt{(2S)^2 + (2nw)^2}} \right] \quad (12)$$

Likewise, the floating potential at electrodes 3 can be obtained and

$$V_{f_3} = \frac{\rho I}{2\pi} \left[ \frac{1}{S} + \sum_{n=1}^{\infty} (-1)^n \frac{4}{\sqrt{S^2 + (2nw)^2}} - \sum_{n=1}^{\infty} (-1)^n \frac{4}{\sqrt{(2S)^2 + (2nw)^2}} \right] \quad (13)$$

The resistivity then becomes

$$\rho = \frac{\rho_0}{G_6 \left( \frac{w}{s} \right)} \quad (14)$$

Where resistivity  $\rho$  is computable from  $\rho_0$  and  $\rho_0$  can be used if the point spacing is different, but approximately equal. The function  $G_6(w/s)$  is computed from

$$G_6 \left( \frac{w}{s} \right) = 1 + 4 \frac{S}{W} \sum_{n=1}^{\infty} (-1)^n \left[ \frac{1}{\sqrt{\left(\frac{S}{W}\right)^2 + (2n)^2}} - \frac{1}{\sqrt{\left(2\frac{S}{W}\right)^2 + (2n)^2}} \right] \quad (15)$$

Which is tabulated in table II and plotted in Fig.8

TABLE II

W/s	G <sub>6</sub> (W/S)	G <sub>7</sub> (W/S)
0.100	0.0000019	13.863
0.141	0.00018	9.704
0.200	0.00342	6.931
0.333	0.0604	4.159
0.500	0.228	2.780
1.000	0.683	1.504
1.414	0.848	1.223
2.000	0.933	1.094
3.333	0.9838	1.0228
5.000	0.9948	1.0070
10.000	0.9993	1.00045

### Case 3. RESISTIVITY MEASUREMENT ON THIN SLICE NON- CONDUCTING BOTTOM SURFACE.

The model for these measurements are like for case 2, except that the bottom surfaces of the slice is non-conducting. This means that all the images of Fig. 7 have the same charge as the current source. Thus all the images on a row have equal charges and it describes the potential difference across the inner pair of probes if  $(-1)^n$  is removed from the equation. Then,

$$\rho = \frac{\rho_0}{G_7 \left( \frac{W}{S} \right)} \quad (16)$$

where,

$$G_7 \left( \frac{W}{S} \right) = 1 + 4 \frac{S}{W} \sum_{n=1}^{\infty} \left[ \frac{1}{\sqrt{\left( \frac{S}{W} \right)^2 + (2n)^2}} - \frac{1}{\sqrt{\left( 2 \frac{S}{W} \right)^2 + (2n)^2}} \right] \quad (17)$$

This function  $G_7 (w/s)$  is tabulated in Table II and plotted in Fig. 9. For smaller values of w/s the function  $G_7 (w/s)$  approaches the case for an infinitely thin slice, or

$$G_7(w/s) = \frac{2S}{w} \log_e 2 \quad (18)$$

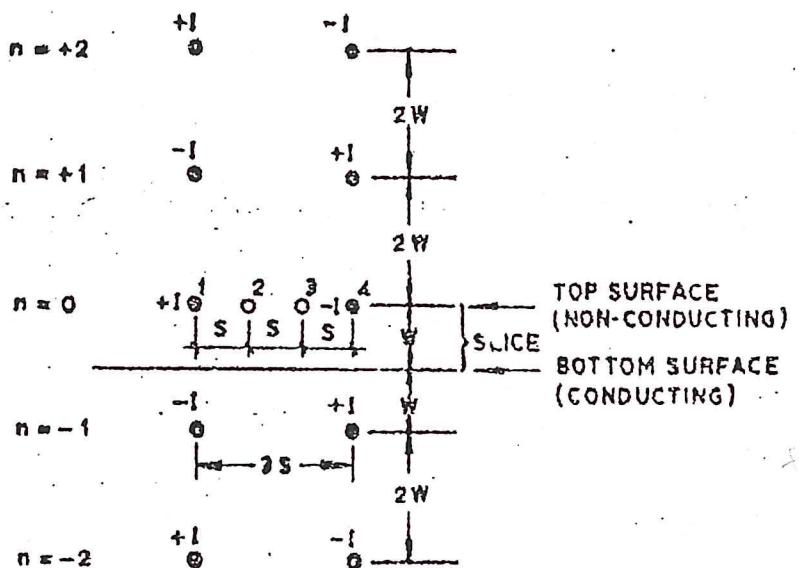


FIG.7 IMAGES FOR THE CASE OF THE RESISTIVITY PROBES  
ON A THIN SLICE WITH A CONDUCTING BOTTOM  
SURFACE

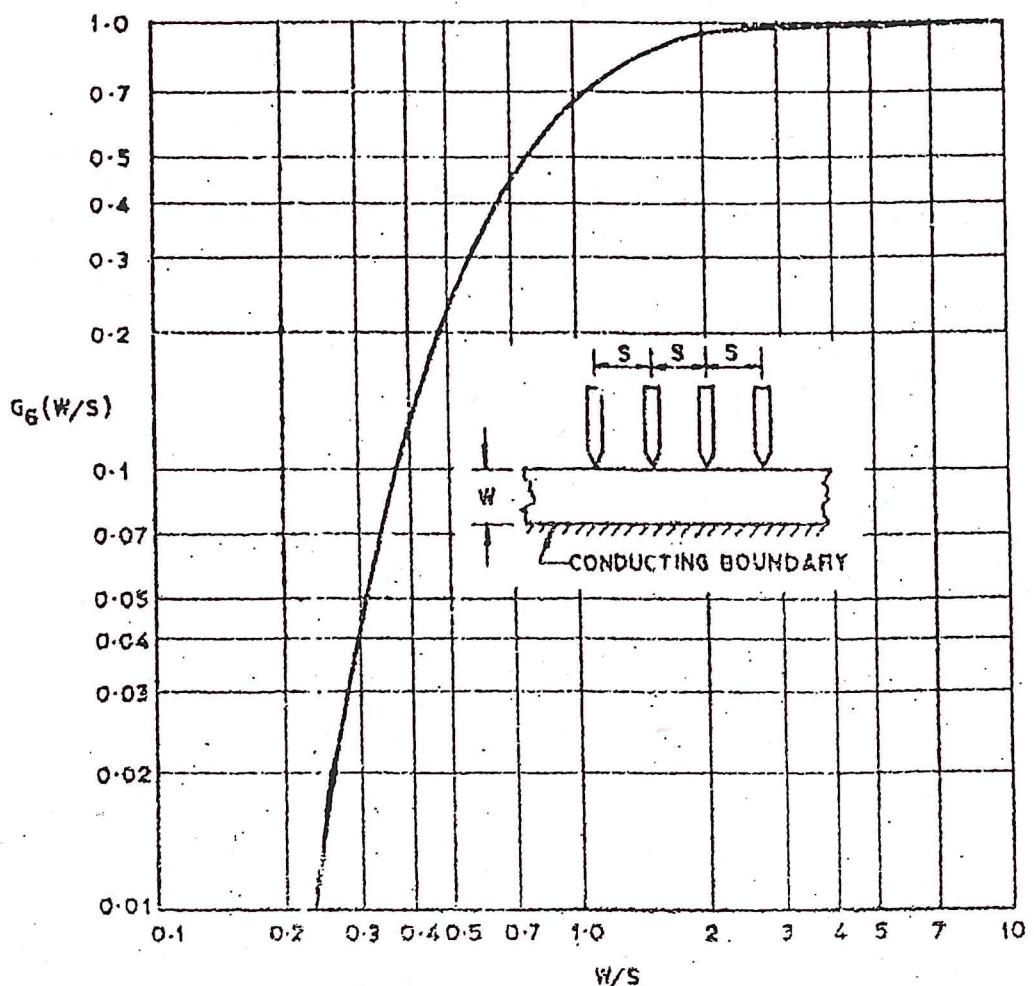


FIG.8 CORRECTION DIVISOR FOR PROBES ON A SLICE  
WITH A CONDUCTING BOTTOM SURFACE

8/15

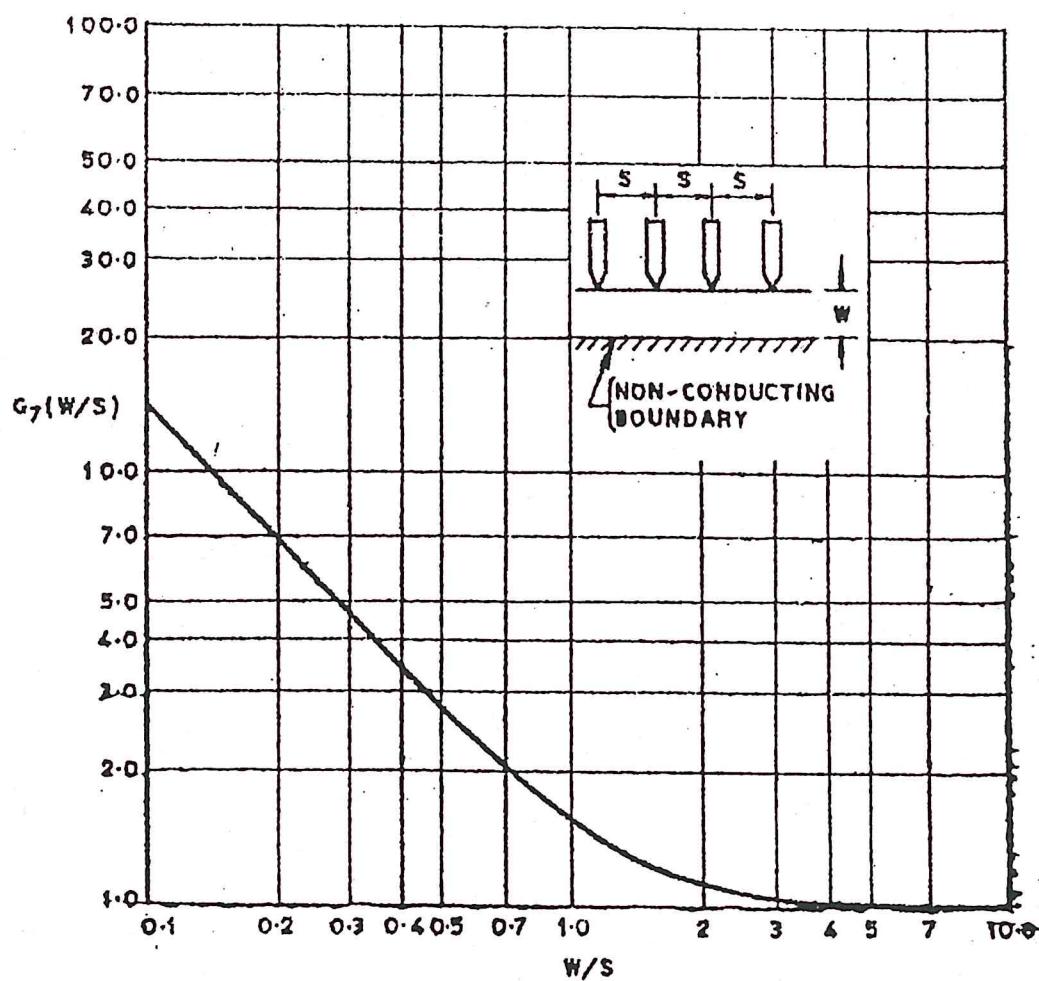


FIG. 9 CORRECTION DIVISOR FOR PROBES ON A THIN SLICE WITH A NONCONDUCTING BOTTOM SURFACE

## BRIEF DESCRIPTION OF THE SET-UP:

### 1. Four Probe Arrangement (Spring type) :

It has four individually spring loaded probes coated with Zn at the tips. The probe are collinear and equally spaced. The Zn coating & individual spring ensure good electrical contacts with the sample. The probes are mounted in a teflon bush, which ensure a electrical insulation between the probes. A teflon spacer near the tips is also provided to the probes at equal distance. The whole arrangement is mounted on a suitable stand and are provided for the voltage and current measurement.

Green pair of leads : For current

Red pair of leads : For Voltage

2. Sample : Ge crystal in the form of chip.

3. Oven : It is a small oven for the variation of temperature of the crystal from the room temperature to about 200 °C.

4. Thermometer : 0 - 200°C .

5. Four Probe Set-up Model DFR-02: The set-up consists of three units in the same cabinet.

(i) *Digital Multirange Electronic Millivoltmeter*: It is specially designed for Four Probe but can be used separately as a measuring device for the measurement of d.c. voltage 100 µV to 2V with a resolution of 100 µV. The circuit is designed using a buffer amp and intersil 3 ½ digit single chip A/D converter ICL 7107. It has accuracy upto zero less than 10 µV, zero drift-less than 1 µV/C, input bias of 10 pA and roll over error of less one count. An impedance balance control (marked 'ZERO ADJ.') enables the millivolt to be used with sources having different impedance ranges. The main advantages over the potentiometric system is that the present unit gives direct reading with a minimum of external adjustment.

## SPECIFICATIONS :

Range	: X 1 ( 0 -200 mV) and X 10 (0 - 2.0V)
Resolution	: 100 µV on X 1 range
Accuracy	: $\pm 0.1\%$ of the reading $\pm 1$ digit
Impedance	: 1 M Ohm
Display	: 3 ½ digit 7 segment LED with auto polarity and decimal indication

Overload Indication : Sign of 1 on the left and blacking on the other digit.

(ii) *Constant current Source*: It is an IC regulated constant current generator to provide a constant current to the outer probes irrespective of the changing of resistance of the sample due to change in temperature. The basic scheme is to use the feed back principle to limit load current of the supply to the preset maximum value. Variation in the current are achieved by varying the feedback voltage.

by a potentiometer included for that purpose. The supply is a highly regulated and practically ripple free d.c. source. The current is measured by the digital panel meter.

### SPECIFICATIONS :

<b>Open circuit voltage</b>	: 18 V
<b>Current Range</b>	: 0 - 20 mA. Or as required.
<b>Resolution</b>	: $10 \mu\text{A}$ .
<b>Accuracy</b>	: $\pm 0.25\%$ of the reading $\pm 1$ digit
<b>Load Regulation</b>	: 0.03% from zero to full load
<b>Line regulation</b>	: 0.05% for 10% change in mains voltages

**Oven Power Supply** : Suitable voltage for oven is obtained through a step down transformer with a provision for low and high rate of heating. A glowing LED indicate, when the oven power supply is ON.

### EXPERIMENTAL PROCEDURE:

#### (A) Initial adjustment of the set-up:

First take out the Four Probe Arrangement and put it on a plane surface. Now put the sample on the circular base plate of the Four Probe Arrangement so that the bottom surface non-conducting and four probe are in the middle of the crystal. Apply some pressure slightly on the pipe so that it clearly makes the contacts with the sample and tighten the screw. Check the continuity between the four probes with the sample by the multimeter. If the contacts are loose tighten the three screw provided on the top of the base plate. The resistance between the Green and Red pairs of leads must nearly be of the order of 2 to 4 Kohm. Put the Four Probe Arrangement in the oven and check the continuity again. Put the thermometer in the hole provided in four probe arrangement to measure the oven temperatures.

Connect the outer pair of probes (Green pair) to the constant current source marked current and inner pair of probes (Red pair of leads) to DMRV marked voltage.

### METHOD:

Connect the set-up with the mains and switch it 'ON'.

Put the selector switch in voltage position and range multiplier switch at X1 position. Adjust zero in DVM with the help of ZERO ADJ keeping either the current terminals in open or the voltage terminal short.

Change the selector switch in current position and apply some current say 2 mA. constant for whole set of readings and note this value of current. Change the selector switch in voltage position with range switch at X 10 position and note the corresponding voltage in DVM and temperature. Records these readings in table.

7. Put the oven switch at L position. Connect the oven with the oven supply and switch ON. A glowing LED will show the ON position of the supply. Measure the Voltage at different values of temperature keeping the current constant. Record these readings in table.
8. Repeat the experiment for other values of current.

### OBSERVATIONS AND TABULATIONS:

Current I = .... mA. (Constnat for whole set of readings)

S. No	Temperature T°C	Voltage V volt	Temperature T°K	$\rho$ ohm-cm	$1/T \times 10^3$ T°K	$\log_{10} \rho$
1.						
2.						
3.						
4.						

Distance between the probes (s) = 2.0 mm

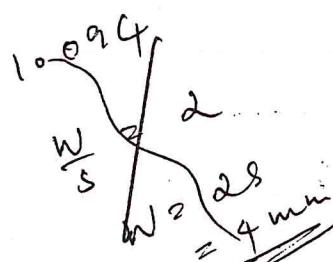
Thickness of the crystal (w) = ....

### CALCULATIONS:

From equation 11

$$\rho_o = \frac{V}{I} 2\pi s \quad \text{in cm}$$

$s = 2 \text{ mm} = 0.2 \text{ cm}$



NB

Since the thickness of the crystal is small compared to the probe distance a correction factor for it is has to be applied. Further, the bottom source is non conducting in the present case, equation (17) will apply to

$$\rho_s = \frac{\rho_o}{G_7(w/s)}$$

The function  $G_7(w/s)$  may be obtained from table -I or fig. 9 for the appropriate Value of (w/s). Thus  $\rho$  may be calculated for various temperatures.

Plot a graph for

$\log_{10} \rho$  Vs  $1000/T(K)$

Using equation (7)

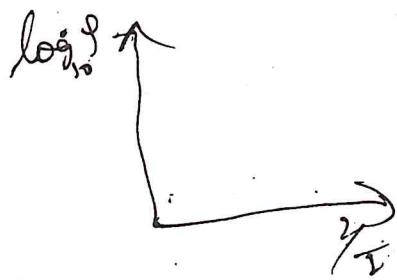
$$\log_e \rho = E_g / 2kT - \log_e K$$

ON.  
rent

and slope of the curve is given by

$$\frac{dy}{dx} \left( \log_e \rho \right) = \frac{E_g}{2k}$$

Where  $k$  is the Boltzmann's constant  $= 8.6 \times 10^{-5}$  eV/deg.  
 $T$  is the temperature in degree kelvin



Thus  $E_g$  can be calculated from the slope of the curve. It should be noted that

$$\log_e \rho = 2.3026 \log_{10} \rho$$

and equation (7) is applicable only in the intrinsic region of the semiconductor. A typical graph is shown in Fig. 10.

#### RESULTS:

1. Table shows the variation of resistivity of the Ge sample with temperature.
2. Graph shows the variation of resistivity of the Ge sample with temperature
3. The experimental observed value of  $E_g = \dots$  eV
4. The standard value of  $E_g = 0.70$  eV

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East West Press.

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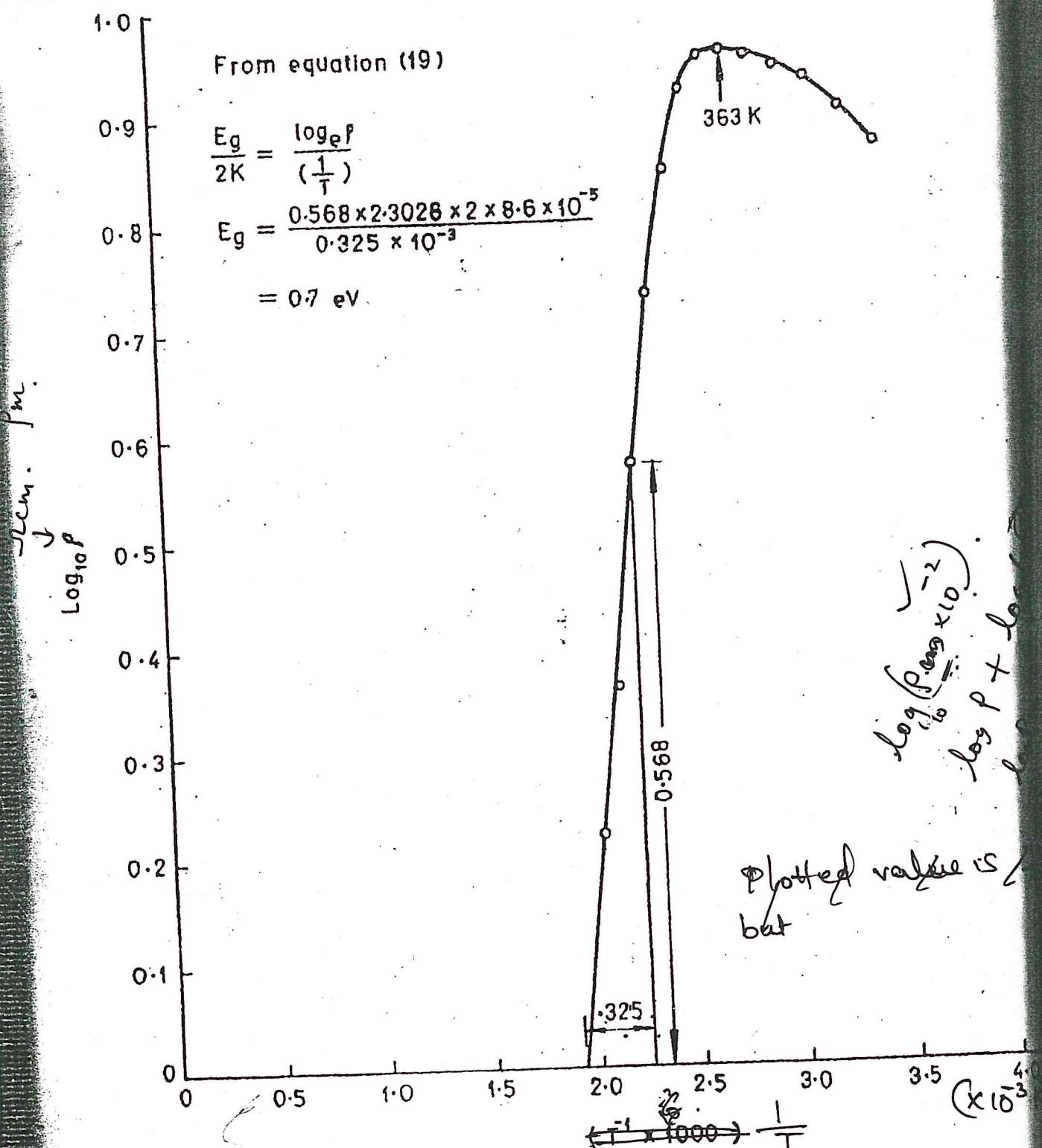


FIG. 10 THE RESISTIVITY OF A GERMANIUM CRYSTAL AS A FUNCTION OF TEMPERATURE. FOR THIS SAMPLE  $T < 363 \text{ K}$ , CONDUCTION IS DUE IN THE IMPURITY CARRIERS (EXTRINSIC REGION); FOR  $T > 363$ , CONDUCTION IS DUE TO ELECTRONS TRANSFERRED TO THE CONDUCTION BAND (AND CORRESPONDING HOLES CREATED IN THE VALENCE BAND): THIS EXTRINSIC REGION.

# Four Probe method to measure the band gap of a semiconductor

## AIM

To determine the resistivity of semiconductors by Four probe Method.

## APPARATUS

The experimental set up consists of probe arrangement, sample , oven 0-200°C, constant current generator , oven power supply and digital panel meter(measuring voltage and current).

Four probe apparatus is one of the standard and most widely used apparatus for the measurement of resistivity of semiconductors. This method is employed when the sample is in the form of a thin wafer, such as a thin semiconductor material deposited on a substrate. The sample is millimeter in size and having a thickness  $w$ . It consists of four probe arranged linearly in a straight line at equal distance  $S$  from each other. A constant current is passed through the two probes and the potential drop  $V$  across the middle two probes is measured. An oven is provided with a heater to heat the sample so that behavior of the sample is studied with increase in temperature.

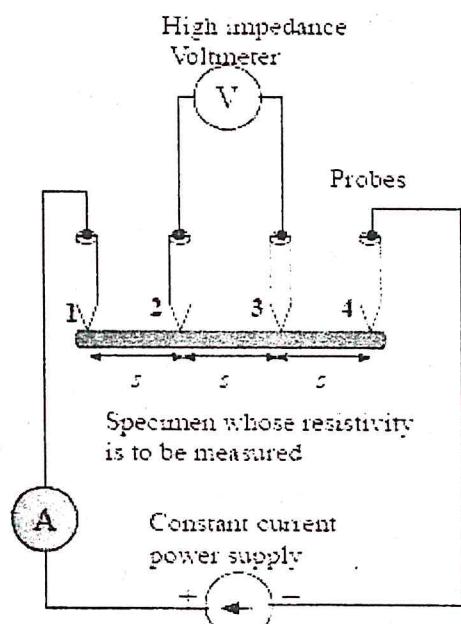


Fig.1.

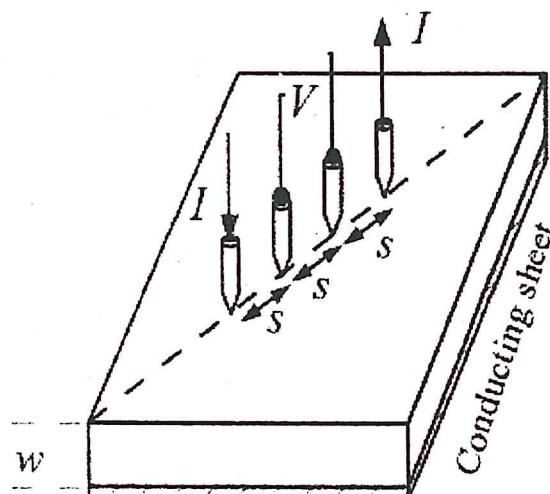


Fig.2.

The figure shows the arrangements of four probes that measure voltage ( $V$ ) and supply current ( $A$ ) to the surface of the crystal.

## THEORY

At a constant temperature, the resistance, R of a conductor is proportional to its length L and inversely proportional to its area of cross section A.

$$R = \rho \frac{L}{A} \quad (1)$$

where  $\rho$  is the resistivity of the conductor and its unit is ohmmeter.

A semiconductor has electrical conductivity intermediate in magnitude between that of a conductor and insulator. Semiconductor differs from metals in their characteristic property of decreasing electrical resistivity with increasing temperature.

According to band theory, the energy levels of semiconductors can be grouped into two bands, valence band and the conduction band. In the presence of an external electric field it is electrons in the valence band that can move freely, thereby responsible for the electrical conductivity of semiconductors. In case of intrinsic semiconductors, the Fermi level lies in between the conduction band minimum and valence band maximum. Since conduction band lies above the Fermi level at 0K, when no thermal excitations are available, the conduction band remains unoccupied. So conduction is not possible at 0K, and resistance is infinite. As temperature increases, the occupancy of conduction band goes up, thereby resulting in decrease of electrical resistivity of semiconductor.

Resistivity of semiconductor by four probe method

1. The resistivity of material is uniform in the area of measurement.
2. If there is a minority carrier injection into the semiconductor by the current- carrying electrodes most of the carriers recombine near electrodes so that their effect on conductivity is negligible.
3. The surface on which the probes rest is flat with no surface leakage.
4. The four probes used for resistivity measurement contact surface at points that lie in a straight line.
5. The diameter of the contact between metallic probes and the semiconductor should be small compared to the distance between the probes.
6. The boundary between the current carrying electrodes and the bulk material is hemispherical and small in diameter.
7. The surface of semiconductor material may be either conducting and non-conducting. A conducting boundary is one on which material of much lower resistivity than semiconductor has been plated. A non-conducting boundary is produced when the surface of the semiconductor is in contact with insulator.

Fig: 2 show the resistivity probes on a die of material. If the side boundaries are adequately far from the probes, the die may be considered to be identical to a slice. For this case of a slice of thickness w and the resistivity is computed as

$$\rho = \frac{\rho_0}{f\left(\frac{w}{s}\right)} \quad (2)$$

The function,  $f(w/S)$  is a divisor for computing resistivity which depends on the value of  $w$  and  $S$

We assume that the size of the metal tip is infinitesimal and sample thickness is greater than the distance between the probes.

$$\rho_0 = \frac{V}{I} \times 2\pi S \quad (3)$$

Where  $V$  – the potential difference between inner probes in volts.

$I$  – Current through the outer pair of probes in ampere.

$S$  – Spacing between the probes in meter.

### Temperature dependence of resistivity of semiconductor

Total electrical conductivity of a semiconductor is the sum of the conductivities of the valence band and conduction band carriers. Resistivity is the reciprocal of conductivity and its temperature dependence is given by,

$$\rho = A \exp \frac{E_g}{2KT} \quad (4)$$

where  $E_g$  – band gap of the material

$T$  – Temperature in kelvin

$K$  – Boltzmann constant,  $K = 8.6 \times 10^{-5}$  eV/K

The resistivity of a semiconductor rises exponentially on decreasing the temperature.



## CORNU'S METHOD

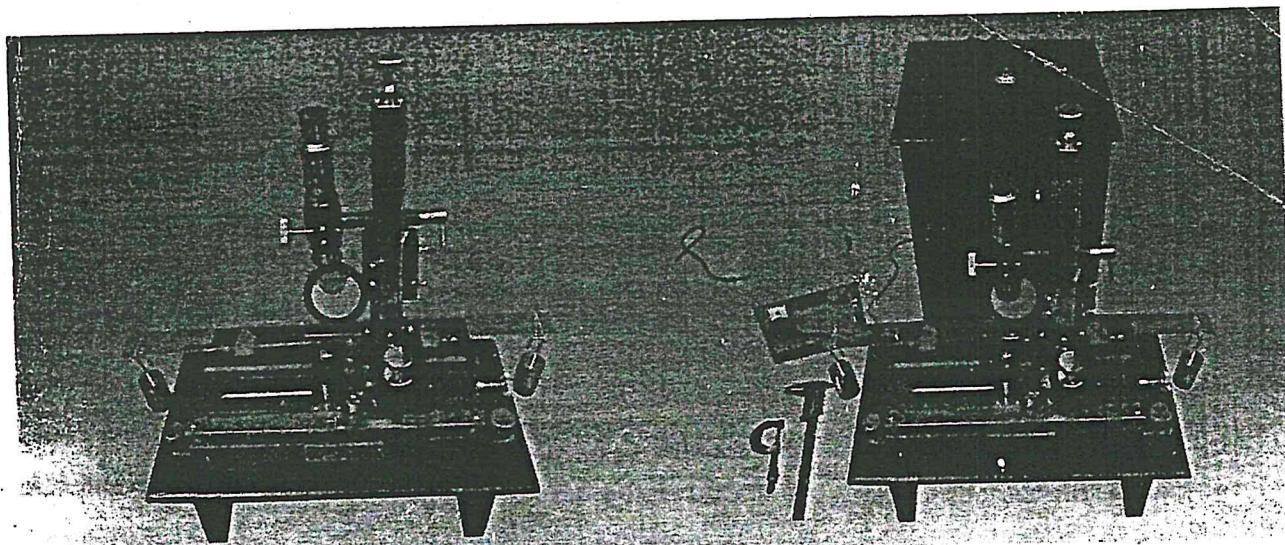


Fig. 1 Complete Cornu's Set-up

**Object:** To determine Young's modulus and Poisson's ratio of glass by Cornu's method.

**Apparatus used:** Traveling microscope with X-Y-Z motion specially designed for Cornu's experiment. Cornu's assembly fitted with glass plate inclined at an desired angle say  $45^{\circ}$  in this case, The experimental glass beam, A small rectangular glass plate, Set of 50 gm weight with hanger, Sodium Vapor lamp, screw gauge ,Vernier Callipers. Wooden plane to level the Cornu's set-up with light source.

**Principle of the Experiment:** If a rectangular glass beam is deformed in the form of a curvature under the action of bending moment, the longitudinal as well as the transverse filaments of the beam on the either side of the neutral surface change in length. In this method the curvature is produced by supporting the glass beam on two knife edges placed near the ends of the beam with their edges normal to the axis of the beam, and suspending weights from the ends.

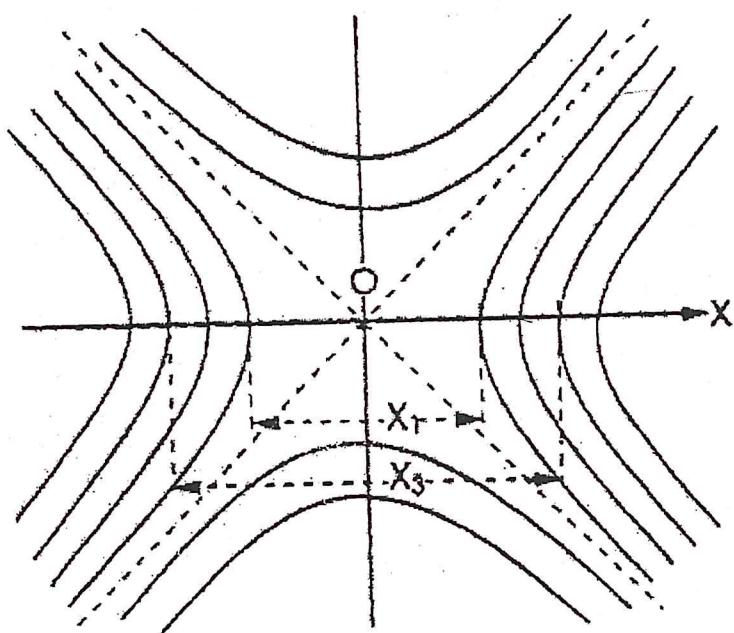


Fig. 2

A plane small glass plate rests on the curved beam to form an air film between the two plates. On illuminating the air film with sodium light we obtain two conjugate systems of hyperbolic interference fringes round the point of contact of two glass plates as shown in fig 2. By measuring longitudinal curvature,  $Y$  can be determined. In addition to longitudinal, if we measure transverse curvature, we can find Poisson's ratio ( $\sigma$ ).

#### Description of the apparatus:

The experimental glass beam AB is supported on two knife edges made of glass  $K_1$  and  $K_2$  as in Fig. 3. Two hangers made of thread loops are suspended symmetrically near both ends of the glass plate. A small rectangular glass plate P is placed in the middle of the beam AB. Light from a monochromatic source (sodium lamp) is made to fall on the cover glass plate P and the beam AB by means of the another glass plate G arranged at an angle of  $45^\circ$  with the horizontal such that the parallel beam is reflected from the lower surface. The interference hyperbolic fringes formed between the lower face of the cover plate and the upper curved surface of the beam can be viewed in a traveling microscope M.

**Theory and Derivation of Formula:** The experimental beam AB is bent under the action of the loads W, hanging at both the ends. The bending moment  $G$  acting at all the transverse sections of the central span is related to the longitudinal radius of curvature  $R_l$  by

$$G = \frac{Y I}{R_l} \quad (1)$$

Where  $Y$  is the Young's modulus for the material of the beam and  $I$  is the geometrical moment of inertia of cross section of the beam about an axis passing through its centroid and perpendicular to the plane of bending. If  $b$  is the breadth and  $t$  the thickness of the beam, then

$$I = \frac{1}{12} b t^3 \quad (2)$$

If  $d$  is the distance between the point of support of the load  $W$  and knife edge nearer to it, then

$$G = W d \quad (3)$$

From the eq. (1) and (3)

$$\frac{Y I}{R_l} = W d$$

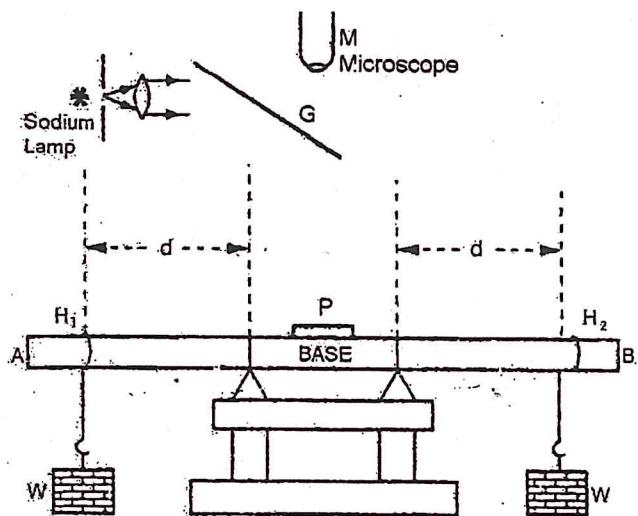


Fig. 3

If the beam has an initial radius of curvature  $R_0$  due to its own weight, then

$$\frac{Y I}{R_l} - \frac{Y I}{R_0} = W d \quad (4)$$

If for a load  $W'$  the radius of curvature (longitudinal) of the beam is  $R_l'$ , then in analogy to eq. (4), we have

$$\frac{Y I}{R_l'} - \frac{Y I}{R_0} = W' d \quad (5)$$

Subtracting eq (5) from (4), we have

$$Y I \left[ \frac{1}{R_l} - \frac{1}{R_l'} \right] = (W - W')d$$

Therefore

$$Y = \frac{(W - W')d}{I \left( \frac{1}{R_l} - \frac{1}{R_l'} \right)}$$

Substituting the value of  $I$  from eq.(2), we have

$$\text{Formula 1}^{\text{st}} \quad Y = \frac{12(W - W')d}{b t^3 \left[ \frac{1}{R_l} - \frac{1}{R_l'} \right]} \quad (6)$$

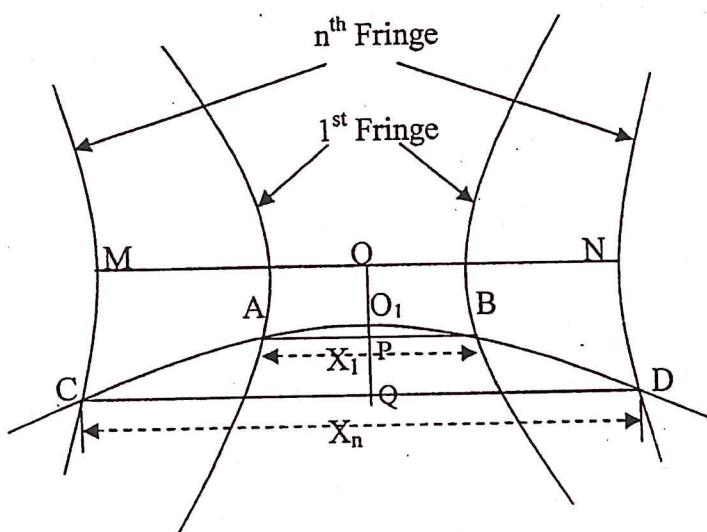


Fig. 4

In order to determine  $R_l$  and  $R_l'$  let us refer to Fig 4. Let MN be the lower surface of the cover plate and AB the curved section of the upper surface of the curved beam for weight W. Let  $O_1$  be the centre of the fringes system and A, B the position of the 1<sup>st</sup> pair of fringes in the longitudinal direction (along X-axis). Let  $AB=X_1$ . From the geometry of the fig. 7, we have

$$(2R_l - O_l P) O_l P = \left( \frac{X_1}{2} \right)^2$$

But  $O_l P \ll 2R_l$ , hence neglecting  $(O_l P)^2$  we have

$$2R_l (O_l P) = \left( \frac{X_1}{2} \right)^2$$

Similarly, if C and D represent the position of nth pair of fringes, separated by  $X_n$ , then

$$2R_l (O_l Q) = \left( \frac{X_n}{2} \right)^2$$

Subtracting these two equations we have

$$2 R_l (O_l Q - O_l P) = \frac{1}{4}(X_n^2 - X_1^2)$$

But  $(O_l Q - O_l P) = \frac{(n-1)\lambda}{2}$  for a bright fringe, where  $\lambda$  is the wavelength of Sodium light.

#### Formula 2<sup>nd</sup>

$$\text{Therefore, } R_l = \frac{(X_n^2 - X_1^2)}{4\lambda(n-1)} \quad (7)$$

Similarly, if  $X_n$  and  $X_1$  corresponds to  $R_l$  i.e. to weight  $W$  then

$$\text{Formula 3<sup>rd</sup>} \quad R_l = \frac{(X_n^2 - X_1^2)}{4\lambda(n-1)} \quad (8)$$

Formula 1<sup>st</sup> is requisite formula for determining the value of Young's modulus Y for glass, the values of  $R_l$  and  $R_l'$  being given by formula 2<sup>nd</sup> and 3<sup>rd</sup> respectively.

Similarly, if  $Y_n$  and  $Y_1$  referred to fringes in the transverse direction along Y-axis and  $R_l$  is the radius of anti-elastic curvature (as there will be contraction in the perpendicular direction of the beam) then

$$\text{Formula 4<sup>th</sup>} \quad R_l = \frac{(Y_n^2 - Y_1^2)}{4\lambda(n-1)} \quad (9)$$

Hence, the Poisson's ratio

$$\sigma = \frac{\text{longitudinal radius of curvature}}{\text{transverse radius of curvature}} = \frac{R_l}{R_t}$$

#### Formula 5<sup>th</sup>

$$\sigma = \frac{(X_n^2 - X_1^2)}{(Y_n^2 - Y_1^2)} \quad (10)$$

#### Procedure:

- The experiment is arranged as shown in figure 1. Weights of certain mass, say 100 gm or 200 gm are placed on each hanger at both the ends of the experimental beam AB. The air film between the rectangular plate P and curved experimental beam is illuminated by means of an extended source of monochromatic light (sodium lamp), so that hyperbolic fringes are formed. The traveling microscope is focused on the fringes.

2. The distance between the 1<sup>st</sup> pair of fringes and the nth (say 4<sup>th</sup>) pair of fringes is determined in the longitudinal direction by means of microscope. These readings give  $X_1$  and  $X_n$  respectively, similarly by measuring the distance in the transverse direction  $Y_1$  and  $Y_n$  are determined. The Poisson's ration is then calculated from  $\sigma \doteq \frac{(X_n^2 - X_1^2)}{(Y_n^2 - Y_1^2)}$
3. Different sets of reading  $X_1$ ,  $X_n$  and  $Y_1$ ,  $Y_n$  are taken for different weights finally the mean value of  $\sigma$  is determined.
4. The values of  $R_l$  and  $R_l'$  are obtained by means of equation 7<sup>th</sup> and 8<sup>th</sup>, using the values of  $X_1$ ,  $X_n$ ,  $X_1'$ ,  $X_n'$  as measured above in two sets for weights  $W$  and  $W'$  (say 200 and 250 gms). Here  $n$  is 4 and  $\lambda$  for sodium light is  $5893 \times 10^{-8}$  cm.
5. The thickness  $t$  of the experimental beam is measured by screw gauge and the breadth  $b$  by vernier caliper. To measure the distance  $d$ , we measure the distance of the knife edges  $K_1$  and  $K_2$  from their respective end A and B (figure 6) and the average of two readings is taken.
6. On substituting the values of  $R_l$  and  $R_l'$  (as determined above),  $W = 200$  gm,  $W' = 250$  gm, the constants  $b$ ,  $t$  and  $d$  in equation 6, the Young's modulus  $Y$  of the glass is calculated.

### Observations:

#### (1) Distances $X_1$ and $X_n$ measured in longitudinal direction, $n = 4$

Least Count of vernier of traveling microscope = ..... cm.

S.No.	Weight $W$ in gm.	No. of fringes	Distance (Diameter)			Distance $X^2$ cm <sup>2</sup>
			Left end cm.	Right end cm.	Difference $X$ cm.	
1 <sup>st</sup>	200	1 n (say 4 <sup>th</sup> )	:		$X_1 = \dots$ $X_n = \dots$	$(X_1)^2 = \dots$ $(X_n)^2 = \dots$
					$X'_1 = \dots$ $X'_n = \dots$	$(X'_1)^2 = \dots$ $(X'_n)^2 = \dots$
					$X''_1 = \dots$ $X''_n = \dots$	$(X''_1)^2 = \dots$ $(X''_n)^2 = \dots$
	300	1 n (say 4 <sup>th</sup> )			$X'''_1 = \dots$ $X'''_n = \dots$	$(X'''_1)^2 = \dots$ $(X'''_n)^2 = \dots$

### Calculations:

$$R_l = \frac{(X_n^2 - X_1^2)}{4\lambda(n-1)}$$

$$\frac{1}{R_l} = \frac{4\lambda(n-1)}{(X_n^2 - X_1^2)} = \dots \text{ cm}^{-1} \text{ for } 150 \text{ gm}$$

Similarly

$$\frac{1}{R'_l} = \frac{4\lambda(n-1)}{(X'_{n})^2 - (X'_{1})^2} = \dots \text{cm}^{-1} \text{ for } 200 \text{ gm}$$

$$\frac{1}{R''_l} = \frac{4\lambda(n-1)}{(X''_{n})^2 - (X''_{1})^2} = \dots \text{cm}^{-1} \text{ for } 250 \text{ gm}$$

$$\frac{1}{R'''_l} = \frac{4\lambda(n-1)}{(X'''_{n})^2 - (X'''_{1})^2} = \dots \text{cm}^{-1} \text{ for } 300 \text{ gm}$$

(2) Distances  $Y_1$  and  $Y_n$  measured in transverse direction,  $n = 4$

S.No.	Weight W in gm.	No. of fringes	Distance (Diameter)			Distance $Y^2 \text{ cm}^2$
			Left end cm.	Right end cm.	Difference Y cm.	
1 <sup>st</sup>	200	1 n (say 4 <sup>th</sup> )			$Y_1 = \dots$ $Y_n = \dots$	$(Y_1)^2 = \dots$ $(Y_n)^2 = \dots$
					$Y'_1 = \dots$ $Y'_n = \dots$	$(Y'_1)^2 = \dots$ $(Y'_n)^2 = \dots$
					$Y''_1 = \dots$ $Y''_n = \dots$	$(Y''_1)^2 = \dots$ $(Y''_n)^2 = \dots$
	300	1 n (say 4 <sup>th</sup> )			$Y'''_1 = \dots$ $Y'''_n = \dots$	$(Y'''_1)^2 = \dots$ $(Y'''_n)^2 = \dots$

Calculations:

$$R_l = \frac{(Y_n^2 - Y_1^2)}{4\lambda(n-1)}$$

$$\frac{1}{R_l} = \frac{4\lambda(n-1)}{(Y_n^2 - Y_1^2)} = \dots \text{cm}^{-1} \text{ for } 150 \text{ gm}$$

Similarly

$$\frac{1}{R'_l} = \frac{4\lambda(n-1)}{(Y'_{n})^2 - (Y'_{1})^2} = \dots \text{cm}^{-1} \text{ for } 200 \text{ gm}$$

$$\frac{1}{R''_l} = \frac{4\lambda(n-1)}{(Y''_{n})^2 - (Y''_{1})^2} = \dots \text{cm}^{-1} \text{ for } 250 \text{ gm}$$

$$\frac{1}{R'''_l} = \frac{4\lambda(n-1)}{(Y'''_{n})^2 - (Y'''_{1})^2} = \dots \text{cm}^{-1} \text{ for } 300 \text{ gm}$$

$$\text{dyne/cm}^2 = M L^{-1} T^{-2} \rightarrow 10^3 \times (10^2)^{-1} T^{-2} \\ \rightarrow 10^3 \times 10^2 \rightarrow \underline{\underline{10}}$$

(3) Calculation of Poisson's Ratio  $\sigma$  :

S.No.	Poisson's Ratio $\sigma$	Mean $\sigma$
1 <sup>st</sup>	$\sigma = \frac{R_l}{R_t} = \frac{X_n^2 - X_1^2}{Y_n^2 - Y_1^2} = \dots$	
2 <sup>nd</sup>	$\sigma = \frac{R_l}{R_t} = \frac{X_n^2 - X_1^2}{Y_n^2 - Y_1^2} = \dots$	
3 <sup>rd</sup>	$\sigma = \frac{R_l}{R_t} = \frac{X_n^2 - X_1^2}{Y_n^2 - Y_1^2} = \dots$	

(4) Constant of the apparatus :

L.C. of screw gauge = .....cm.

L.C. of vernier callipers = .....cm.

(i) Thickness t of the beam (by screw gauge) for W= 0.

S.No.	Zero error (cm)	Main scale reading (cm)	Vernier scale reading (cm).	Total reading (cm).	Mean t (cm).
1.					
2.					
3.					

(ii) Breadth b of the beam (By vernier callipers) for W= 0.

S.No.	Zero error (cm).	Main scale reading (cm).	Vernier scale reading (cm).	Total reading (cm).	Mean b (cm).
1.					
2.					
3.					

(iii) Distance d between knife edge and hanger for W=0.

S.No.	Distance K <sub>1</sub> A <sub>1</sub> (cm).	Distance K <sub>2</sub> B <sub>2</sub> (cm).	$d = \frac{K_1 A_1 + K_2 B_2}{2}$	Mean d cm.
1.				
2.				
3.				

**NOTE:**

Approx. dimension of experimental plate: Length = 50 cm, Breath 3.7 cm and Width = 1.85 mm.

Distance between the knife edge ; 23 cm.

'd' can be calculated by measuring the position of hanger

**(5) Calculations of Young modulus:**

$\lambda$  for sodium light is  $5893 \times 10^{-8}$  cm.

$$\text{Set 1}^{\text{st}} \quad Y = \frac{12(W-W')d}{bt^3 \left( \frac{1}{R_l} - \frac{1}{R_l'} \right)} , \text{ where } W=150\text{gm and } W'=200\text{ gm}$$

$$= \dots \times 10^{11} \text{ dynes/cm}^2$$

$$\text{Set 2}^{\text{nd}} \quad Y = \frac{12(W''-W''')d}{bt^3 \left( \frac{1}{R_l''} - \frac{1}{R_l'''} \right)} , \text{ where } W''=250\text{gm and } W'''=300\text{gm}$$

$$= \dots \times 10^{11} \text{ dynes/cm}^2$$

Similarly take other pairs e.g  $(W - W')$ ,  $(W - W'')$ ,  $(W' - W'')$ ,  $(W' - W''')$  and find the value of Y for each set.

Mean value of Y = .....  $\times 10^{11}$  dynes/cm<sup>2</sup>

**Result:** (i) Poisson's Ratio  $\sigma$  for glass = .....  
(ii) Young's Modulus Y for glass = .....  $\times 10^{11}$  dynes/cm<sup>2</sup>

**Standard Result:** (i) Poisson's Ratio  $\sigma$  for glass = .....  
(ii) Young's Modulus Y for glass = .....  $\times 10^{11}$  dynes/cm<sup>2</sup>

**Sources of error and precautions:**

1. The beam should be placed symmetrically on two knife edges. ( i.e  $K_1 A$  and  $K_2 B$  must be equal ).
2. Constants of the apparatus should be determined only when the beam is unloaded.
3. The experimental beam should not touch the base of traveling microscope.
4. All the glass plates should be optically plane and clean.
5. If the first few fringes are blurred then we take the reading for s (say 4<sup>th</sup>) and n + s (say 11); and Y,  $\sigma$  are determined by using the relations  

$$R_l = \frac{(X_{n+s}^2 - X_s^2)}{4\lambda n}$$
 and  $R_l = \frac{(Y_{n+s}^2 - Y_s^2)}{4\lambda n}$  ✓  
i.e if n=1 is not clear  
R<sub>l</sub> =  $\frac{X_n^2 - X_s^2}{4\lambda(n-2)}$
6. To eliminate effects arising due to imperfect flatness in the experimental beam and cover plate, the beam is turned over and the loads applied as usual. It is however necessary to place the cover plate below the beam and bring their surface together with the help of three leveling screws which are used to support the cover glass thus the mean value of  $X_n$  or  $Y_n$  is determined.

## EXPERIMENT – 2

### ELASTIC CONSTANTS OF A TRANSPARENT MATERIAL – CORNU'S METHOD

**AIM :** To determine Young's modulus and Poisson ratio of a transparent material (glass) by Cornu's method.

**APPARATUS :** Travelling microscope with X-Y-Z motion, Cornu's assembly fitted with a glass plate inclined at  $45^\circ$ , glass beam under investigation, a small rectangular glass plate, a set of 50 gm weights with weight hanger, Sodium vapour lamp, screwgauge, Vernier Callipers, wooden plane to level the Cornu's setup with the light source.

**PRINCIPLE :** The Cornu's method is an *interference fringes* based method to determine elastic constants of a transparent beam (such as a glass/perspex beam) with a rectangular cross-section. Consider a beam of cross-sectional area  $A=ab$ , placed uniformly on two knife edges, and loaded uniformly with two identical masses  $m$  at distances  $L$  from the two knife edges as shown in figure.

The internal bending moment generated is given by

$$\frac{YbaK^2}{R_1} \quad (4.1)$$

where  $K$  is the radius of gyration of the beam about the bending axis. For the rectangular bar  $K = b/\sqrt{12}$ . At equilibrium the internal and external bending moments are equal and we have,

$$mgL = \frac{YbaK^2}{R_1} \quad (4.2)$$

thus,

$$Y = \frac{12mgLR_1}{ab^3} \quad (4.3)$$

where  $Y$  is the Young's modulus of the material and  $R_1$  is the radius of curvature of the beam in the longitudinal direction. If  $\varepsilon_x$  and  $\varepsilon_y$  are the strains produced in the longitudinal and transverse directions of the beam, then the Poisson ratio of the material of the beam is given by  $\sigma = \varepsilon_y / \varepsilon_x$ .

Let  $c$  be the unloaded length of the beam (length of the neutral plane). The filaments above the neutral plane get elongated and those below get compressed, developing restoring stresses in the beam in the direction indicated by arrows. Let  $\Delta c$ , and  $-\Delta c$  be the elongation and compression of the filament as shown in figure. Let  $R_1$  be the longitudinal radius of curvature of the beam and  $\phi$  be the angle subtended by the arc, then we can write,

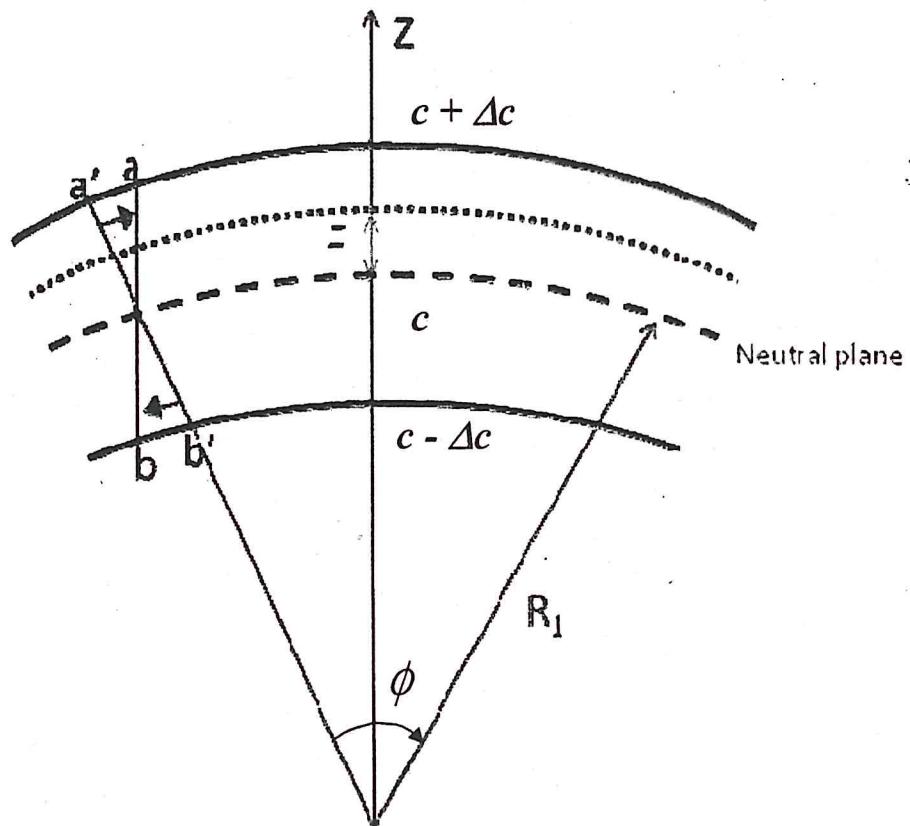
$$\phi = \frac{c + \Delta c}{R_1 + b/2} = \frac{c - \Delta c}{R_1 - b/2} \quad (4.4)$$

i.e., 
$$\frac{\Delta c}{c} = \frac{b}{2R_1} \equiv \varepsilon_x \quad (4.5)$$

Where  $\varepsilon_x$  is longitudinal strain and  $b$  is the thickness of the beam. The expression for the lateral strain  $\varepsilon_y$ , can be similarly derived to be,

$$\varepsilon_y = \frac{a}{2R_2} \quad (4.6)$$

Where  $a$  is width of the beam and  $R_2$  is the Radius of curvature of the beam in the lateral direction. The Poisson ratio ( $\sigma$ ) for the material of the beam is given by



**Fig. 1.** Showing longitudinal bending of the beam. The arrows indicate the direction of stresses developed in the beam. The *neutral plane* is neither extended nor compressed.

$$\sigma = \frac{\text{lateral strain } (\varepsilon_y)}{\text{longitudinal strain } (\varepsilon_x)} = \frac{R_1}{R_2} \begin{matrix} \leftarrow \text{longitudinal} \\ \leftarrow \text{transverse} \end{matrix} \quad (4.7)$$

The lateral deformation of the beam can be visualized using Fig. 2. To measure  $R_1$  and  $R_2$ , Cornu's method can be used, which involves the placing a glass plate on the deformed beam and observing the interference pattern formed between the light rays reflected from the upper surface of the glass plate and those from the upper surface of the beam. If  $d$  represents the distance from the upper surface of the glass plate to the upper surface of the glass beam at a given position  $(x, y)$ , then the loci of points of constant  $d$  can be shown to be,

$$\frac{x^2}{R_1} - \frac{y^2}{R_2} = 2(d - d_o) \quad (4.8)$$

where  $d_o$  is the distance between the upper surface of the glass plate and the upper

**PROCEDURE :** The experimental glass beam (rectangular) is loaded symmetrically (ensure symmetry along both, the  $x$ - and the  $y$ -directions) on the knife-edges provided as shown in Fig. 1. Two equal masses  $m$  are suspended, using a string such that the distance of each mass from the nearest knife-edge is  $L$  (Fig. 1.). A rectangular glass slide is placed on the glass beam such that, the centre of the glass slide is exactly over the center of the glass beam and the long edges of the slit is exactly parallel to the long edges of the rectangular glass beam.

The sodium vapour lamp ( $\lambda=5893 \text{ \AA}$ ) is switched on. It acts as the extended source of monochromatic light for our experiment. Light from the sodium vapour lamp is reflected on to the glass-slide *normally* using a glass plate inclined at  $45^\circ$  to the incident light as indicated in Fig. 1. The travelling microscope is adjusted to be exactly over the centre of the glass beam using X, Y translation and interference fringes formed brought into sharp focus using the Z-translation. Notice that the interference fringes obtained are hyperbolic. The symmetry of the hyperbolic pattern is an indicator of how symmetrically, the glass beam and the glass slide has been arranged. Small adjustments to the glass beam or glass slide (or both) are made in order to obtain a completely symmetric hyperbolic fringe system. Count and ensure that about 10 – 12 fringes are visible on either sides of the center in both the  $x$ - and the  $y$ -directions. Ensure that the microscope travels exactly over the  $x$ - and  $y$ -axes when the X or Y translation screws are operated.

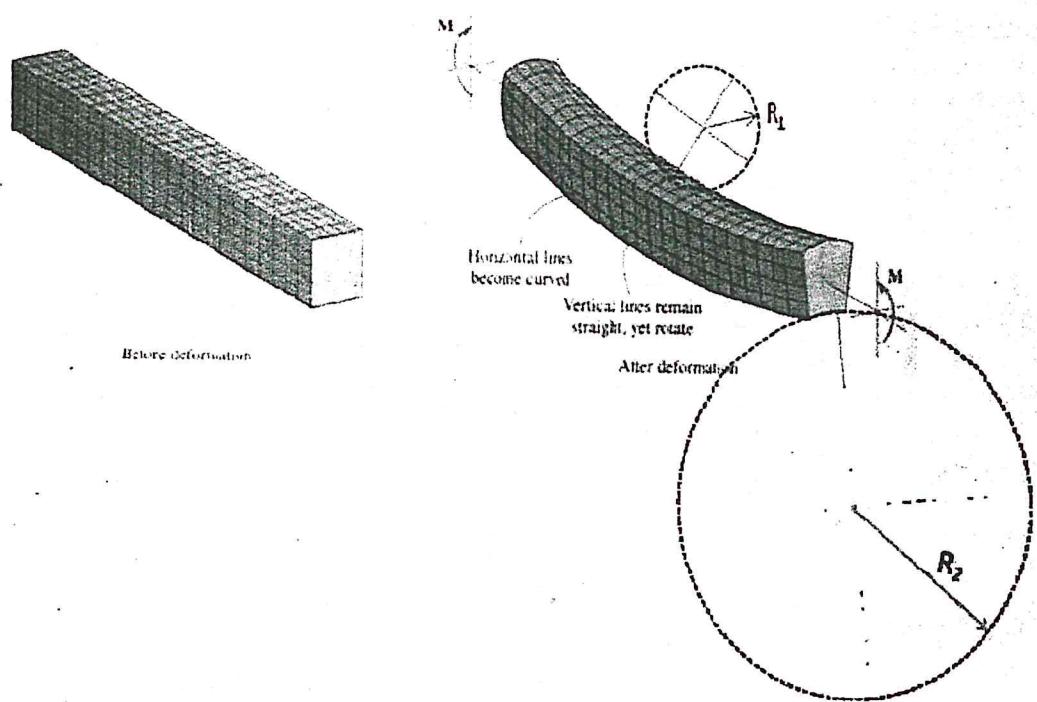
Place the vertical crosswire of the microscope exactly tangential to the 12<sup>th</sup> fringe on the left of the origin (i.e., center of fringe system). Note the reading of the microscope. Next rotate the X-translation screw such that the crosswire is tangential to the 11<sup>th</sup> fringe and so on until the 1<sup>st</sup> fringe is reached, noting down the readings of the microscope for each fringe. Next, cross the origin and place the crosswire on tangential to the 1<sup>st</sup> fringe on the right of the origin and thereafter on the 2<sup>nd</sup> fringe and so on till the 12<sup>th</sup> fringe on the right side is reached. Note microscope readings in each case. Throughout the entire process above, the X-translation screw must be rotated only in one direction (either *clockwise* or *anti-clockwise*) in order to avoid backlash errors.

Next, place the crosswire at the origin, and move to the 12<sup>th</sup> fringe along the positive  $y$ -direction. Repeat the experiment exactly as described above, right up to the 12<sup>th</sup> fringe in the negative  $y$ -direction.

Measure the thickness of the glass beam using a screw gauge and the breadth using a Vernier caliper, taking into account any zero-errors present in the instrument. The length of the beam is measured using a scale.

The entire experiment is repeated for two sets of masses. A better estimate of the elastic constants can be obtained by reversing the glass beam such that the original upper surface now becomes the lower one and vice-versa. This will average out any artifacts resulting from differences in uniformity or flatness of the experimental glass beam or the glass slide.

surface of the glass beam at the point  $(x,y) = (0,0)$ . Eqn. 4.8 implies that the observed interference fringes will be hyperbolic in shape.



The hyperbola Eqn. 4.8 has common asymptotes given by  $x^2 R_2 = y^2 R_1$ , that make an angle  $\theta$  given by

$$\cot^2 \theta = \frac{R_1}{R_2} = \sigma \quad (4.9)$$

At  $y = 0$ , Eqn. 4.8 gives  $x^2 = 2R_1(d - d_o)$ . Since on a fringe we must have  $2d = N\lambda$ , where  $\lambda$  is the wavelength of light used and  $N$  is an integer, we get

$$x^2 = R_1(N\lambda - 2d_o) \quad (4.10)$$

A similar argument leads to the eqn.

$$y^2 = R_2(N\lambda - 2d_o) \quad (4.11)$$

Thus the  $x^2$  vs.  $N\lambda$  graph is linear with slope  $R_1$  and  $y^2$  vs.  $N\lambda$  graph is linear with a slope  $R_2$ . Thus the Young's modulus ( $Y$ ) and Poisson ratio ( $\sigma$ ) can be calculated out. The bulk modulus of the material of the beam is then obtained as,

$$K = \frac{Y}{3(1-2\sigma)} \quad (4.12)$$

**OBSERVATIONS :**

**(a) Breadth of the beam (*b*)**

Least count of the vernier calipers =

Zero error =

Dimension	Trial No.	M. S. R.	V. S. R.	Observed Reading	Corrected Reading	Mean
1	1					
	2					
	3					
	4					
	5					
	6					

**(b) Thickness of the beam (*c*) using screw gauge**

Pitch of the screw gauge =

No. of divisions on the head scale =

$$L.C. = \frac{\text{pitch}}{\text{No. of divisions on Head Scale}} =$$

Zero Correction =

Dimension	Trial No.	P. S. R.	H. S. R.	Observed Reading	Corrected Reading	Mean
1	1					
	2					
	3					
	4					
	5					
	6					

**(c) Length of the beam**



3)

Fringes in longitudinal direction ( $x$ -axis)				L. C. =		$m =$	$L =$						
Ring position ( $N$ )	$x$	$x'$	centre ( $c$ )	MSR	VSR	Reading	MSR	VSR	Reading	( $x - c$ )	( $x' - c$ )	$\frac{(x - c) + (x' - c)}{2}$	$R_I$
1													
2													
3													
4													
5													
6													
7													
8													
9													
10													

Fringes in transverse direction ( $y$ -axis)

L. C. =

Ring position ( $N$ )	$y$	$y'$	centre ( $c$ )	$(y - c)$	$(y' - c)$	$\frac{(y - c) + (y' - c)}{2}$	$R_2$
	MSR	VSR	Reading	MSR	VSR	Reading	
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

Young's Modulus for Glass = 50-90 GPa

Poisson's Ratio for Glass = 0.18 - 0.22