svm

January 27, 2021

0.1 This is the svm workbook for ECE C147/C247 Assignment #2

Please follow the notebook linearly to implement a linear support vector machine.

Please print out the workbook entirely when completed.

We thank Serena Yeung & Justin Johnson for permission to use code written for the CS 231n class (cs231n.stanford.edu). These are the functions in the cs231n folders and includes code to preprocess and show the images. The classifiers used are based off of code prepared for CS 231n as well.

The goal of this workbook is to give you experience with training an SVM classifier via gradient descent.

0.2 Importing libraries and data setup

[33]:
```
import numpy as np # for doing most of our calculations
import matplotlib.pyplot as plt# for plotting
from cs231n.data_utils import load_CIFAR10 # function to load the CIFAR-10␣
 ↪dataset.
import pdb

# Load matplotlib images inline
%matplotlib inline

# These are important for reloading any code you write in external .py files.
# see http://stackoverflow.com/questions/1907993/
 ↪autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

[34]:
```
# Set the path to the CIFAR-10 data
cifar10_dir = 'C:/Users/Ashwin/Desktop/UCLA/current classes/ece 247/hw2/
 ↪hw2_code/cifar-10-batches-py/'
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape:  (50000, 32, 32, 3)
Training labels shape:  (50000,)
Test data shape:  (10000, 32, 32, 3)
Test labels shape:  (10000,)
```

[35]:
```
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse',␣
 ↪'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```

[36]:
```python
# Split the data into train, val, and test sets. In addition we will
# create a small development set as a subset of the training data;
# we can use this for development so our code runs faster.
num_training = 49000
num_validation = 1000
num_test = 1000
num_dev = 500

# Our validation set will be num_validation points from the original
# training set.
mask = range(num_training, num_training + num_validation)
X_val = X_train[mask]
y_val = y_train[mask]

# Our training set will be the first num_train points from the original
# training set.
mask = range(num_training)
X_train = X_train[mask]
y_train = y_train[mask]

# We will also make a development set, which is a small subset of
# the training set.
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# We use the first num_test points of the original test set as our
# test set.
mask = range(num_test)
X_test = X_test[mask]
y_test = y_test[mask]

print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('Dev data shape: ', X_dev.shape)
print('Dev labels shape: ', y_dev.shape)
```

```
Train data shape:  (49000, 32, 32, 3)
Train labels shape:  (49000,)
Validation data shape:  (1000, 32, 32, 3)
Validation labels shape:  (1000,)
Test data shape:  (1000, 32, 32, 3)
Test labels shape:  (1000,)
```

```
Dev data shape:  (500, 32, 32, 3)
Dev labels shape:  (500,)
```

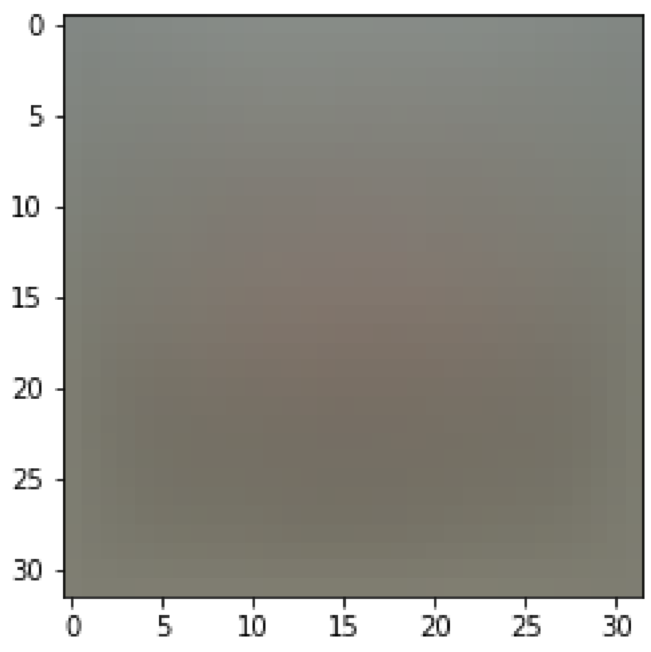
[37]:
```python
# Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# As a sanity check, print out the shapes of the data
print('Training data shape: ', X_train.shape)
print('Validation data shape: ', X_val.shape)
print('Test data shape: ', X_test.shape)
print('dev data shape: ', X_dev.shape)
```

```
Training data shape:  (49000, 3072)
Validation data shape:  (1000, 3072)
Test data shape:  (1000, 3072)
dev data shape:  (500, 3072)
```

[38]:
```python
# Preprocessing: subtract the mean image
# first: compute the image mean based on the training data
mean_image = np.mean(X_train, axis=0)
print(mean_image[:10]) # print a few of the elements
plt.figure(figsize=(4,4))
plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the mean␣
 ↪image
plt.show()
```

```
[130.64189796 135.98173469 132.47391837 130.05569388 135.34804082
 131.75402041 130.96055102 136.14328571 132.47636735 131.48467347]
```



[39]:
```
# second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image
```

[40]:
```
# third: append the bias dimension of ones (i.e. bias trick) so that our SVM
# only has to worry about optimizing a single weight matrix W.
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

print(X_train.shape, X_val.shape, X_test.shape, X_dev.shape)
```

```
(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)
```

0.3 Question:

(1) For the SVM, we perform mean-subtraction on the data. However, for the KNN notebook, we did not. Why?

0.4 Answer:

(1) SVM's can converge much faster on data with mean 0. However, KNN calculates based on other points, not on the overall distribution, and hence would not speed up if we made this change. Hence, this change is not needed for KNN. Additionally, mean-subtraction helps

reduce features with large magnitudes, which could potentially dominate other features when calculating the distance between the separating plane and the support vectors for SVM.

The reason we don't do normalization is since image pixels are usually homogeneous; hence, data normalization is not really needed.

0.5 Training an SVM

The following cells will take you through building an SVM. You will implement its loss function, then subsequently train it with gradient descent. Finally, you will choose the learning rate of gradient descent to optimize its classification performance.

[41]:
```
from nndl.svm import SVM
```

[42]:
```
# Declare an instance of the SVM class.
# Weights are initialized to a random value.
# Note, to keep people's initial solutions consistent, we are going to use a␣
 ↪random seed.

np.random.seed(1)

num_classes = len(np.unique(y_train))
num_features = X_train.shape[1]

svm = SVM(dims=[num_classes, num_features])
```

SVM loss

[43]:
```
## Implement the loss function for in the SVM class(nndl/svm.py), svm.loss()

loss = svm.loss(X_train, y_train)
print('The training set loss is {}.'.format(loss))

# If you implemented the loss correctly, it should be 15569.98
```

```
The training set loss is 15569.977915410193.
```

SVM gradient

[44]:
```
## Calculate the gradient of the SVM class.
# For convenience, we'll write one function that computes the loss
#   and gradient together. Please modify svm.loss_and_grad(X, y).
# You may copy and paste your loss code from svm.loss() here, and then
#   use the appropriate intermediate values to calculate the gradient.

loss, grad = svm.loss_and_grad(X_dev,y_dev)

# Compare your gradient to a numerical gradient check.
# You should see relative gradient errors on the order of 1e-07 or less if you␣
↪implemented the gradient correctly.
svm.grad_check_sparse(X_dev, y_dev, grad)
```

```
numerical: 1.139153 analytic: 1.139153, relative error: 1.205192e-07
numerical: 3.136700 analytic: 3.136700, relative error: 6.917619e-08
numerical: -6.469076 analytic: -6.469076, relative error: 1.453638e-08
numerical: 8.848929 analytic: 8.848930, relative error: 2.128730e-08
numerical: -4.041552 analytic: -4.041553, relative error: 7.274893e-08
numerical: -4.804845 analytic: -4.804845, relative error: 1.561937e-08
numerical: 1.420281 analytic: 1.420281, relative error: 1.180564e-07
numerical: -4.114251 analytic: -4.114252, relative error: 3.423090e-08
numerical: -3.228525 analytic: -3.228525, relative error: 4.994892e-08
numerical: -13.368588 analytic: -13.368587, relative error: 3.684456e-08
```

0.6 A vectorized version of SVM

To speed things up, we will vectorize the loss and gradient calculations. This will be helpful for stochastic gradient descent.

[45]:
```
import time
```

[48]:
```
## Implement svm.fast_loss_and_grad which calculates the loss and gradient
#    WITHOUT using any for loops.

# Standard loss and gradient
tic = time.time()
loss, grad = svm.loss_and_grad(X_dev, y_dev)
toc = time.time()
print('Normal loss / grad_norm: {} / {} computed in {}s'.format(loss, np.linalg.
↪norm(grad, 'fro'), toc - tic))

tic = time.time()
loss_vectorized, grad_vectorized = svm.fast_loss_and_grad(X_dev, y_dev)
toc = time.time()
print('Vectorized loss / grad: {} / {} computed in {}s'.format(loss_vectorized,␣
↪np.linalg.norm(grad_vectorized, 'fro'), toc - tic))

# The losses should match but your vectorized implementation should be much␣
↪faster.
print('difference in loss / grad: {} / {}'.format(loss - loss_vectorized, np.
↪linalg.norm(grad - grad_vectorized)))

# You should notice a speedup with the same output, i.e., differences on the␣
↪order of 1e-12
```

```
Normal loss / grad_norm: 15264.509941347484 / 2005.2446248571475 computed in
0.09596419334411621s
```

```
Vectorized loss / grad: 15264.509941347476 / 2005.2446248571475 computed in
0.0070035457611083984s
difference in loss / grad: 7.275957614183426e-12 / 3.2481758762084007e-12
```

0.7 Stochastic gradient descent

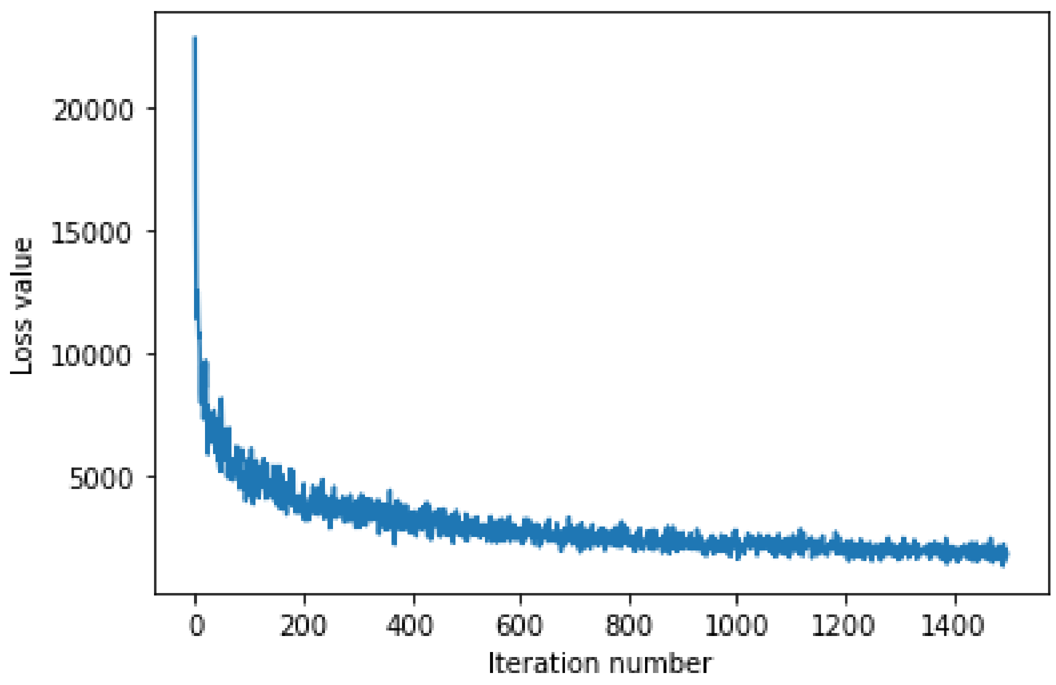
We now implement stochastic gradient descent. This uses the same principles of gradient descent we discussed in class, however, it calculates the gradient by only using examples from a subset of the training set (so each gradient calculation is faster).

[51]:
```
# Implement svm.train() by filling in the code to extract a batch of data
# and perform the gradient step.

tic = time.time()
loss_hist = svm.train(X_train, y_train, learning_rate=5e-4,
                      num_iters=1500, verbose=True)
toc = time.time()
print('That took {}s'.format(toc - tic))

plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```

```
iteration 0 / 1500: loss 22770.756607864787
iteration 100 / 1500: loss 4505.767134293777
iteration 200 / 1500: loss 3196.6474088671994
iteration 300 / 1500: loss 3179.892535847857
iteration 400 / 1500: loss 3120.17599694821
iteration 500 / 1500: loss 3413.5936203839497
iteration 600 / 1500: loss 2331.54384795122
iteration 700 / 1500: loss 2235.9108382401664
iteration 800 / 1500: loss 2519.3944914825474
iteration 900 / 1500: loss 2618.8846084258166
iteration 1000 / 1500: loss 2322.2631218538977
iteration 1100 / 1500: loss 1752.6067446319437
iteration 1200 / 1500: loss 2077.292143244585
iteration 1300 / 1500: loss 1704.1347667002772
iteration 1400 / 1500: loss 1858.1419171516927
That took 14.698964595794678s
```



0.7.1 Evaluate the performance of the trained SVM on the validation data.

[61]:
```
## Implement svm.predict() and use it to compute the training and testing error.

y_train_pred = svm.predict(X_train)
print('training accuracy: {}'.format(np.mean(np.equal(y_train,y_train_pred), )))
y_val_pred = svm.predict(X_val)
print('validation accuracy: {}'.format(np.mean(np.equal(y_val, y_val_pred)), ))
```

```
training accuracy: 0.28489795918367344
validation accuracy: 0.292
```

0.8 Optimize the SVM

Note, to make things faster and simpler, we won't do k-fold cross-validation, but will only optimize the hyperparameters on the validation dataset (X_val, y_val).

[67]:
```
# ================================================================ #
# YOUR CODE HERE:
#   Train the SVM with different learning rates and evaluate on the
#     validation data.
#   Report:
#     - The best learning rate of the ones you tested.
#     - The best VALIDATION accuracy corresponding to the best VALIDATION error.
#
```

```
#   Select the SVM that achieved the best validation error and report
#     its error rate on the test set.
#   Note: You do not need to modify SVM class for this section
# ================================================================ #
learning_rates = [0.0001, 0.00025, 0.001, 0.0025, 0.01, 0.025, 0.1, 0.5, 1]

#generates error based on learning rate on validation test
def generateError (rate):
    svm.train(X_train, y_train, rate)
    y_test_pred = svm.predict(X_test)
    y_val_pred = svm.predict(X_val)

    test_error_rate = 1 - np.mean(np.equal(y_test,y_test_pred))
    validation_accuracy = np.mean(np.equal(y_val, y_val_pred))
    print("learning rate:", rate, "validation_accuracy", validation_accuracy,␣
 ↪"test_error_rate", test_error_rate)
    return (rate, validation_accuracy, test_error_rate)

tuples = [generateError(x) for x in learning_rates]

print(max(tuples, key = lambda x : x[1]))

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
```

```
learning rate: 0.0001 validation_accuracy 0.204 test_error_rate 0.787
learning rate: 0.00025 validation_accuracy 0.245 test_error_rate 0.8
learning rate: 0.001 validation_accuracy 0.252 test_error_rate 0.761
learning rate: 0.0025 validation_accuracy 0.23 test_error_rate 0.767
learning rate: 0.01 validation_accuracy 0.271 test_error_rate 0.748
learning rate: 0.025 validation_accuracy 0.276 test_error_rate 0.727
learning rate: 0.1 validation_accuracy 0.271 test_error_rate 0.732
learning rate: 0.5 validation_accuracy 0.237 test_error_rate 0.758
learning rate: 1 validation_accuracy 0.274 test_error_rate 0.757
(0.025, 0.276, 0.727)
```

As we can see, the best validation_accuracy is 0.276, which means the best validation error is 1 - 0.276 = 0.724. This corresponds to a learning rate of 0.025 and a test error rate of 0.727.

0.9 SVM Notebook Code:

[ ]:
```
import numpy as np
import pdb

"""
This code was based off of code from cs231n at Stanford University, and␣
 ↪modified for ECE C147/C247 at UCLA.
```

```
"""
class SVM(object):

  def __init__(self, dims=[10, 3073]):
    self.init_weights(dims=dims)

  def init_weights(self, dims):
    """
        Initializes the weight matrix of the SVM.  Note that it has shape (C, D)
        where C is the number of classes and D is the feature size.
        """
    self.W = np.random.normal(size=dims)

  def loss(self, X, y):
    """
    Calculates the SVM loss.

    Inputs have dimension D, there are C classes, and we operate on minibatches
    of N examples.

    Inputs:
    - X: A numpy array of shape (N, D) containing a minibatch of data.
    - y: A numpy array of shape (N,) containing training labels; y[i] = c means
      that X[i] has label c, where 0 <= c < C.

    Returns a tuple of:
    - loss as single float
    """

    # compute the loss and the gradient
    num_classes = self.W.shape[0]
    num_train = X.shape[0]
    loss = 0.0

    for i in np.arange(num_train):
    # ================================================================ #
    # YOUR CODE HERE:
          #   Calculate the normalized SVM loss, and store it as 'loss'.
    #   (That is, calculate the sum of the losses of all the training
    #   set margins, and then normalize the loss by the number of
          #           training examples.)
    # ================================================================ #

    #using https://mlxai.github.io/2017/01/06/
↪vectorized-implementation-of-svm-loss-and-gradient-update.html for a␣
↪reference (Piazza @200)
```

```
        scores = np.dot(self.W, X[i,:]).T
        correct_class_score = scores[y[i]]

        for j in np.arange(num_classes):
            #there will be no loss if we are in the same class
            if j == y[i]:
                continue
            loss += max(0, scores[j] - correct_class_score + 1) #delta = 1 in␣
↪this case

    #normalize
    loss = loss / num_train

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #


    return loss

  def loss_and_grad(self, X, y):
    """
        Same as self.loss(X, y), except that it also returns the gradient.

        Output: grad -- a matrix of the same dimensions as W containing
                the gradient of the loss with respect to W.
        """

    # compute the loss and the gradient
    num_classes = self.W.shape[0]
    num_train = X.shape[0]
    loss = 0.0
    grad = np.zeros_like(self.W)

    for i in np.arange(num_train):
    # ================================================================ #
    # YOUR CODE HERE:
        #   Calculate the SVM loss and the gradient.  Store the gradient in
    #   the variable grad.
    # ================================================================ #
        scores = np.dot(self.W, X[i,:]).T
        correct_class_score = scores[y[i]]

        for j in np.arange(num_classes):
            #there will be no loss if we are in the same class
            if j == y[i]:
                continue
            #delta = 1 in this case; according to https://cs231n.github.io/
↪linear-classify/, it should always be 1?
            margin = scores[j] - correct_class_score + 1

            if margin > 0:
                loss += margin
                grad[y[i],:] -= X[i,:]
                grad[j,:] += X[i,:]

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    loss /= num_train
    grad /= num_train

    return loss, grad

  def grad_check_sparse(self, X, y, your_grad, num_checks=10, h=1e-5):
    """
    sample a few random elements and only return numerical
    in these dimensions.
    """

    for i in np.arange(num_checks):
      ix = tuple([np.random.randint(m) for m in self.W.shape])

      oldval = self.W[ix]
      self.W[ix] = oldval + h # increment by h
      fxph = self.loss(X, y)
      self.W[ix] = oldval - h # decrement by h
      fxmh = self.loss(X,y) # evaluate f(x - h)
      self.W[ix] = oldval # reset

      grad_numerical = (fxph - fxmh) / (2 * h)
      grad_analytic = your_grad[ix]
      rel_error = abs(grad_numerical - grad_analytic) / (abs(grad_numerical) +␣
↪abs(grad_analytic))
      print('numerical: %f analytic: %f, relative error: %e' % (grad_numerical,␣
↪grad_analytic, rel_error))

  def fast_loss_and_grad(self, X, y):
    """
    A vectorized implementation of loss_and_grad. It shares the same
        inputs and ouptuts as loss_and_grad.
    """
    loss = 0.0
    grad = np.zeros(self.W.shape) # initialize the gradient as zero

    # ================================================================ #
    # YOUR CODE HERE:
        #   Calculate the SVM loss WITHOUT any for loops.
    # ================================================================ #

    #using the same reference as above

    num_classes = self.W.shape[0]
    num_train = X.shape[0]

    scores = X.dot(self.W.T)
    yi_scores = scores[np.arange(scores.shape[0]), y]
    margins = np.maximum(0, (scores.T - np.matrix(yi_scores)).T + 1) #delta = 1␣
↪always
    margins[np.arange(num_train),y] = 0
    loss = np.mean(np.sum(margins, axis=1))
    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #




        # ================================================================ #
    # YOUR CODE HERE:
        #   Calculate the SVM grad WITHOUT any for loops.
    # ================================================================ #
     #use Guangyuan's discussion as reference:https://colab.research.google.com/
↪drive/1ugOJc2qZHMQYh2sKf3QXz9-QbH5t5FIW and the other reference mentioned␣
↪above

    binary = margins
    binary[margins > 0] = 1
    row_sum = np.sum(binary, axis=1)
    binary[np.arange(num_train), y] = -row_sum.T
    grad = np.dot(binary.T, X)

    #average
    grad /= num_train

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #
    return loss, grad

  def train(self, X, y, learning_rate=1e-3, num_iters=100,
            batch_size=200, verbose=False):
    """
    Train this linear classifier using stochastic gradient descent.

    Inputs:
    - X: A numpy array of shape (N, D) containing training data; there are N
      training samples each of dimension D.
    - y: A numpy array of shape (N,) containing training labels; y[i] = c
      means that X[i] has label 0 <= c < C for C classes.
    - learning_rate: (float) learning rate for optimization.
    - num_iters: (integer) number of steps to take when optimizing
    - batch_size: (integer) number of training examples to use at each step.
    - verbose: (boolean) If true, print progress during optimization.

    Outputs:
    A list containing the value of the loss function at each training iteration.
    """
    num_train, dim = X.shape
    num_classes = np.max(y) + 1 # assume y takes values 0...K-1 where K is␣
↪number of classes

    self.init_weights(dims=[np.max(y) + 1, X.shape[1]])        # initializes␣
↪the weights of self.W

    # Run stochastic gradient descent to optimize W
    loss_history = []

    for it in np.arange(num_iters):
      X_batch = None
      y_batch = None

      # ================================================================ #
      # YOUR CODE HERE:
      #   Sample batch_size elements from the training data for use in
      #          gradient descent.  After sampling,
      #     - X_batch should have shape: (dim, batch_size)
            #     - y_batch should have shape: (batch_size,)
            #   The indices should be randomly generated to reduce correlations
            #   in the dataset.  Use np.random.choice.  It's okay to sample with
            #   replacement.
      # ================================================================ #

      #pick batch_size random indices from {0, num_train)
      indices = np.random.choice(np.arange(num_train), batch_size)
      X_batch = X[indices]
      y_batch = y[indices]

      # ================================================================ #
      # END YOUR CODE HERE
      # ================================================================ #

      # evaluate loss and gradient
      loss, grad = self.fast_loss_and_grad(X_batch, y_batch)
      loss_history.append(loss)

      # ================================================================ #
      # YOUR CODE HERE:
      #   Update the parameters, self.W, with a gradient step
      # ================================================================ #
      self.W = self.W - learning_rate * grad

            # ================================================================ #
      # END YOUR CODE HERE
      # ================================================================ #

      if verbose and it % 100 == 0:
        print('iteration {} / {}: loss {}'.format(it, num_iters, loss))

    return loss_history

  def predict(self, X):
    """
    Inputs:
    - X: N x D array of training data. Each row is a D-dimensional point.

    Returns:
    - y_pred: Predicted labels for the data in X. y_pred is a 1-dimensional
      array of length N, and each element is an integer giving the predicted
      class.
    """
    y_pred = np.zeros(X.shape[1])


    # ================================================================ #
    # YOUR CODE HERE:
    #   Predict the labels given the training data with the parameter self.W.
    # ================================================================ #

    #np.dot(X, W^T) = np.matmul(XW)^T
    #argmax = indices of max values along axis
    product = np.dot(self.W, X.T)
    y_pred = np.argmax(product, axis=0)

    # ================================================================ #
    # END YOUR CODE HERE
    # ================================================================ #

    return y_pred
```