Convolution using FFT

Given two n-dimensional vectors $a = [a_0, \dots, a_{n-1}]$ and $b = [b_0, b_1, \dots, b_{n-1}]$ over complex numbers, use FFT and its inverse to output the convolution $c = a \otimes b$, where, $c_k = \sum_{j=0}^{k} a_j b_{k-j}$, for $k = 0, 1, \dots, 2n-2$.

Let F_n denote the $n \times n$ DFT matrix. That is,

$$F_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \omega_n & \omega_n^2 & \dots & \omega_n^{n-1} \\ 1 & \omega_n^2 & \omega_n^4 & \dots & \omega_n^{2(n-1)} \\ & \vdots & \vdots & \vdots & \\ 1 & \omega_n^{n-1} & \omega_n^{2(n-1)} & \dots & \omega_n^{(n-1)^2} \end{bmatrix}$$

Recall by taking inner-product of any two columns that $F_n^* F_n = nI$. Hence, $F_n^{-1} = \frac{1}{n} F_n^*$ and therefore,

$$(DFT)_n^{-1}(y) = (1/n) F_n^*(y) \ .$$

We obtain two ways of computing DFT_n^{-1}. From definition of F_n^*, we have that F_n^* is the same as that of F_n^* with ω_n replaced by $\overline{\omega_n} = \omega_n^{-1} = e^{-2\pi i/n}$. So, in the computation of $F_n y$, if we replace the role of ω_n by ω_n^{-1} appropriately throughout, and divide by n, we should obtain $DFT^{-1}(y)$. The second method comes by observing the rows of F_n^* and relating them to rows of F_n. Note that $\overline{\omega_n}^k = \omega_n^{-k} = \omega_n^{n-k}$, for $0 \le k \le n-1$.

$$F_n^* = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \overline{\omega_n} & \overline{\omega_n}^2 & \dots & \overline{\omega_n}^{n-1} \\ 1 & \overline{\omega_n}^2 & \overline{\omega_n}^4 & \dots & \overline{\omega_n}^{2(n-1)} \\ & \vdots & \vdots & \vdots & \\ 1 & \overline{\omega_n}^{n-1} & \overline{\omega_n}^{2(n-1)} & \dots & \overline{\omega_n}^{(n-1)^2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \omega_n^{n-1} & \omega_n^{2(n-1)} & \dots & \omega_n^{(n-1)^2} \\ 1 & \omega_n^{n-2} & \omega_n^{2(n-2)} & \dots & \omega_n^{(n-2)(n-1)} \\ & \vdots & \vdots & \vdots & \\ 1 & \omega_n & \omega_n^2 & \dots & \omega_n^{n-1} \end{bmatrix}$$

Thus row indexed 0 of F_n^* is the same as row indexed 0 of F_n. Row 1 of F_n^* is same as row $n-1$ of F_n, row 2 of F_n^* is same as row $n-2$ of F_n, row $n-i$ of F_n^* is same as row i of F_n, for $i = 1, 2, \dots, n-1$. This same relation therefore holds between $F_n^* y$ and $F_n y$.

Since, a and b are both n dimensional, first pad a and b each with n zero new coefficients $a_n, \dots, a_{2n-1}$ and $b_n, \dots, b_{2n-1}$ that are all zeros to make them $2n$ dimensional vectors. Now compute $c = a \otimes b$ as follows. Let N be the closest power of 2 that is equal to or larger than $2n$.

$$c = DFT_N^{-1}(FFT_N(a) \bullet FFT_N(b))$$

where, for any k-dimensional vectors u and v, $(u \bullet v)_j = u_j \cdot v_j$, for $j = 0, \dots, k-1$.

Input Format

First line of each input is a positive integer t - number of test cases.

For each test case -

1. First line contains n - number of coefficients of input polynomials
2. Following $1 \le k \le n$ lines contains the components of vector **a** as a pair pq for $p + iq$

3. Next $1 \leq k \leq n$ lines contains the components of vector **b** as a pair rs for $r + is$

Constraints

- $1 \leq t \leq 100$
- $1 \leq n \leq 1000$

Output Format

For each test case output the vector **c**. Let N be the closest power of 2 that is equal to or larger than $2n$

For $1 \leq j \leq N$, j^{th} line will contain $c_{j-1} = (x + iy)$ as a tuple (x, y)

Sample Input 0

```
2
2
2 0
3 0
1 0
4 0
3
2 0
4 0
3 0
3 0
1 0
7 0
```

Sample Output 0

```
(2.000,0.000)
(11.000,0.000)
(12.000,0.000)
(0.000,0.000)
(6.000,0.000)
(14.000,0.000)
(27.000,0.000)
(31.000,0.000)
(21.000,0.000)
(0.000,0.000)
(0.000,0.000)
(0.000,0.000)
```