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# The Wind Farm Layout Optimization Problem

Michele Samorani

**Abstract** An important phase of a wind farm design is solving the Wind Farm Layout Optimization Problem (WFLOP), which consists in optimally positioning the turbines within the wind farm so that the wake effects are minimized and therefore the expected power production maximized. Although this problem has been receiving increasing attention from the scientific community, the existing approaches do not completely respond to the needs of a wind farm developer, mainly because they do not address construction and logistical issues. This paper describes the WFLOP, gives an overview on the existing work, and discusses the challenges that may be overcome by future research.

*Keywords:* wind farm layout, wake effects, wind turbine, optimization, genetic algorithms.

## 1 Introduction

Wind energy is the fastest growing source of renewable energy, as the worldwide production has doubled between 2005 and 2008, reaching 121.2 GW of total installed capacity. The transformation of wind power into electrical power is performed by wind turbines, which are usually grouped into wind farms in order to exploit considerations relative to economies of scale, such as lower installation and maintenance costs. But as costs decrease, grouping turbines leads to a reduction in the power produced because of the presence of wake effects within the wind farm. When a turbine extracts power from the wind, it generates a "wake" of turbulence that propagates downwind, so that the wind speed and therefore the power extracted by the turbines affected are reduced. In large wind farms wake effects lead to considerable power loss [22], and thus it is desirable to minimize them in order to maximize the expected power output. The wind farm layout optimization problem (WFLOP) consists of finding the turbine positioning (wind farm layout) that

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maximizes the expected power production. Finding high quality solutions may ultimately lead to high profits for the wind farm developers. Currently, this problem is usually solved using very simple rules that lead to rectilinear layouts, where turbines are often organized in identical rows that are separated by a conveniently large distance (<http://www.offshorewindenergy.org>). Recently, a few papers showed that irregular layouts result in a higher expected energy production than regular grids [2, 12, 13, 19, 23, 25, 26].

The existing work that has tackled this problem is very limited and has been carried out by the wind engineering and wind energy communities, whereas no effort has been done by the optimization community. Existing algorithms include only genetic algorithms and simulated annealing. There is therefore potential for improvement by using other optimization techniques, such as mixed-integer programming, dynamic programming, stochastic programming, etc... As it will be clear in the following, the main reasons why this problem has been largely disregarded by the operations research community are its nonlinearity and the difficulty in obtaining data about the problem instances.

In this paper we present the process of building a wind farm, discuss the problems caused by the wake effects and how they impact energy production and maintenance costs, review the existing literature on the WFLOP, comment on the advantages and shortcomings of the existing methods, and lay out the lines of research that can be developed.

## 2 Construction of a wind farm

This section briefly describes the phases of a wind farm development project. The first step is finding a windy site to ensure the economic profitability of the project. Sites are usually classified in 7 different wind power classes (<http://www.awea.org>) that correspond to 7 different average wind speed intervals. Usually, sites with wind power class 4 or higher are considered potentially profitable for large size projects; nevertheless, not all sites belonging to high wind power classes are feasible sites for a project. In fact, some sites can be very far from the electrical grid or reachable only by roads that are not wide enough to transport very long trucks. In the first case, the site is not profitable because of the high cost to connect it to the electrical grid, in the second case, because of the high cost of the necessary road work.

Once a suitable site is found, land owners are contacted and asked if they are interested in hosting wind turbines on their land. Land owners that participate in the project usually receive a percentage of the profit generated by the turbines on their land, and extra money if roads or other infrastructure are built on their terrain. The process of contacting land owners and agreeing on the lease terms is not straightforward though, and usually takes a few months. In the meantime, the wind developer installs measurement towers in order to assess the wind distribution (or wind rose) of all the parts of the site. This measurement may take as short as one month for sites where the wind is known to blow from the same direction all year around, or as

long as 2 years for sites where seasonal winds blow from different directions. The accuracy of these measurements is critical for the project because they are used to find the optimal layout and to assess the expected annual profit of the wind farm. Nevertheless, the measurements may be taken at significantly lower heights than the hub height of the wind turbines because of the high cost of installing tall towers. In these cases, atmospheric models such as the "power law model" [24] use the wind speed at measurement height to extrapolate the wind speed at hub height. Alternative measurement methods are Doppler SODARs and Doppler LIDARs. As described in [18], Doppler SODARs "measure the wind from acoustic energy that is backscattered from small-scale turbulent fluctuations of temperature (density)," whereas Doppler LIDARs measure the wind "using the light energy backscattered from microscopic particulates or aerosols being transported by the wind". Both technologies have the advantage of accurately computing the wind distribution in all parts of the site with no need to install measurement towers. Since it is so accurate [9], LIDAR is becoming a more and more popular tool used by wind farm developers. When the land that can be used for the project is known and the wind distribution has been obtained, the WFLOP is solved.

How to choose the number and the model of the turbines to install depends on a variety of factors. First, it is important to note that a more powerful turbine is usually preferred to a less powerful one since both the cost of a turbine and the energy it generates are usually proportional to its nominal power. Thus, the net profit generated by the turbine is also proportional to its nominal power. Obviously this property does not hold for extremely powerful (and therefore state-of-the-art) turbines because spare parts may be very expensive and maintenance costs very high. Since the trend is to build turbines that are more and more powerful, the cost of unused smaller turbines dramatically decreases. Therefore turbine manufacturers may offer advantageous discounts on small turbines in order to reduce their inventory. Other times, large wind farm development companies have their own turbine inventory. In this case, they may have extra turbines in stock that need to be used before they become obsolete. Given these considerations, the existing works on the WFLOP assume that the type and number of the turbines to install is predetermined.

### 3 Wind turbines and wake effects

The characteristics of a wind turbine that are related to wind farm layout optimization are the following:

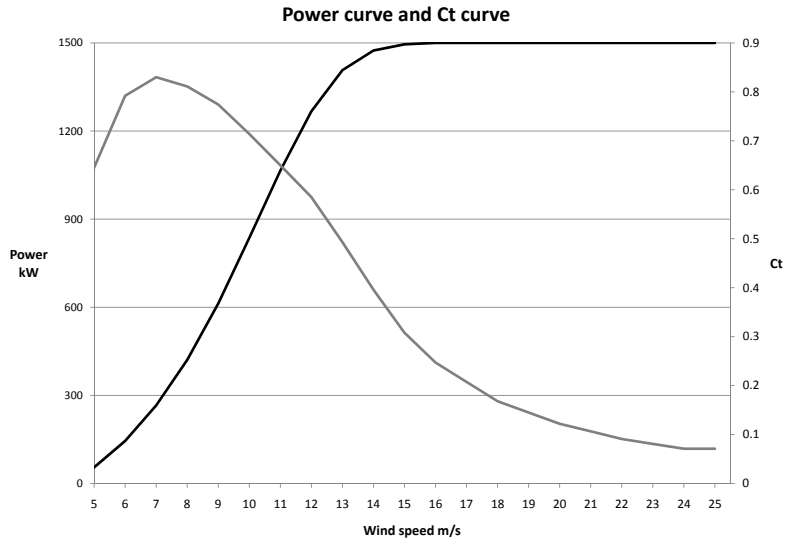
- Cut-in speed  $c_i$
- Cut-out speed  $c_o$
- Nominal speed
- Nominal power
- Power curve
- Thrust coefficient curve

- Rotor diameter  $d$
- Hub height  $z$

When the wind speed is greater than  $c_i$ , the blades of the turbine start spinning and therefore the turbine generates power. The power produced roughly increases with the cube of the wind speed until the *wind speed* reaches the nominal speed, at which point the control system of the turbine modifies the pitch of the blades so that the power produced is constant and equal to the *nominal power*. When the speed reaches  $c_o$ , it is considered too high, and the turbine is shut down to avoid damaging the blades.

Other important characteristics of interest are the *power curve* and the *thrust coefficient curve*. They report respectively the power produced and the value of the thrust coefficient ( $C_t$ ) at every wind speed included between  $c_i$  and  $c_o$ . Roughly speaking, the thrust coefficient measures the proportion of energy captured when the wind passes through the blades of the turbine [1]. For both power curve and thrust coefficient curve, manufacturers usually provide a few data points, which need to be interpolated to obtain the intermediate points. For example, Fig. 1 shows the two curves of the Vestas turbine V63 ( $c_i = 5$  m/s,  $c_o = 25$  m/s, nominal power = 1.5 MW, nominal speed = 16 m/s). The data points, which are provided only for 1 m/s, 2 m/s,... 25 m/s, have been linearly interpolated.

By extracting energy from the wind, a turbine creates a cone (*wake*) of slower and more turbulent air behind it. This phenomenon, which is referred to as *wake ef-*

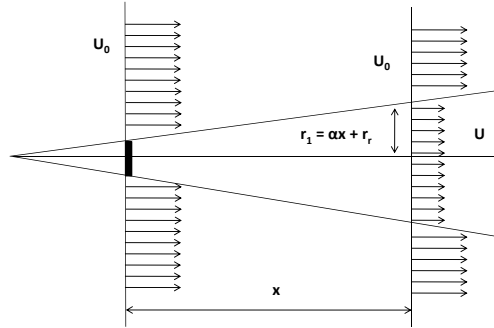


**Fig. 1** Power curve (black line) and thrust coefficient curve (grey line) of the turbine Vestas V63

*fect* (see Fig. 2), has been studied by several authors in the fluid-aerodynamic field. In their survey of these studies, Vermeer et al. [27] focus on the experiments aimed at identifying mathematical models that accurately describe the wake effect, both in terms of wind speed reduction and turbulence intensity. Some of these models are only valid near the turbine that generates the wake (near wake models), and others are only valid far from the turbine that generates the wake (far wake models). Beyond 3 rotor diameters distance, Vermeer et al. [27] suggest to use the far wake models. In the near wake, the turbulence intensity is so large that the affected turbines must be shut down in order to avoid blade damage. Although more accurate computational methods have been proposed (in particular CFD – see [28]), the existing works on wind farm layout optimization use the model proposed by [14, 16] because its simplicity makes it more practical to embed within optimization procedures and in computer programs, such as PARK [15]. Although simple, this wake model has been shown to accurately compute the wind speed reduction in the far wake case [3, 4].

Let us explain the Jensen model through the example in Fig. 2. The wind blows from left to right at speed  $u_0$  and hits a turbine (represented as a black rectangle on the left) whose rotor radius is  $r_r$ . At a distance  $x$  downwind, the wind speed is  $u$  and the wake radius (initially equal to  $r_r$ ) becomes  $r_1 = \alpha x + r_r$ . The  $\alpha$ -dimensional scalar  $\alpha$  determines how quickly the wake expands with distance and it is defined as:

$$\alpha = \frac{0.5}{\ln \frac{z}{z_0}} \quad (1)$$



**Fig. 2** Schematic representation of the wake effect [23]

where  $z$  is the hub height of the turbine generating the wake and  $z_0$  is a constant called surface roughness, which depends on the terrain characteristics.

Let  $i$  be the position of the turbine that generates the wake,  $j$  the position affected by it,  $u_0$  the ambient wind speed, and  $u_j$  the wind speed at  $j$ . Then:

$$u_j = u_0(1 - vd_{ij}) \quad (2)$$

where  $vd_{ij}$  is the *velocity deficit* induced on position  $j$  by the wake generated by  $i$ .  $vd_{ij}$  is computed as follows:

$$vd_{ij} = \frac{2a}{1 + \alpha \left( \frac{x_{ij}}{r_d} \right)^2} \quad (3)$$

The term  $a$  that appears in the numerator is called *axial induction factor* and is computed by the following expression:

$$a = 0.5 \left( 1 - \sqrt{1 - C_T} \right) \quad (4)$$

The term  $r_d$  that appears in the denominator is called *downstream rotor radius* and is equal to:

$$r_d = r_r \sqrt{\frac{1-a}{1-2a}} \quad (5)$$

The term  $x_{ij}$  is the distance between positions  $i$  and  $j$ . The notation we propose is coherent to the ones used in [2, 12, 13, 23, 26], and it is equivalent to the one proposed by [20].

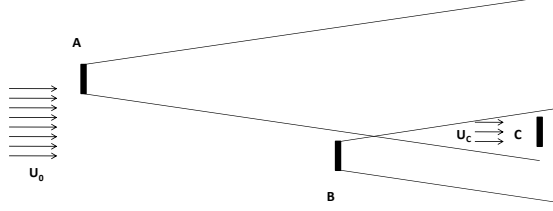
Since many turbines are installed in a wind farm, wakes can intersect and affect turbines downwind at the same time. In the Jensen model, the total velocity deficit  $v_{def}(j)$  at a location  $j$  that is affected by more wakes is obtained as follows:

$$v_{def}(j) = \sqrt{\sum_{i \in W(j)} vd_{ij}^2} \quad (6)$$

where  $W(j)$  is the set of turbines affecting position  $j$  with a wake.  $v_{def}(j)$  is then used in (2) in place of  $vd_{ij}$  to compute  $u_j$ . Let us illustrate this with the example reported in Fig. 3.

In this example the wind blows from left to right at a speed equal to  $U_0 = 12m/s$  and the wakes generated by turbines  $A$  and  $B$  affect position  $C$ . We are interested in computing the wind speed  $U_C$ . The data of the problem are:

- $x_{AC} = 500m$
- $x_{BC} = 200m$
- $z = 60m$
- $z_0 = 0.3m$
- $r_r = 20m$  (i.e.  $D = 40m$ )
- $C_T = 0.88$



**Fig. 3** Example of multiple wakes affecting a position

In order to solve the problem, we use (1), (3), (4), (5) to compute  $vd_{AC}$  and  $vd_{BC}$  and obtain:

- $vd_{AC} = 0.0208$
- $vd_{BC} = 0.1116$

These partial results can be interpreted as follows:

- if  $C$  was affected only by  $A$ , then the wind speed at  $C$  would be reduced by 2.08% from the ambient wind speed
- if  $C$  was affected only by  $B$ , then the wind speed at  $C$  would be reduced by 11.16% from the ambient wind speed

Then we use (6) to compute the total velocity deficit  $v_{def}(C) = 0.1135$  (wind speed at  $C$  reduced by 11.35%) and (2) to compute  $U_C = U_0(1 - v_{def}(C)) = 10.64m/s$ . Note that the computations would be the same even if  $B$  was inside the wake created by  $A$ .

This example highlights a very important property of multiple wake combinations: the total velocity deficit mostly depends on the closest turbine that generates a wake. In the example, the total velocity deficit  $v_{def}(C)$  is very close to the velocity deficit  $vd_{BC}$  caused by turbine  $B$ . In other words, the presence of turbine  $A$  does not substantially affect the wind speed in  $C$  since  $U_C$  would be equal to  $10.66m/s$  if  $A$  was not present. However, the interaction of multiple wakes is not fully understood and is subject of many studies in the aerodynamics field. Vermeer et al. [27] report that recent studies highlighted significant discrepancies between the real and the estimated energy production in large wind farms, where this type of interaction has a big impact.

Under scenario  $s$ , characterized by a wind direction and an ambient wind speed, the power produced by the wind farm can be obtained by computing the wind speed  $v_j^s$  at each turbine location  $j \in L$ . Let  $P(v)$  be the function, which we assume known,



that computes the power generated by one turbine if the wind speed is  $v$  at the turbine location. The total power produced is obtained by summing up the contribution of all the turbines. If more scenarios are present, we are interested in the expected power produced, which is calculated by summing up the power produced in each scenario  $s$  weighted by the probability of its realization  $r_s$ . In the existing literature, the objective is to maximize the total power (T) function:

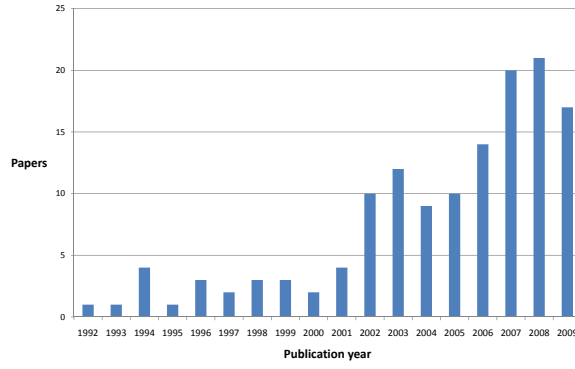
$$T = \sum_{s \in S} r_s \sum_{j \in L} P(v_j^s) = \sum_{s \in S} r_s \sum_{j \in L} P \left[ U_s \cdot \left( 1 - \sqrt{\sum_{i \in W^s(j)} v d_{ij}^2} \right) \right] \quad (7)$$

where  $W^s(j)$  is the set of turbines affecting position  $j$  with a wake under scenario  $s$ . Although the decision variables (i.e., the locations of the turbines) do not explicitly appear in (7), the sets  $W^s(j)$  directly depend on them. Obviously, a mathematical model should also include the constraints on the  $W^s(j)$ , in order to guarantee the correctness of the model. For example, if under scenario  $s$  position  $a$  affects position  $b$  and  $b$  affects position  $c$ , then  $a$  must affect also  $c$ . Since this is outside the scope of this survey, we leave the definition of a complete mathematical model to future work.

#### 4 Work on wind farm layout optimization

The WFLOP has been largely neglected by the operations research and operations management communities. To date, most of the published works that address this problem appear in journals whose topics are related to energy and wind engineering. These articles apply existing optimization methodologies to solve different versions of this problem without focusing on the solution method itself. However, this topic has been receiving increasing attention, as reported in Fig. 4, which shows the number of papers published from 1992 to 2009 retrieved through a google scholar search with the following keywords: "*wind farm*", *wake*, *turbine*, *position*, *optimization*. Note that some of the retrieved papers focus on wake effect modeling and not on wind farm layout optimization. Nevertheless, the increasing presence of this type of papers indicates an increasing interest in accurately assessing the energy production of a wind farm so that it can be designed more carefully.

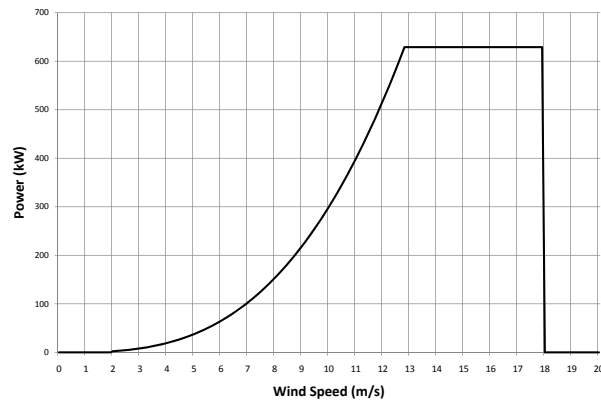
In the following discussion we identify and review the most prominent published works on this topic, with a particular emphasis on both their shortcomings and on the research opportunities that can be addressed by the optimization community. Mosetti et al. [23] were the first to take into consideration the WFLOP. They model the wind farm site as a 10x10 square grid, where the centers of the 100 squares are the possible positions of the turbines. To ensure the validity of the Jensen model, the side of each cell is  $5D$  – although  $3D$  would be enough [27]. The turbine used has a hub height  $z = 60m$ , diameter  $D = 40m$ , and a constant thrust coefficient  $C_T = 0.88$ . The power curve, depicted in 5, is expressed by the following:



**Fig. 4** Number of papers published from 1992 to 2009, retrieved through a google scholar search with the following keywords: "wind farm", wake, turbine, position, optimization

$$P(U) = \begin{cases} 0 & \text{for } U < 2 \\ 0.3U^3 & \text{for } 2 \leq U < 12.8 \\ 629.1 & \text{for } 12.8 \leq U < 18 \\ 0 & \text{for } 18 \leq U \end{cases}$$

where the wind speed  $U$  is expressed in m/s and the power in kW.



**Fig. 5** Power curve used in [23]

Mosetti et al. [23], who do not assume a predefined number of turbines to install, define their goal as maximizing the power produced  $P_{tot}$  while minimizing the installation cost  $cost_{tot}$ . The power produced is derived as explained in the previous section, whereas the installation cost is defined as:

$$cost_{tot} = N_t \left( \frac{2}{3} + \frac{1}{3} e^{-0.00174 N_t^2} \right) \quad (8)$$

where  $N_t$  is the number of turbines installed. The cost of a turbine, which is the expression in parenthesis in the formula above, decreases as  $N_t$  increases, which reflects the economies of scale considerations reported in the introduction. The adopted objective function is:

$$Obj_{MOS} = \frac{1}{P_{tot}} w_1 + \frac{cost_{tot}}{P_{tot}} w_2 \quad (9)$$

where the weight  $w_1$  has been kept small compared to  $w_2$ .

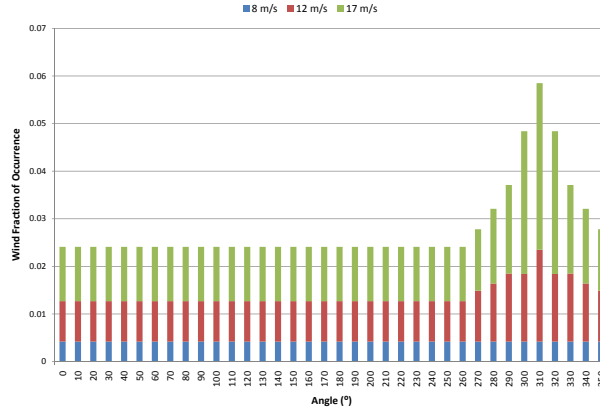
The solution method they use is based on Genetic Algorithms (GAs). GAs keep a population of solutions which iteratively evolves through combinations and selections. At each iteration, solutions are combined and a new solution is obtained whose components are inherited from one of two parents. A solution is represented by a vector of 100 binary variables  $x_i$  (with  $i = 1, \dots, 100$ ), each indicating the presence of a turbine in position  $i$ . Therefore, combining two solutions effectively consists of generating a new solution that has some turbines in the same positions of the first parent and the others in the same positions of the second parent. After generating a new solution, some of its components may be changed in order to introduce diversity in the population. This mechanism is called mutation, given its resemblance to the genetic changes that are involved in the evolution of the species. The reader should consult [11] for an overview of GAs. Mosetti et al. [23] let a population of 200 individuals evolve for 400 iterations.

They introduce 3 problem instances: A, B, and C. In A the wind constantly blows from North at 12m/s; in B the wind speed is 12m/s but the direction is uniformly distributed across 36 angles having the same angular sectors width  $10^\circ$ ; in C the wind blows at 3 possible different speeds (8 m/s, 12 m/s, 17 m/s) from one the 36 directions described above. The probability distribution that describes the occurrence of each wind speed and direction is reported in Fig. 6.

A computational study shows that the solutions obtained by GAs have a higher objective function value and a higher efficiency than solutions obtained by installing turbines in random positions. The efficiency, which is a very common way of evaluating and comparing solutions, is defined in [23] as

the ratio between the total energy extracted by the windfarm having  $N_t$  turbines and  $N_t$  times the energy extracted by an isolated turbine with the same undisturbed wind

Grady et al. [12] replicated the experiments presented in [23] by modifying the settings of the GA. In particular, they show that by letting 20 subpopulations evolve for 3000 iterations one can achieve better solutions. More recently, Housheng [13] improved upon these results using a distributed GA where a small fraction of the high-



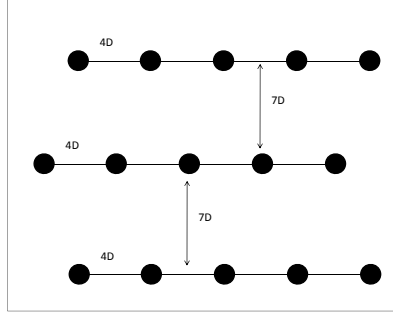
**Fig. 6** Wind distribution in C [23]

est quality individuals of each subpopulation periodically migrates to another subpopulation. Sisbot et al. [26] proposed a multi-objective genetic algorithm, where two objectives are the one of maximizing the power produced and the one of minimizing the cost. Although interesting, their method has only been tested on the case where wind blows from a constant direction at a constant speed. They claim that this assumption makes it possible to have rectangular cells instead of square ones, so that the minimum distance between two turbines is  $8D$  in the prevailing wind direction and  $2D$  in the crosswind direction. This type of consideration is also the basis of the following rule of thumb that can be used to design a wind farm layout (<http://guidedtour.windpower.org/en/tour/wres/park.htm>, last access on 01/06/2010):

turbines in wind parks are usually spaced somewhere between 5 and 9 rotor diameters apart in the prevailing wind direction, and between 3 and 5 diameters apart in the direction perpendicular to the prevailing winds (see Fig. 7)

This simple approach ignores all wind directions except the prevailing one, and it is therefore likely to lead suboptimal wind farm layouts. Furthermore, it fails to describe how to compute the power produced when the wind blows from a crosswind direction, in which case the Jensen model is not valid because the distance between the turbines that generate a wake and the turbines affected by it is too short.

All these GA-based approaches share an evident shortcoming: the solution space is discrete. In other words, there is a predefined set of possible positions - the centers of the cells - of which a subset must be chosen as installation positions of the turbines. Since consecutive positions are spaced by a distance of several rotor diameters and it is not possible to choose intermediate positions, it follows that potentially better positions are not even considered. To solve this problem one may consider a



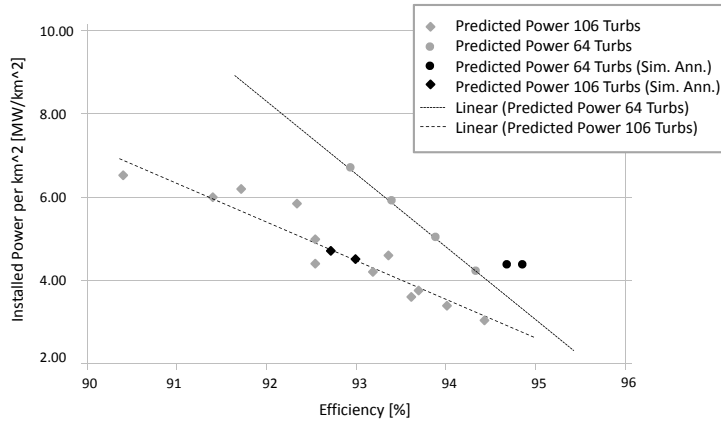
**Fig. 7** Three rows of five turbines each, installed according to the rule of thumb

finer grid, i.e., a grid whose cell sides are shorter, as long as proximity constraints are introduced to avoid infeasible solutions. These constraints forbid turbines to be installed too close to each other (at less than  $3D$  distance). Unfortunately GAs do not offer a natural way to embed constraints, which would have to be enforced by introducing a feasibility check in the objective function evaluation routine, making the search significantly slower. Nevertheless, the computational impact of introducing proximity constraints is unknown because it has not been tried yet.

The first attempt to address the limitation of a discrete space was made by Aytun and Norman [2], who proposed a local search that iteratively considers the operations of adding, removing, or moving turbines in an attempt of improving the objective function value. The add operation randomly generates a set of locations, which are individually considered as potential installation positions of new turbines; the remove operation considers the removal of each existing turbine; the move operation attempts to move each existing turbine by  $4D$  along a set of predefined directions. Whenever the add operation is considered, a new set of candidate locations is randomly generated and evaluated, and therefore turbines can be potentially placed in any position of the site. Nevertheless, it would be incorrect to say that this approach considers a continuous solution space; rather, it considers a discrete space where the possible positions are randomly generated during the search instead of being predefined.

A similar work, developed by Rivas et al. [25], consists of a simulated annealing procedure that uses the same set of moves (add, remove, move). Since simulated annealing is a neighborhood search that may accept non-improving moves [5], it overcomes the limitation of the search proposed in [2], which was purely local. Besides improving the solution method, Rivas et al. [25] performed a relevant computational study to assess the difference between the quality of solutions obtained by their approach and the quality of solutions obtained by the rule of thumb. They consider two different problems: one where we must install 106 3MW turbines and one where we must install 64 5MW turbines. As they note, the total power installed is similar for the two problems. Each problem is solved by imposing a predefined

geographical extension (or site area) of the wind farm, which is equivalent to imposing a predefined density of installed power (the smaller the area, the higher the density). For all site areas considered, Rivas et al. [25] solved the 2 problems by either using the proposed method or the rule of thumb. Fig. 8 summarizes their findings by reporting the efficiency obtained for both problems and for the considered site areas. The light dots represent the results obtained by the rule of thumb, while the dark ones the results obtained by simulating annealing. Obviously, when using the rule of thumb to solve either problem (106 or 64 turbines), if the site area increases, so does the efficiency of the wind farm. This trend is highlighted by the two lines in Fig. 8. The relevant finding that is worthwhile noting here is that by using their method one finds higher quality solutions for the 64 turbine problem (they are on the upper half-plane of the line) but not for the 106 turbine problem (they lie on the line).



**Fig. 8** Study by Rivas et al. [25] on the impact that site area and number of turbines have on efficiency

In other words, the potential improvement due to their method over the rule of thumb is more evident if the turbines are few and big (e.g. 64 5MW turbines), whereas it tends to disappear if they are many and small (e.g. 106 3MW turbines). Although this property may hold only for their method, it may hide a more general property, which is valid for any method: the chosen layout strongly impacts the quality of the solution if we install few turbines or, equivalently, the chosen layout does not strongly impact the quality of the solution if we install many turbines. The argument supporting this idea is based on the property of multiple wake superimposition, according to which *the total velocity deficit mostly depends on the closest*

*turbine that generates a wake* (Sect. 3). If we are installing many turbines, we expect that most of them are impacted by at least one wake, regardless to how they are positioned; conversely, if we are installing few turbines, it is possible that only few or none of them are affected by a wake, for example if they are aligned in one row that is perpendicular to the wind direction. In the first case (many turbines) optimizing the layout may reduce the average number of wakes that affect the turbines at the same time, but this will have a little impact on the objective function value; in the second case (few turbines), optimizing the layout may prevent some turbines to be affected by a wake, which will have a great impact on the objective function. Nevertheless, further studies need to be carried out in order to validate this idea.

Let us now consider the first approach that actually considers a continuous solution space: the one proposed by [20]. They aim at finding the optimal wind farm layout for an offshore site where the number of turbines is predefined. An offshore scenario differs from an onshore one in that both costs and energy production strongly depend on the turbine positions. Installation and maintenance costs increase with the depth of the water, and so does the energy production because the wind speed becomes higher as we go farther from shore. Hence, there is an optimal trade-off between these two contrasting effects that determines how far from shore to install the turbines. The objective used is the one of minimizing the levelized cost of energy (LCOE), defined in [21] as:

$$LCOE = \frac{C_C \times FCR + C_{O\&M}}{AEP} \quad (10)$$

where  $C_C$  is the total *installed capital cost* of the wind farms (turbine, infrastructure, and transmission costs),  $FCR$  is the *fixed charge rate*, a present value factor that considers debt and equity costs, taxes, and insurance,  $C_{O\&M}$  is the annual operations and maintenance cost, and  $AEP$  is the Annual Energy Production. The objective function is similar to  $OBJ_{MOS}$ , but it also includes also operations and maintenance costs, which make it more complete. The  $AEP$  is computed as explained in Sect. 3, except that the wind distribution is described through a continuous probability distribution instead of a set of scenarios. In particular, in agreement with [10, 20] fit the wind data with a parametric Weibull distribution that depends on the wind direction, the wind speed, and the position. The variables of the optimization problem are the coordinates of the turbines to install, whose number is fixed, and the optimization is performed through a gradient search that proceeds towards the steepest ascent direction of the objective function. They test their procedure by solving a two-turbine positioning problem in a real world site, obtaining a final solution where one turbine is as close as possible to the connection point of the grid (as to minimize the connection costs to the grid) and the other one is in the position that minimizes the mutual wake effects and is as close as possible to the first turbine (separated by 3.5D) so as to minimize turbine interconnection costs. Clearly, their example is too small to show the validity of their method on real world problems, which usually involve tens of turbines. Nevertheless, their purpose is to provide a framework on top of which more effective optimization procedures can be executed. They achieve their goal by considering an objective function that is realistic and complete, by accurately

modeling the wind characteristics through a Weibull distribution, and by considering aspects that are ignored by other works, particularly the connection costs to the grid and the interconnection costs among the turbines. Their objective function can be readily embedded in heuristic solution methods, but on the other hand it is very complex and nonlinear; therefore, it is not suitable to be embedded within exact solution methods.

## 5 Conclusions

In this paper we describe the WFLOP, which is a crucial problem that needs to be solved during the design of a wind farm. Being able to find a better layout leads to higher energy production and profit. This problem has only recently been given attention by the scientific community, even if it has been neglected by the operations research area. We have described the mathematical model used to compute the impact of wake effects on energy production, noting that despite its simplicity it has been proven effective and precise.

The main works that have been carried out to date are certainly a good starting point for further research on more effective solution methods, but they cannot be considered satisfactory for several reasons. First, none of the solution methods proposed is able to assess the quality of the solution found. In other words, none of the existing works computes an upper bound on the power produced - with the exception of the power produced if no wake effect is present. The algorithms proposed in these works find a possibly good solution, but none of them can indicate how far it is from optimality. A wind farm developer needs to know this to decide if it is worth spending more time looking for a higher quality layout. Second, all the proposed algorithms are heuristic. An exact solution method, on the other hand, would allow one to find the global optimum or, possibly, to obtain tighter upper bounds. The only attempt in this direction was made by Donovan [8], who formulates the WFLOP as an integer program but does not take into account the wake effects, which are in fact forbidden in his model.

An aspect that should be considered is the topography of the territory for the computation of the wind speed and the wake effects. The existing works always consider a flat area and assume, with the exception of [20], that the wind distribution is identical throughout the entire site. The flat area assumption certainly holds for offshore sites, but it is very unrealistic for onshore sites, where the terrain is rarely flat and uniform; the presence of hills, rivers, forests, roads, or buildings significantly impacts the wind distribution and the behavior of the wakes. All these elements have not been considered so far. One of the reasons for ignoring them is that it is hard to implement a routine that takes into account this information when computing the objective function value. This difficulty is not only technical but it is also caused by the lack of published works that describe how to implement it. Nevertheless, there exist software packages, such as WaSP©, that can compute the objective function value by taking into account the onshore topographical ele-



ments. WaSP© is a computer aided design program used to manually design wind farms. A WaSP© user defines a wind distribution, the turbine type to use, the details of the site (which include the presence of natural elements), and the turbine positions. Then, the program computes the expected AEP by taking into account all these inputs. Although WaSP© is not free, its implementation details are available at <http://www.wasp.dk/Products/wat/WAThelp/>. To the best of our knowledge, this website provides one of the most exhaustive descriptions of how to compute the AEP.

Even if wake effects cause a decrease in wind speed, this is not their only negative consequence. Besides being slower, the air in the wake is also more turbulent, which, in the long run, may lead to blade damage and high maintenance costs. The existing approaches ignore this aspect, although turbulence intensity has been the subject of several studies [27, 7]. The impact of turbulence on maintenance costs has been disregarded because it cannot be described accurately. Although some effort has been done [17], there is no method to measure the cost of the turbulence. Nonetheless, multi-objective optimization techniques could include the additional objective of minimizing the turbulence intensity (besides the traditional objective of maximizing the power produced). Or, alternatively, a constraint may be included to prevent too large values of turbulence intensity.

The last and most important aspect that has been ignored to date is the installation phase. Particularly for onshore projects, even if one finds the optimal solution to the WFLOP as defined in this paper, that solution is not necessarily a feasible one or the one that minimizes the construction costs. In fact, there are three aspects that should be considered: landowners, road construction, and setback constraints. Some people may own the land under consideration, and, as mentioned in the introduction, they must be actively involved in the project. Some landowners can be easily convinced to host turbines on their land, while others may be noncooperative. In the latter case, the wind farm developer may offer them more money if their terrain is considered particularly important or strategic. The importance of an area depends on the strength of the wind that blows there and on other considerations related to the second aspect we discuss, the road construction. One fundamental constraint that determines many decisions during the project is that each turbine location must be connected to a road; otherwise, it would be impossible to transport the necessary construction material for its installation. If a road is not present (which is usually the case), the wind farm developer needs to build one. Oftentimes, land owners, who are usually farmers, allow roads to be built only along crop boundaries, so as to minimize the impact on their activity, although in other cases this constraint is not present (for example if an area is dedicated to livestock holdings). Here the purpose of the wind farm developer is to minimize the construction of roads, but also to have a road network that is completely connected, i.e., such that from each point one can reach any other point without passing through public roads. In this way, the cranes used to install the turbines can be easily moved throughout the road network without being disassembled. If, on the other hand, the road network is composed by subnetworks that are separated by traits of public road, the cranes need to be moved through the public road to reach the next location. Unfortunately, an assembled crane cannot

be transported on public roads, and therefore it must be disassembled, transported through the public road to another subnetwork, and reassembled. Although it is hard to obtain the cost of this operation (wind farm developers do not release information on costs), it is estimated to be tens of thousands of dollars. Finally, there are other setback constraints that usually impose restrictions on the turbine locations. For example, turbines cannot be installed too close to houses, military facilities, airports, or boundaries of a noncooperative landowner. Furthermore, turbines cannot be installed along the migration path of birds, in locations that may visually impact the landscape, and so on. We refer to [6] for a more complete list of setback constraints.

To the best of our knowledge, all these logistical aspects have been completely ignored in the existing formulations of the WFLOP. We believe that the main reason is the unavailability of published material that treats them extensively. In fact, this type of information is kept secret by wind farm developers, who are not willing to share it. Nevertheless, without taking into account these aspects, the WFLOP risks remaining an abstract mathematical exercise.

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## References

1. Ainslie, JF (1988) Calculating the flowfield in the wake of wind turbines. *J of Wind Engineering and Industrial Aerodynamics* 27:213–224.
2. Aytun Ozturk, U, Norman B (2004) Heuristic methods for wind energy conversion system positioning. *Electric Power Systems Research* 70:179–185.
3. Barthelmie R, Larsen G, Pryor H, et al (2004) ENDOW (efficient development of offshore wind farms): modelling wake and boundary layer interactions. *Wind Energy* 7:225–245
4. Barthelmie R, Folkerts L, Larsen GC, et al (2006) Comparison of Wake Model Simulations with Offshore Wind Turbine Wake Profiles Measured by Sodar. *J of Atmospheric and Oceanic Technology* 23:888–901
5. Brooks SP, Morgan BJ (1995) Optimization Using Simulated Annealing. *Journal of the Royal Statistical Society. Series D (The Statistician)* 44:241–257
6. Burton T, Sharpe D, Jenkins N, et al (2001) *Wind Energy Handbook*. Wiley, New York
7. Crespo, A. and Hernandez J (1996) Turbulence characteristics in wind-turbine wakes. *J of Wind Engineering and Industrial Aerodynamics* 61:71–85.
8. Donovan S (2005) *Wind Farm Optimization*. University of Auckland, New Zealand
9. Frehlich R, Kelley N (2010) Applications of scanning Doppler Lidar for the wind energy industry. The 90th American Meteorological Society Annual Meeting. Atlanta, GA
10. Garcia A, Torres JL, Prieto E, et al (1998) Fitting wind speed distributions: a case study. *Solar Energy* 62:139–144
11. Goldberg DE (1989) *Genetic Algorithms in Search, Optimization and Machine Learning*. Addison-Wesley Longman Publishing Co., Inc., Boston, MA

12. Grady SA, Hussaini MY, Abdullah MM (2005). Placement of wind turbines using genetic algorithms. *Renewable Energy* 30:259–270
13. Hou-Sheng H (2007) Distributed Genetic Algorithm for Optimization of Wind Farm Annual Profits. International Conference on Intelligent Systems Applications to Power Systems, Kaohsiung, Taiwan
14. Jensen NO (1983) A note on wind generator interaction, Risø DTU National Laboratory for Sustainable Energy
15. Katic I (1993) Program PARK, Calculation of Wind Turbine Park Performance, Release 1.3++. Risø National Laboratory, Roskilde
16. Katic I, Højstrup J, Jensen NO (1986) A Simple Model for Cluster Efficiency. European Wind Energy Association Conference and Exhibition. Rome, Italy
17. Kelley ND, Sutherland HJ (1997) Damage estimates from long-term structural analysis of a wind turbine in a U.S. wind farm environment. Prepared for the 1997 ASME Wind Energy Symposium, Reno, Nevada. NREL/CP-440-21672, Golden, CO: National Renewable Energy Laboratory. 12 pp.
18. Kelley ND, Jonkman BJ, Scott GN, et al (2007) Comparing Pulsed Doppler LIDAR with SODAR and Direct Measurements for Wind Assessment. American Wind Energy Association WindPower 2007 Conference and Exhibition. Los Angeles, California.
19. Kusiak A, Song Z (2009) Design of wind farm layout for maximum wind energy capture. *Renewable Energy* 35:685–694
20. Lackner MA, Elkinton CN (2007) An Analytical Framework for Offshore Wind Farm Layout Optimization. *Wind Engineering* 31:17–31
21. Manwell JF, McGowan JG, Rogers AL (2002) *Wind Energy Explained*. John Wiley & Sons Ltd, West Sussex, England
22. Méchali M, Barthélemy R, Frandsen S, Jensen L, Réthoré P (2006) Wake effects at Horns Rev and their influence on energy production. European Wind Energy Conference and Exhibition, Athens
23. Mosetti G, Poloni C, Diviacco D (1994) Optimization of wind turbine positioning in large windfarms by means of a genetic algorithm. *J of Wind Engineering and Industrial Aerodynamics* 51:105–116
24. Petersen EL, Mortensen NG, Landberg L, et al (1998) Wind power meteorology. Part I: climate and turbulence. *Wind Energy* 1:25–45
25. Rivas RA, Clausen J, Hansen KS, et al (2009) Solving the Turbine Positioning Problem for Large Offshore Wind Farms by Simulated Annealing. *Wind Engineering* 33:287–297
26. Sisbot S, Turgut O, Tunç M, et al (in press). Optimal positioning of wind turbines on Gökçeada using multi-objective genetic algorithm. *Wind Energy*
27. Vermeer LJ, Sørensen JN, Crespo A (2003) Wind turbine wake aerodynamics. *Progress in Aerospace Sciences* 39:467–510
28. Wilcox DC (1998) *Turbulence Modeling for CFD*, La Canada, DCW Industries, CA