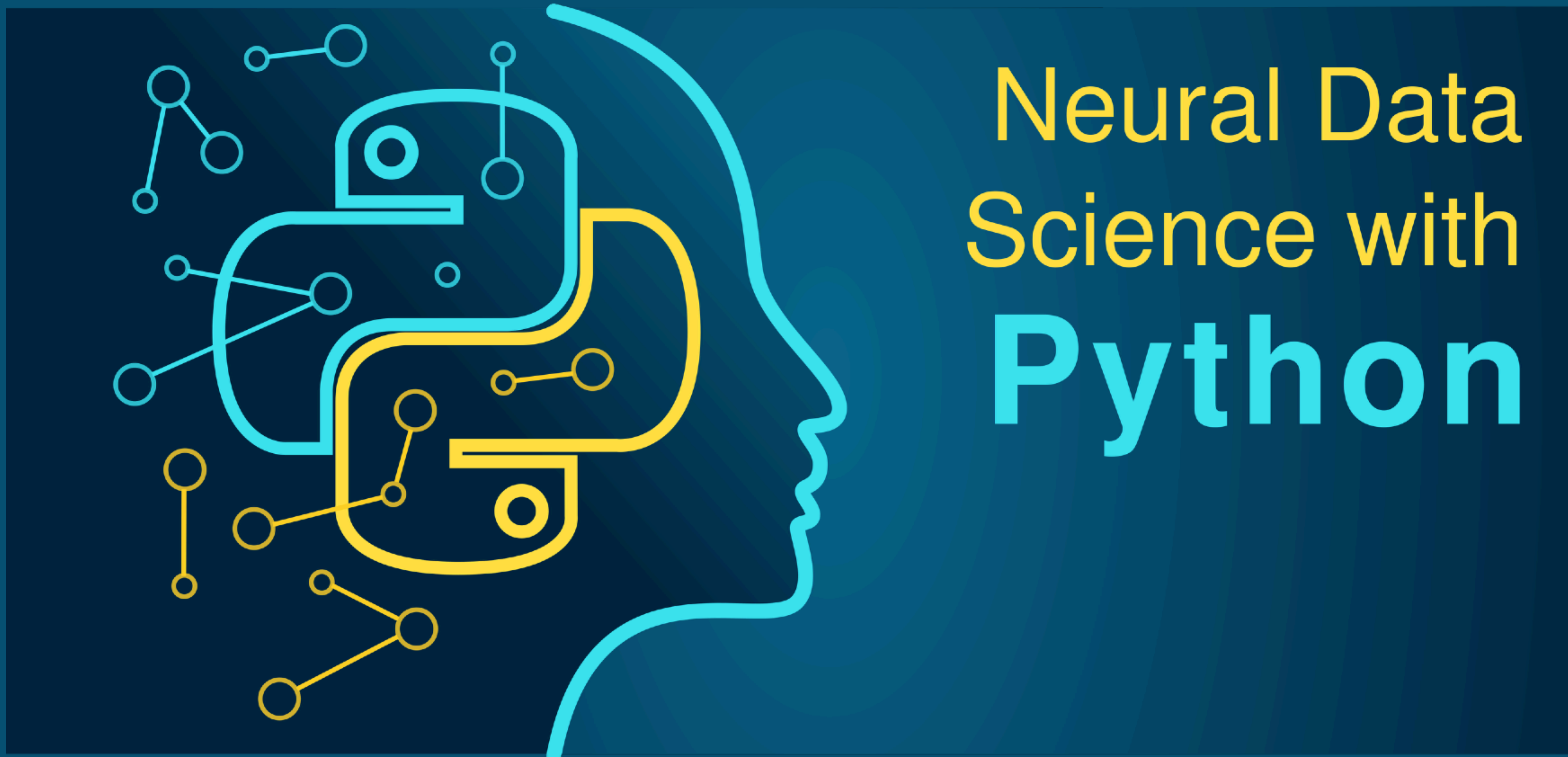

Biophysical neuron models: The Integrate-and-Fire neuron

Jonas Ranft
Institut de Biologie de l'ENS (IBENS)

What is a biophysical model?

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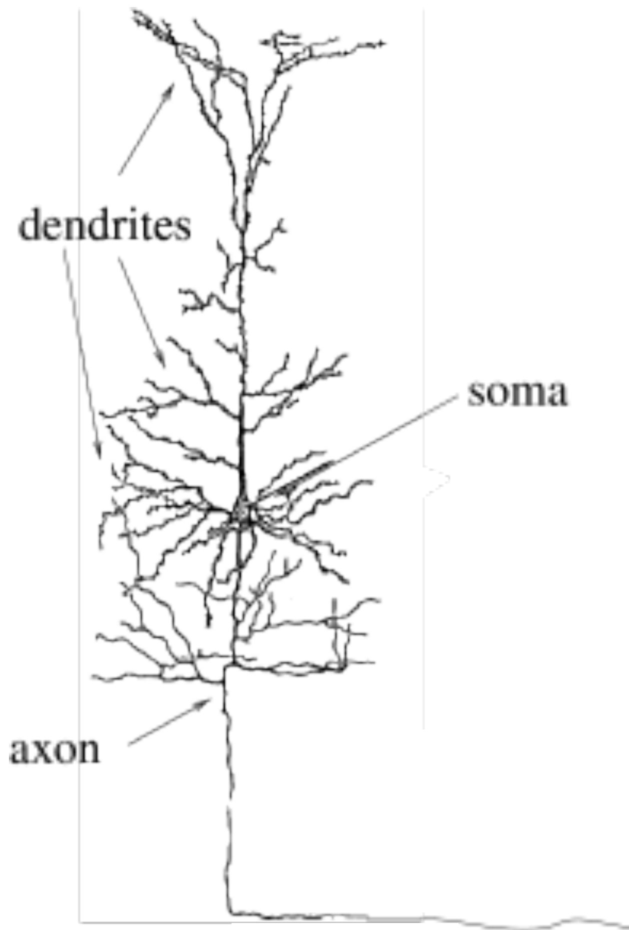

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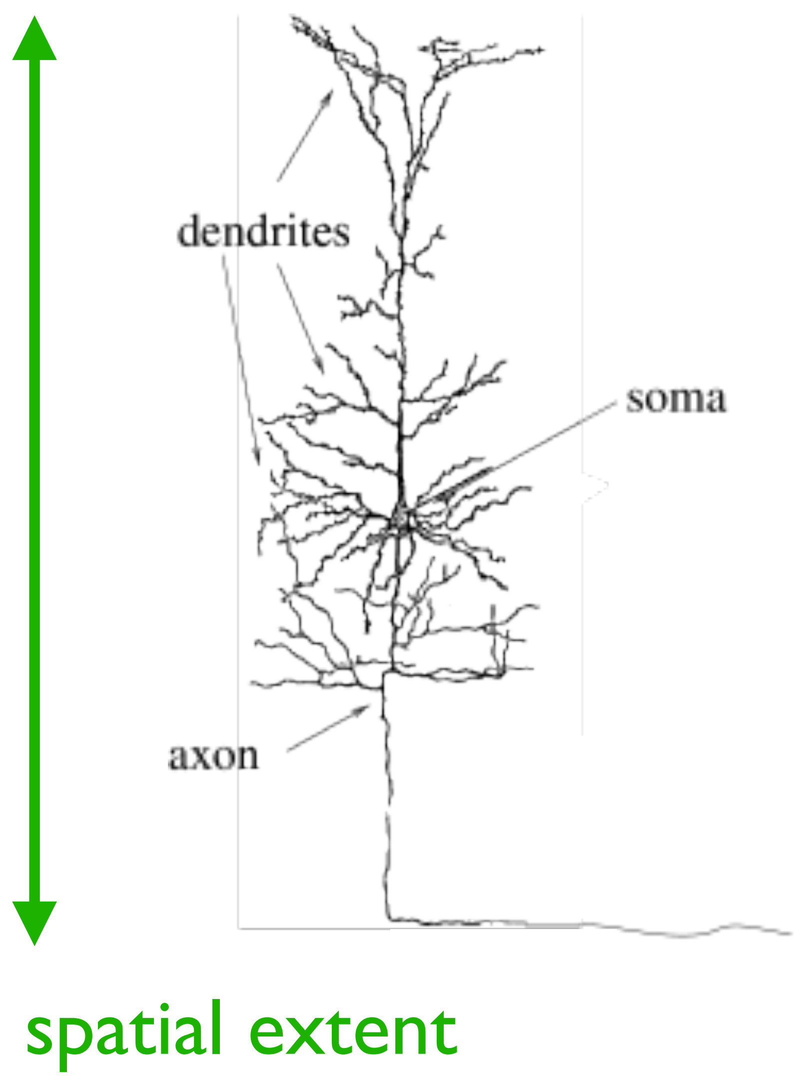

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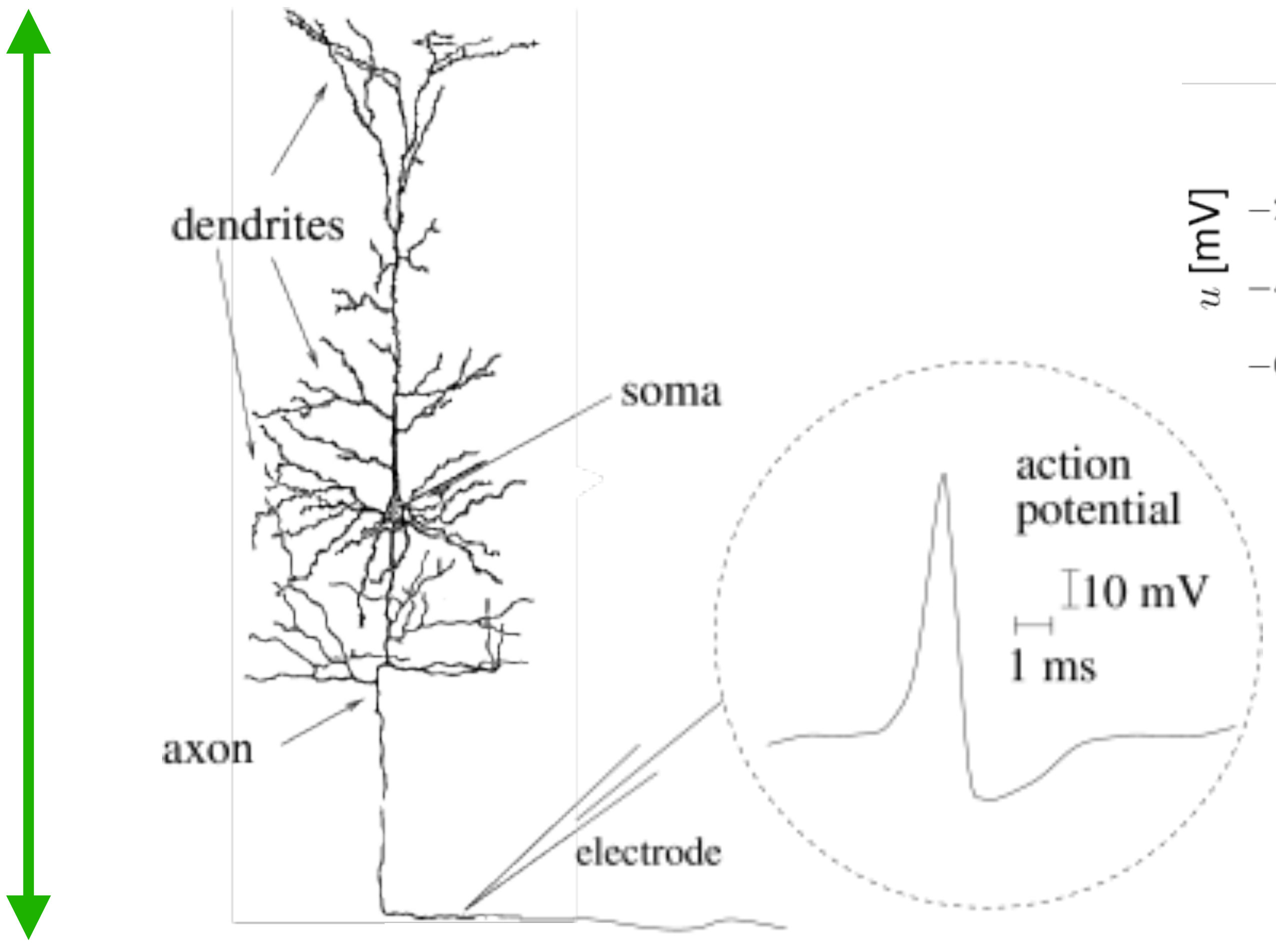

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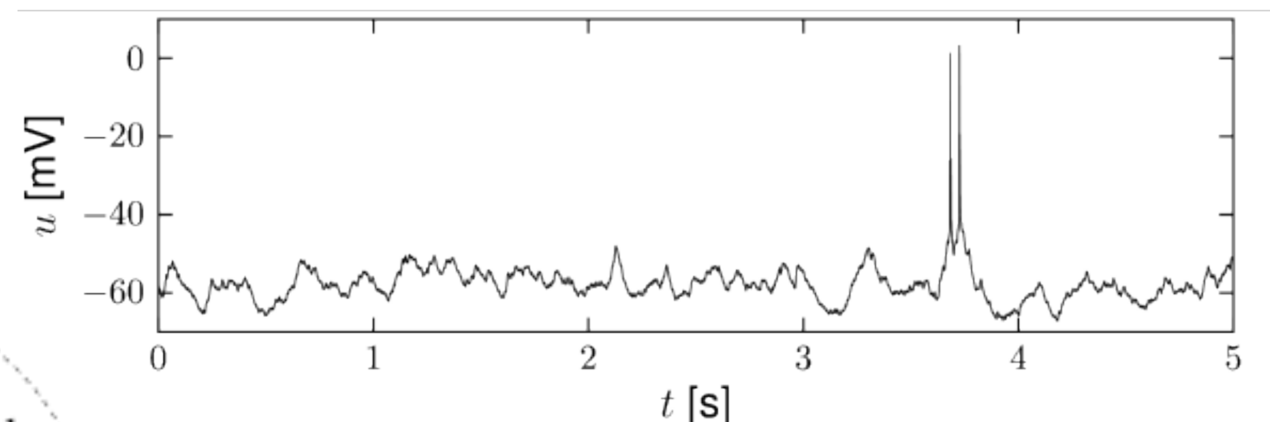

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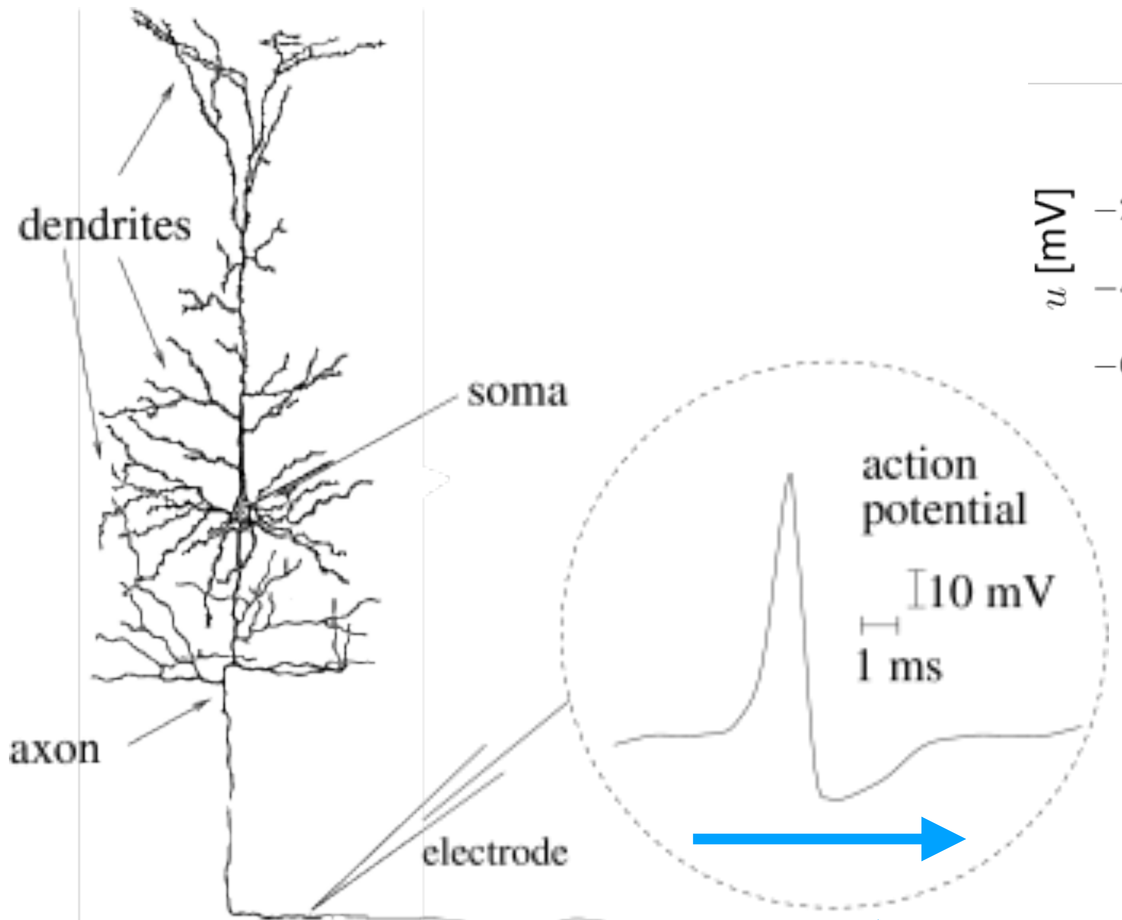

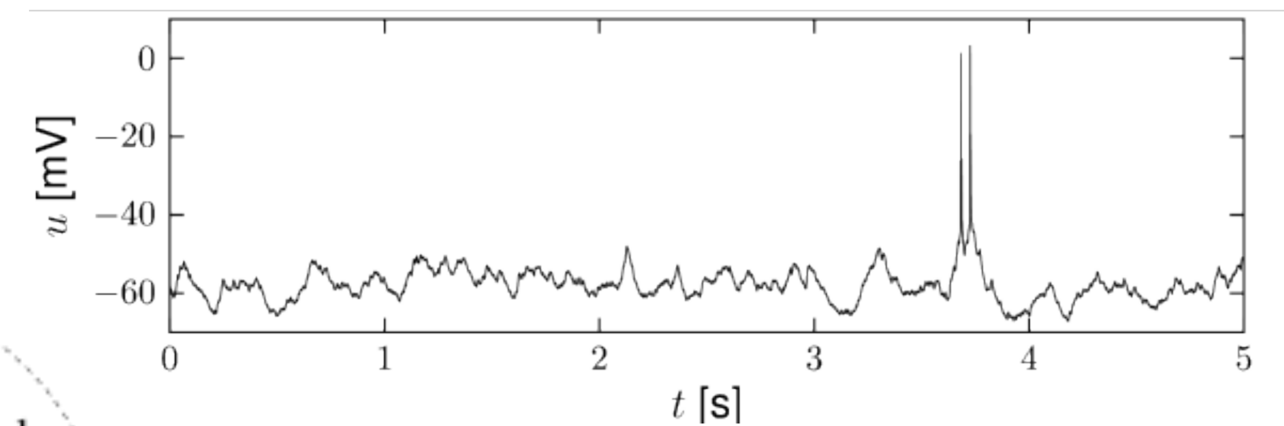

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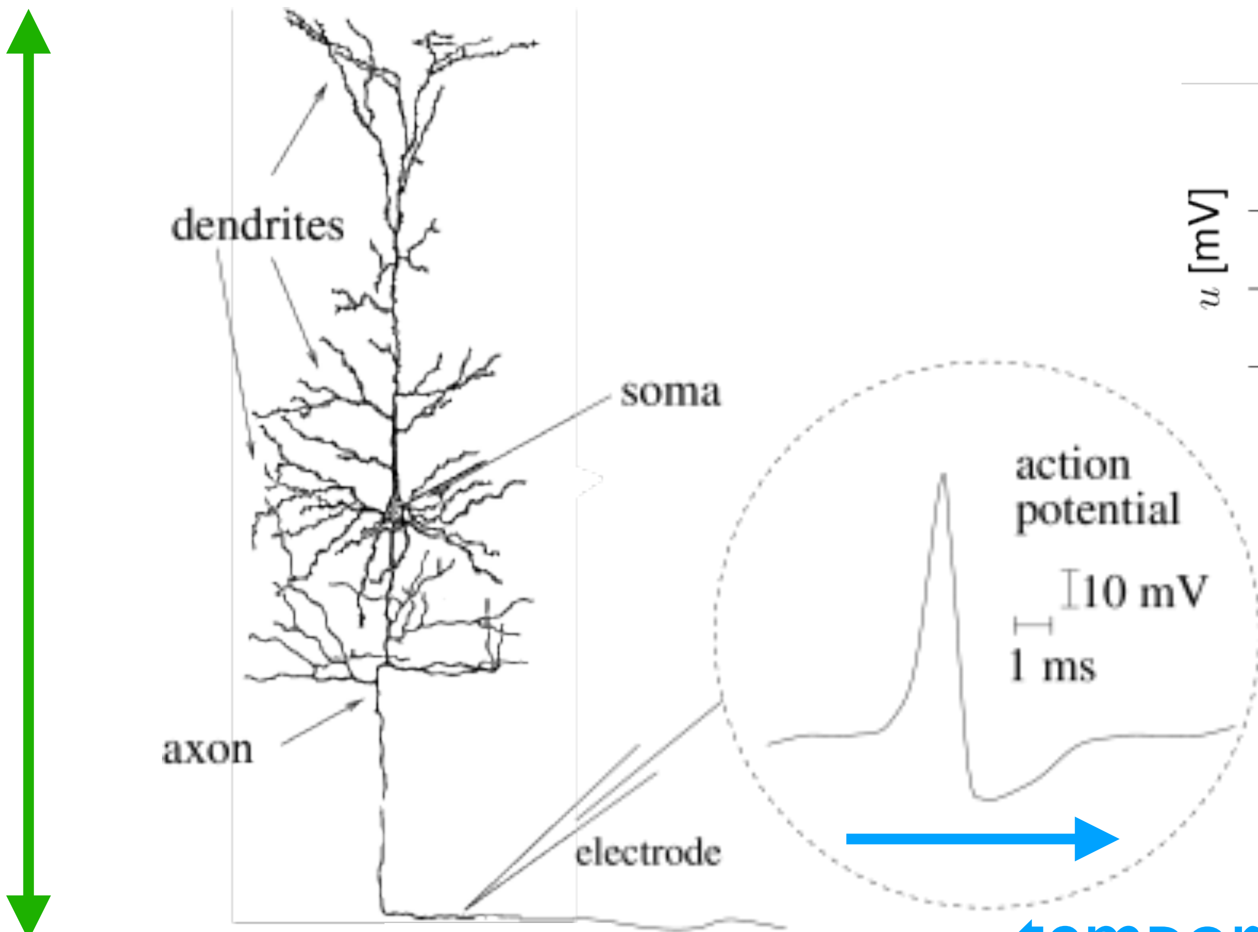

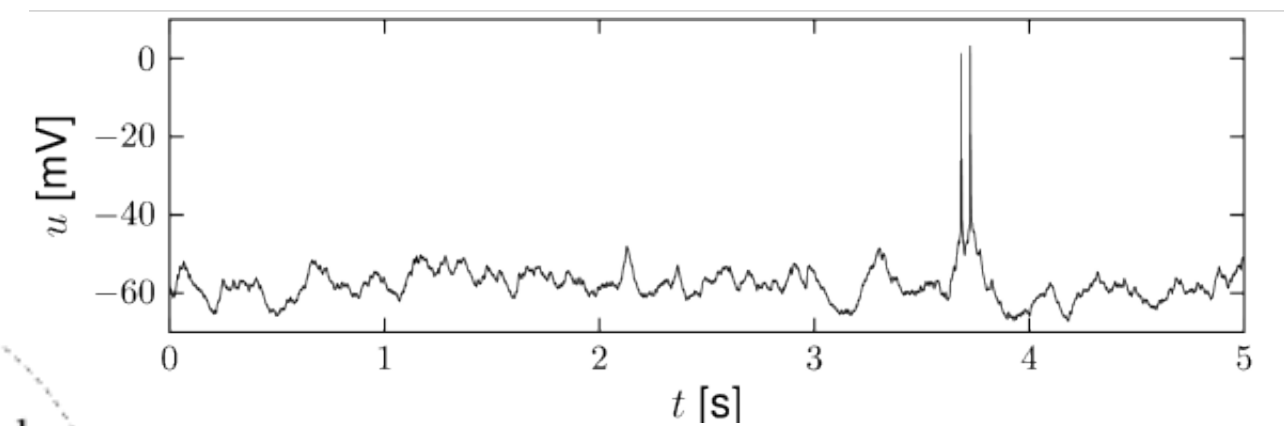

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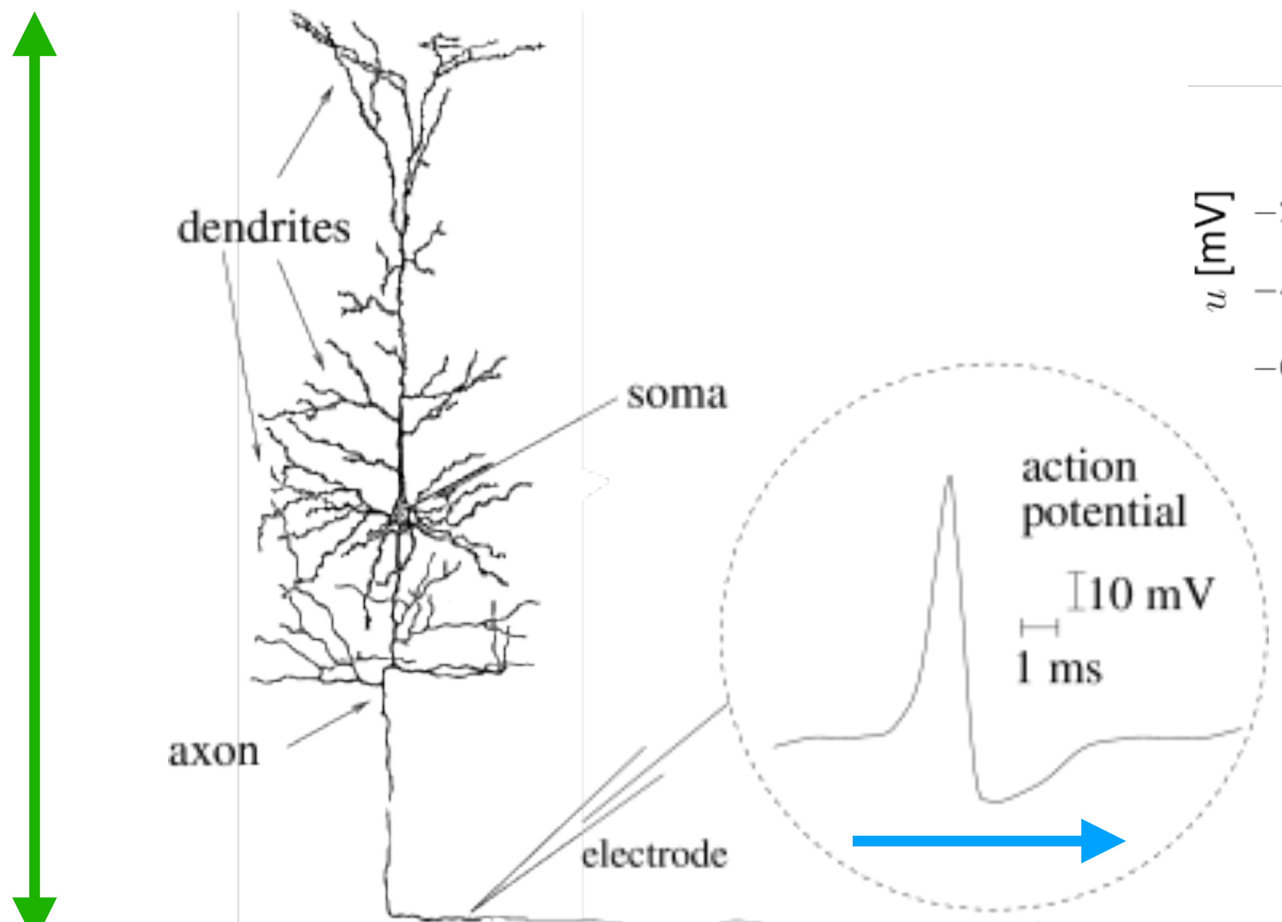

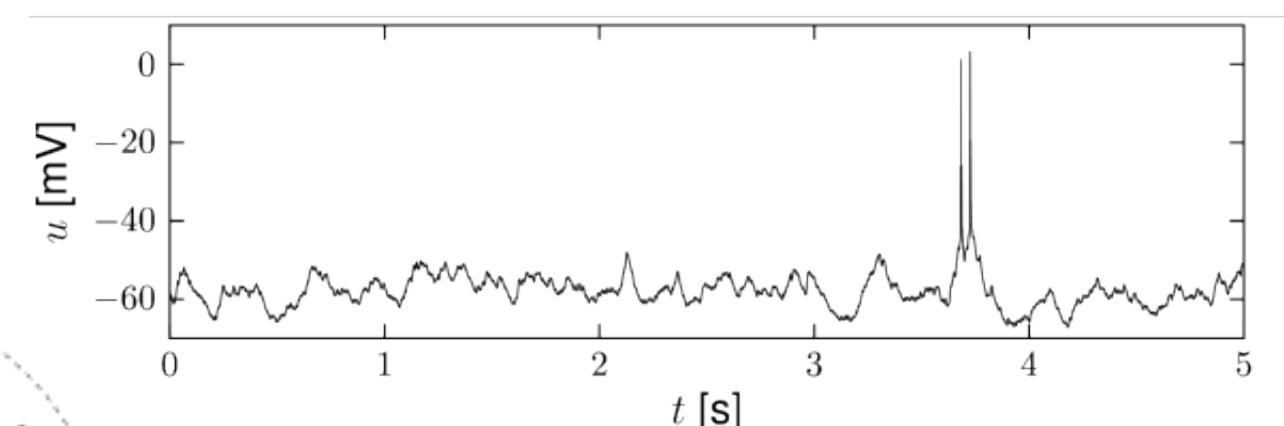

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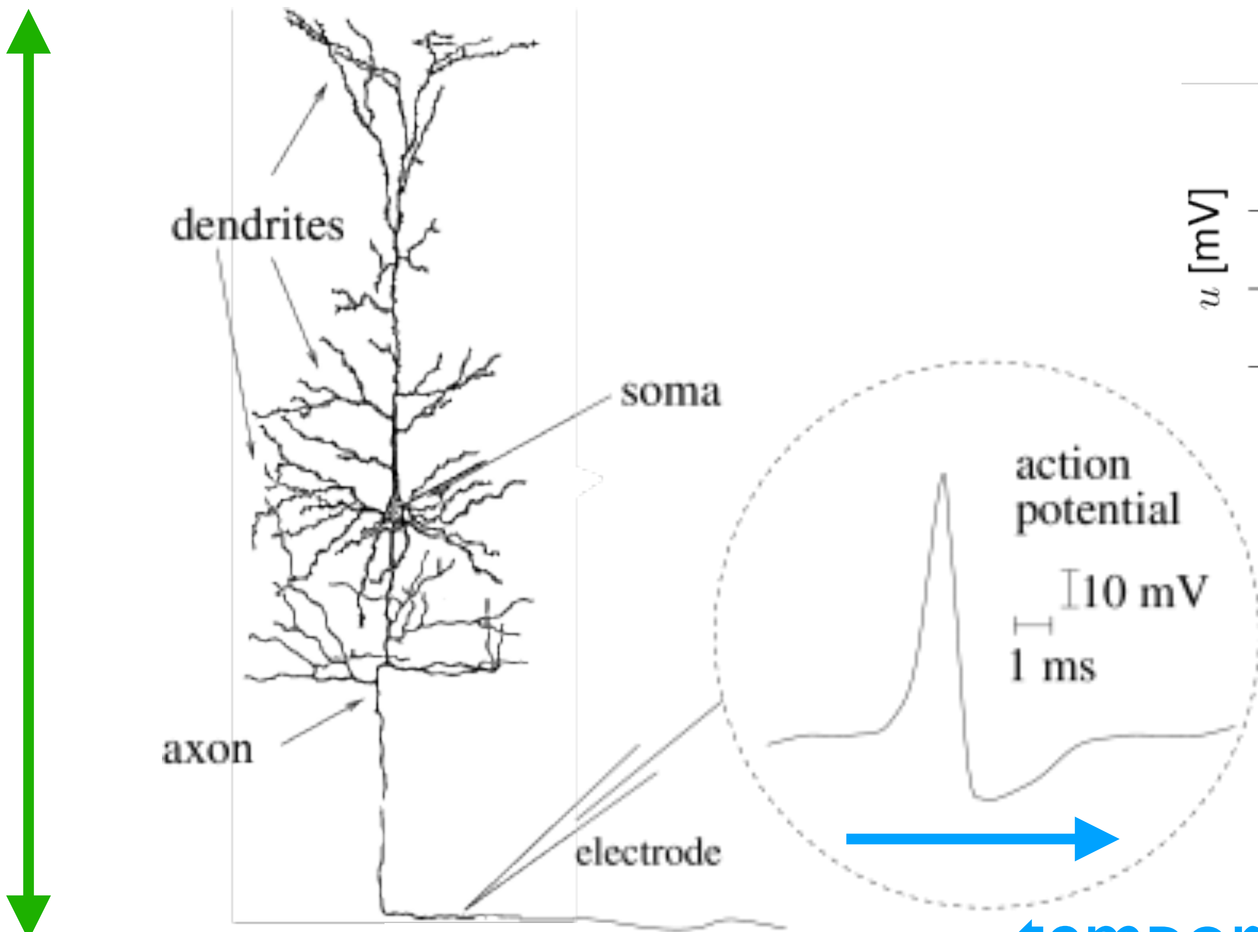

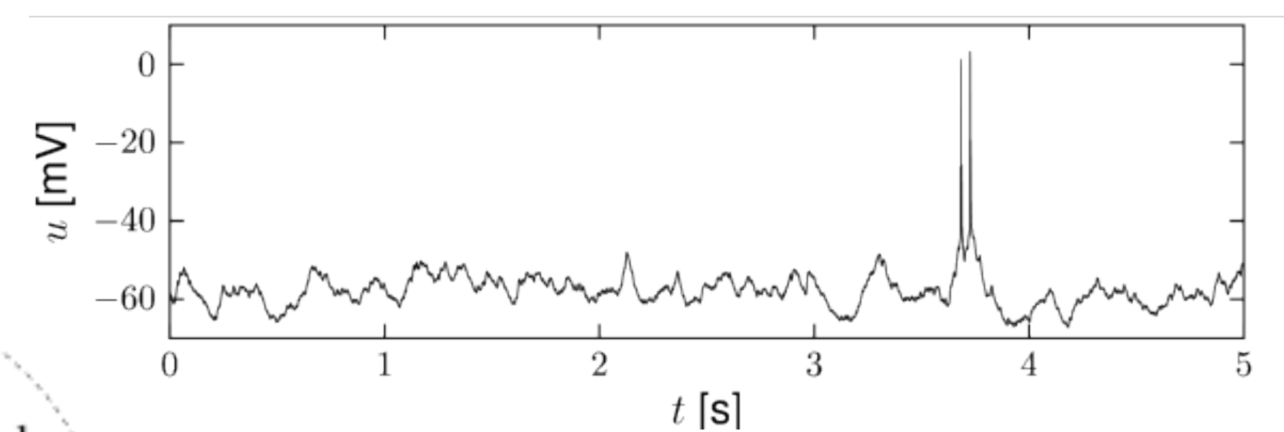

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Why would we try to do so?

The power of models

1. Explain ("make sense of observations")
2. Predict ("let's test the theory")

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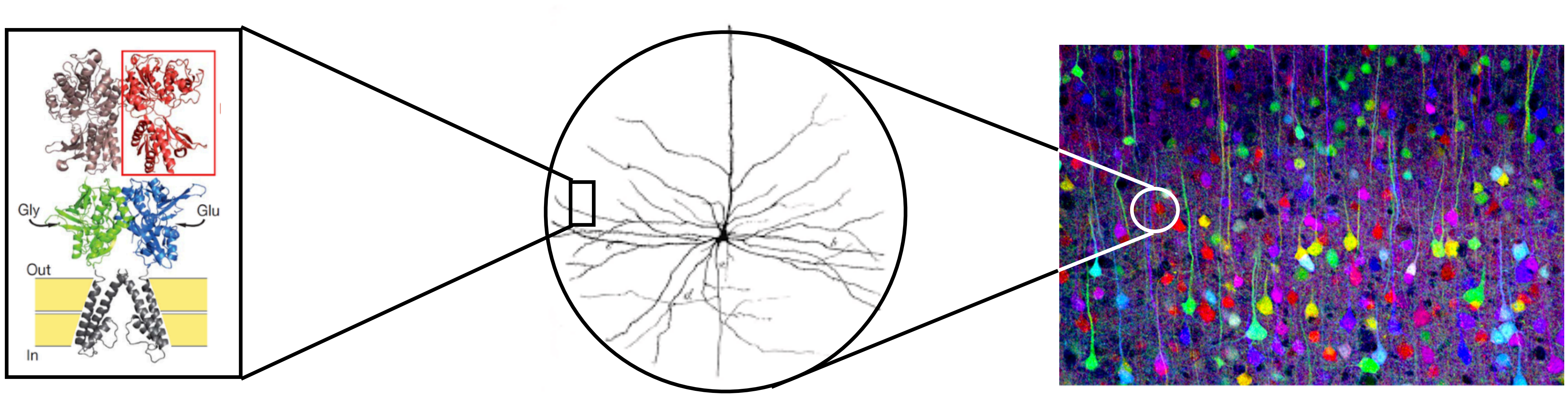

NMDA receptor   pyramidal cell   cortical network

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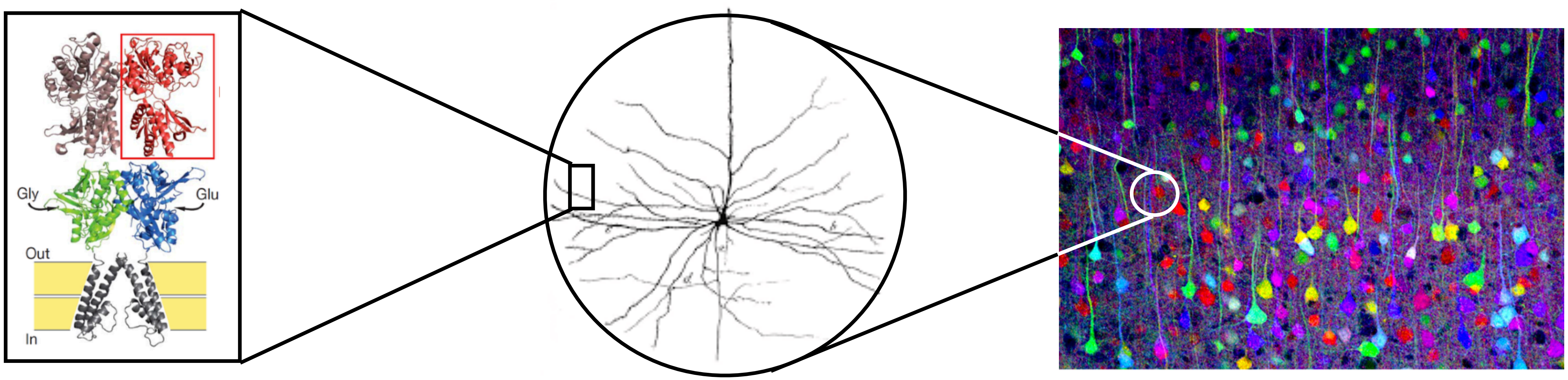

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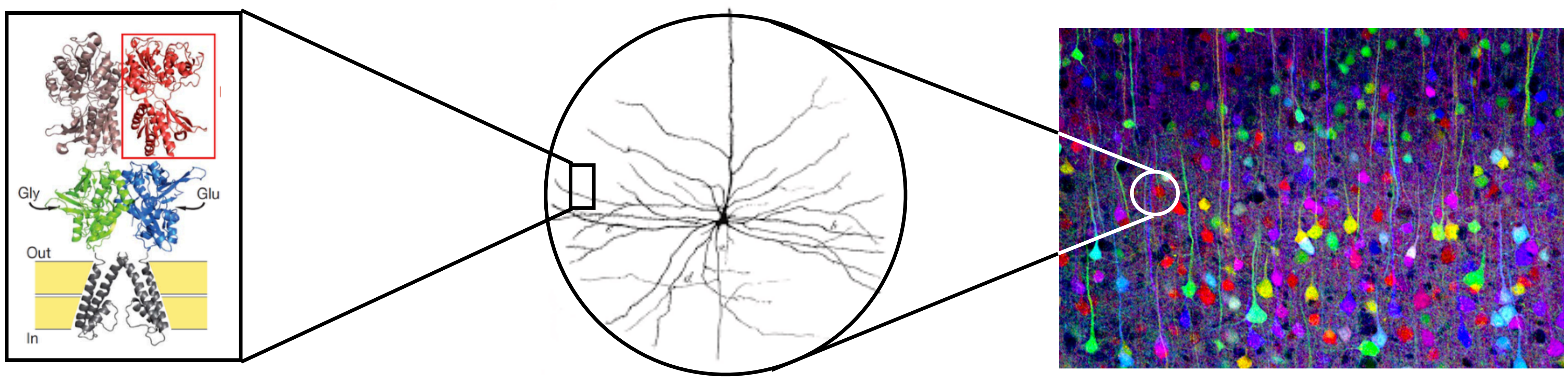

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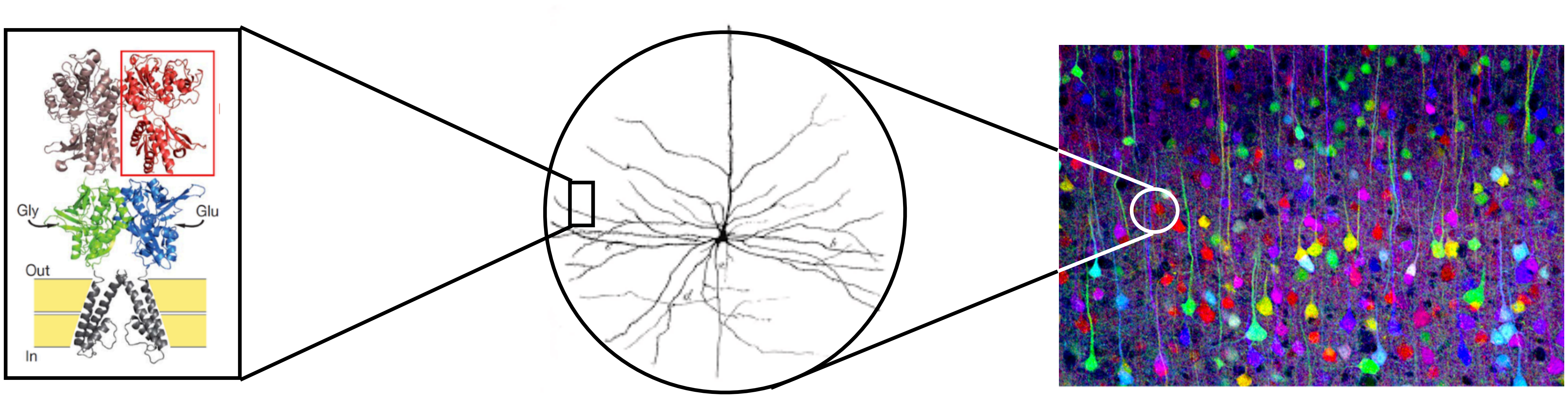

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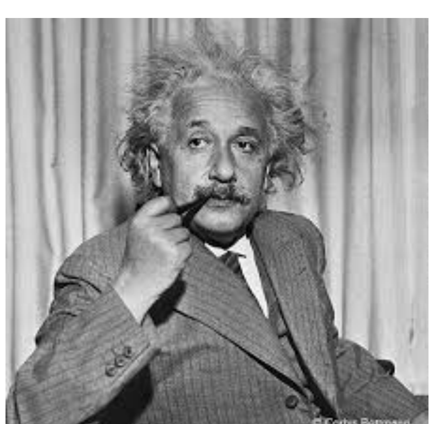
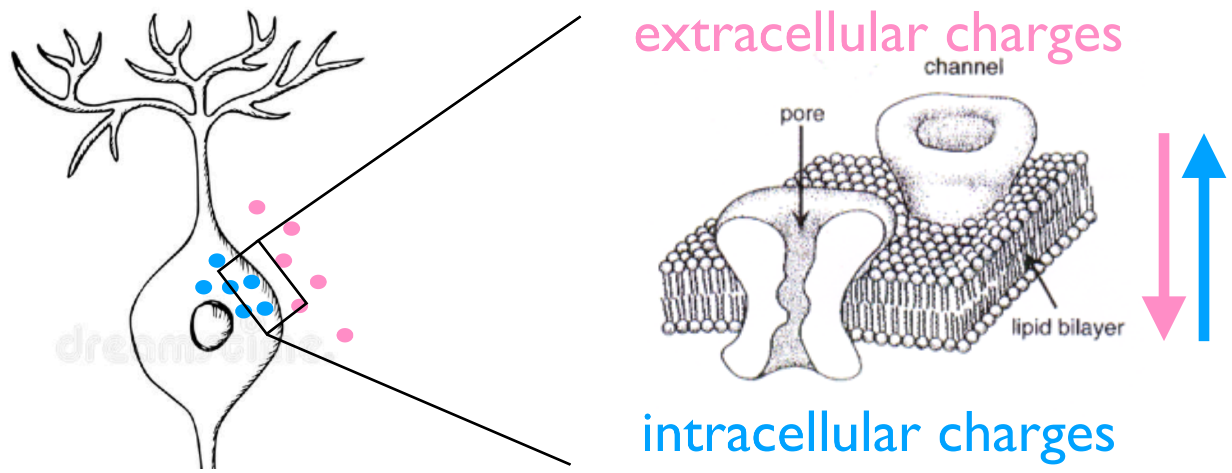


"Everything should be made as simple as possible, but no simpler."

Biophysical neuron models

- <u>Physical quantities</u>: voltages, currents (electric circuits)



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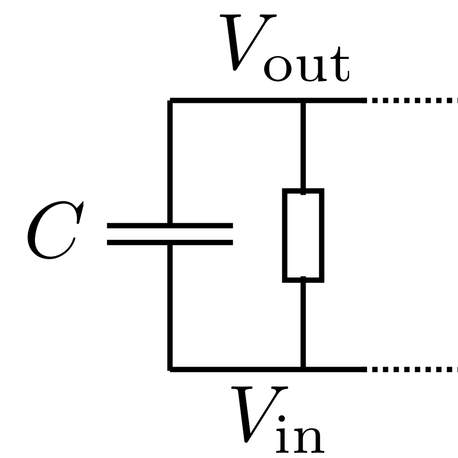

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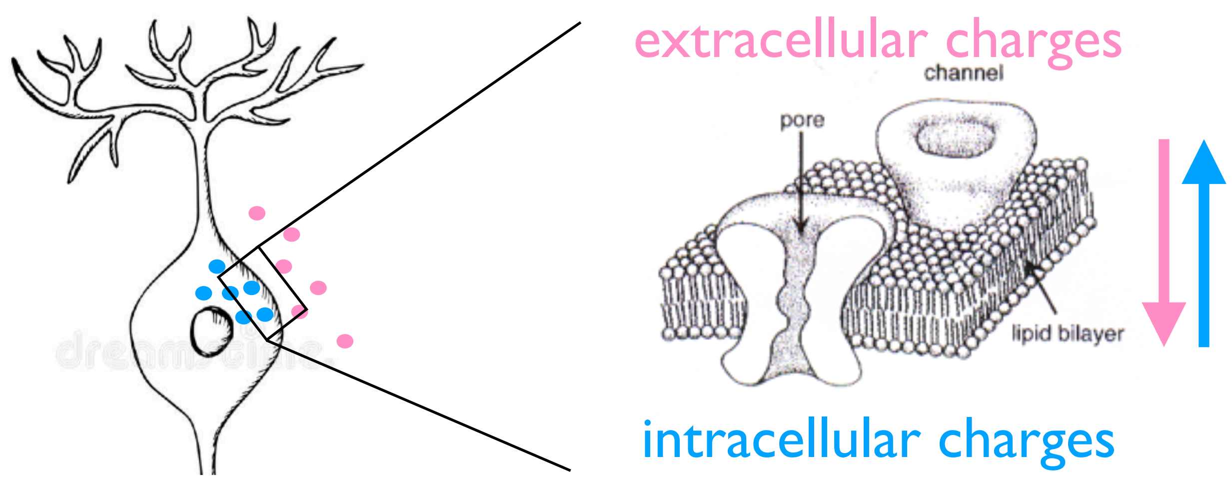

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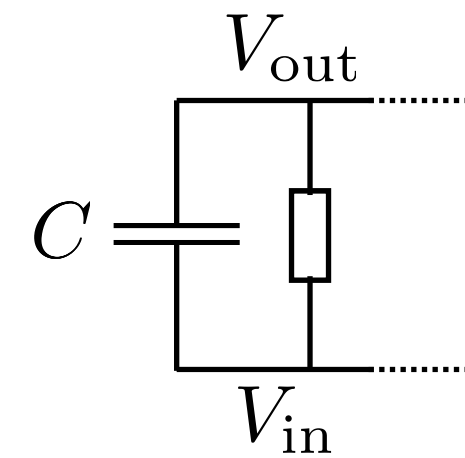

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Ohm's law: $I_R = \dfrac{V}{R} = gV$

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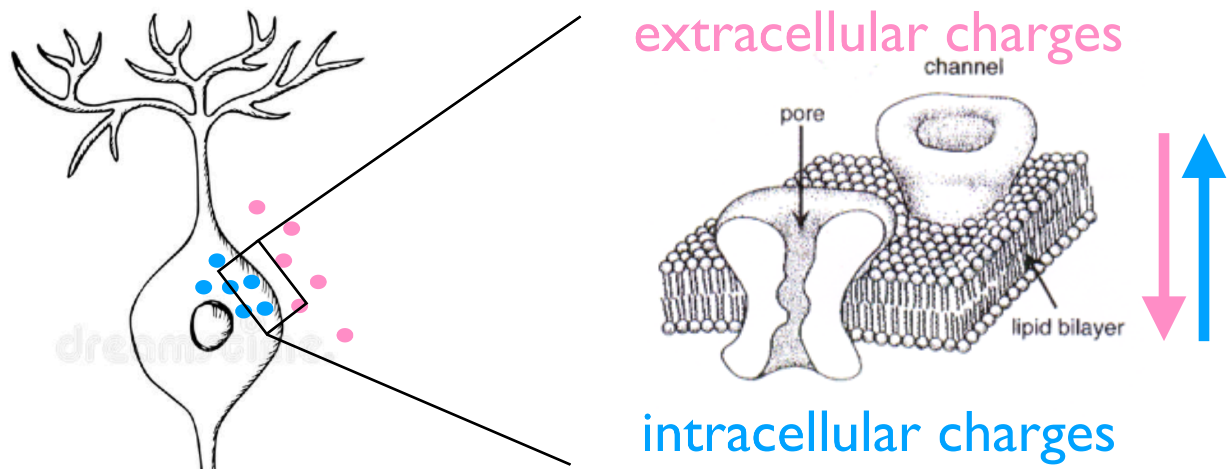

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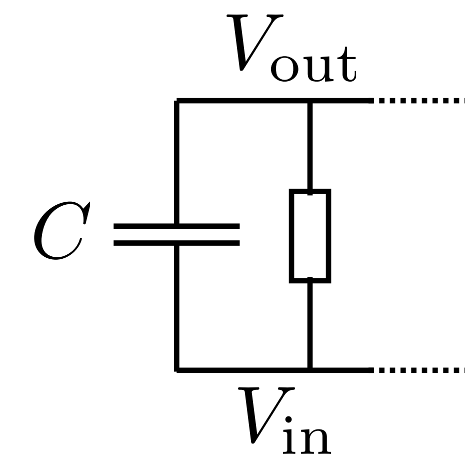

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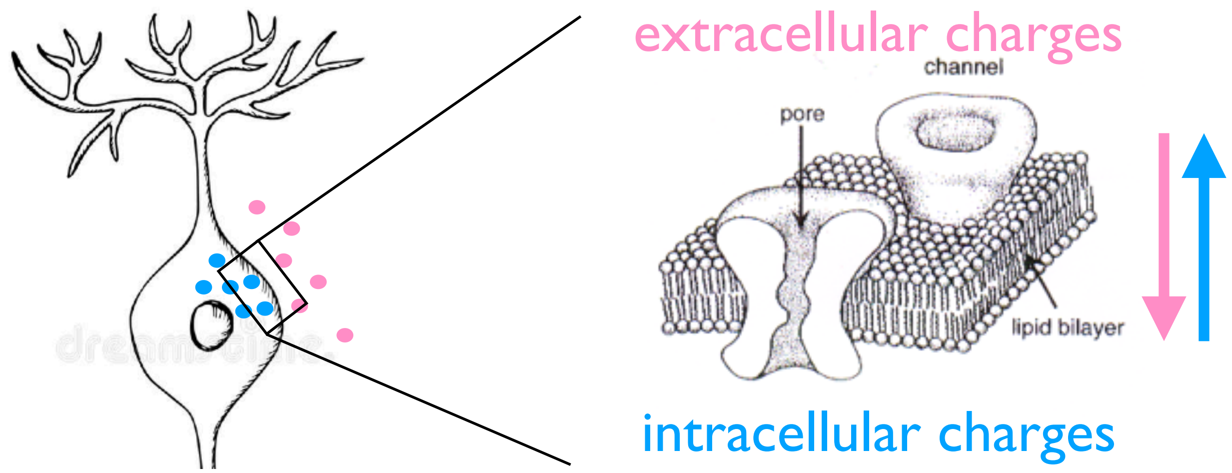

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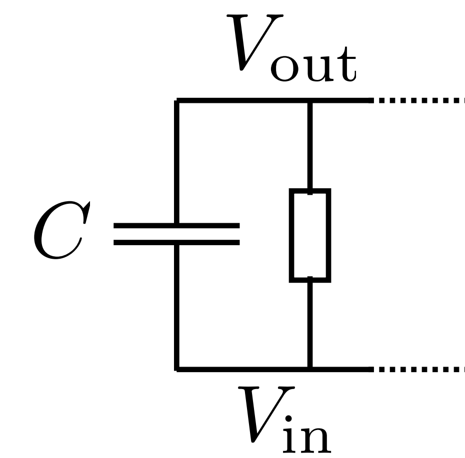

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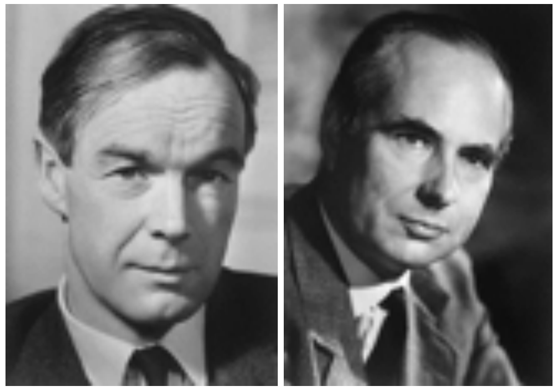

Hodgkin Huxley

1. Hodgkin-Huxley model

Nobel prize 1963

Lapicque

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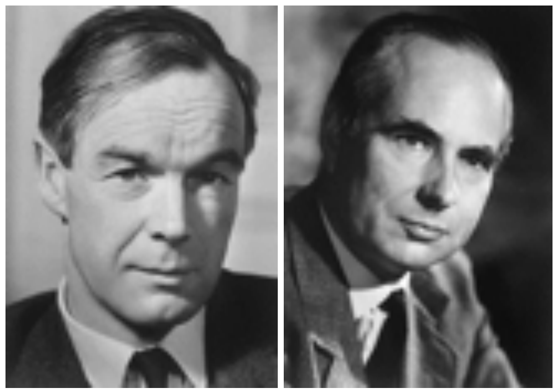

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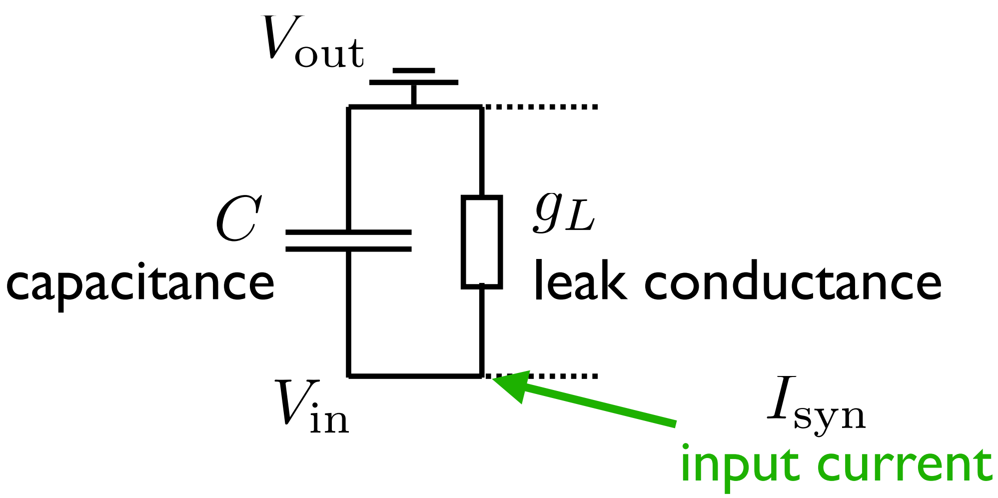

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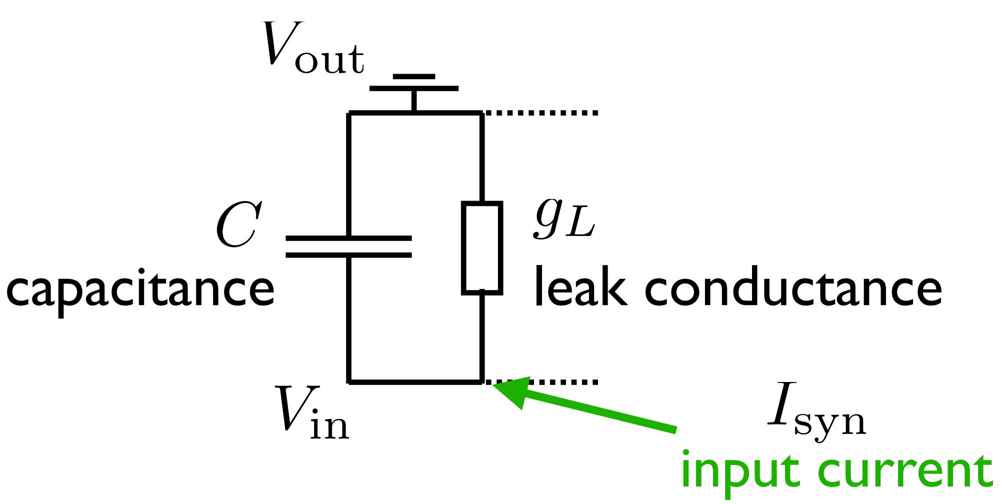

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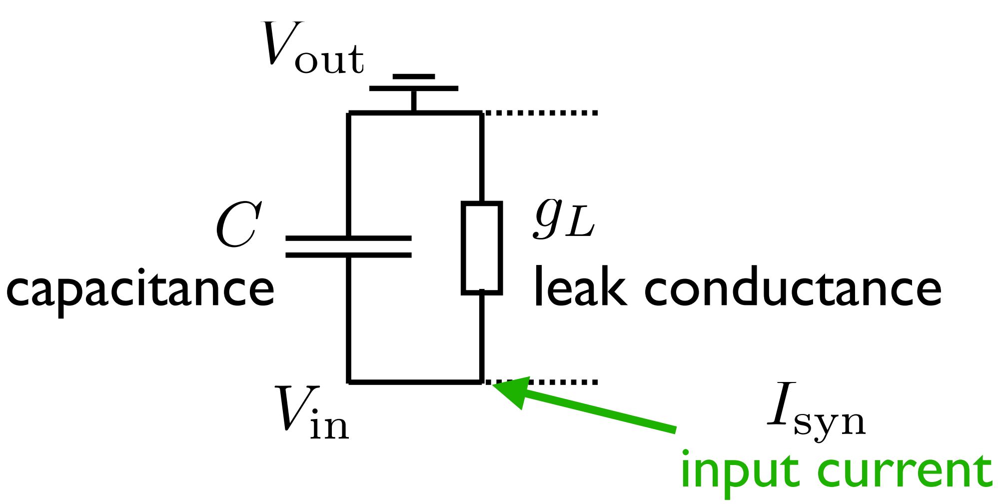

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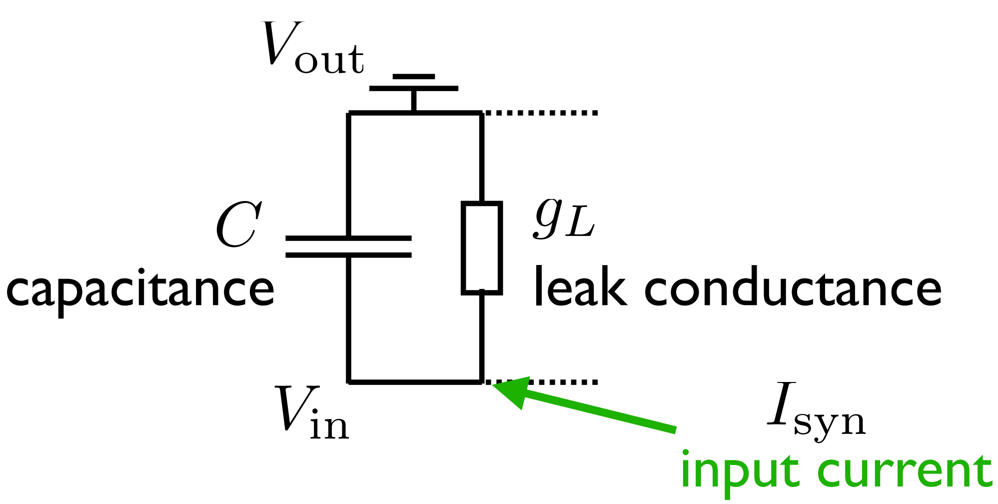

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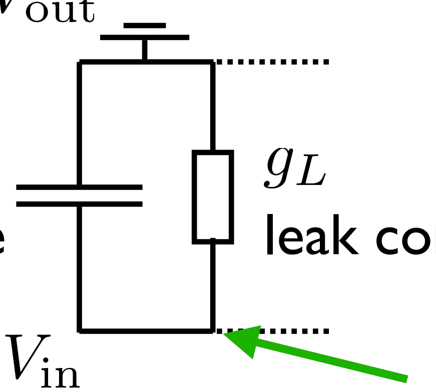

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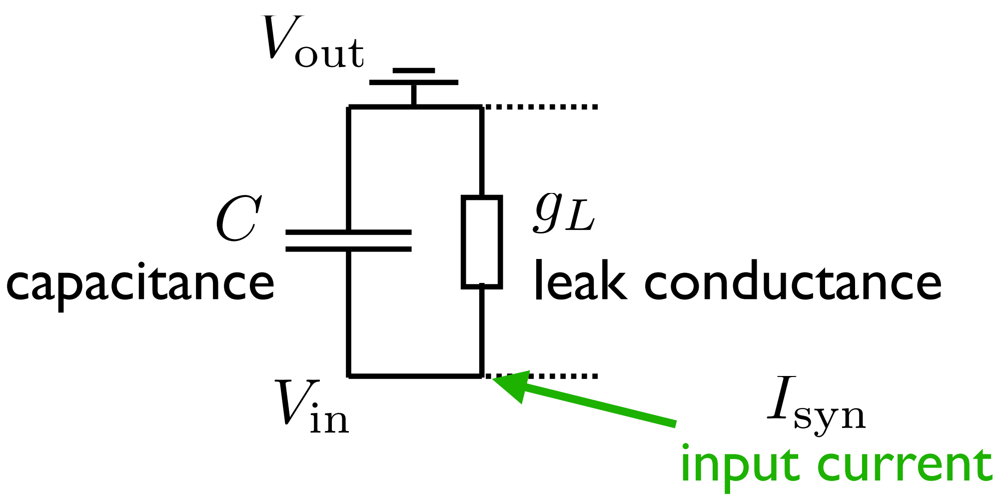

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- *Spike emission* whenever the voltage crosses a threshold, followed by a reset:

$$\boxed{V(t) > V_{\text{threshold}} \quad \rightarrow \quad \text{spike} + V(t) = V_{\text{reset}}}$$

Analyzing the LIF model

LIF = Leaky Integrate-and-Fire

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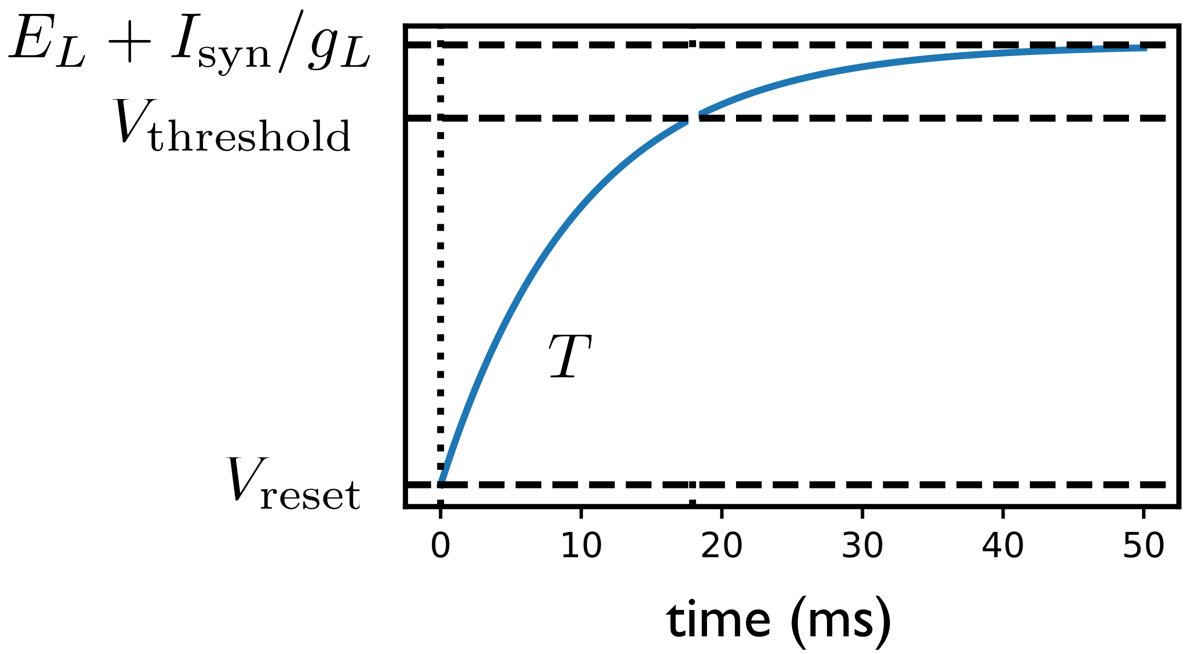

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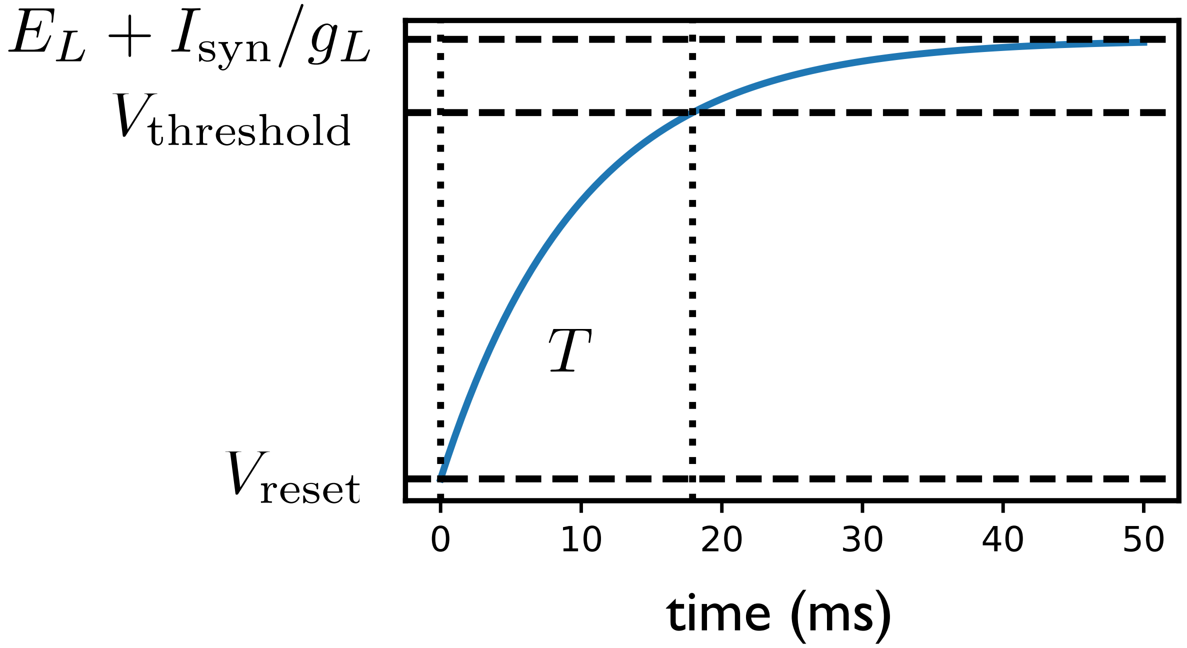

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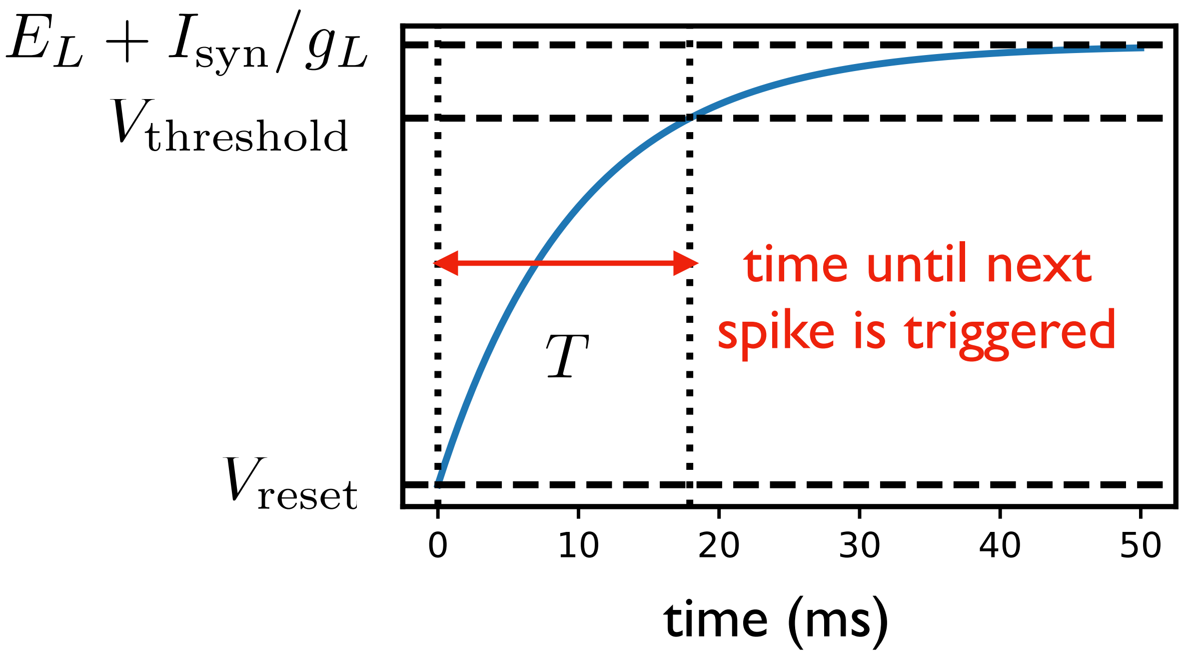

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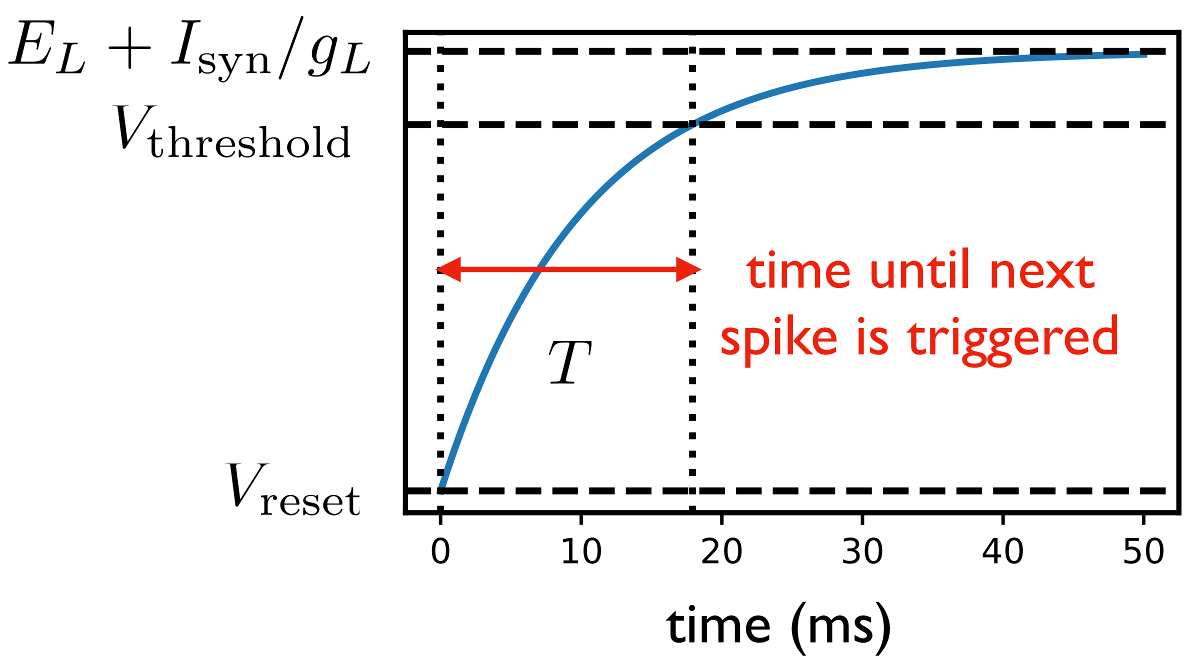

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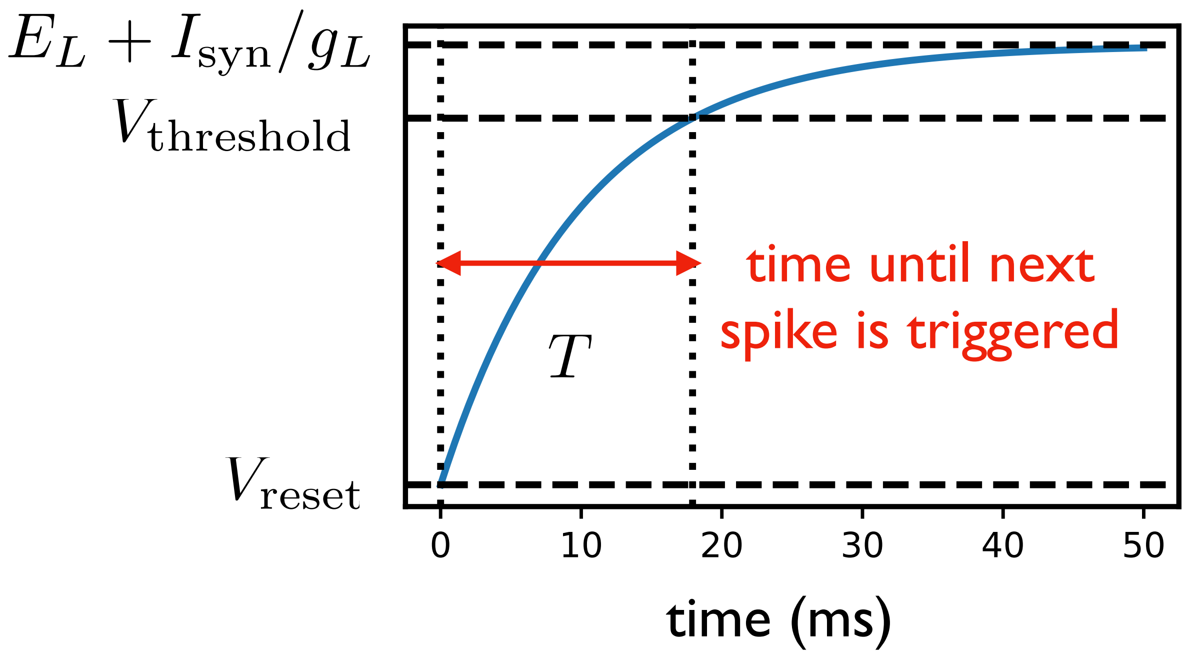

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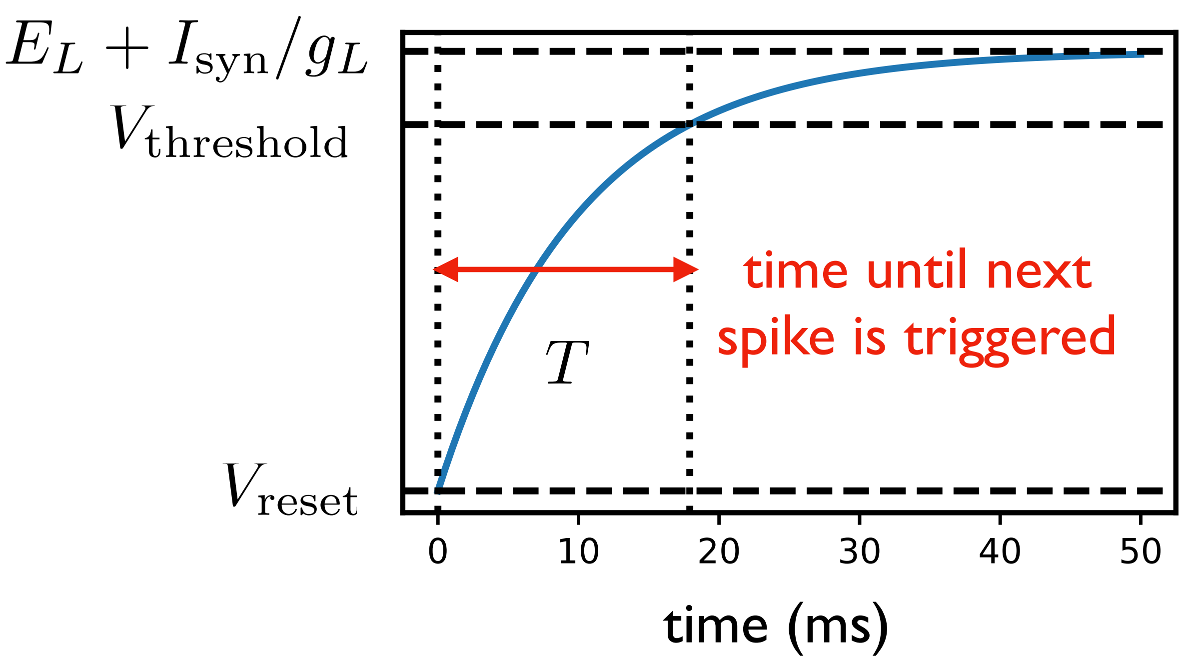

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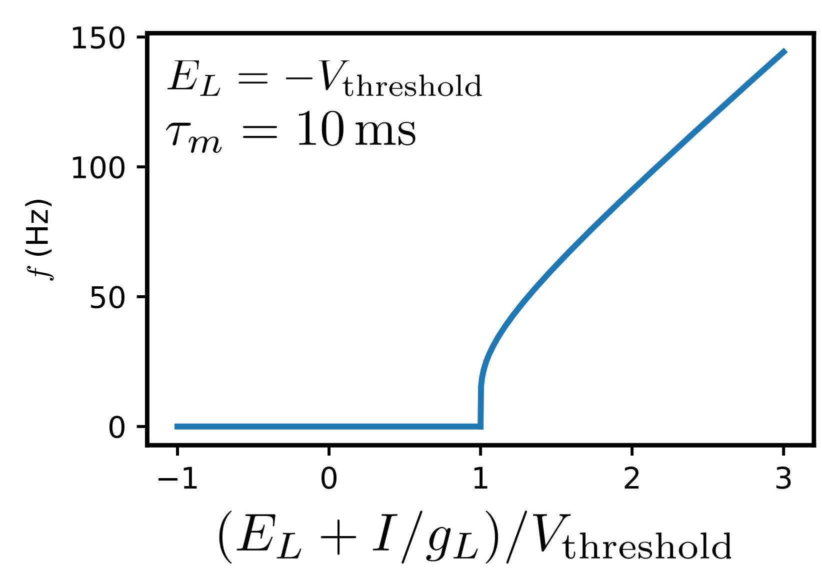

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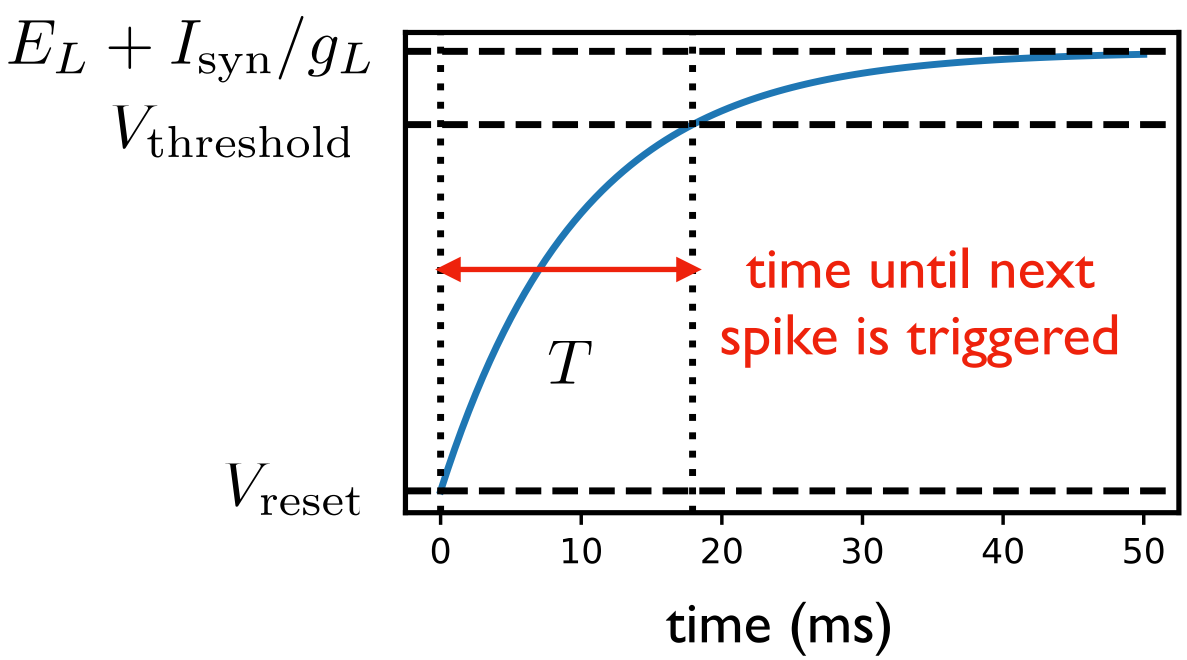

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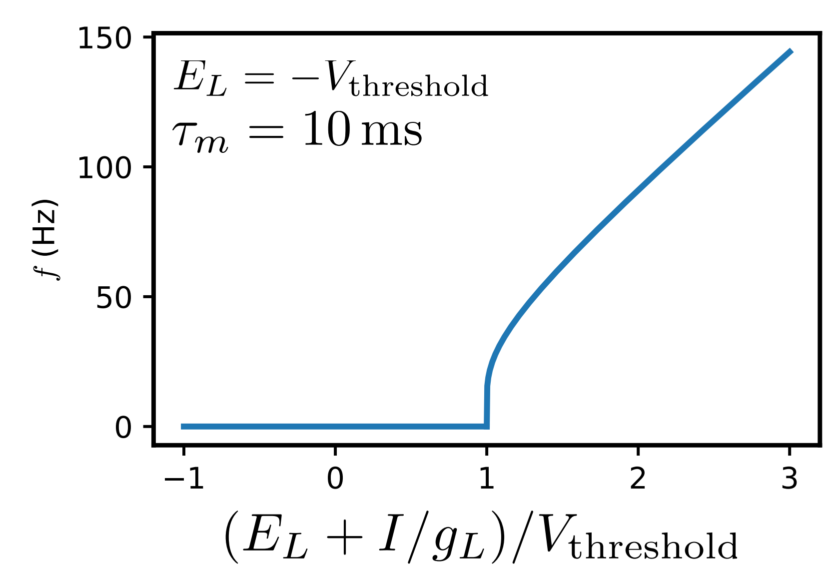
Here, we considered a constant current, but of course input is typically time-dependent.



Where LIFs are useful

- Because of their relative simplicity, we can use many connected LIF neurons to model networks of spiking neurons:

$$C\frac{dV_i}{dt} = g_L(E_L - V_i) + I_{\mathrm{syn},i} \qquad I_{\mathrm{syn},i}(t) = \sum_j I_{ij}(t) \qquad i = 1, \ldots, N$$

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- Neuron integrates excitatory and inhibitory post-synaptic currents of ***several 1000 synapses***.

LIF allows further mathematical analysis when approximating I(t) with mean + noise term:

$$I(t) = I_0 + \sigma\xi(t)$$

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$$C\frac{dV_i}{dt} = g_L(E_L - V_i) + I_{\mathrm{syn},i} \qquad I_{\mathrm{syn},i}(t) = \sum_j I_{ij}(t) \qquad i = 1, \ldots, N$$

sum over presynaptic neurons

- Different possibilities to generate I_{syn} from presynaptic activity.

  <u>*Simplest choice*</u>*: increment postsynaptic voltage after each spike*

$$I_{ij}(t) = J_{ij} S_j(t)$$

weight matrix   spike train of neuron *j*

- Neuron integrates excitatory and inhibitory post-synaptic currents of ***several 1000 synapses***.

  LIF allows further mathematical analysis when approximating I(t) with mean + noise term:

$$I(t) = I_0 + \sigma\xi(t)$$

- For general, time-dependent *I(t)*   numerical integration!

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