

Assignment 3

AI1110: Probability and Random Variables

Indian Institute of Technology Hyderabad

Ashwin Agrawal
CS22BTECH11009

10.15.1.25 Question: Which of the following arguments are correct and which are not correct? Give reasons for your answer.

- 1) If two coins are tossed simultaneously there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.
- 2) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Solution:

- 1) Let, X be a random variable, denoting the number of heads in two coin tosses.

$$X \sim \text{Bin}(n, p) \quad (1)$$

where, p be the probability of getting head on one toss of coin. n is the number of coins tossed.

$$\implies p = \frac{1}{2} \quad (2)$$

$$\implies n = 2 \quad (3)$$

$$\Pr(X = i) = \binom{n}{i} \times p^i \times (1 - p)^{n-i} \quad (4)$$

$$\therefore \Pr(X = i) = \binom{2}{i} \times \frac{1}{2^2} \quad (5)$$

There are 3 possible outcomes i.e $X = 0, 1, 2$
For 1st outcome (Two Heads) $X=2$,

$$\Pr(X = 2) = \binom{2}{2} \times \frac{1}{2^2} = \frac{1}{4} \quad (6)$$

For 2nd outcome (Two Tails) $X=0$,

$$\Pr(X = 0) = \binom{2}{0} \times \frac{1}{2^2} = \frac{1}{4} \quad (7)$$

For 3rd outcome (One Head and One Tail) $X=1$,

$$\Pr(X = 1) = \binom{2}{1} \times \frac{1}{2^2} = 2 \times \frac{1}{4} = \frac{1}{2} \quad (8)$$

From (??) , (??) and (??) , the probabilities are not equal to $\frac{1}{3}$. Hence, the statement is false.

- 2) Let Y be a random variable.

$$Y = \begin{cases} 1, & \text{if an odd number comes on the die} \\ 0, & \text{if an even number comes on the die} \end{cases}$$

There are 2 possible outcomes i.e $Y = 0, 1$

$$\Pr(Y = 1) = \frac{\text{Odd Numbers on Die}}{\text{Total numbers on Die}} \quad (9)$$

$$\therefore \Pr(Y = 1) = \frac{3}{6} = \frac{1}{2} \quad (10)$$

Hence, the statement is true.