Assignment 3

AI1110: Probability and Random Variables
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10.15.1.25 Question: Which of the following arguments are correct and which are not correct? Give reasons for your answer.

1) If two coins are tossed simultaneously there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.
2) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Solution:

1) Let, X be a random variable, denoting the number of heads in two coin tosses.

$$X \sim Bin(n, p) \tag{1}$$

where,p be the probability of getting head on one toss of coin. n is the number of coins tossed.

$$\implies p = \frac{1}{2} \tag{2}$$

$$\implies n = 2 \tag{3}$$

$$\Pr(X = i) = \binom{n}{i} \times p^i \times (1-p)^{n-i} \tag{4}$$

$$\therefore \Pr(X = i) = \binom{2}{i} \times \frac{1}{2^2} \tag{5}$$

There are 3 possible outcomes i.e $X = 0, 1, 2$
For 1st outcome (Two Heads) X=2,

$$\Pr(X = 2) = \binom{2}{2} \times \frac{1}{2^2} = \frac{1}{4} \tag{6}$$

For 2nd outcome(Two Tails) X=0,

$$\Pr(X = 2) = \binom{2}{0} \times \frac{1}{2^2} = \frac{1}{4} \tag{7}$$

For 3rd outcome (One Head and One Tail) X=1,

$$\Pr(X = 1) = \binom{2}{1} \times \frac{1}{2^2} = 2 \times \frac{1}{4} = \frac{1}{2} \tag{8}$$

From (6) , (7) and (8) , the probabilities are not equal to $\frac{1}{3}$. Hence, the statement is false.

2) Let Y be a random variable.

$$Y = \begin{cases} 1, & \text{if an odd number comes on the die} \\ 0, & \text{if an even number comes on the die} \end{cases}$$

There are 2 possible outcomes i.e $Y = 0, 1$

$$\Pr(Y = 1) = \frac{\text{Odd Numbers on Die}}{\text{Total numebers on Die}} \tag{9}$$

$$\therefore \Pr(Y = 1) = \frac{3}{6} = \frac{1}{2} \tag{10}$$

Hence, the statement is true.