

Assignment 3

AI1110: Probability and Random Variables

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10.15.1.25 Question: Which of the following arguments are correct and which are not correct? Give reasons for your answer.

- 1) If two coins are tossed simultaneously there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.
- 2) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Solution: Let us define the random variables X and Y:

| Parameter | Value | Description |
|-----------|---------------|--|
| X | {0,1,2} | Number of Heads in two coin tosses |
| Y | {1,2,3,4,5,6} | Number appearing on a Die throw |
| n | 2 | Number of coin tossed |
| p | $\frac{1}{2}$ | Probability of getting heads on a coin |

PMF of X:

$$X \sim \text{Bin}(n, p) \quad (1)$$

$$\Pr(X = i) = \binom{n}{i} \times p^i \times (1 - p)^{n-i} = \binom{2}{i} \times \frac{1}{2^2} \quad (2)$$

PMF OF Y:

$$\Pr(X = i) = \frac{1}{6}, i = 1, 2, 3, 4, 5, 6 \quad (3)$$

- 1) There are 3 possible outcomes i.e $X = 0, 1, 2$
For 1st outcome (Two Heads) $X=2$,

$$\Pr(X = 2) = \binom{2}{2} \times \frac{1}{2^2} = \frac{1}{4} \quad (4)$$

For 2nd outcome(Two Tails) $X=0$,

$$\Pr(X = 2) = \binom{2}{0} \times \frac{1}{2^2} = \frac{1}{4} \quad (5)$$

For 3rd outcome (One Head and One Tail) $X=1$,

$$\Pr(X = 1) = \binom{2}{1} \times \frac{1}{2^2} = 2 \times \frac{1}{4} = \frac{1}{2} \quad (6)$$

From (4) , (5) and (6) , the probabilities are not equal to $\frac{1}{3}$. Hence, the statement is false.

- 2) There are 2 possible outcomes i.e $Y=\text{Odd, Even}$

$$\Pr(Y = \text{Odd}) = \Pr(Y = 1) + \Pr(Y = 3) + \Pr(Y = 5) = \frac{1}{2} \quad (7)$$

Hence, the statement is true.