COMPUTATIONAL PHYSICS LAB

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Bisection Method

Aim

To implement the Bisection method to numerically compute the roots of non-trivial equations using MATLAB.

Algorithm

- Step 1: Read the range to work with, say x_1 and x_2
- Step 2: Read the tolerance limit, say e
- Step 3: Compute the no of iterations using the formula $\frac{\log(|x_2 - x_1|/e)}{\log(2)}$
- Step 4: If $f(x_1) \times f(x_2)$ is greater than 0 prompt the user about the invalid range and terminate the program
- Step 5: If $f(x_1) \times f(x_2)$ is less than 0 compute $x = \frac{(x_1+x_2)}{2}$
- Step 6: If $|((x_1 - x_2)/x)|$ is greater than or equal to e proceed to step 7 else goto step 10
- Step 7: Calculate the percentage error value using $\left|\frac{x_i - x_{i-1}}{x_i}\right| \times 100$
- Step 8: If $f(x_1) \times f(x)$ is less than 0 update x_2 as x and goto step 5 else goto step 9
- Step 9: Update x_1 as x and goto step 5
- Step 10: Plot percentage error vs no of iterations graph
- Step 11: Display the root of the equation in given interval, no of iterations computed earlier and actual no of iterations

Code

```
function outputnum = func(x)
outputnum = x.^2 - 4;
end

function root = bisection()

er = zeros(100);
co = 0;

x1 = input('Enter value of x1: ');
x2 = input('Enter value of x2: ');
e = input('Enter value tolerance limit: ');

nTh = log( abs(x2 - x1) / e ) / log(2);

if func(x1) * func(x2) > 0
    disp('Wrong Choice of Range, please check')
    return;
else
    x = (x1 + x2) / 2;
    while ( abs((x1 - x2) / x ) >= e)
        co = co + 1;
        er(co) = x;
```

```
        x = (x1 + x2) / 2;
        er(co) = 100 * abs(( x - er(co) ) / x);
        if ((func(x) * func(x1)) < 0)
            x2 = x;
        else
            x1 = x;
        end
    end
end

r = 1:co;
plot(r(2:co),er(2:co));
xlabel('No of Iterations');
ylabel('Percentage Error');
title('Bisection Method - % Error vs Iterations');

fprintf('\nTheoretical value of No of Iterations: %i', floor(nTh));
fprintf('\nTotal no. of Iterations:  %i', co);
fprintf('\nRoot of the given function: %f', x);

root = x;
end
```

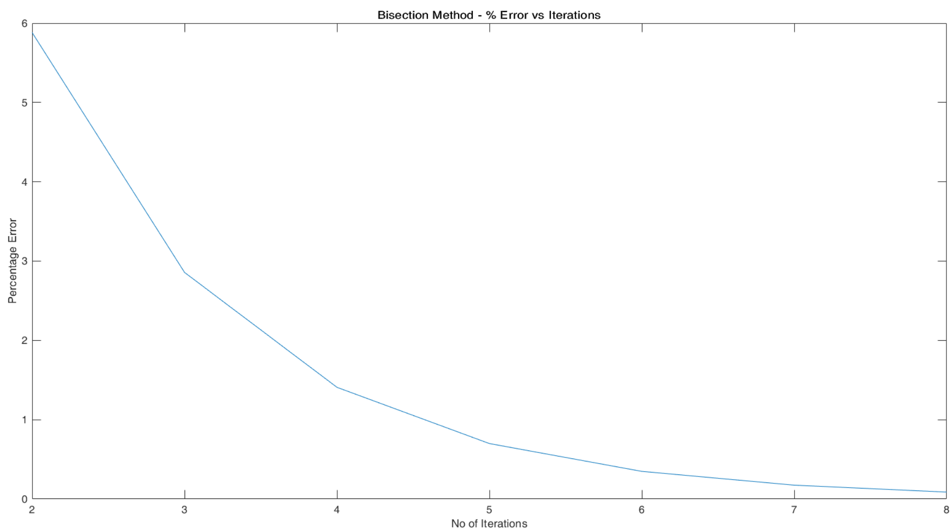
Output

```
Command Window
>> bisection()
Enter value of x1: 2.0
Enter value of x2: 2.5
Enter value tolerance limit: 0.001

Theoretical value of No of Iterations: 8
Total no. of Iterations:  8
Root of the given function: 2.236328
ans =

    2.2363

fx >>
```



Newton Raphson Method

Aim

To implement the Newton Raphson method to numerically improve the accuracy of the roots of non-trivial equations using MATLAB.

Algorithm

- Step 1: Read the approximate root value to the given function to work with, say x
- Step 2: Read the tolerance limit, say e
- Step 3: Compute a better approximation using Newton Raphson formula
  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
- Step 4: $f'(x_n)$ could be computed by directly providing the derivative manually to MATLAB or by a simple approximation
  $f'(x_n) = \frac{f(x_n+0.001)-f(x_n-0.001)}{0.002}$
- Step 5: Calculate the percentage error value using
  $\left|\frac{x_i - x_{i-1}}{x_i}\right| \times 100$
- Step 6: If $|(x_{n+1} - x_n)/x_n|$ is greater than or equal to e proceed to step 7 else goto step 3
- Step 7: Plot percentage error vs no of iterations bar graph
- Step 8: Display the root value computed earlier

Code

```
function outputnum = func(x)
outputnum = x.^2 - 5;
end

function root = newtonRaphson(x, e)

tmp = 0.0;
er = zeros(100);
count = 1;

while (abs((x - tmp) / x ) >= e)
    tmp = x;
    x = x - 2 * 0.001 * func(x) / (func(x + 0.001) - func(x - 0.001));
    er(count) = abs( (x - tmp) / x ) * 100;
    count = count + 1;
end

n = count;
r = 1:n;
bar(r(1:n-1),er(2:n));
xlabel('No of Iterations');
ylabel('Percentage Error');
title('Newton-Raphson Method - % Error vs Iterations');

root = vpa(x);
```

```
end
```

Output

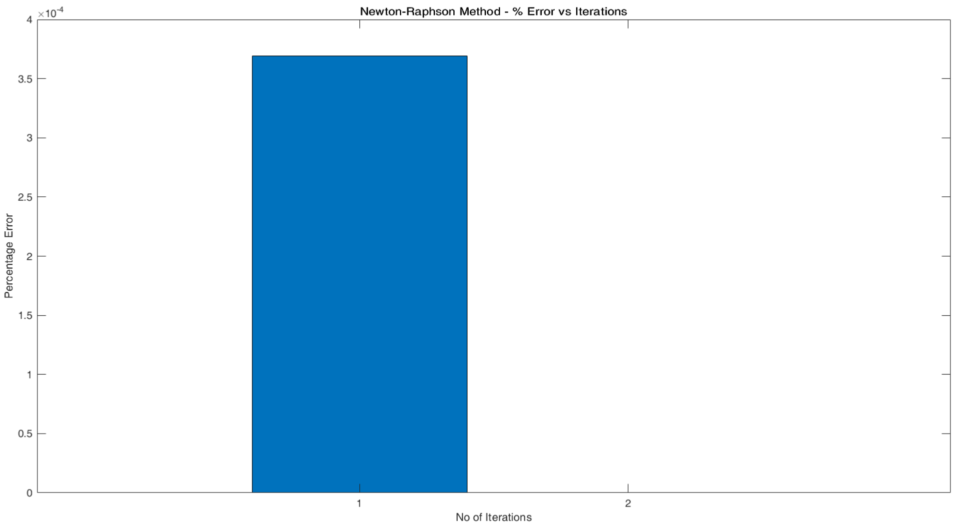
Command Window

```
>> newtonRaphson(2.23, 0.0001)

ans =

2.2360679775150300585551121912431

>>
```



Secant Method

Aim

To implement the Secant method to numerically improve the accuracy of the roots of non-trivial equations using MATLAB.

Algorithm

- Step 1: Read the approximate root value to the given function to work with, say x
- Step 2: Read the tolerance limit, say e
- Step 3: Compute a better approximation using Secant method formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
- Step 4: Calculate the percentage error value using $\left|\frac{x_i - x_{i-1}}{x_i}\right| \times 100$
- Step 5: If $|(x_{n+1} - x_n)/x_n|$ is greater than or equal to e proceed to step 6 else goto step 3
- Step 6: Plot percentage error vs no of iterations bar graph
- Step 7: Display the root value computed earlier
- Step 8: Compare percentage error vs no of iterations graph with Newton-Raphson's results

Code

```
function outputnum = func(x)
outputnum = x.^2 - 5;
end

function root = secant(x1, x2, e)

x = 0.0;
er = zeros(100);
count = 1;

while (abs((x1 - x2) / x2 ) >= e)
    x = x2;
    x2 = ( x1 * func(x2) - x2 * func(x1) ) / ( func(x2) - func(x1) );
    x1 = x;
    er(count) = abs( (x1 - x2) / x2 ) * 100;
    count = count + 1;
end

n = count;
r = 1:n;
bar(r(1:n-1),er(2:n));
xlabel('No of Iterations');
ylabel('Percentage Error');
title('Secant Method - % Error vs Iterations');

root = vpa(x2);

end
```

Output

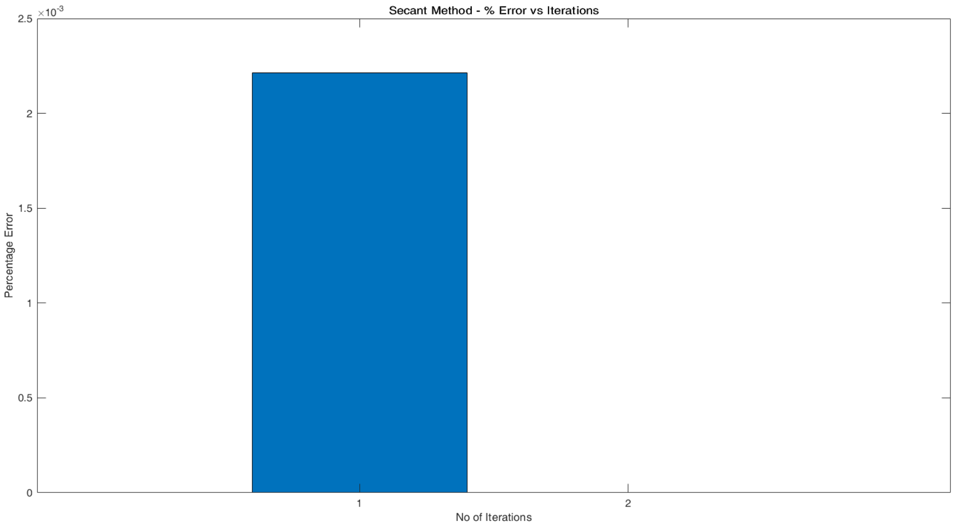
Command Window

```
>> secant(2.2, 2.23, 0.001)

ans =

2.2360679103760947583623419632204

>>
```



Gauss Elimination Method

Aim

To implement Gauss Elimination method to solve a system of linear equations exactly in MATLAB.

Algorithm

- Step 1: Check for proper dimensions of Matrix A and Vector b
- Step 2: Compute the formula for iterations of
  $k = 1 \ldots n-1, i = k+1 \ldots n \text{ and } j = k+1 \ldots n$
  $a_{ij} = a_{ij} - \frac{a_{ik} \times a_{kj}}{a_{kk}}$
- Step 3: Check a_{ii} is not equal to zero. If zero, swap rows $i \text{ and } i-1$. This is called partial pivoting.
- Step 4: Now for the back substitution, use
  $x_n = b_n / a_{nn}$
- Step 5: Compute remaining solutions by iterating $k = n-1 \ldots 1$ the below relation
  $x_k = \frac{a_{kn+1} - \sum_{j=k+1}^{n} a_{kj} \times x_j}{a_{kk}}$
- Step 6: Return/Print the vector x to display the solutions for given system of linear equations represented by A and b.

Code

```
function [ x ] = gaussElimination( A, b)

sz = size(A);
if sz(1) ~= sz(2)
    fprintf('A is not n by n\n');
    clear x;
    return;
end

n = sz(1);

if n ~= sz(1)
    fprintf('b is not 1 by n.\n');
    return
end

x = zeros(n,1);
aug = [A b];
tempmatrix = aug;

for i= 2:sz(1)
    tempmatrix(1,:) = tempmatrix(1,:) / max(tempmatrix(1,:));
    temp = find(abs(tempmatrix) - max(abs(tempmatrix(:,1))));
    if length(temp) > 2
        for j = 1:length(temp)-1
            if j ~= temp(j)
                maxi = j;
```

```
                break;
            end
        end
    else
        maxi = 1;
    end
    if maxi ~= 1
        temp = tempmatrix(maxi,:);
        tempmatrix(maxi,:) = tempmatrix(1,:);
        tempmatrix(1,:) = temp;
    end
    for j = 2:length(tempmatrix)-1
        tempmatrix(j,:) = tempmatrix(j,:) - tempmatrix(j,1) ...
            / tempmatrix(1,1) * tempmatrix(1,:);
        if tempmatrix(j,j) == 0 || ...
                isnan(tempmatrix(j,j)) || abs(tempmatrix(j,j)) == Inf
            fprintf('Error: Matrix is singular.\n');
            clear x;
            return
        end
    end
    aug(i-1:end,i-1:end) = tempmatrix;
    tempmatrix = tempmatrix(2:end,2:end);
end

x(end) = aug(end,end) / aug(end,end-1);

for i = n-1:-1:1
    x(i) = (aug(i,end) - dot(aug(i,1:end-1),x)) / aug(i,i);
end

end
```

Output

Command Window

```
>> A = [-3,2,-6;5,7,-7;1,4,-2]

A =

    -3     2    -6
     5     7    -7
     1     4    -2

>> b = [6;6;8]

b =

     6
     6
     8

>> gaussElimination( A, b)

ans =

   -1.6491
    2.7895
    0.7544
```

Gauss Jordan Method

Aim

To implement Gauss Jordan method to solve a system of linear equations exactly in MATLAB.

Algorithm

- Start from $k = 0$ and $l = 0$.
- Increment k by one unit.
- Increment l by one unit.
- Stop the algorithm if $l > L$. Else proceed to the next step.
- If $A_{il} = 0$ for $i = k, \cdots, K$, return to step 3. Else proceed to the next step.
- Interchange the k-th equation with any equation i (with $i > k$) such that $A_{il} \neq 0$ (if $i = k$ there is no need to perform an interchange).
- Divide the k-th equation by A_{kl}.
- For $i = 1, \cdots, k-1$ and $i = k+1, \cdots, K$, subtract the k-th equation multiplied by A_{il} from the i-th equation.
- If $k < K$, return to step 2. Else stop the algorithm.

Code

```
function solution = gauss_jordan(A, b, n)
     M = [A, b];
    for i = 2:n
        for j = 1:i-1
            M(i,:) = M(i,:) - M(j,:)*(M(i,j)/M(j,j));
        end
    end

   for i = n-1:-1:1
        for j = n:-1:i+1
            M(i,:) = M(i,:) - M(j,:)*(M(i,j)/M(j,j));
        end
   end

   A = M(1:n,1:n);
   b = M(:,n+1);

   disp(A)
    for i = 1:n
        solution(i,1) = b(i)/A(i,i);
    end
end
```

Output

Command Window

```
>> n = 3;

rnum = randi(20,n,n);
rnum2 = randi(20,n,1);

disp(rnum);
disp(rnum2);

sol = gauss_jordan(rnum, rnum2, n);
disp(sol);
    17    19     6
    19    13    11
     3     2    20

    20
     4
    20

   17.0000         0         0
         0   -8.2353         0
         0         0   18.2357

   -2.3141
    2.7857
    1.0685
```

Trapezoidal Rule

Aim

To implement Trapezoidal Rule in MATLAB and compute integrals numerically.

Algorithm

- Step 1: Generate a vector x such that it contains domain values in desired integration range, say $[x_0, x_n]$ for given function $f(x)$
- Step 2: Generate a vector y for any desired function $f(x)$ such that $y = f(x)$
- Step 3: Determine step size h and number of points n using x
- Step 4:Compute the integral $\int_{x_0}^{x_n} ydx$ using the \textbf{Trapezoidal Rule} formula $\int_{x_0}^{x_n} ydx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n]$
- Step 5: Display the numerical value of the given definite integral
- Step 6: Compare the numerical accuracy of Simpson's 1/3 Rule with Trapezoidal Rule for different functions

Code

```
function numInt = trapezoidal(x, y)

h = x(2) - x(1);
[m, n] = size(y);
sum = 0.000;

for count = 1:n
    sum = sum + y(count);
end

sum = 2.0 * sum - (y(1) + y(n));
sum = h * sum / 2.0;

numInt = vpa(sum);
end
```

Output

Command Window

```
>> x = 1:0.01:10;
y = x.^4 + exp(-x);
>> trapezoidal(x, y)

ans =

20000.20113710354053182527422905

>>
```

Simpson's 1/3 Rule

Aim

To implement Simpson's 1/3 Rule in MATLAB and compute integrals numerically.

Algorithm

- Step 1: Generate a vector x such that it contains domain values in desired integration range, say $[x_0, x_n]$ for given function $f(x)$
- Step 2: Generate a vector y for any desired function $f(x)$ such that $y = f(x)$
- Step 3: Determine step size h and number of points n using x
- Step 4:Compute the integral $\int_{x_0}^{x_n} ydx$ using the **Simpson's 1/3 Rule** formula $\int_{x_0}^{x_n} ydx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2}) + y_n]$
- Step 5: Display the numerical value of the given definite integral

Code

```
function numInt = simpson13(x, y)

h = x(2) - x(1);
[m, n] = size(y);
sum = 0.000;

for count = 1:n
    if ( count >= 2 ) & ( count <= n-1 ) & ( mod(count, 2) ==  0)
        sum = sum + 4 * y(count);
    end
    if ( count >= 3 ) & ( count <= n-2 ) & ( mod(count, 2) ==  1)
        sum = sum + 2 * y(count);
    end
end

sum = sum + y(1) + y(n);
sum = h * sum / 3.0;

numInt = vpa(sum);
end
```

Output

Command Window

```
>> x = 1:0.01:10;
>> y = x.^4 + exp(-x);
>> simpson13(x, y)

ans =

20000.16783405328897060826420784

>>
```

Romberg integration

Aim

To implement romberg integration recursively in MATLAB to compute definite integrals numerically with varying accuracy.

Algorithm

To compute
$\int_a^b f(x)dx$
using **Romberg's Integration** Method, define a **recursive function** using the below definitions

$h_n = \frac{b-a}{2^n}$

$R(0,0) = \frac{f_b - f_a}{h_1}$

$R(n,0) = \frac{R(n-1,0)}{2} + h_n \sum_{k=1}^{2^{n-1}} f\left(a + \frac{2k-1}{h_n}\right)$

$R(n,m) = R(n,m-1) + \frac{R(n,m-1) - R(n-1,m-1)}{4^m - 1}$

where $n \geq m$ and $m \geq 1$. Here,

- The zeroth extrapolation, R(n, 0), is equivalent to the trapezoidal rule with $2^n + 1$ points
- The first extrapolation, R(n, 1), is equivalent to Simpson's rule with $2^n + 1$ points
- The second extrapolation, R(n, 2), is equivalent to Boole's rule with $2^n + 1$ points

Code

```
function [ Result ] = romberg( fun, x1, x2, n, m )

func = inline(fun);
sum = 0.0;
h = @(t) ((x2 - x1) / (2^t));

if( (n == 0) & (m == 0) )
    Result = h(1) * (func(x1) + func(x2));
elseif ( m == 0)
    for k = 1:(2^(n-1))
        sum = sum + func(x1 + (2*k - 1)*h(n));
    end
    Result = ( 0.5 * romberg(fun, x1, x2, n-1, 0) + sum*h(n));
else
    Result = ((4^m)*romberg(fun, x1, x2, n, m-1) - romberg(fun, x1, x2, n-1, m-
1)) / (4^m - 1);
end

end
```

Output

Command Window

```
>> romberg('x.^2', 0, 5, 1, 0)

ans =

   46.8750

>> romberg('x.^2', 0, 5, 1, 1)

ans =

   41.6667

>> romberg('x.^2', 0, 5, 3, 1)

ans =

   41.6667

>>
```

Euler Method

Aim

To implement Euler Method in MATLAB to solve the differential equation $f'(x) = f(x, y)$ for it's particular solution numerically.

Algorithm

- Define function $f(x)$
- Get the values of x_0, y_0, h and x_n. Here x_0 and y_0 are the initial conditions, h is the interval, x_n is the required value
- Define $n = \frac{(x_n - x_0)}{h} + 1$
- Start loop from $i = 1$ to n
- $y = y_0 + h \times f(x_0, y_0)$
- $x = x + h$
- Print values of y_0 and x_0
- Check if $x \leq x_n$
- If yes, assign $x_0 = x$ and $y_0 = y$ and continue loop
- If no, End loop i

Code

```
function euler(fun, x0, y0, h, fname)

func = inline(fun);

x = zeros(1001);
y = x;

x(1) = x0;
y(1) = y0;

fid = fopen(fname,'w');

i = 0;
n = 1/h;

for i = 2:n
    x(i) = x(i-1) + h;
    y(i) = y(i-1) + h * func(x(i-1),  y(i-1));

    fprintf(fid,'%8.4f %8.4f \n', x(i), y(i));
end

plot(x,y);

end
```

Output

Command Window

```
>> euler( '(x^2)/3 + 0*y', 0, 0, 0.01, 'eulerxyza')
>>
```

eulerxyza - Notepad

File Edit Format View Help

```
0.7500   0.0459
0.7600   0.0478
0.7700   0.0497
0.7800   0.0517
0.7900   0.0537
0.8000   0.0558
0.8100   0.0580
0.8200   0.0601
0.8300   0.0624
0.8400   0.0647
0.8500   0.0670
0.8600   0.0694
0.8700   0.0719
0.8800   0.0744
```

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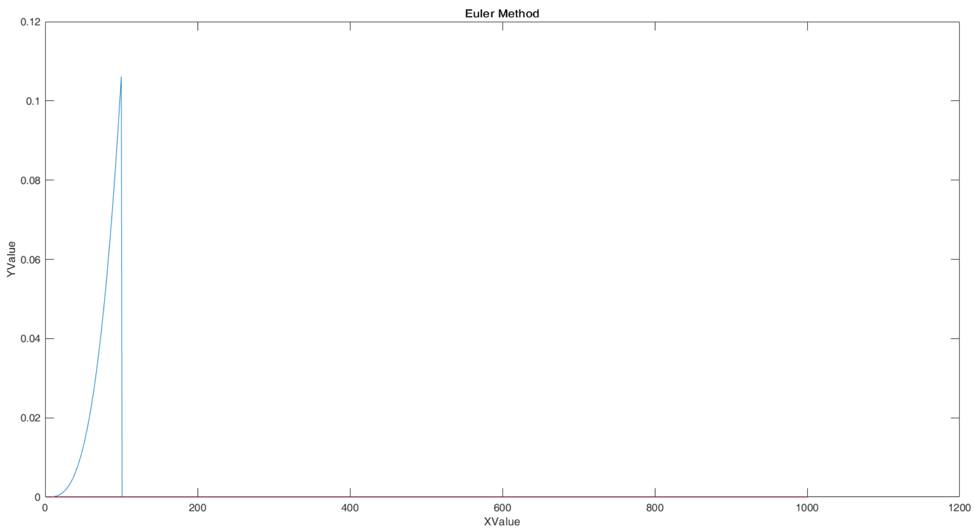


Runge Kutta Method

Aim

To implement 2nd and 4th order Runge Kutta Methods to solve a Differential Equation for it's particular solution numerically using MATLAB for the derivative function $f(x, y)$ in the domain $[a, b]$

Algorithm

- Step 1: Define the function $f(x, y)$
- Step 2: Determine step size h for desired accuracy
- Step 3: Compute y_1 from (x_0, y_0) and $x_1 = x_0 + h$ using the following formulas

For 2nd Order:
$y_{i+1} = y_i + \frac{k_1+k_2}{2}h$

where

$k_1 = f(x_i, y_i) \text{ and } k_2 = f(x_i + h, k_1 h)$
and for 4th Order:

$y_{i+1} = y_i + \frac{k_1+2k_2+2k_3+k_4}{6}$

where
$k1 = hf(x_i, y_i),\ k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}),\ k_4 = hf(x_i + \frac{h}{2}, y_i + \frac{k_3}{2})$

- Step 5: Check if x_n is equal to b, if yes then, plot the result else continue.
- Step 6: Save the vector x and y to a file for further reference.

Code

2nd Order Implementation

```
function rkutta2(x0, y0, h, fname)

func = @(x,y) 4 * x ^ 3 + 0 * y;

fid = fopen(fname,'w');

i = 0;
n = 1/h;

x = zeros(n);
y = x;

x(1) = x0;
y(1) = y0;
```

```
k2 = @(x,y) func((x + h), (y + (func(x, y) * h)));

for i = 2:n
    x(i) = x(i-1) + h;
    y(i) = y(i-1) + 0.5 * (func(x(i-1), y(i-1)) ...
        + k2(x(i-1), y(i-1))) * h;

    fprintf(fid,'%8.4f %8.4f \n', x(i), y(i));
end

plot(x,y);

end
```

4th Order Implementation

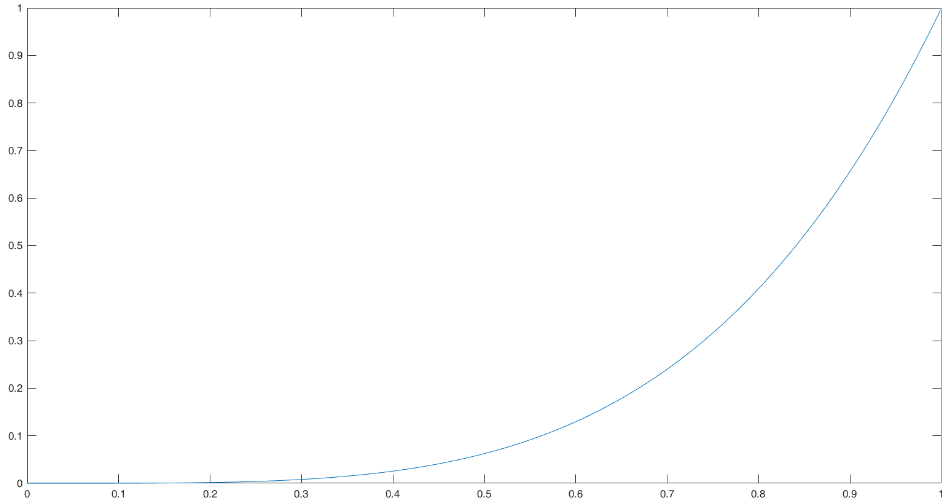
```
function rkutta4(x0, y0, h, fname)

func = @(a, b) 4 * a ^ 3 + 0 * b;

x = zeros(10001);
y = x;

x(1) = x0;
y(1) = y0;

fid = fopen(fname,'w');

i = 0;
n = 1/h;

k1 = @(a,b) func(a, b);
k2 = @(a,b) func((a + h / 2.0), (b + (k1(a,b) / 2.0)));
k3 = @(a,b) func((a + h / 2.0), (b + (k2(a,b) / 2.0)));
k4 = @(a,b) func((a + h), (b + k3(a,b)));

for i = 2:n
    x(i) = x(i-1) + h;
    y(i) = y(i-1) + h * (k1(x(i-1),y(i-1)) + ...
        2 * k2(x(i-1),y(i-1)) + ...
        2 * k3(x(i-1),y(i-1)) + k4(x(i-1),y(i-1))) / 6.0;

    fprintf(fid,'%8.4f %8.4f \n', x(i), y(i));
end

plot(x,y);

end
```

Output

Runge Kutta 2nd Order:

Command Window

```
>> rkutta2(0, 0, 0.001, 'rkutta2.txt')
>> rkutta2(0, 0, 0.001, 'rkutta2.txt')
fx >>
```



rkutta2.txt - Notepad

File Edit Format View Help

```
0.5170   0.0714
0.5180   0.0720
0.5190   0.0726
0.5200   0.0731
0.5210   0.0737
0.5220   0.0742
0.5230   0.0748
0.5240   0.0754
0.5250   0.0760
0.5260   0.0765
0.5270   0.0771
0.5280   0.0777
0.5290   0.0783
0.5300   0.0789
```

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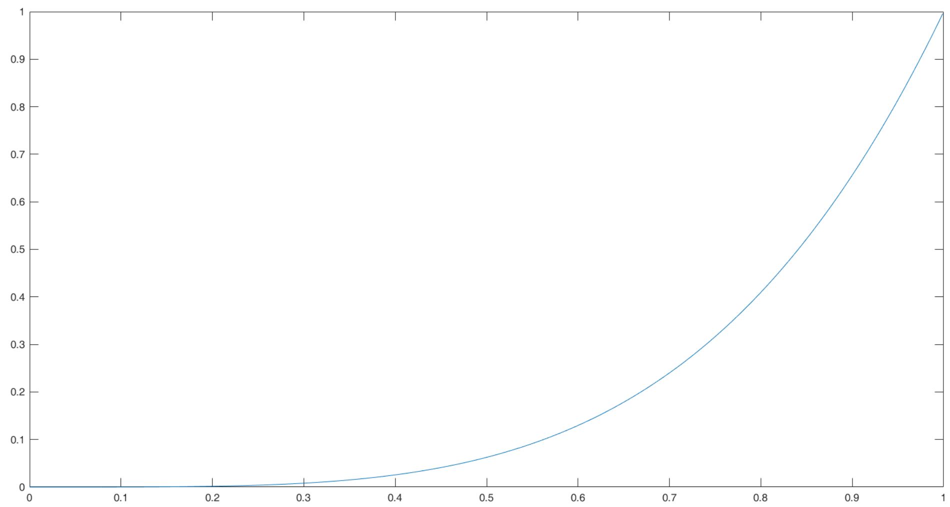
Runge Kutta 4th Order:

Command Window

```
>> rkutta4(0, 0, 0.001, 'rkutta4.txt')
fx >>
```



rkutta4.txt - Notepad

File Edit Format View Help

```
  0.0010   0.0000
  0.0020   0.0000
  0.0030   0.0000
  0.0040   0.0000
  0.0050   0.0000
  0.0060   0.0000
  0.0070   0.0000
  0.0080   0.0000
  0.0090   0.0000
  0.0100   0.0000
  0.0110   0.0000
  0.0120   0.0000
  0.0130   0.0000
  0.0140   0.0000
```

Unix (LF) Ln 1, Col 1 100%

Least Square Method

Aim

To implement Least Square Method in MATLAB to find the least error fit for the given data $(x_1, y_1) \to (x_n, y_n)$.

Algorithm

- Step 1: Compute $\sum_{i=1}^{n} x_i$, $\sum_{i=1}^{n} x_i^2$, $\sum_{i=1}^{n} x_i y_i$ and $\sum_{i=1}^{n} y_i$
- Step 2: Define a matrix $\mathcal{A}$

$$\mathcal{A} = \begin{pmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{pmatrix}$$

and a vector v

$$v = \begin{pmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{pmatrix}$$

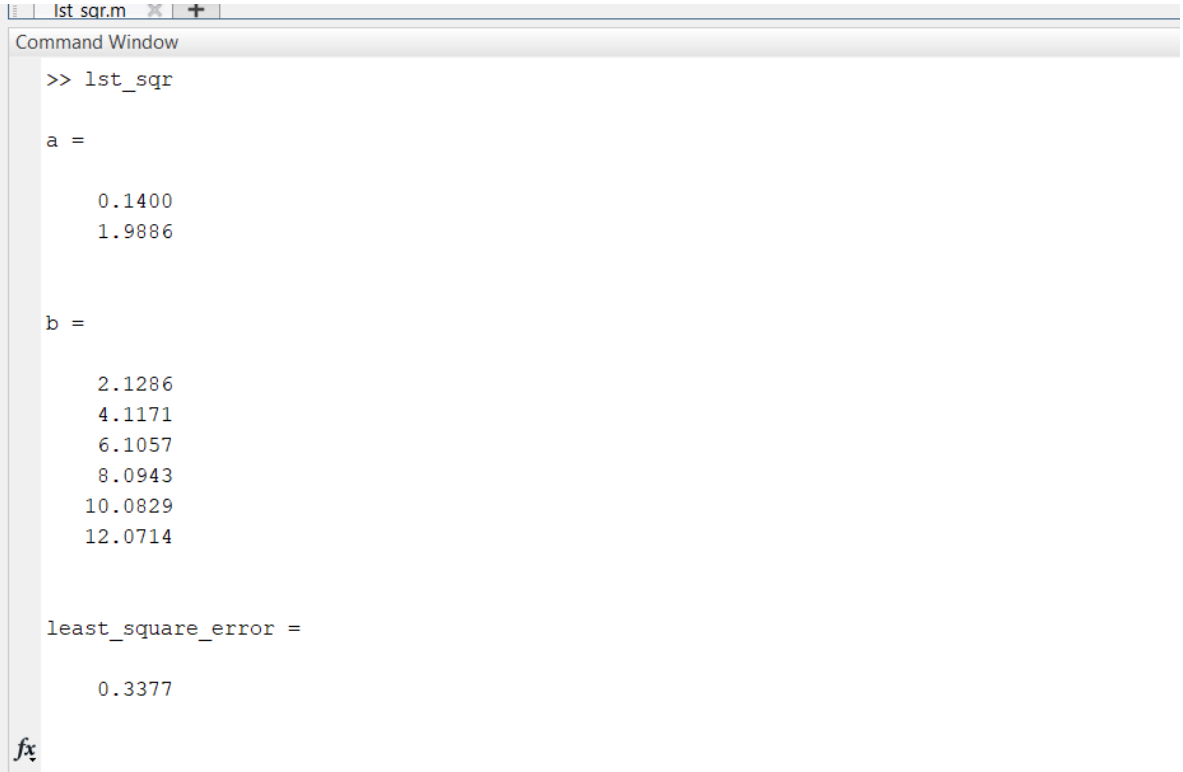
- Use Gauss Elimination to solve $\mathcal{A}\Phi = v$
- The components of Φ gives us the slope and intercept of the best fit line.

Code

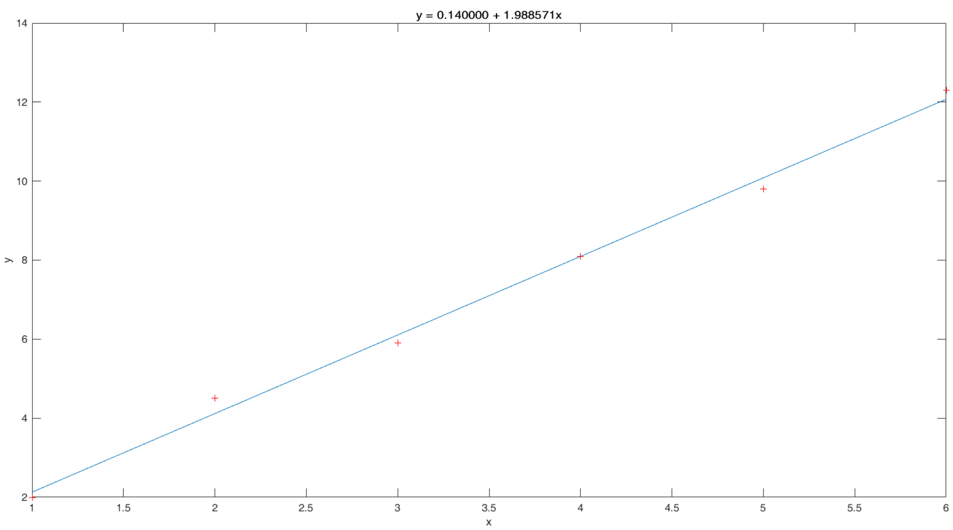
```
% input in the form of matrix, each row is a (x, y).
input = [...
  1, 2;...
  2, 4.5;...
  3, 5.9;...
  4, 8.1;...
  5, 9.8;...
  6, 12.3];

m = size(input, 1);
n = size(input, 2);
x = input(:,1:n-1);
y = input(:,n);

% The first column of matrix X is populated with ones,
% and the rest columns are the x columns of the input.
X = ones(m, n);
X(:,2:n) = input(:,1:n-1);

% Try to find the a that minimizes the least square error Xa - y.
% Project y onto the C(X) will give us b which is Xa.

% The relationship is X'Xa = X'b

% Use left division \ to solve the equation, which is equivalent
% to a = inverse(X'*X)*X'*y, but computationally cheaper.
a = (X' * X) \ (X' * y)
b = X*a
```

```
least_square_error = sum((b - y) .^ 2)

% Plot the best fit line.
plot(x, b);
title(sprintf('y = %f + %fx', a(1), a(2)));
xlabel('x');
ylabel('y');

hold on;
% Plot the input data.
plot(x, y, '+r');
hold off;
pause;
```

Output



```
lst_sqr.m
Command Window
>> lst_sqr

a =

    0.1400
    1.9886


b =

    2.1286
    4.1171
    6.1057
    8.0943
   10.0829
   12.0714


least_square_error =

    0.3377

fx
```



Hermite Cubic Interpolation

Aim

To implement Cubic Hermite Interpolator spline to fit a cubic polynomial to given data points numerically using MATLAB.

Algorithm

- Form a linear system of equations for input values of y and a dy sampled at points x (for y) and dx (for dy)
- Form a Matrix A for all value for x and dx. Form a (column) vector b for corresponding values of y and dy
- Use Gauss Elimination or \ operator to solve the system

Code

```
function [a, b, c, d] = hermite_cubic_interpolate(x,y,dx,dy)

% hermite_cubic_interpolate: Function to compute a Cubic Hermit Polynomial
%
% Assume a cubic polynomial of the form
%
%  y = a*x*x*x + b*x*x + c*x + d
%
%  with derivative:
%
%  dy = 3A*x*X + 2*b*x + c
%
% Solution is quite simple: Form a linear system of equations for input
% values of y and a dy sampled at points x (for y) and dx (for dy)
%
% Form a MATRIX A for all value for x and dx. Form a (column) vector b for
% corresaponding values of y and dy
%
% Use MATLAB \ operator to solve the system
%
% Inputs: x --- values of x where y samples are given
%         y --- values of y at given x values
%         dx --- values of dx where dy samples are given
%         dy --- values of dy at given dx values
%
% All inputs assumed row vectors.
%
% Outputs: a,b,c,d --- parameters of Hermite Cubic


% Get number of x data points
[m n] = size(x);

% Form matrix A for X values
```

```
A = [x.*x.*x; x.*x; x; ones(1,n)]';


% Get number of x data points
[m n] = size(dx);

% Form matrix dA for X values

dA = [3*dx.*dx; 2*dx; ones(1,n); zeros(1,n)]';

% Concatentate A and da  for final A matrix
A = [A ; dA];

% Concatentate y and dy  for b COLUMN vector ---
%   so columnise y and dy (use ')
b = [y' ; dy'];

% solve linear system of equations

p = A\b;

% output a,b,c,d

a = p(1);
b = p(2);
c = p(3);
d = p(4);
```

Output

Command Window

```
>> x = 0:0.01:1;
>> y = x.^3 - 4*x;
>> dx = 0.01 * ones(101,1);
>> dx = dx';
>> dy = 3*x.^2 -4;
>> hermite_cubic_interpolate(x,y,dx,dy)

ans =

    2.4630

>> hermite_cubic_interpolate(x,y,dx,dy)

a =

    2.4630


b =

   -2.3600


c =

   -2.9495


d =

   -0.1040

fx
```