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DEPARTMENT OF COMPUTER SCIENCE

The Deeply Reinforced Trader

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A dissertation submitted to the University of Bristol in accordance with the requirements of the degree of Master of Engineering in the Faculty of Engineering.

Tuesday 4th August, 2020

Declaration

This dissertation is submitted to the University of Bristol in accordance with the requirements of the degree of MEng in the Faculty of Engineering. It has not been submitted for any other degree or diploma of any examining body. Except where specifically acknowledged, it is all the work of the Author.

Ashwinder Khurana, Tuesday 4$^{\text{th}}$ August, 2020

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Executive Summary

One of the primary goals of artificial intelligence is the solve complex tasks that rely on highly dimensional, unprocessed and noisy input. Namely, recent advances in Deep Reinforcement Learning (DRL) has resulted in the "Deep Q Network [?], which is capable of human level performance on several Atari video games - using unprocessed pixels for input. A similar "gamification" of the process of a continuous double auction limit-order-book (LOB), can be an application of DRL.

DRL requires for an agent to interact with an environment and to derive actions from the state of the environment at each timestep. The environment in this case can be a market session, in which several sales traders interact and utilise the LOB to communicate orders to buy or sell an asset, and the agent that interacts with the environment is the proprietary trader, which aims to seek profit for itself rather than on the behalf of a client. Information available from the LOB can be used as combinations of inputs to represent the current state of the 'game', from which the trader can learn which actions to take during the learning process. This is done by using a policy which determines what action to take depending on the current state of the LOB and a reward system which provides rewards depending on the action.
Falcao had used a Policy Gradient agent (PG Agent) in conjunction with Cliff's ZIP to solve the sales trader problem. In comparison this thesis attempts to solely use state of the art DRL techniques to solve the proprietary trader problem, namely via models such as Advantage Actor-Critic, Policy Gradient and DDPG.

Whilst most algorithms are trained and analysed with data from public exchanges collected from sites such as finance.yahoo.com, this thesis focuses on using Cliff's Bristol Stock Exchange [?] and Falcao's Bristol Stock Gym [?] to simulate market sessions of robot traders to train and compare the effectiveness of the agent against other traditional machine learning based algorithms.

This is a hugely challenging task as the mapping between the current state of the system and the action the agent takes in such an environment is a problem all financial institutions are aiming to tackle in the most sophisticated ways to gain an overall competitive advantage. The results of this thesis are not a success story, but it details the journey of iterating and analysing results to create a profitable trader using the techniques outlined above and the vast challenges that were presented.

> My research hypothesis is that the latest developments in machine learning, more specifically, a trader that uses Deep Reinforcement Learning techniques alone provide a tool set to create a proprietary trader that outperforms humans.

This project explores the hypothesis

- I spent 100+ hours researching prior work on the field and learning about Deep Reinforcement Learning techniques.
- I wrote a total of ? lines of source code, comprising of the merger of capabilities of BSE into Falcao's BristolStockGym, several DRL models, Variational Auto-Encoder and Regression Models.
- Spent 100+ hours training the various models and running experiments on Blue Crystal 4 [?]

Supporting Technologies

- All the codebase is written using Python version 3.6.7 and the Python Package Installer PIP tool in its 19.0.3 version.
- Several Python libraries were used extensively throughout the project, namely: PyTorch(1.3.0), NumPy(1.18.1), Matplotlib(3.1.3), Sklearn(0.22)
- I used a Falcao's version of Cliff's Bristol Stock Exchange that is refactored to be compatible for Deep Reinforcement Learning analogous to OpenAI's Gym

Notation and Acronyms

All of the acronyms used in this document are explained on their first usage and no more. However, several acronyms are used across multiple chapters throughout the thesis, they are as follows:

Acronyms

AI	:	Artificial Intelligence
BSE	:	Bristol Stock Exchange
BSG	:	Bristol Stock Gym
LOB	:	Limit Order Book
CDA	:	Continuous Double Auction
MDP	:	Markov Decision Process
DRL	:	Deep Reinforcement Learning
PG	:	Policy Gradient
DDPG	:	Deep Deterministic Policy Gradient
ZI-C	:	Zero Intelligence - Constrained
ZIP	:	Zero Intelligence Plus
DRT	:	Deeply Reinforced Trader
	$\vdots$	
$\mathcal{H}(x)$	:	the Hamming weight of x
$\mathbb{F}_q$	:	a finite field with q elements
x_i	:	the i-th bit of some binary sequence x, st. $x_i \in \{0, 1\}$

Acknowledgements

An optional section, of at most 1 page

It is common practice (although totally optional) to acknowledge any third-party advice, contribution or influence you have found useful during your work. Examples include support from friends or family, the input of your Supervisor and/or Advisor, external organisations or persons who have supplied resources of some kind (e.g., funding, advice or time), and so on.

Chapter 1

Contextual Background

1.1 Introduction

The underlying context behind this thesis is the comprehension, analysis and exploitation of the mechanisms that underpin the real-world financial markets, from a Computer Scientist's perspective. The infrastructure and mechanism of interest are major stock exchanges. Stock exchanges are where individuals or institutions come to a centralised market to buy and sell stocks, where the price of these stocks are set by the supply and demand in the market as the buyers and sellers place their orders on the exchange. There are various types of orders that can be used to trade a stock, however, this thesis focuses on a specific type of order, the Limit Order. A limit order is a type of order to purchase or sell a security at a specified price or better. For buy limit orders, the order will be executed only at the limit price or a lower one, while for sell limit orders, the order will be executed only at the limit price or a higher one [**?**]. A collection of orders of this kind formulates the Limit-Order-Book (LOB) , which is a book, maintained by the exchange, that records the outstanding limit orders made by traders.

The role of a sales trader is to trade on behalf of a client, which means that they do not own a stock of their own, but instead buys and sells to secure the best price for their client. A proprietary trader, or a "prop" trader on the other hand buys and sells stocks on their own behalf with their own capital with the aim of seeking profitable returns. Both types of traders have to constantly adapt to the market conditions and react through the prices they quote for their orders.

Originally, the job of trading was fulfilled by humans who reacted to the market conditions using their intuition and their knowledge of the financial markets, however the job was hit with a wave of automation after 2001, when IBM researchers published results that indicated that two algorithms (ZIP by Prof Dave Cliff and MGD, a modification of GD by Steven Gjerstad & John Dickhaut) consistently outperformed humans in a experimental laboratory version of financial markets [**?**].

Since then, there has been relatively little research done in this field within academia, and subsequent solutions have built upon the traditional heuristic alongside 'old school' machine learning approaches. Considering the explosion of Deep Learning as a field in this last decade, this thesis attempts to explore the application of these state-of-the-art techniques, in particular Deep Reinforcement Learning, to the problem of the adaptive prop trader.

1.2 History of Research

1.2.1 Experimental Economics to Algorithmic Trading

Analysis of the sales trader problem can be seen as early as the 1960's in Vernon Smith's experiments at Purdue University, which led to the birth of the field of Experimental Economics and a Nobel Prize for Smith.

Smith's motivation behind these experiments was to find empirical evidence for the credibility of basic microeconomic theory of Continuous Double Auction's (CDA's) to generate competitive equilibria [**?**] . He formalised this experiment structure as a Double-oral-Auction in which he would a split a group of humans up into two subgroups: Buyers and Sellers. He gave each human a private limit price, where buyers would not buy above that price and sellers would not sell below that price. Prices quoted by the buyer's are known as the bid prices and the prices quoted by the sellers are known as the ask prices. The buyers and sellers would interact within small time based sessions, otherwise known as trading sessions.

After running repeated experiments of the trading sessions, Smith empirically demonstrated that the prices quoted by the traders demonstrated rapid equilibration, where the prices would converge to the theoretical 'best' price of an asset, despite a small number of traders used.

These sets of experiments were repeated by Gode and Sunder [?] in the 1990's. Crucially however, the human traders that were the core of Smith's experiments were replaced with computer based algorithmic traders. Gode and Sunder introduced two types of *zero-intelligence* algorithmic traders for CDA markets: Zero Intelligence - Unconstrained (ZI-U) and Zero Intelligence - Constrained (ZI-C). ZI-U traders are traders that generated a uniformly random bid/ask price, whilst ZI-C traders were traders that also generated a random price, however it was constrained between the private limit price of the trader and the lowest/highest possible price w.r.t to whether it was a buyer or seller - therefore it could never sell at a loss.

The purpose of Gode and Sunder replicating the experiments by Smith with their adaptation from humans to computers was to figure out if the allocative efficiency of a market was down to the intelligence of the traders or the organisation of the market. They ran several experiments with 5 different types of market structures whilst monitoring their allocative efficiency. They concluded that the ZI-U traders were essentially useless in that they did not gravitate to an equilibrium price, however, the surprising result was that the ZI-C traders were surprisingly human-like as the prices quoted by these traders eventually did gravitate to the equilibrium price over the course of each trading session. This allowed them to conclude that most of the 'intelligence' was in the market structure rather than the traders.

This conclusion was critiqued by Cliff in 1996 [?] hypothesised that the ZI-C traders would fail to equilibrate under a set of market conditions and them empirically demonstrated that they did indeed fail to equilibrate. He also developed a new algorithmic trader coined as zero-intelligence plus (ZIP), which quoted prices based on a profit margin the algorithm adjusts using a learning rule (Widrow-Huff with momentum). This algorithm succeeded in the market conditions in which ZI-C failed in.

Around the same time, in 1998 Gjerstad and Dickhaut developed the GD algorithm [?]. The core of this algorithm is that the traders compute a belief function that an order will be accepted by the opposing trader types (buyer vs seller) based on the information extracted from the market. This belief function seeks to maximise the trader's expected gain over the course of the session.

Prior to 2001, these algorithms had solely been pit against other algorithms, however in 2001, Tesauro and Das from IBM [?] ran numerous experiments where they would pit algorithms like ZIP, MGD (a modified GD) against humans as well as agent vs agent trading contests. The results of these experiments were groundbreaking as they demonstrated that ZIP and MGD were dominant in the agent vs agent contents, but more crucially their performance surpassed that of human traders.

Despite the promising results and the wide press received from the IBM publications, relatively little experimentation has been conducted of this type. Meanwhile, Tesauro and Bredin published an improved version GD called GD-Extended (GDX) in 2002, and Vytelingum published the Adaptive-Aggressiveness Algorithm in 2006 [?].

1.2.2 Developments in Artificial Intelligence

Despite the the longstanding theory behind Deep Learning, it has been an explosive field in the last decade, with notable achievements including: enabling google translate to become significantly more accurate, furthering NLP research massively and key factor behind the driverless car. It has been labelled the 'Deep Learning Revolution' by Terry Sejnowski [?].

Similarly, Reinforcement Learning has been an idea coined for a long time[?], however it was applied to very limited and relatively simple problems.

The two ideas silo'd have had their respective achievements, however recently there have been unprescedented achievements in the field of artificial intelligence with the combination of the two fields predictably called Deep Reinforcement Learning. Some notable achievements including Robotics [?] that maps raw input pixels to actions from a robot and AlphaZero from Google-owned startup DeepMind which is a Deep Reinforcement Learning successor to their own AlphaGo, who beat the world champion at the board game 'Go'.

1.3 Motivations

The motivations behind this project are three fold.

1.3.1 Finance perspective

The first motivation is in the fact that any project that invents or improves upon current solutions within this domain is solving an extremely complex problem in the world of Finance.

Huge institutions such as banks and hedge funds are at a constant arms race with one another to see who can create the most robust, sophisticated and profitable algorithms to enact their trades, thus an arms race for keeping their clients and staying in business. In doing so, these institutions are hiring the most able Computer scientists, and Quantitative developers to create optimal solutions in house in order to deliver this competitive edge against the rest of the market.

The heavy investment in this jobs and the development of trading algorithms comes from the big Financial institutions relatively late adoption of surging technologies, mainly driven by small FinTech companies eating from the established companies' profits with their innovative tech solutions to an otherwise very traditional industry. MX reports [?] that as an example, a breakdown of JP Morgan's revenue indicates that roughly 15% of JP Morgan's interest net income ($9b) stems from their trading of assets and investing securities, which highlights their interest in optimising the entire process of trading.

The aforementioned IBM experiment 1.2.1 also indicated that the ZIP and MGD algorithms outperformed their human counterparts in the sales trader job, albeit in a much more simplified environment, the claim still holds true for real world applications where algorithmic approaches outperform the previously hired traditional human trader.

There are several reasons for this, as computers are able to outperform a traders intuition and can analyse large sets of data very quickly to produce fast and appropriate action. Another aspect is more so to do with human psychology. Research indicates[?] that trading and money management is akin to human survival, which directly affects the part of the brain called the amygdala. This is an evolutionary knee-jerk reaction that triggers pain or pleasure. A successful trade creates a huge sense of euphoria, but a bad trade equally creates a sense of pain, regret and frustration. This can lead to inappropriate action taken from the human trader, for example revenge trading. The objective perspective from an individual not involved in trading suddenly becomes hard to translate when there is a stake involved whilst trading when the animalistic sense of survival kicks in, leaving very little room for subjectivity. This aspect of human psychology makes it more desirable for firms to trust algorithms that are not prone to this downfall. This has resulted in 35% of the U.S's equities being managed by automatised processes [?].

On the flip side, despite the removal of human error, Torsten Slok, chief international economist at Deutsche Bank, has named it the number 1 risk to markets in 2019[?]. This is down to the potential of concentrated sell offs causing a downward spiral of markets. This is a credible argument made and raises questions for the future of AI within finance and its integration within society as whole.

Despite this, the reliance on state of the art technology and research to propel financial services into the fifth industrial revolution is undeniable which presents an opportunity for curious computer scientists and academics to tackle these complex problems.

1.3.2 Computer Science perspective

As mentioned in section ?? DRL has recently scaled to previously unsolvable problems. This is down to the ability of DRL to learn from raw sensors or input images as input, without the intervention of human, in an unsupervised machine learning manner.

AlphaGo Zero surpassed its predecessor AlphaGo in an unprecedented manner because the knowledge of the AI is not bounded by human knowledge[?]. This is achieved by an initialised neural network that has no knowledge about the game of Go. The AI then plays games against itself via the combination of its network plus a powerful search algorithm. As more iterations of the games are completed, the network is tuned and better predictions are made. This updates the neural network and combined with the search algorithm to create a more 'powerful' version of itself, a new AlphaGo Zero. This process repeats to continuously improve its performance. AlphaGo in the other hand, used a more traditional approach to learn how to play the game, because it "watched" many different exemplar games and learnt based in a supervised machine learning manner.

AlphaGo Zero differs from its predecessor in many ways. The most apparent difference is that rather than the hand-engineered feature extraction process that AlphaGo used, AlphaGo Zero used the raw images of the black and white stones from the Go board as its input. Also, AlphaGo used a more traditional approach to learn how to play the game, because it "watched" many different exemplar games and learnt based in a supervised machine learning manner, whilst AlphaGo Zero learnt by playing itself.

Whilst AlphaGo Zero as an example is a holy grail within the DRL field, the potential for this process to applied to varying problem is one of the key motivations behind trying to apply the technique to this problem domain.

1.3.3 Furthering previous work

Falcao's thesis [?] also alluded to the excitement of applying DRL techniques to this domain, but after proving unsuccessful, opted to direct his attention to combining DRL techniques with existing more traditional techniques.

Furthermore, Falcao built an excellent framework BSG (Bristol Stock Gym) and interface to train DRL within the problem definition as it is a framework that is refactored from Cliff's Bristol Stock Exchange to simulate trade experiments as well training the network whilst iterating through episodes of the 'game' which are analogous to market sessions in BSE.

Whilst Falcao's thesis provides a very good overview of his journey exploring DRL, this project focuses on applying several more techniques within the field to see if they alone can solve the problem of a prop trader

1.4 Central Challenges

The challenges in this project are vast and unsolved for this domain.

One of the main challenges is to extract the key information available from the LOB at every time step. The word 'key' is ambiguous here because that is a challenge within itself, to decide what is useful information for the networks to use as input.

Another challenge is to design a network architecture that is fit to tackle the prop trader problem, including all its hyperparameters and model layer choices. This is a common problem within machine learning, and despite resources being available as well as exemplar material, the choices made w.r.t to these factors have to be bespoke to the data available from BSG, which would take a considerable amount of time experimenting and training.

Research indicates that one of the most complex aspects of DRL is to create the optimal reward function so that the DRL agent learns the desired behaviour, an example is in the OpenAI gym environment 'Lunar Landing' game, in which the agent has to land the spacecraft safely. A well intentioned reward system may include punishing the agent with negative rewards for landing the spacecraft incorrectly, however, can result in the agent not wanting to land the spacecraft at all, which is not desirable behaviour. This raises in question the reward function itself as well as the choices behind the magnitude of reward for each action.

1.5 Conclusion

The high-level objective of this project is to use state-of-art Deep Reinforcement Learning techniques to develop an adaptive proprietary trader. More specifically, the concrete aims are:

1. To develop a concrete understanding of Deep Reinforcement Learning techniques from scratch as they are not a core part of the university's machine learning curriculum.
2. To develop a feature extraction mechanism to extract the most crucial information available from the public market session at every time step.
3. Experiment and explore different DRL models with a varying set of hyper-parameters and layer choices
4. Most importantly, develop the optimal reward function for the agent to exhibit the most desirable behaviour

Chapter 2

Technical Background

2.1 The Overarching Problem

2.1.1 The Limit Order Book

The Limit Order Book (LOB) is a standardised view of all relevant/key market data within a Continuous Double Auction (CDA), and they serve as the backbone for most modern electronic exchanges within financial markets. It summarises all the live limit orders that are currently outstanding on the exchange with bid orders – offers to buy an asset, and ask orders – offers to sell an asset, both displayed on either side of the LOB. Each side is called a book, ergo the bid book is on one side, and the ask book is on the other.

Figure 2.1: A graphical representation of a Limit Order Book (LOB), reproduced from Cliff (2018)

The left hand side of figure ?? is a mockup of a graphical representation of BSE, whilst in its original form, is in a python dictionary data structure. As seen in the figure, the bid book lists all the bid orders with their prices and the respective quantities available at that price as a key/value pair in descending order to represent all the outstanding orders from the buyers. Conversely, the ask book lists the price-quantity pair for all the ask orders in ascending order. In this order, the highest bid price is at the top of the bid book alongside the lowest ask price: the two prices are called the *best bid* and the *best ask* respectively.

This graphical representation allows for easy feature extraction for some key elements that may not explicitly stated in the figure.

- The *Time Stamp* for the current time in the trading session, shown post-facing the ”T” in the top left corner
- The *Bid-Ask Spread* which is the difference between the *best ask* and the *best bid*
- The *Midprice* is the arithmetic mean of the *best ask* and the *best bid*

- The *Tape* shows a record of all the executed trades and cancellations of orders
- The *Microprice* is a cross volume weighted average of the *best ask* and *best bid*, prefaced by the green "M" below the time stamp
- The latest limit order sent to the exchange by a trader

The *midprice* and *microprice* are values that are used to approximate the value of the asset and attempt to summarise the market. In this example snapshot figure ?? the *best ask* is \$1.77 and the *best bid* is \$1.50 so the midprice is $(\$1.77 + \$1.50)/2 = \$1.66$. The *microprice* is a more intricate calculation because it is a cross volume weighted average:

$$\frac{BestBidQty * BestAskPrice + BestAskQty * BestBidPrice}{BestBidQty + BestAskQty} = Microprice \tag{2.1}$$

$$\frac{5 * \$1.77 + 13 * \$1.50}{5 + 18} = \$1.58 \tag{2.2}$$

The right hand side of the figure represent the supply and demand curves curated from the ask and bid books respectively, where the orange line is the demand curve and the blue line is the supply curve. Following the orange or blue step functions from left to right, at each step, the height represents the price and the width represents the available quantity at the price point. Lastly, the green cross represents the microprice.

A trade occurs when a bid offer's price is greater than the *best ask* or when an ask offer's price is less than the *best bid* on the LOB, this is known as *crossing the spread*. This process is quantity invariant, that is if the latest order price crosses the spread but the requested quantity exceeds the quantity available at that price, all the quantities that are available are sold crossed price, and the unfulfilled quantities are then placed on the LOB as an outstanding order.

As seen in the figure ??, the supply and demand curves do not cross, which therefore represents the fact that no trader is willing to trade at the current prices given in the exchange. The supply and demand curves not crossing means that the LOB is currently at equilibrium, as the best bid is not greater than the best ask.

Figure 2.2: A graphical representation of a Limit Order Book (LOB), reproduced from Cliff (2018)

The figure ?? indicates that a new order has been placed to the exchange. Interestingly, this order crosses the spread because the ask price \$1.48 was less than the *best bid* in figure ?? so therefore a transaction occurs. Incidentally, the quantity requested was less than the quantity available at \$1.50 so the entire order is fulfilled, and the quantity remaining is updated from 10 to 5. For arguments sake, if the quantity requested was 20 rather than 5, all the quantities that are available at \$1.50 (10) would be used to fulfil the new order and then the LOB would have a new *best bid* of \$1.48 with a qty of 10.

2.1.2 Proprietary trader's Objective

The agents that are taking part in the 'gamification' of the trading session are sales traders. The aim of the sales trader is to maximise their consumer surplus relative to their client's order. As mentioned in the previous chapter, the sales trader does not hold a stock of their own, they trade on behalf of their *client*, who provides them with a limit price that they cannot buy or sell above or below respectively. The goal of the sales trader is to maximise the difference between the limit price and the trade price, which is the consumer surplus. In the real world, the sales trader makes commission based on the consumer surplus and the client pockets the rest.

The proprietary trader's objective is that to maximise their own capital buy buying and selling stocks for themselves. Whilst initially it may seem like the objective of the prop trader is different to the sales trader and therefore cant be compared, but the proprietary traders ability to maximise the difference between the price they buy and sell a stock at is analogous to a sales trader maximising the difference between its limit price and its trade price. This rephrasing of the problem allows this project to directly compare the traditional machine learning approaches to the prop trader developed in this project.

2.2 Reinforcement Learning

This project transforms the current Prop trader problem to a problem that is well suited to a Reinforcement Learning paradigm, which will be the main avenue of solutions this project explores, so it is paramount to provide the technical background for reinforcement learning, which is the basis of this project.

2.2.1 Markov Decision Process

Chapter 3

Project Execution

A topic-specific chapter, of roughly 15 pages

This chapter is intended to describe what you did: the goal is to explain the main activity or activities, of any type, which constituted your work during the project. The content is highly topic-specific, but for many projects it will make sense to split the chapter into two sections: one will discuss the design of something (e.g., some hardware or software, or an algorithm, or experiment), including any rationale or decisions made, and the other will discuss how this design was realised via some form of implementation.

This is, of course, far from ideal for *many* project topics. Some situations which clearly require a different approach include:

- In a project where asymptotic analysis of some algorithm is the goal, there is no real "design and implementation" in a traditional sense even though the activity of analysis is clearly within the remit of this chapter.
- In a project where analysis of some results is as major, or a more major goal than the implementation that produced them, it might be sensible to merge this chapter with the next one: the main activity is such that discussion of the results cannot be viewed separately.

Note that it is common to include evidence of "best practice" project management (e.g., use of version control, choice of programming language and so on). Rather than simply a rote list, make sure any such content is useful and/or informative in some way: for example, if there was a decision to be made then explain the trade-offs and implications involved.

3.1 Example Section

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Table 3.1: This is an example table.

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3.1.1 Example Sub-section

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for $i = 0$ **upto** n **do**
  $t_i \leftarrow 0$
end

Algorithm 3.1: This is an example algorithm.

```
for( i = 0; i < n; i++ ) {
  t[ i ] = 0;
}
```

Listing 3.1: This is an example listing.

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Example paragraph. This is an example paragraph; note the trailing full-stop in the title, which is intended to ensure it does not run into the text.

Chapter 4

Critical Evaluation

A topic-specific chapter, of roughly 15 pages

This chapter is intended to evaluate what you did. The content is highly topic-specific, but for many projects will have flavours of the following:

1. functional testing, including analysis and explanation of failure cases,
2. behavioural testing, often including analysis of any results that draw some form of conclusion wrt. the aims and objectives, and
3. evaluation of options and decisions within the project, and/or a comparison with alternatives.

This chapter often acts to differentiate project quality: even if the work completed is of a high technical quality, critical yet objective evaluation and comparison of the outcomes is crucial. In essence, the reader wants to learn something, so the worst examples amount to simple statements of fact (e.g., "graph X shows the result is Y"); the best examples are analytical and exploratory (e.g., "graph X shows the result is Y, which means Z; this contradicts [1], which may be because I use a different assumption"). As such, both positive *and* negative outcomes are valid *if* presented in a suitable manner.

Chapter 5

Conclusion

A compulsory chapter, of roughly 5 pages

The concluding chapter of a dissertation is often underutilised because it is too often left too close to the deadline: it is important to allocation enough attention. Ideally, the chapter will consist of three parts:

1. (Re)summarise the main contributions and achievements, in essence summing up the content.
2. Clearly state the current project status (e.g., “X is working, Y is not”) and evaluate what has been achieved with respect to the initial aims and objectives (e.g., “I completed aim X outlined previously, the evidence for this is within Chapter Y”). There is no problem including aims which were not completed, but it is important to evaluate and/or justify why this is the case.
3. Outline any open problems or future plans. Rather than treat this only as an exercise in what you *could* have done given more time, try to focus on any unexplored options or interesting outcomes (e.g., “my experiment for X gave counter-intuitive results, this could be because Y and would form an interesting area for further study” or “users found feature Z of my software difficult to use, which is obvious in hindsight but not during at design stage; to resolve this, I could clearly apply the technique of Smith [7]”).

Bibliography

Appendix A

An Example Appendix

Content which is not central to, but may enhance the dissertation can be included in one or more appendices; examples include, but are not limited to

- lengthy mathematical proofs, numerical or graphical results which are summarised in the main body,
- sample or example calculations, and
- results of user studies or questionnaires.

Note that in line with most research conferences, the marking panel is not obliged to read such appendices.