Course Name	**PROBABILITY AND STATISTICS**
Course Code	AHS010
Programme	B.Tech
Semester	II
Course Coordinator	Mr. J Suresh Goud
Course Faculty	Ms. P Srilatha
Lecture Number	43
Topic Covered	Test of hypothesis for single mean-2
Course Learning Outcome's	Understand the foundation for hypothesis testing

Test of hypothesis for Single Mean:

Consider a Sample of size n with mean $\bar{x}$ from the population with mean μ and standard deviation σ then the test statistic z is given as

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Note:

Confidence interval $= \left(\bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}}, \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}} \right)$

Problems:

1.A sample of 400 items whose standard deviation is 10 is taken from the population the mean of sample is 40. Test whether the sample come from population with mean 38 at level of significance 0.05. Also Calculate the confidence interval for the single mean.

Solution:

Given $n=400$, $s=10$, $\bar{x}=40$, $\mu=38$

Step 1: Null Hypothesis: $\mu=38$

Step 2: Alternative Hypothesis: $\mu\neq38$

Step 3: Level of Significance:

$$z_\alpha=1.96 \quad at \quad \alpha=0.05$$

step4: Test Statistics: $Z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

$$Z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}} = \dfrac{40 - 38}{\dfrac{10}{\sqrt{400}}} = 4$$

$$|Z| = 4$$

Step 5: Conclusion:

$$|Z| > Z_{\alpha}$$

$\therefore$ We reject the Null hypothesis.

$$\text{Confidence interval} = \left(\bar{x} - Z_\alpha \frac{\sigma}{\sqrt{n}}, \bar{x} + Z_\alpha \frac{\sigma}{\sqrt{n}} \right)$$

$$= \left(40 - 1.96 \frac{10}{\sqrt{400}}, 40 + 1.96 \frac{10}{\sqrt{400}} \right)$$

$$= \left(39.02, 40.98 \right)$$

2. A sample of 900 members has a mean of 3.4 cms and standard deviation 2.61 cms. Is this sample has been taken from a large population of mean 3.25 cm and standard deviation 2.61 cms. If the population is normal and its mean is unknown find the 95% confidential limits of true mean.

Solution:

Given $n=900, \sigma=2.61, \bar{x}=3.4, \mu=3.25$

Step 1: Null Hypothesis: $\mu=3.25$

Step 2: Alternative Hypothesis: $\mu \neq 3.25$

Step 3: Level of Significance:

$$z_\alpha = 1.96 \quad \text{at} \quad \alpha = 0.05$$

Step4: Test Statistics: $Z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

$$Z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}} = \dfrac{3.4 - 3.25}{\dfrac{2.61}{\sqrt{900}}} = 4$$

$$|Z| = 1..724$$

Step 5: Conclusion:

$$|Z| < Z_\alpha$$

$\therefore$ We reject the Null hypothesis.

$$\text{Confidence interval} = \left(\bar{x} - Z_\alpha \frac{\sigma}{\sqrt{n}}, \bar{x} + Z_\alpha \frac{\sigma}{\sqrt{n}} \right)$$

$$= \left(3.4 - 1.96 \frac{2.61}{\sqrt{900}}, 3.4 + 1.96 \frac{2.61}{\sqrt{900}} \right)$$

$$= \left(3.57, 3.22 \right)$$

Exercise:

1. The mean of random sample is 11,795 and standard deviation is 14,054. If sample size is 50 then find 95% confidence interval for the mean.

2. A sample of 100 workers in a large plant gave a mean assembly time of 294 seconds with a standard deviation of 12 seconds in a time and motion study. Find a 95% confidence interval for the mean assembly time for all workers in the plant.