INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad -500 043

COMPUTER SCIENCE AND ENGINEERING

COURSE HANDOUT

Course Name	**PROBABILITY AND STATISTICS**
Course Code	AHS010
Programme	B.Tech
Semester	II
Course Coordinator	Mr. J Suresh Goud
Course Faculty	Ms. P Srilatha
Lecture Number	38
Topics Covered	Problems on Estimation
Course Learning Outcome's	Understand the concept of estimation for classical inference involving confidence interval.

Problems:

1. What is the maximum error can to make with probability 0.95 when using a mean of random sample of size 64 to estimate the mean of population with variance 2.56.

Solution:

Given $n = 64$, $\sigma^2 = 2.56, \sigma = 1.6$

$$Z_{\alpha/2} = 1.96 \quad at \quad 95\%$$

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 1.96 \times \frac{1.6}{8}$$

$$= 0.392$$

2. Assuming that σ =20 how large a random sample to be taken with probability 0.95 that the sample mean will not differ from the population mean by more than 3 points.

Solution:

Given

$\sigma = 0.3, E = 3, n = ?$

$$Z_{\alpha/2} = 1.96 \quad at \quad 95\%$$

$$n = \left[\frac{Z_{\alpha/2}\sigma}{E}\right]^2$$

$$= \left[\frac{1.96 \times 20}{3}\right]^2$$

$$= 171$$

3. Determine a 95% confidence interval for the mean of normal distribution with variance 0.25, using a sample of size 100 values with mean 212.3.

Solution:

Given Sample size (n) = 100

Standard deviation of sample (σ) = $\sqrt{0.25} = 0.5$

Mean of sample ($\bar{x}$) = 212.3 and $Z_{\alpha/2}$ = 1.96(for 95%)

$\therefore$ Confidence interval = $\left(\bar{x} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$

$$= \left(212.3 - 1.96.\frac{0.5}{\sqrt{100}},\ 212.3 + 1.96.\frac{0.5}{\sqrt{100}}\right)$$
= (212.202, 212.39)

2. The mean of random sample is an unbiased estimate of the mean of the population 3,6,9,15,27.

   i) List of all possible samples of size 3 that can be taken without replacement from the finite population.

   ii) Calculate the mean of each of the samples listed in (i) And assigning each sample a probability of 1/10. Verify that the mean of these $\bar{x}$ is equal to 12. Which is equal to the mean of the population θ i.e., E($\bar{x}$)= θ i.e., prove that $\bar{x}$ is an unbiased estimate of θ.

Solution:

i) The possible samples of size 3 taken from 3,6,9,15,27 without replacement are ${}^{5}C_{3}$=10 samples i.e., (3,6,9), (3,6,15), (3,6,27), (6,9,15), (6,9,27), (3,9,15), (3,9,27), (9,15,27), (6,15,27), (3,15,27).

ii) Mean of the population $\theta = \dfrac{3+6+9+15+27}{5} = 12$

Mean samples are 6, 8,12,10,14,9,13,17,16,15.
Probability assigned to each one is 1/10 each

$\bar{x}$	6	8	12	10	14	9	13	17	16	15
P($\bar{x}$)	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10

$$E(\bar{x}) = 6.\frac{1}{10} + 8.\frac{1}{10} + 12.\frac{1}{10} + 10.\frac{1}{10} + 14.\frac{1}{10} + 9.\frac{1}{10} + 13.\frac{1}{10} + 17.\frac{1}{10} + 16.\frac{1}{10} + 15.\frac{1}{10}$$
$$= \frac{1}{10} \times 120$$
$$= 12$$
$$= \theta$$

$\therefore E(\bar{x}) = \theta$

$\therefore$ $\bar{x}$ is an unbiased estimate of θ.

$\therefore$ The mean of a random sample is an unbiased estimator of the mean of the population.

Exercise:

1. Assuming that $\sigma = 48$ hours, how large the sample is needed that one will be able to assert with 95% confidence level that the sample mean is of by utmost 10 hours.

2. A random sample of size 100 has standard deviation 5 what can you say with 95% confidence level.

3. A research worker wants to determine the average time it takes a mechanic to rotate the types of car and he wants to be able to assert with 95%. Confidence that the mean of his sample is off by at most 0.5 minutes. If he can presume from presume from past experience that σ =1.6 minutes, how large a sample will have to take?

4. Random samples of size 81 were taken whose variance is 20.25 and mean is 32, construct 98% confidence interval.

5. In a random sample of 100 packages shipped by air freight 13 had some damage. Construct 95% confidence interval for the true proportion of damage package.